Optimum two-dimensional beamforming employing sum-and-difference technique

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A 2-D adaptive beamforming scheme using a quadruplet array is presented. The scheme is developed based on the sum-and-difference technique employed in conventional monopulse trackers. The construction of the proposed beamformer involves a joint linearly constrained minimum variance procedure on the sum-and-difference processors. With the difference beam used as an auxiliary to suppress the desired signal, the resulting sum beam exhibits robustness to pointing errors in acquiring the desired signal. To maintain an accurate look direction, a simple method for desired source localization is developed using beamspace MUSIC. The proposed beamformer working in conjunction with the angle estimator provides a much better solution to 2-D beamforming than the MVDR beamformer, as confirmed by analysis results and numerical examples.

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INTRODUCTION

The conventional minimum variance distortionless response (MVDR) beamformer[1] using the ensemble correlation matrix and true look direction is known to achieve good performance in terms of output signal-to-interference-plus-noise ratio (SINR) for uncorrelated signals. However, a small perturbation in the look direction vector, such as that incurred with pointing errors, results in severe performance degradation. The effect of errors in the look direction vector on the performance of the MVDR-type beamformer has been studied in the literature.[2–5] These results indicate that the MVDR beamformer suffers desired signal cancellation due to imperfect knowledge of the look direction vector. Another effect detrimental to the MVDR beamformer is the mutual cancellation due to the presence of coherent interference. In this case, it not only fails to suppress the interference, but also tends to eliminate the desired signal in the beamformer output. A method for alleviating desired signal cancellation was suggested by Duvall,[6] wherein the signal component is removed from the adaptive processor, then reinserted to the array to form the final output. The Duvall beamformer was shown to exhibit a better SINR performance than the MVDR beamformer in the presence of pointing errors and/or coherent interference. Research results have been proposed to further improve the robustness of the Duvall beamformer.[7,8]

Motivated by the concept of desired signal suppression, we propose in this paper a new adaptive 2-D beamforming scheme based on the sum-and-difference technique[9] for arrays consisting of quadruplets. In conventional monopulse target localization, a sum beam is formed with the maximum gain in the look direction close to the target. A difference beam is formed accordingly with a null in the same direction, such that the target can be localized via the ratio of the difference beam output to the sum beam output. This is similar to the Duvall beamformer wherein a "difference beam" with a null in the look direction is formed using the minimum variance technique, and a "sum beam" is formed accordingly with its weights copied from the difference beamformer. The Duvall beamformer was developed originally for uniform, linear arrays for which successive subtraction can be performed to construct the difference beamformer. Nevertheless, it was not used to localize the desired source. In the proposed method, the difference beam is used as an "auxiliary" beam to eliminate the desired signal, and the sum beam is used as a "primary" beam to acquire the desired signal. The realization of the proposed scheme involves a joint linearly constrained minimum variance (LCMV)[10] procedure on the sum-and-difference processors, followed by a translation of weights to construct the sum beamformer. Due to the quadruplet structure of the array, the resulting sum-and-difference patterns share a common factor. We demonstrate that this common pattern factor exhibits interference cancellation such that the sum-and-difference beams share a set of deep nulls in the interfering directions. As with the Duvall beamformer, these nulls are in fact generated by the difference beamformer first, then "copied" to the sum beamformer via the common pattern factor.

To avoid performance degradation due to a large pointing error, it is necessary to develop a scheme for estimating the desired source direction of arrival (DOA). It is demonstrated that applying the beamspace MUSIC technique[11] on a set of judiciously formed sum-and-difference data leads to a closed-form desired source DOA estimator which provides an accurate look direction in the presence of interference. For practical implementations, an iterative procedure is suggested for preliminary localization of the desired source to avoid performance breakdown due to initial beam squint. The proposed beamformer working in conjunction with the DOA estimator thus provides a much better solution to 2-D adaptive beamforming than the MVDR beamformer.

