Analysis of Plane-Wave Scattering by Bigratings

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Abstract

We present here a rigorous analysis of the plane-wave scattering by a class of bigratings, each of which consists of two separated gratings that may have different periods and different orientations. Inherently, this is a three-dimensional boundary value problem, regardless of the incidence condition. An exact formulation based on the method of mode matching is carried out, taking into account the effect of polarization couplings. Numerical results are shown for identifying physical phenomena associated with the class of bigrating structures. In particular, Wood's anomalies in the presence of a bigrating are carefully examined and the results are interpreted in terms of those well known for single gratings.

Inroduction

The analysis of a planar structure consisting of multiple gratings has been presented by many authors [1-4]. All the analyses in the literature have been carried out under certain special conditions, to that the boundary-value problem can be treated as a scalar one. On the other hand, in the case of a bigrating with an arbitrary crossing angle between the two constituent gratings, the overall diffraction phenomenon has to be treated as a three-dimensional boundary-value problem requiring the coupling of TE and TM polarized waves to satisfy the boundary conditions. Based on the method of mode matching, such a boundary-value problem has been formulated rigorously and numerical results has been obtained, showing a host of new physical phenomena.

Formulation

A typical configuration of a bigrating structure is shown in Fig. 1. The two grating are in parallel and their rulings cross each other at an angle φ. We choose the coordinate system such that the upper grating (Grating A) is periodic in the x-direction and uniform in the y-direction, while the lower grating (Grating B) is periodic in the u-direction and uniform in the v-direction. Thus, (x,y,z) and (u,v,z) are two rectangular coordinate systems that are related to each other by a rotation of the angle φ about the z-axis. Specifically, Grating A has a thickness t_a and a

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dielectric constant $\varepsilon_a(x)$ that is periodic in the x-direction with a period a, while Grating B has a thickness t_b and a dielectric constant $\varepsilon_b(u)$ that is periodic in the u-direction with a period b. The uniform layer separating the two gratings has a dielectric constant ε_u and a thickness t_u. The whole structure is bound by two half spaces of the dielectric constant ε_c and ε_s, for the cover and substrate regions, respectively. With respective to the xyz-coordinate system, such a bigrating structure as a whole is more than a two-dimensional periodic structure; the structure is doubly periodic in the x-direction, with the two periods a and $b/\cos\varphi$, and singly periodic in the y-direction, with the period $b/\sin\varphi$. With the two constituent gratings physically separated, the plane-wave scattering by the bigrating may be described in terms of the multiple scattering between the two single gratings, and such a view point will greatly simplify the formulation of the bigrating structure, as explained below.

As indicated in Fig. 2 a uniform plane wave is obliquely incident from the cover region above Grating A. In view of the fact that the structure as a whole is doubly periodic in the x-direction and singly periodic in the y-direction, we expect that the fields may be represented by a double Fourier series in x and a single Fourier series in y. Thus, the propagation constants of the incident plane wave are related to the incident angles θ_0, and ϕ_0 by:

$$k_{x00}=k_c\sin\theta_0\cos\phi_0 \qquad (1.a)$$

$$k_{y0}=k_c\sin\theta_0\sin\phi_0 \qquad (1.b)$$

$$k_z=k_c\cos\theta_0 \qquad (1.c)$$

where k_c is the propagation constant of the plane-wave in the cover region. Here we shall take the z-direction as the longitudinal direction, since we are interested in the reflection and transmission of energy in that direction. Thus, the transverse propagation constants of the mn^{th} space harmonic are related to those of the incident plane wave by

$$k_{xmn}=k_{x00}+m\frac{2\pi}{a}+n\frac{2\pi}{b}\cos\varphi \qquad (2.a)$$

$$k_{yn}=k_{y0}+n\frac{2\pi}{b}\sin\varphi \qquad (2.b)$$

It is noted that between the two indices, m and n, the first one refers to the harmonics of Grating A, while the second one refers to those of Grating B.

