

# Self-Phase-Modulation-Limited Transmission Distance of Repeaterless 1.55 $\mu\text{m}$ Multi-channel AM-VSB External Modulation systems

W. H. Chen, M. C. Wu, C. T. Chang\* and W. I. Way

Department of Communication Engineering  
National Chiao-Tung University  
Hsinchu, Taiwan, Republic of China

\*Department of Electrical and Computer Engineering  
San Diego State University  
San Diego, CA92182

Phone:886-3-572138 ; Fax:886-3-5718870 ; E-mail:waway@cc.nctu.edu.tw

## ABSTRACT

The repeaterless transmission distance of a 1.55  $\mu\text{m}$  external modulation system using an erbium-doped fiber power amplifier is generally believed to be limited by stimulated-Brillouin-scattering (SBS). In this paper, by using both analytical and numerical methods, we find that even when the SBS threshold can be increased well beyond  $\sim 24$  mW, the maximum repeaterless transmission distance will still be limited by self-phase-modulation (SPM)-induced CSOs. Our analysis is based on the nonlinear amplitude envelope equation in a lossy, dispersive and nonlinear single-mode-fiber (SMF) link at 1.55  $\mu\text{m}$ , and our numerical approach uses split-step Fourier method.

## 1. INTRODUCTION

The transmission distance of multichannel AM-VSB optical fiber links is limited to about 30 km for 1.3  $\mu\text{m}$  direct modulation systems (when no 1.3  $\mu\text{m}$  optical amplifiers is used). Recently, however, the combination of 1.55  $\mu\text{m}$  externally-modulated transmitters and high-power erbium-doped fiber amplifiers (EDFAs) (with output power in the range of 40 to 200 mW) have been used to extend the transmission distance. Although linear optical fiber dispersion is not a problem for 1.55  $\mu\text{m}$  external modulation systems, optical fiber nonlinearity-induced degradation factors such as SBS and SPM can still limit the transmission distance. The repeaterless (i.e., without in-line EDFAs) transmission distance limited by SBS can be derived from a recent report<sup>1</sup> to be  $\sim 65$  km, under the condition that the SBS-limited input optical power in a long-distance link is  $\sim 25$  mW. However, when more sophisticated SBS suppression techniques, such as combining phase modulation (via an external modulator<sup>2</sup>) and frequency modulation (via a  $> 20$  mW 1.55  $\mu\text{m}$  DFB laser)<sup>3</sup>, are applied to a 1.55 external-modulator-based transmitter, SBS threshold can be increased to beyond 63 mW ( $\sim 18\text{dBm}$ ). In this case, the power-limited transmission distance is about  $(18-0)/0.22 = 82$  km, which is longer than

what can be observed experimentally for a 1.55  $\mu\text{m}$  AM external modulation system. Then, what fundamental mechanism limits the repeaterless transmission distance when SBS is not the major limiting factor? In this paper, we answer this question through theoretical analysis and numerical calculations, we provide an estimation of the SPM-limited transmission distance.

## 2. ANALYSIS

We begin our analysis by assuming the electric field in a subcarrier multiplexed lightwave system to be

$$A = A_0 \sqrt{X(z,t)} e^{jY(z,t)} \quad (1)$$

In Eq. (1),  $A_0$  represents the input optical field amplitude,  $X(z,t)$  represents the intensity modulation, and  $Y(z,t)$  represents the phase modulation.

The wave-envelope equation of the electrical field  $A$  propagating in the  $z$  direction of a lossy, dispersive, nonlinear optical fiber link is given by<sup>4</sup>

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} = \frac{j}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} - \frac{\alpha}{2} A - jkn_2 |A|^2 A \quad (2)$$

where  $\alpha$  is the intensity loss ( $5.1 \times 10^{-2}$  nepers/km at 1.55  $\mu\text{m}$ ),  $\beta_1$  and  $\beta_2$  are the first and second derivatives of the propagation constant  $\beta$  with respect to the angular optical frequency  $\omega$ ,  $k = \frac{2\pi}{\lambda}$ , and  $n_2$  is the nonlinear refractive index. With both intensity  $X(z,t)$  and phase  $Y(z,t)$  assumed to be real, we substitute Eq. (1) into Eq. (2) and separate real part from imaginary part to obtain