I. SIGNAL MODEL AND PROBLEM FORMULATION

Some of the notations frequently used in the paper are defined as follows:

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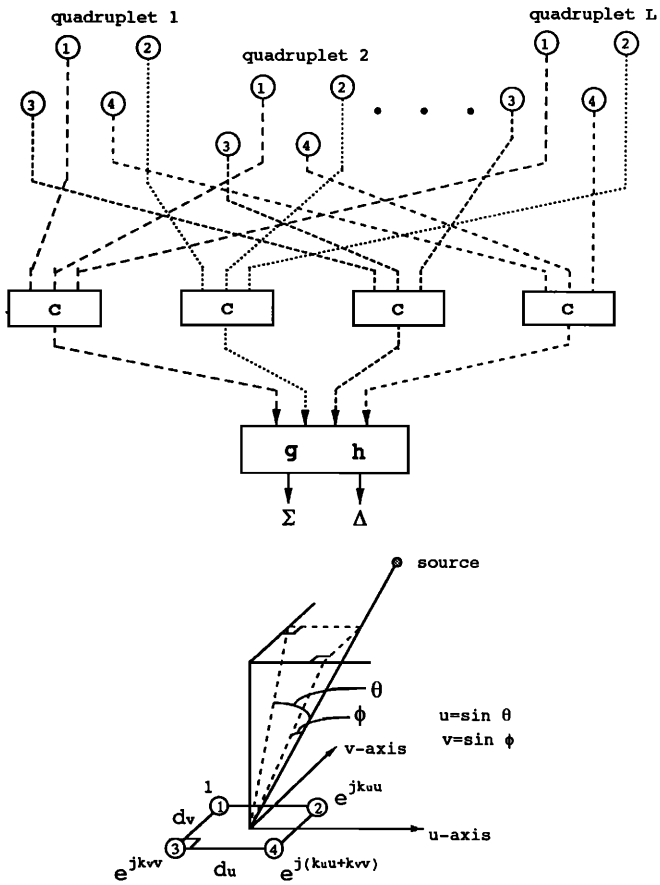

FIG. 1. Geometry of the quadruplet array and its coordinate system.

$\mathbf{M}^T$: transpose of $\mathbf{M}$, $\mathbf{M}^H=(\mathbf{M}^T)^*$: conjugate transpose of $\mathbf{M}$, $\mathbf{I}_n$: $n\times n$ identity matrix, $\mathbf{0}_n$: $n\times 1$ all-zero vector, and $E\{x\}$: expected value of x.

Consider the scenario of a desired source and K interferers detected by an M-element narrow-band sensor array configured in L quadruplets, as shown in Fig. 1. Each quadruplet is composed of four identical elements, and the elements in different quadruplets need not be the same. The elements in each quadruplet are arranged in a rectangle with interelement spacings d_u and d_v along two orthogonal directions. Alternatively, the array can be viewed as consisting of four identical subarrays of size L as illustrated. Note that M is less than $4L$ if the subarrays overlap. The desired source and interferers are assumed to be in the far field of the array such that the signal received at the array can be modeled as plane waves. For the convenience of notation, we define the $4L\times 1$ augmented array data vector (in complex envelops):

$$\mathbf{x}=\begin{bmatrix}\mathbf{x}_1\\ \mathbf{x}_2\\ \mathbf{x}_3\\ \mathbf{x}_4\end{bmatrix}=\xi_d\mathbf{a}(u_d,v_d)+\sum_{k=1}^{K}\xi_k\mathbf{a}(u_k,v_k)+\mathbf{n}_w, \tag{1}$$

