An improved method for measuring refractive-index of a medium

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ABSTRACT

An improved method for measuring the refractive index of a medium is presented. First, the light coming from a circular heterodyne source is incident on the test medium. And, the reflected light passes through an analyzer with the transmission axis at a moderate azimuth angle. Then, the phase variation of the reflected light is measured by the heterodyne interferometric technique. Finally, substituting this phase variation into the equations derived from Fresnel's equations and Jones calculus, we can obtain the refractive index of the test medium.

Keywords: index measurements, heterodyne interferometry, Fresnel's equation.

1. INTRODUCTION

Almost all of the common techniques for measuring refractive index are related to the measurement of light intensity variation[1-5], their accuracy of measurements is limited. To overcome this drawback, Chiu et al.[6] proposed a method to evaluate the refractive index by measuring the phase difference variation instead of light intensity variation. It has many merits, such as simple optical setup, easy operation, high stability, high measurement accuracy, and rapid measurement. However, its measurable range is limited by the refractive index of the prism in which the total internal reflection is occurred.

In this paper, we modified Chiu's method and present a more feasible method to measure the refractive index of a medium. Instead of using the effect of the total internal reflection, the effect of a simple reflection from the surface of the test medium is used. And, similar processes are performed to measure the phase variation. Then, by substituting this phase variation into the equations derived from Fresnel's equations[7] and Jones calculus[8], we can obtain the refractive index of the test medium. This improved method has all merits as those of Chiu's method. Moreover, it is more feasible and its measurable range is not limited. We demonstrate its feasibility.

2. PRINCIPLE

The schematic diagram of this new method is shown in Fig. 1. A linearly polarized light passing through an electro-optic modulator EO being modulated by the electric signal generated by a function generator FG and a quarter-wave plate Q, is incident on the test medium S. The light reflected by this medium passes through an analyzer AN, and enters the photodetector D. If the amplitude of the light detected by D is E_t, then the intensity measured by D is $I_t = |E_t|^2$. Here it is the test signal. On the other hand, the electrical signal generated by the function generator is filtered and becomes the reference signal. Finally, these two signals are sent to a phase meter PM to evaluate the phase difference between them.

2.1. The intensity of the test signal

For convenience, the +z axis is chosen along the propagation direction and the y -axis is along the vertical direction. Let the light coming from a laser be linearly polarized at 45 ° with respective to the x- axis, then its Jones vector[8] can be written

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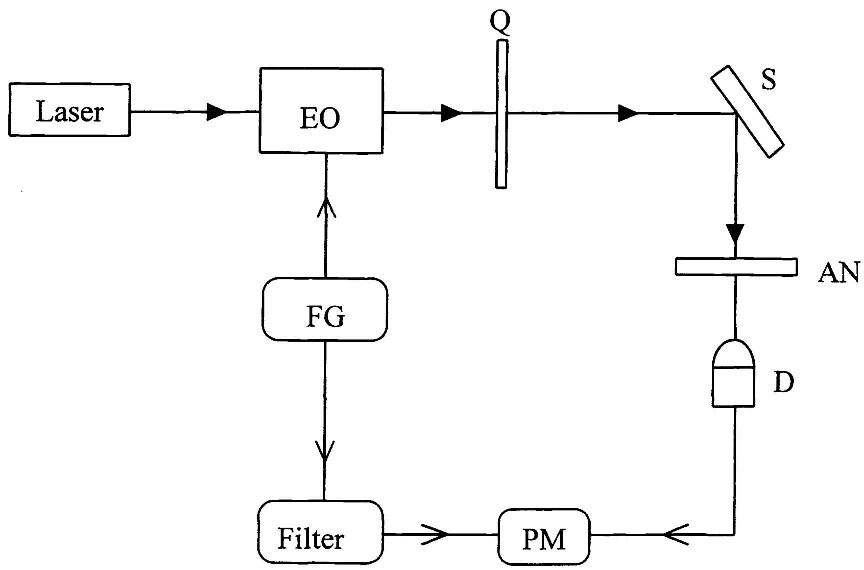