On the other hand, with respective to the uvz-coordinate system, the transverse propagation constants of the mn^{th} space harmonic can be obtained from (2) via the formulas for coordinate rotation, and the results are:

$$k_{umn}=k_{u00}+m\frac{2\pi}{a}\cos\varphi+n\frac{2\pi}{b} \qquad (3.a)$$

$$k_{vm} = k_{v0} - m\frac{2\pi}{a}\sin\varphi \tag{3.b}$$

where the fundamental components are related to those of the xyz-coordinate system by the coordinate rotation formula:

$$\begin{pmatrix} k_{u00} \\ k_{v0} \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} k_{x00} \\ k_{y0} \end{pmatrix} \tag{4}$$

Thus, when the incident angles are given, the transverse propagation constants of all the space harmonics can be easily be determined in either coordinate system, and the fields in every subregion of the structure can then be determined.

The bigrating structure under consideration is composed of a stack of uniform and periodic layers. We defined the input-output relations [5,6] for a periodic layer, including its input admittance matrix and the transfer matrix, for a given output admittance matrix. These results can be simplified for the case of uniform layer which is taken as a limiting case of vanishing periodic variation. Thus, the input-output relation of a periodic layer can be used successively for the construction of those of the whole bigrating structure, as is done in this work. With such an approach, the boundary value problem of the bigrating structure is reduced to the performance of two important operations on the admittance and transfer matrices: one is the rotation of the coordinate system because of differing grating orientations, and the other is the regrouping of space harmonics because of differing periodicities and orientations of the two gratings.

Numerical Results

Fig.3 shows the transmitted intensity versus elevation angle θ_o of the incident wave. There exist regions of rapid variation, which can be explained as Wood's anomalies. To substantiate such an explanation, the normalized transverse propagation constant is plotted against the elevation angle in solid curves in Fig. 4, for various harmonics. The effective indices of surface waves are included as the dashed lines. An intersection of a solid curve with a dashed curve determine the condition on the phase matching between the diffracted wave and a guided wave; consequently, this determines the region of rapid variation of the diffraction efficiency. The two sets of curves in Fig. 3 and 4 confirms such a relationship on physical basis.

Conclusion

We have presented a module approach to the three-dimensional boundary-value problem of plane-wave scattering by a bigrating. The formulation of the problem is based on the rigorous method of modal matching, which is applicable to gratings of any profile. Numerical results are given to illustrate the effect of interactions between the constituent gratings of the structure; in particular, Wood's anomalies associated with bigratings are carefully examined and are explained on the physical basis.

References

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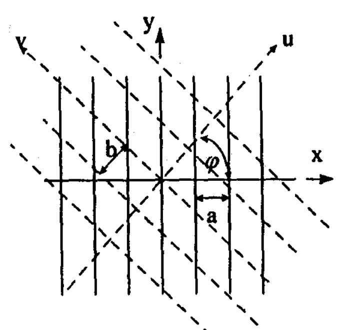

Fig. 1 Structure configuration of a bigrating

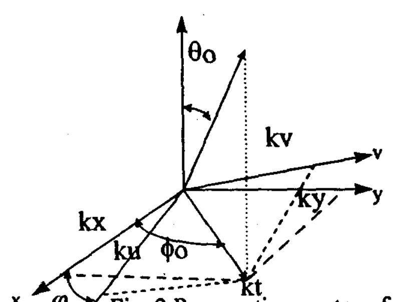

Fig. 2 Propagation vector of a uniform plane wave

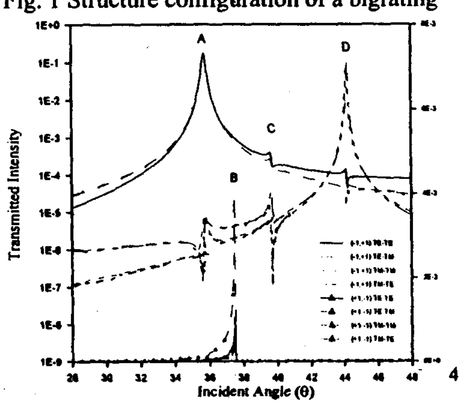

Fig. 3 Effect of the elevation angle on the

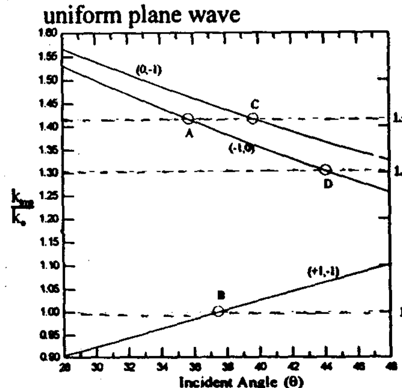

Fig. 4 Normalized transverse