which is composed of the $L\times 1$ data vectors $\mathbf{x}_i$, $i=1,\ldots,4$, received at the four subarrays at a certain sampling instant. The various quantities in (1) are defined as follows. The complex scalar ξ_d represents the desired source signal with power $\sigma_d^2=E\{|\xi_d|^2\}$. The real scalars $u_d=\sin(\theta_d)$ and $v_d=\sin(\phi_d)$ represent the sine-space angles of the desired source measured with respect to the quadruplet axes, as depicted in Fig. 1. The $4L\times 1$ vector $\mathbf{a}(u_d,v_d)$ is the direction vector accounting for the gain/phase variation across the four subarrays due to the desired signal wavefront. Similarly, ξ_k represents the kth interfering signal from direction (u_k,v_k) with power $\sigma_k^2=E\{|\xi_k|^2\}$, and $\mathbf{a}(u_k,v_k)$ is the corresponding direction vector. Finally, the $4L\times 1$ vector $\mathbf{n}_w$ is composed of the complex envelops of the additive noise presents at the four subarrays. We assume that the noise component is uncorrelated from element to element with the same power σ_n^2, and is uncorrelated with the desired and interfering signals. It should be noted that $\mathbf{x}$ has some repeated components if the subarrays overlap.

The general structure of $\mathbf{a}(u,v)$ can be easily derived by considering the array as a "superquadruplet," and the four subarrays as four "superelements" characterized by the same $L\times 1$ direction vector $\mathbf{a}_s(u,v)$. A plane wave from direction (u,v) induces the same gain/phase response within each subarray according to $\mathbf{a}_s(u,v)$, and pure phase shifts among different subarrays according to the geometry of the quadruplet. These are described mathematically by[12]

$$\mathbf{a}(u,v)=\begin{bmatrix}\mathbf{a}_s(u,v)\\ e^{jk_uu}\mathbf{a}_s(u,v)\\ e^{jk_vv}\mathbf{a}_s(u,v)\\ e^{j(k_uu+k_vv)}\mathbf{a}_s(u,v)\end{bmatrix}, \tag{2}$$

where $k_u=2\pi d_u/\lambda$, $k_v=2\pi d_v/\lambda$, and λ is the source wavelength. The exponential terms in (2) account for those pure phase shifts and are denoted here in the vector form

$$\mathbf{a}_q(u,v)=[1,\quad e^{jk_uu},\quad e^{jk_vv},\quad e^{j(k_uu+k_vv)}]^T. \tag{3}$$

The subscript q indicates that $\mathbf{a}_q(u,v)$ is in fact the direction vector associated with the quadruplets.

A. Sum-and-difference beamformers

In conventional amplitude-comparison source localization, the sum-and-difference beams are formed in the vicinity of the desired source DOA by applying two sets of beamforming weights on the array data:

$$\Sigma=\mathbf{s}^H\mathbf{x},\quad \Delta=\mathbf{d}^H\mathbf{x}, \tag{4}$$

where $\mathbf{s}$ and $\mathbf{d}$ are the $4L\times 1$ sum-and-difference beamforming weight vectors, respectively. In general, the difference beam produces a null in the look direction where the sum beam has the maximum gain. The source DOA is determined via the ratio Δ/Σ, indicating the normalized off-boresight error. In this regard, the difference beam is employed as an auxiliary beam to localize the desired signal, whereas the sum beam works as the primary beam to acquire the desired signal. This is similar in principle to the Duvall beamformer[6] in which a "master" beam is formed with a null in the look direction via subtractive preprocessing, and a "slave" beam is formed accordingly with its weights copied from the master beam so as to produce a high gain in the same direction. Much work has been reported on the modifications of the Duvall beamformer.[7,8,13] We here demonstrate that the Duvall scheme can be readily applied to the quadruplet array described previously via the sum-and-difference processing.

We first examine the distinctive structure of the sum-and-difference weight vectors.