Fig. 1: Schematic diagram for this improved method for measuring refractive index of the test medium: EO, electro-optic modulator; FG, function generator; Q, quarter-wave plate; S, test medium; AN, analyzer; D, photodetector; PM, phase meter.

$$E_{in} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}. \tag{1}$$

If the fast axis of EO is along the x-axis, and an external sawtooth voltage signal with angular frequence ω and amplitude $V_{\lambda/2}$, the half-voltage of EO, is applied to EO, then the phase retardation produced by EO can be expressed as ωt[9]. And if the fast axis of Q is 45° with respective to the x-axis, then we have

$$\begin{aligned} E'_{in} &= Q(45^\circ)\cdot EO(\omega t)\cdot E_{in} \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}e^{i\frac{\omega t}{2}} & 0\\ 0 & e^{-i\frac{\omega t}{2}}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix}1\\ i\end{pmatrix}e^{i\frac{\omega t}{2}} + \frac{1}{2}\begin{pmatrix}1\\ -i\end{pmatrix}e^{-i\frac{\omega t}{2}+i\frac{\pi}{2}} \end{aligned} \tag{2}$$

From Eq. (2), it is obvious that this is a circular heterodyne light source with angular frequence ω and phase difference $-\pi/2$ between left- and right- circular polarizations. If the transmission axis of AN is at 45° with respective to x-axis, we have

$$\begin{aligned} E_t &= AN(45^\circ)\cdot S\cdot E'_{in} \\ &= \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}r_p & 0\\ 0 & r_s\end{pmatrix}\left[\frac{1}{2}\begin{pmatrix}1\\ i\end{pmatrix}e^{i\frac{\omega t}{2}} + \frac{1}{2}\begin{pmatrix}1\\ -i\end{pmatrix}e^{-i\frac{\omega t}{2}+i\frac{\pi}{2}}\right] \end{aligned}$$

$$= \frac{1}{4}\left[(r_p + ir_s)e^{i\omega t/2} + (ir_p + r_s)e^{-i\omega t/2}\right]\begin{pmatrix}1\\1\end{pmatrix}, \tag{3}$$

where S is the Jones matrix for the test medium as the light is reflected from it, and r_p and r_s are its reflection coefficients for p- and s- polarizations, respectively. Hence, the intensity of the test signal is

$$I_t = |E_t|^2 = \frac{(r_p^2 + r_s^2)}{4}\left[1 + \cos(\omega t - \phi)\right], \tag{4}$$

where

$$\phi = \tan^{-1}\left[\frac{r_p^2 - r_s^2}{2r_p r_s}\right]. \tag{5}$$

On the other hand, the reference signal has the form of

$$I_r = |E_r|^2 = \frac{I_0}{2}(1 + \cos\omega t). \tag{6}$$

Thus from Eq.(4) and Eq.(6), it is seen that both the test signal and reference signal are sinusoidal signal. When they are sent to a phase meter PM, the phase variation ϕ can be measured.

2.2. Measurement of the refractive index of the medium

Rearranging Eq.(5), we have

$$\frac{r_p}{r_s} = \tan\phi \pm \sqrt{\tan^2\phi + 1}\cdot \tag{7}$$

To understand the meaning of signs "±", we write r_p and r_s according to Fresnel's equations[7]:

$$r_p = \frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}, \tag{8}$$

and

$$r_s = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}; \tag{9}$$

where θ is the incident angle. For easy understanding, n=1.5 is chosen as an example to clarify the relations among r_p, r_s, ϕ, and θ. They are depicted in Fig. 2 by using Eqs.(5)-(9). The relation curves of r_p and r_s versus θ in Fig. 2(a) show that both r_p and r_s are negative as θ is larger than the Brewster angle θ_B. So when $\theta > \theta_B$, the plus sign is taken in Eq.(7), and when $\theta < \theta_B$, the minus sign is taken in Eq.(7). If the range of ϕ defined in Eq.(5) is chosen within -90° and 90° as in common mathematical calculation, the relation curve of ϕ versus θ behaves as shown in Fig. 2(b). It is obvious that there is a discontinuity near $\theta = \theta_B$, hence it becomes difficult to evaluate ϕ accurately. On the other hand, if the measurable range ϕ is moved into the range between -180° and 0°, then the relation curve of ϕ versus θ is changed to that shown in Fig. 2(c)