$$\frac{\partial X}{\partial z} + \beta_1 \frac{\partial X}{\partial t} + \alpha X = -\beta_2 \left[ \frac{\partial X}{\partial t} \frac{\partial Y}{\partial t} + X \frac{\partial^2 Y}{\partial t^2} \right] \quad (3a)$$

and

$$\frac{\partial Y}{\partial z} + \beta_1 \frac{\partial Y}{\partial t} = \beta_2 \left[ -\frac{1}{8X^2} \left( \frac{\partial X}{\partial t} \right)^2 + \frac{1}{4X} \frac{\partial^2 X}{\partial t^2} - \frac{1}{2} \left( \frac{\partial Y}{\partial t} \right)^2 \right] - NX \quad (3b)$$

where  $N = kn_2 A_0^2$ . To solve Eqs (3a) and (3b), we use the perturbation theory and assume the solutions as

$$X(z,t) = e^{-\alpha z} x_0(\mu) + x_1(\mu, z) + x_2(\mu, z) + x_3(\mu, z) + x_4(\mu, z) + \dots \quad (4a)$$

$$Y(z,t) = y_0(\mu) + y_1(\mu, z) + y_2(\mu, z) + y_3(\mu, z) + y_4(\mu, z) + \dots \quad (4b)$$

where  $u = t - \beta z$ ;  $x_0, y_0$  are the unperturbed intensity and phase modulation propagating through the optical fiber link without distortion; and  $x_n$  and  $y_n$  are the  $n$ -th order perturbation of intensity and phase modulation, respectively, containing the distortion part of the resultant signal. For SCM systems,  $x_0(u)$  is given by  $1 + m \sum_i \cos(\omega_i u)$ ,  $y_0(u)$  is given by  $2\pi\gamma m \int \sum_i \cos(\omega_i u) du$  where  $m$  is the intensity modulation index, and  $\gamma$  is the laser peak chirped frequency. For multichannel AM video signal-modulated semiconductor lasers,  $\gamma$  can be represented as  $G(I_b - I_{th})$ , where  $G$  is the laser chirp parameter (GHz/mA), and  $I_b$  and  $I_{th}$  are the bias and threshold currents of the laser, respectively. For external modulation systems,  $\gamma$  approaches zero. By substituting Eqs (4a) and (4b) into Eqs (3a) and (3b), and solve for  $x_1$  and  $x_2$ , we can obtain

$$x_1 = -\beta_2 z e^{-\alpha z} \frac{d}{d\mu} \left[ x_0 \frac{dy_0}{d\mu} \right] \quad (5a)$$

and

$$x_2 \approx \beta_2 N \bar{z}^2 e^{-\alpha z} \frac{d}{d\mu} \left[ x_0 \frac{dx_0}{d\mu} \right] \quad (5b)$$

where  $z$  is the transmission distance,  $\bar{z}^2 = \frac{\alpha z - 1 + e^{-\alpha z}}{\alpha^2}$ ,  $\beta_2 = -\frac{\lambda^2}{2\pi c} D$ , and  $D$  is the dispersion coefficient ( $= 17$  ps/nm  $\cdot$  km for  $1.55$   $\mu$ m signal propagating in  $1.3$   $\mu$ m zero-dispersion single-mode fibers). Note that for a transmission distance less than  $100$  km, the contributions from higher orders  $x_i$  ( $i \geq 3$ ) to nonlinear distortions are negligible. By substituting  $x_0$  and  $y_0$  into Eqs (5a) and (5b), and normalizing to fundamental subcarrier power, we can obtain CSOs due to linear dispersion and SPM as

$$CSO_{\text{disp}} = (\pi m \beta_2 z \gamma \Omega)^2 N_{2i} + (2\pi m \beta_2 z \gamma \Omega)^2 N_{i+j} \quad (6)$$

and

$$CSO_{\text{SPM}} = \left( -\frac{1}{4} m \beta_2 N \bar{z}^2 \Omega^2 \right)^2 N_{2i} + \left( -\frac{1}{2} m \beta_2 N \bar{z}^2 \Omega^2 \right)^2 N_{i+j} \quad (7)$$

where  $\Omega$  is the angular frequency at which CSO occurs;  $N_{2i}$  and  $N_{i+j}$  are the product counts of harmonic and inter-modulation products, respectively. If there is only a single input tone, then  $N_{2i} = 1$  and  $N_{i+j} = 0$ .