Consider again the subarray configuration shown in Fig. 1. Suppose that a beamformer is attached to each subarray with the same $L\times 1$ weight vector $\mathbf{c}=[c_1,c_2,\ldots,c_L]^T$. The sum-and-difference beams are then formed with 4×1 weight vectors $\mathbf{g}=[g_1,g_2,g_3,g_4]^T$ and $\mathbf{h}=[h_1,h_2,h_3,h_4]^T$, respectively, treating these subarray beamformers as the elements of a superquadruplet. It can be verified[14] that the resulting sum-and-difference weight vectors exhibit the following factorization:

$$\mathbf{s}=\mathbf{G}\mathbf{c}, \quad \mathbf{d}=\mathbf{H}\mathbf{c}, \tag{5}$$

where

$$\begin{aligned}\mathbf{G}&=\text{Toeplitz}\{[g_1,\mathbf{0}_{L-1}^T,g_2,\mathbf{0}_{L-1}^T,g_3,\mathbf{0}_{L-1}^T,g_4]\}_{4L\times L},\\ \mathbf{H}&=\text{Toeplitz}\{[h_1,\mathbf{0}_{L-1}^T,h_2,\mathbf{0}_{L-1}^T,h_3,\mathbf{0}_{L-1}^T,h_4]\}_{4L\times L},\end{aligned} \tag{6}$$

and Toeplitz $\{[v_1,v_2,\ldots,v_n]\}_{m\times k}$ denotes the $m\times k$ banded Toeplitz matrix formed according to

$$\text{Toeplitz}\{[v_1,v_2,\ldots,v_n]\}_{m\times k}=\begin{bmatrix} v_1 & & \mathbf{0} \\ \vdots & v_1 & \\ v_n & \vdots & \ddots \\ & v_n & \ddots & v_1 \\ & & \ddots & \vdots \\ \mathbf{0} & & & v_n \end{bmatrix}. \tag{7}$$

We here refer to $\mathbf{G}$ and $\mathbf{H}$ as the sum-and-difference "preprocessors," respectively.

On the other hand, invoking the multiplication principle of phased arrays,[15] the sum-and-difference patterns can be expressed as

$$\begin{aligned}s(u,v)&=\mathbf{s}^H\mathbf{a}(u,v)=g(u,v)c(u,v),\\ d(u,v)&=\mathbf{d}^H\mathbf{a}(u,v)=h(u,v)c(u,v),\end{aligned} \tag{8}$$

where

$$c(u,v)=\mathbf{c}^H\mathbf{a}_s(u,v) \tag{9}$$

represents the subarray pattern, and

$$g(u,v)=\mathbf{g}^H\mathbf{a}_q(u,v), \quad h(u,v)=\mathbf{h}^H\mathbf{a}_q(u,v) \tag{10}$$

represent the quadruplet patterns associated with the sum-and-difference beams, respectively. An important point gleaned from (8) is that the sum-and-difference patterns share a common factor $c(u,v)$.

According to the properties of the sum-and-difference patterns stated previously, we impose that $h(u,v)$ has a null at (u_o,v_o), the look direction of the array, and $g(u,v)$ has the maximum gain at (u_o,v_o). Incorporation of these facts leads to the following choice for $\mathbf{g}$ and $\mathbf{h}$:

$$\begin{aligned}\mathbf{g}&=[1, \quad e^{jk_uu_o}, \quad e^{jk_vv_o}, \quad e^{j(k_uu_o+k_vv_o)}]^T,\\ \mathbf{h}&=[1, \quad -e^{jk_uu_o}, \quad -e^{jk_vv_o}, \quad e^{j(k_uu_o+k_vv_o)}]^T.\end{aligned} \tag{11}$$

Substitution of (3) and (11) in (10) yields

$$g(u,v)=g_u(u)g_v(v), \quad h(u,v)=h_u(u)h_v(v), \tag{12}$$

where

$$\begin{aligned}g_u(u)&=1+e^{jk_u(u-u_o)}; \quad g_v(v)=1+e^{jk_v(v-v_o)},\\ h_u(u)&=1-e^{jk_u(u-u_o)}; \quad h_v(v)=1-e^{jk_v(v-v_o)}.\end{aligned} \tag{13}$$

This indicates that the quadruplet patterns become separable in u and v. In particular, we note that $g(u_o,v_o)=4$ and $h(u_o,v_o)=0$.