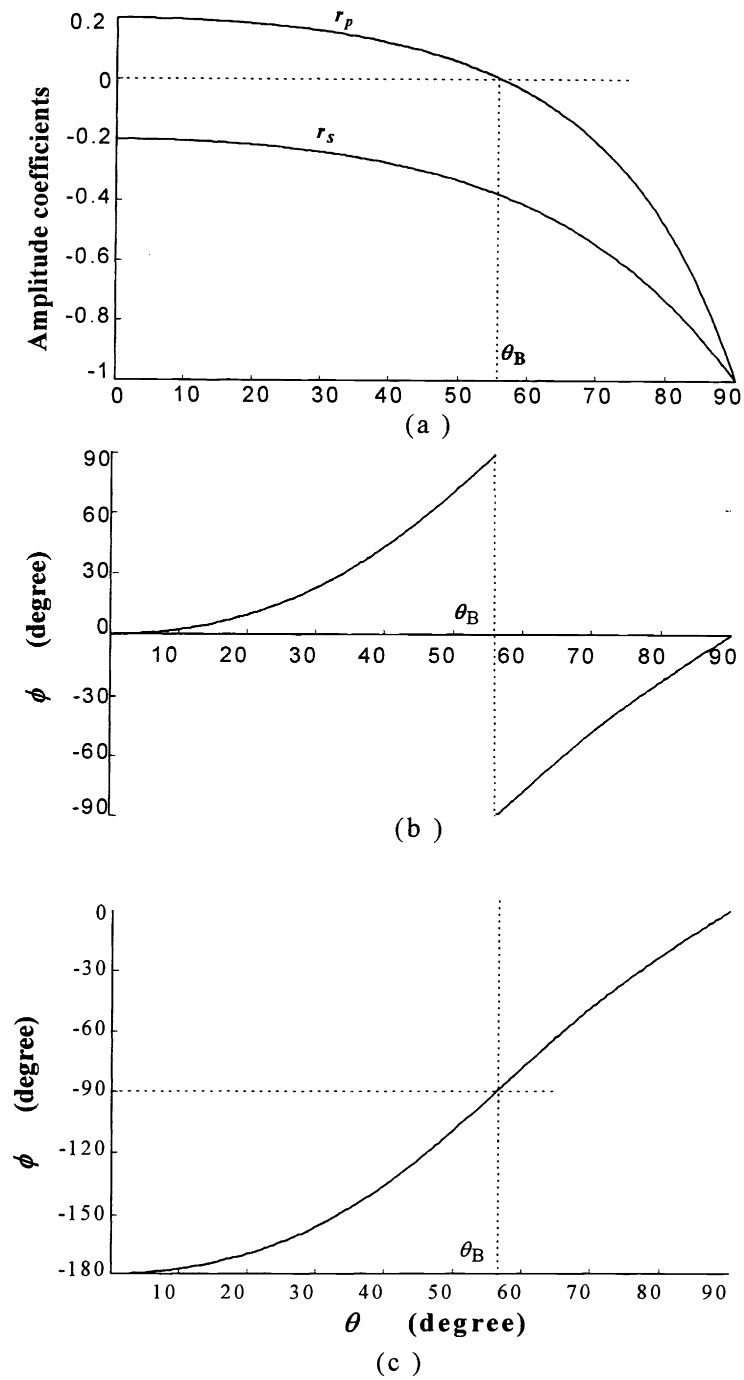

Fig. 2: The relation curves of (a) r_p and r_s versus θ, (b) ϕ versus θ, as ϕ is chosen within -90° and 90°, and (c) ϕ versus θ, as ϕ is chosen within -180° and 0°, for n=1.5.