### 3. NUMERICAL CALCULATIONS - SPLIT-STEP FOURIER METHOD

Eq. (2) can also be solved numerically by using Split-Step Fourier Transform method<sup>4</sup>. We can write Eq. (2) in the form

$$\frac{\partial A}{\partial z} = (\hat{D} + \hat{N})A \quad (8)$$

where  $\hat{D}$  is a differential operator that accounts for dispersion and absorption in a linear medium and  $\hat{N}$  is a nonlinear operator that governs the effect of fiber nonlinearities on signal propagation. In general, dispersion and nonlinear effect act together along the transmission distance. The split-step Fourier method obtains an approximate solution of Eq. (8), by assuming that in propagating the optical field over a small distance  $h$ , the dispersion and nonlinear effect can be assumed to act independently. The procedure to obtain  $A$  can be depicted in symmetric form mathematically as<sup>4</sup>

$$A(z + h, T) = \exp\left[\frac{h}{2}\hat{D}\right] \exp\left[\int_z^{z+h}\hat{N}(z')dz'\right] \exp\left[\frac{h}{2}\hat{D}\right] A(z, T) \quad (9)$$

The integral in the middle exponential is used to include the  $z$  dependence of the nonlinear operator  $\hat{N}$ . We have tried several step sizes to ensure the accuracy of numerical simulations. The phase of each AM carrier is assumed to be random, and each calculated CSO product is an average result of 60 different phase combinations of multiple carriers.

### 4. RESULTS

Fig. 1 shows the ratio of a second harmonic (2HD) and a fundamental carrier as a function of transmission distance, for both direct and external modulation systems.  $\gamma$  is assumed to be  $2\pi \times 20$  GHz for the former, and zero for the latter. Analytical results are presented in terms of dashed and solid lines, and numerical results are presented in terms of solid circles and squares. We can see that both analytical and numerical results completely match. In addition, we see that for a distance up to 500 km, 2HDs caused by linear fiber dispersion is more than 35 dB greater than those caused by SPM. This is the basic reason why the transmission distance (through 1.3  $\mu\text{m}$  zero-dispersion SMFs) of a 1.55  $\mu\text{m}$  direct modulation multichannel AM system is limited to only a few kilometers<sup>5</sup>.

For a direct modulation system with  $\gamma = 20$  GHz, when the modulating signal contains 80 AM-VSB channels and the transmission distance is 10 km, Fig. 2 shows the resultant CSOs as a function of channel frequency due to linear fiber dispersions. We can see that the analytical results, represented by solid lines, match fairly well with the numerical results represented by solid circles.

We also see that the performance of this direct modulation system is severely limited by the poor CSOs which range from  $-50$  to  $-32$  dBc over the entire 550 MHz band, even for this short transmission distance of 10 km.

For a chirp-free ( $\gamma = 0$ ) external modulation system with 80 AM-VSB channels, we assume the transmission distance and the launched optical power into the link to be 100 km and 20 mW, respectively. The reason why the launched optical power is assumed to be 20 mW is to ensure that SBS-induced CSOs do not occur. Fig. 3 shows the SPM-limited CSOs as a function of channel frequency. Note once again we have well-matched analytical and numerical results. We can clearly see that, without the effect of linear fiber dispersions, the SPM-limited CSOs are much lower ( $< -55$  dBc) than those obtained in Fig. 2, even though the transmission distance now is ten times longer. Note also that difference between the low-frequency CSO and the high-frequency CSO can be as high as  $\sim 40$  dB. Fig. 3 also tells us that, for a transmission distance of 100 km and a 20 mW input optical power, the SPM-induced CSO cannot meet the standard CATV requirement. Therefore, the next question we want to answer is, what is the SPM-limited repeaterless transmission distance of a 1.55  $\mu\text{m}$  external modulation AM CATV system? This question can be easily answered by using Eq. (7) and by setting CSO =  $-60$  dBc, so that we can plot in Fig. 4 the SPM-limited transmission distance (for both 1.3  $\mu\text{m}$ -zero-dispersion and dispersion-shifted SMFs) as a function of launched optical power. The values of various parameters in Eq. (7) are given in the figure caption of Fig. 4. The solid line shown in Fig. 4 is simply obtained from  $(\text{launched optical power} - 0 \text{ dBm}) / (0.22 \text{ dB/km})$ , where we have assumed the received optical power is 0 dBm. The optimum transmission distance is given by the intercept point of solid and dashed (or broken) curves, and is  $\sim 63$  km with 24 mW launched power for conventional SMF, and is  $\sim 56$  km with 18 mW launched power for dispersion-shifted SMF. This SPM-limited transmission distance is coincidentally very close to the SBS-limited transmission distance of  $\sim 65$  km conventional SMF.