It is straightforward to verify by substituting (11) in (6) and using (10) that

$$\mathbf{G}^H\mathbf{G}=\mathbf{H}^H\mathbf{H}=4\mathbf{I}_L \tag{14}$$

and

$$\begin{aligned}\mathbf{G}^H\mathbf{a}(u,v)&=g(u,v)\mathbf{a}_s(u,v),\\ \mathbf{H}^H\mathbf{a}(u,v)&=h(u,v)\mathbf{a}_s(u,v).\end{aligned} \tag{15}$$

The second equation is an important property of the quadruplet array, which says that the sum-and-difference preprocessors convert the original direction vector into the corresponding subarray direction vector, with the conversion gain given by the respective quadruplet patterns.

B. Construction of optimum beamformer

The proposed 2-D beamformer is constructed via two steps. First, the subarray weight vector $\mathbf{c}$ is determined by minimizing the output power of the difference (master) beamformer subject to a unit response constraint on the sum beamformer:

$$\begin{aligned}&\min_{\mathbf{c}} \; E\{|\mathbf{d}^H\mathbf{x}|^2\}\equiv\mathbf{c}^H\mathbf{H}^H\mathbf{R}_{xx}\mathbf{H}\mathbf{c},\\ &\text{subject to} \;\; \mathbf{s}^H\mathbf{a}(u_o,v_o)=\mathbf{c}^H\mathbf{G}^H\mathbf{a}(u_o,v_o)\\ &\qquad\qquad\qquad =4\mathbf{c}^H\mathbf{a}_s(u_o,v_o)=\alpha,\end{aligned} \tag{16}$$

where $\mathbf{R}_{xx}=E\{\mathbf{x}\mathbf{x}^H\}$ is the ensemble data correlation matrix, α is a nonzero constant, and we have used (15) in the constraint equation. This is a classical LCMV problem whose solution is given by

$$\mathbf{c}=[\mathbf{H}^H\mathbf{R}_{xx}\mathbf{H}]^{-1}\mathbf{a}_s(u_o,v_o). \tag{17}$$

Note that we have omitted the constant gain in $\mathbf{c}$ since it does not affect the output SINR performance. The resulting $\mathbf{c}$ vector is then "copied" to the sum (slave) beamformer, yielding the sum weight vector

$$\mathbf{s}=\mathbf{G}[\mathbf{H}^H\mathbf{R}_{xx}\mathbf{H}]^{-1}\mathbf{a}_s(u_o,v_o). \tag{18}$$

Because of the minimum variance operation, the subarray beamformer produces nulls in the directions of incoherent interferers. It follows from (8) that the sum pattern exhibits nulls in the same directions as well. These nulls are in fact generated by the difference beamformer first, then transferred to the sum beamformer via the common pattern factor $c(u,v)$. The advantage of working with the sum beamformer instead of the subarray beamformer is that the sum beamformer exhibits a higher gain in signal-to-noise ratio (SNR) against spatially white noise. For nonoverlapping subarrays, the sum beamformer offers approximately $4\times$ the SNR gain of the subarray beamformer.

It is of interest to examine the behavior of the beamformer with a perfect look direction, i.e., $(u_o,v_o)=(u_d,v_d)$.

In this case, the desired signal is completely removed by the difference preprocessor, and the cost function in (16) is rephrased as one of minimizing the output interference-plus-noise power of the difference beamformer. This concludes that the solution in (17) maximizes the ratio of the output signal power of the sum beamformer to the output interference-plus-noise power of the difference beamformer. The optimality of the beamformer is therefore defined in the sense of maximum cross SINR. As a consequence, the proposed beamformer does not constitute a true optimum detector when used in a hypothesis test of the presence of the desired source, even the perfect knowledge of the source localization is given. On the other hand, if the beamformer is employed as a spatial spectrum estimator for localizing the desired source, it does not yield the optimum DOA estimate due to the lack of *a priori* knowledge of the presence of the desired source. These results also apply to the MVDR beamformer.[16] However, it is conceivable that the proposed beamformer is superior to the MVDR beamformer in terms of optimality since the desired signal has been effectively removed from $\mathbf{R}_{xx}$ in (18).