and becomes continuous. Based on this new relation curve, the estimation of ϕ is very convenient. Consequently as ϕ<-90° , minus sign is taken in Eq.(7), and as ϕ> -90° , positive sign is taken in Eq.(7). If Eqs.(8) and (9) are substituted into Eq.(5) and Eq.(7), then we have

$$\phi = \begin{cases} \tan^{-1}\left[\dfrac{2\sin^2\theta\cos\theta\sqrt{n^2-\sin^2\theta}}{\cos^2\theta\cdot n^2-\sin^2\theta}\right]-180^\circ & \text{if } \theta<\theta_B, \\ \tan^{-1}\left[\dfrac{2\sin^2\theta\cos\theta\sqrt{n^2-\sin^2\theta}}{\cos^2\theta\cdot n^2-\sin^2\theta}\right] & \text{if } \theta>\theta_B, \end{cases} \tag{10}$$

and

$$n = \sin\theta\sqrt{1+\left\{\tan\theta\cdot\left[\frac{1-\left(\tan\phi\pm\sqrt{\tan^2\phi+1}\right)}{1+\left(\tan\phi\pm\sqrt{\tan^2\phi+1}\right)}\right]\right\}^2}, \tag{11}$$

respectively. It is obvious from Eq.(11) that n can be calculated with the measurement of ϕ under the experimental condition θ is specified.

3. EXPERIMENTS AND RESULTS

To show the feasibility of this new method, the refractive indices of three kinds of glass, BK7, BaSF2 and SF11, were measured. A He-Ne laser with a 632.8-nm wavelength and an electro-optic modulator(model PC/2, manufactured by England Electro-Optics Developments Ltd.) with a half-wave voltage of 170 V were used in this test. The frequency of the sawtooth signal applied to the electro-optic modulator was 2 kHz. A high-precision rotation stage(PS-θ-90) with the angular resolution of 0.005° manufactured by Japan Chuo Precision Industrial Company Ltd. was used to mount the test medium. At θ=60° , the phase differences of BK7, BaSF2 and SF11 were measured to be -79.30° , -86.70° , and -91.99° , respectively. After introducing these data into Eq.(11), we obtained the refractive indices of these samples as 1.5154, 1.6603 and 1.7788, and their reference refractive indices[10] are 1.51509, 1.66068, and 1.77862, respectively.

Moreover, the theoretical and the experimental curves of ϕ versus θ for these medium are shown in Fig. 3. In this figure, the solid curves represent the theoretical curves that we obtained by introducing the above reference refractive indices into Eq.(10), and the symbols O, +, and * represent the measured values for BK7, BaSF2 and SF11, respectively. It is clear that the three curves show good correspondence.

4. DISCUSSIONS

Since the effect of the total internal reflection is not used in this method, the measurable range is not limited. From Eq.(11), we can get

$$\Delta n \cong \left[\frac{\left(n^2-\sin^2\theta\right)\left(1+\cos^2\theta\right)}{n\cdot\sin\theta\cos\theta}+\frac{\sin\theta\cos\theta}{n}\right]\times\Delta\theta + \left[\frac{2\left(\sin^2\theta-n^2\right)\left(\tan^2\phi+1\right)}{n\cdot\left[1-\left(\tan\phi\pm\sqrt{\tan^2\phi+1}\right)^2\right]}\left(1\pm\frac{\tan\phi}{\sqrt{\tan^2\phi+1}}\right)\right]\times\Delta\phi, \tag{12}$$

where $\Delta\theta$ and $\Delta\phi$ are the errors in the incident angle and the phase variation, respectively. It is obvious that the refractive index measurement is dependent on the incident angle (i.e., the rotation angle of the rotation stage) and the angular resolution of the phase meter. In our experiment, the angular resolutions of the rotation stage and the phase meter are 0.005 ° and 0.01 ° , respectively. Consequently, the curve of Δn versus θ can be obtained by substituting $\Delta\theta$=0.005 ° and $\Delta\phi$ =0.01 ° into Eq.(12) for several different refractive indices *n*, as shown in Fig. 4. Obviously, the refractive index error becomes smaller as *n* decreases. The best resolution can be obtained as the incident angle is in the neighborhood near 60 ° , and it is better than 8×10^{-4}.