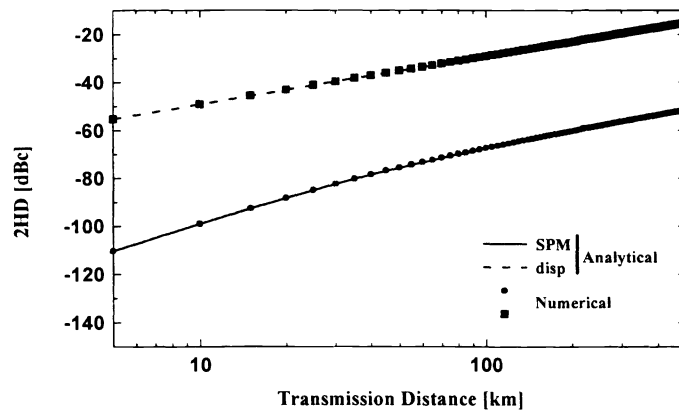
## 5. CONCLUSION

We have carried out both analytical and numerical calculations of CSO distortions due to linear dispersion and SPM, for direct and external modulation systems, respectively. Our results show that, for a 1.55  $\mu\text{m}$  external modulation system with a  $\sim 24$  mW launched power into the optical fiber link, the SPM-limited transmission distance of 63 km is very close to the SBS-limited transmission distance. This result implies that even when SBS threshold can be increased beyond  $\sim 24$  mW, the maximum repeaterless transmission distance will still be limited by SPM-induced CSOs to be around 63 km.

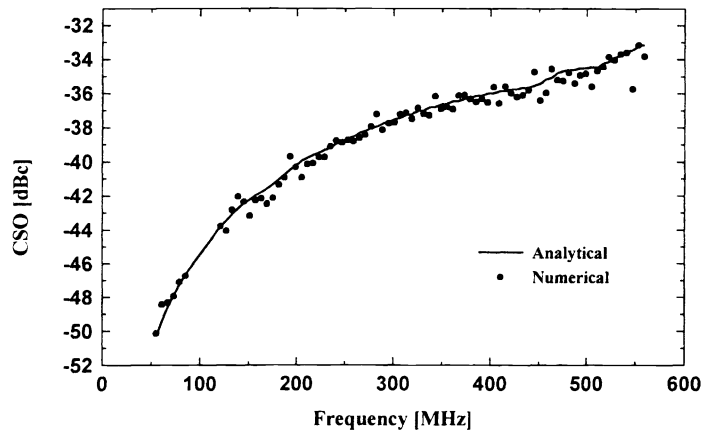
## REFERENCE

1. M. R. Phillips, K. L. Sweeney, "Distortion by Stimulated Brillouin Scattering Effect in Analog Video Lighwave Systems," OFC'96, PD23.

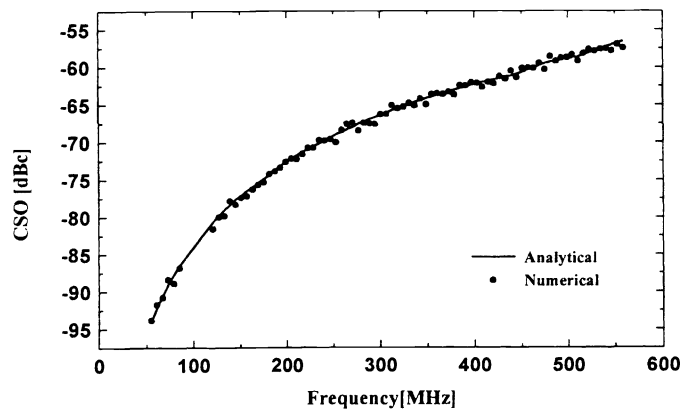
2. F. W. Willems, J. C. van der Plaats, W. Muys, "Harmonic distortion caused by stimulated Brillouin scattering suppression in externally modulated lightwave AM-CATV systems," *Electron. Lett.*, 17<sup>th</sup>, vol. 30, pp. 343-345, Feb. 1994.
3. F. W. Willems, W. Muys, J. S. Leong, "Simultaneous Suppression of Stimulated Brillouin Scattering and Interferometric Noise in Externally Modulated Lightwave AM-SCM Systems," *IEEE Photon. Technol. Lett.*, vol. 6, pp. 1476-1478, Dec. 1994.
4. G. P. Agrawal, "Nonlinear fiber optics," second edition, Academic press, 1995.
5. M. R. Phillips, T. E. Darcie, D. Marcuse, G. E. Bodeep, and N. J. Frigo, "Nonlinear distortion generated by dispersive transmission of chirped intensity-modulated signals," *IEEE Photon. Technol. Lett.*, vol. 3, pp. 481-483, May 1991.



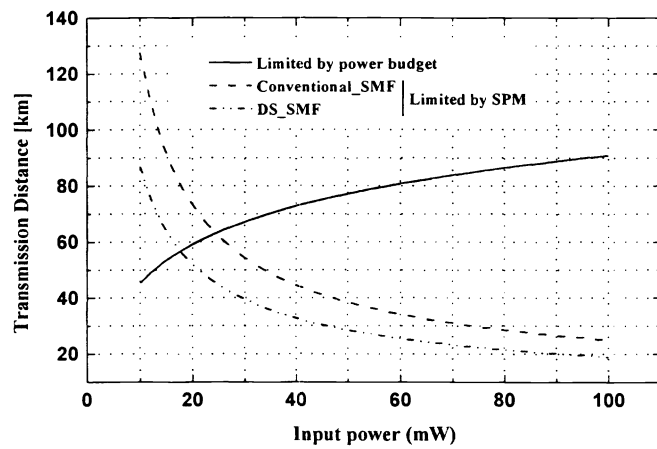
**Figure 1.** Analytical and numerical results for second harmonic distortion versus transmission distance. The following parameters are assumed:  $m = 0.04/\text{ch}$ ,  $\Omega = 2\pi \times 505.25 \text{ MHz}$ ,  $\lambda_0 = 1.55 \mu\text{m}$ ,  $D = 17 \text{ ps/nm} \cdot \text{km}$ ,  $\gamma = 20 \text{ GHz}$ ,  $P = 20 \text{ mW}$ ,  $A_{\text{eff}} = 80 \mu\text{m}^2$ ,  $\alpha = 0.22 \text{ dB/km}$ .



**Figure 2.** Analytical and numerical results for frequency-dependent, linear dispersion-induced CSOs. The following parameters are assumed: transmission distance = 10 km,  $m = 0.04/\text{ch}$ , AM-channel number (NTSC frequency plan) = 80,  $\lambda_0 = 1.55 \mu\text{m}$ ,  $D = 17 \text{ ps/nm} \cdot \text{km}$ ,  $\gamma = 20 \text{ GHz}$ .



**Figure 3.** Analytical and numerical results for frequency-dependent, SPM-induced CSOs. The following parameters are assumed: transmission distance = 100 km,  $m = 0.04/\text{ch}$ , AM-channel number (NTSC frequency plan) = 80,  $\lambda_0 = 1.55 \mu\text{m}$ ,  $D = 17 \text{ ps/nm} \cdot \text{km}$ ,  $n_2 = 3 \times 10^{-20} \text{ m}^2/\text{W}$ ,  $P = 20 \text{ mW}$ ,  $\alpha = 0.22 \text{ dB/km}$ ,  $A_{\text{eff}} = 80 \mu\text{m}^2$ .



**Figure 4.** Transmission distance limited by SPM and optical attenuation loss, plotted as a function of the input optical power into a 1.3  $\mu\text{m}$ -zero dispersion (conventional) or dispersion-shifted (DS) SMFs. Parameters used are:  $m = 0.04/\text{ch}$ , AM-channel number (NTSC frequency plan) = 80,  $n_2 = 3 \times 10^{-20} \text{ m}^2/\text{W}$ ,  $\lambda_0 = 1.55 \mu\text{m}$ ,  $D = 17 \text{ ps}/\text{nm} \cdot \text{km}$ ,  $\alpha = 0.22 \text{ dB}/\text{km}$ ,  $A_{\text{eff}} = 80 \mu\text{m}^2$  for conventional\_SMF and  $A_{\text{eff}} = 50 \mu\text{m}^2$  for DS\_SMF