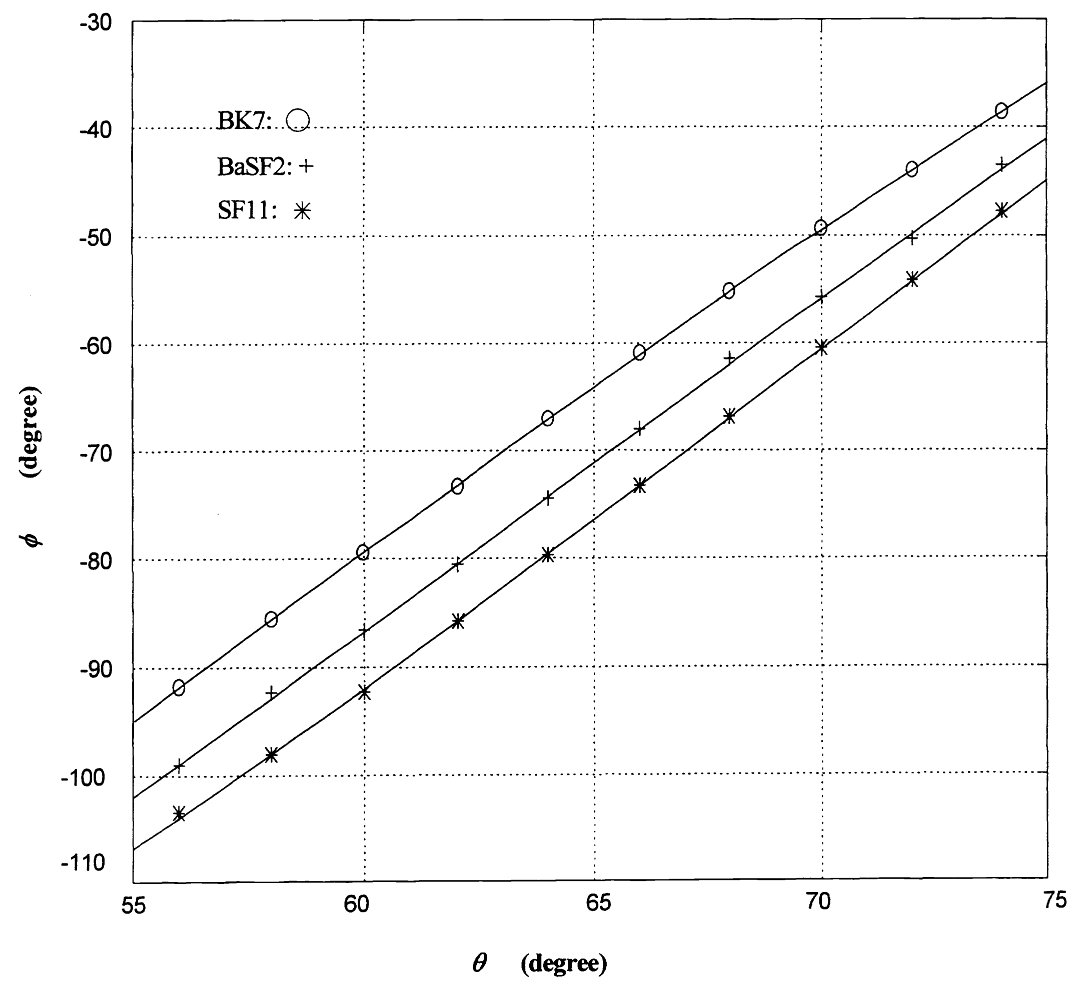

Fig. 3: Theoretical and experimental curves for ϕ versus θ for BK7, BaSF2 and SF11.

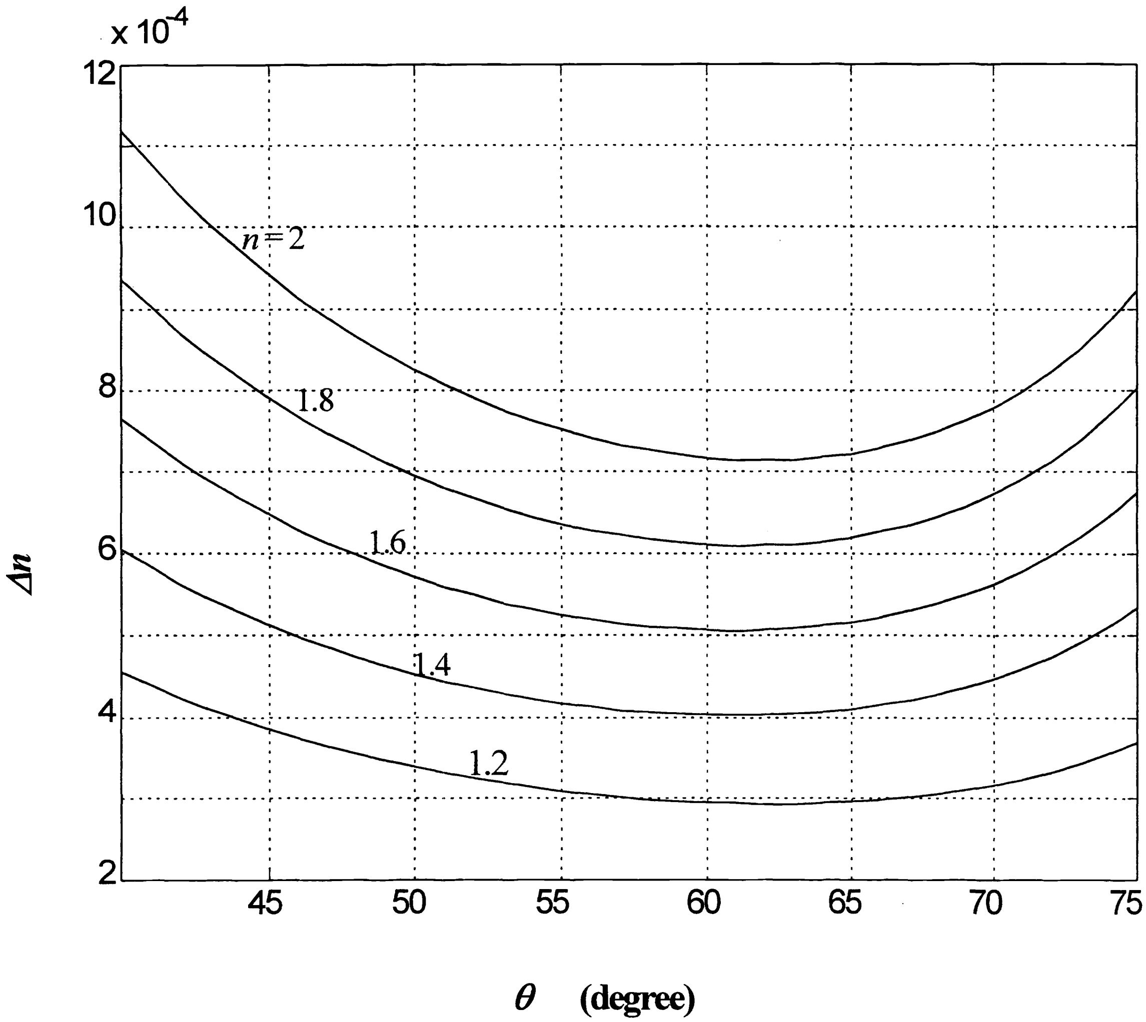

Fig. 4: Relation curves of Δn versus θ for several different n.

5. CONCLUSIONS

An improved method for measuring the refractive index of a medium is proposed. First, the light coming from a circular heterodyne source is incident on the test medium. And the reflected light passes through an analyzer with the transmission axis at a moderate azimuth angle. Then, the phase variation of the reflected light is measured by the heterodyne interferometric technique. Finally, substituting this phase variation into the equations derived from Fresnel's equations and Jones calculus, we can obtain the refractive index of the test medium. This method has many merits, such as simple optical setup, easy operation, high stability, high measurement accuracy, and rapid measurement. Moreover, its measurable range is not limited. Its feasibility has been demonstrated.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

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