As a final remark, we note that in practice, $\mathbf{R}_{xx}$ is not available and an estimated version $\hat{\mathbf{R}}_{xx}$ is used instead. Given N data vectors $\mathbf{x}(n)$, $n=1,\ldots,N$, collected at different times, the sample correlation matrix constructed by

$$\hat{\mathbf{R}}_{xx}=\frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(n)\mathbf{x}^H(n) \tag{19}$$

is a consistent estimate of $\mathbf{R}_{xx}$ under the assumption of ergodicity. Unfortunately, working with (19) results in the loss of "extendibility" associated with $\mathbf{R}_{xx}$.[17] In addition, the nonstationarity of practical data does not always allow sufficient time to obtain an accurate time average.[16] The problem of extendibility is somewhat lessened by the use of a structured estimate,[18] whereas the nonstationarity of environment necessitates an adaptive implementation.

II. PERFORMANCE OF PROPOSED BEAMFORMER

The following sections derive the output SINR of the sum beamformer for some simple cases. The derivations are all based on the true ensemble data correlation matrix, and therefore the effect of finite data sample size is not considered. For brevity, we define below some notations:

$$\mathbf{R}_{nn}=E\left[\left(\sum_{k=1}^{K}\xi_k\mathbf{a}(u_k,v_k)+\mathbf{n}_w\right)\times\left(\sum_{k=1}^{K}\xi_k\mathbf{a}(u_k,v_k)+\mathbf{n}_w\right)^H\right], \tag{20}$$

$$\mathbf{R}_{ww}=E\{\mathbf{n}_w\mathbf{n}_w^H\}, \tag{21}$$

$$\mathbf{Q}_{xx}=\mathbf{H}^H\mathbf{R}_{xx}\mathbf{H}, \tag{22}$$

$$\mathbf{Q}_{nn}=\mathbf{H}^H\mathbf{R}_{nn}\mathbf{H}, \tag{23}$$

$$\mathbf{Q}_{ww}=\mathbf{H}^H\mathbf{R}_{ww}\mathbf{H}, \tag{24}$$

$$\mathbf{a}_x=\mathbf{a}_s(u_x,v_x),\quad x=o,d,1,\ldots,K, \tag{25}$$

$$\tau_{xy}=4\sigma_n^2\mathbf{a}_x^H\mathbf{Q}_{ww}^{-1}\mathbf{a}_y,\quad x,y=o,d,1,\ldots,K, \tag{26}$$

$$\rho_{xy}=\mathbf{a}_x^H\mathbf{a}_y,\quad x,y=o,d,1,\ldots,K, \tag{27}$$

$$\gamma_x=\sigma_x^2/\sigma_n^2,\quad x=d,1,\ldots,K, \tag{28}$$

$$\mu_x=|g(u_x,v_x)|^2,\quad x=d,1,\ldots,K, \tag{29}$$

$$\kappa_x=|h(u_x,v_x)|^2,\quad x=d,1,\ldots,K. \tag{30}$$

Note that $\mathbf{R}_{nn}$ represents the interference-plus-noise correlation matrix, and $\mathbf{R}_{ww}$ represents the noise-only correlation matrix. Due to the spatially white noise assumption, the (i,k)th component of $\mathbf{R}_{ww}$ equals σ_n^2 if the ith and kth components of $\mathbf{x}$ correspond to the same array element, and zero otherwise. The $L\times L$ matrices $\mathbf{Q}_{xx}$, $\mathbf{Q}_{nn}$, and $\mathbf{Q}_{ww}$ represent the "difference preprocessed" versions of $\mathbf{R}_{xx}$, $\mathbf{R}_{nn}$, and $\mathbf{R}_{ww}$, respectively.

A. Ideal case: No pointing errors

In this case, $u_o=u_d$ and the desired signal is completely removed by the difference preprocessor. The sum weight vector is given accordingly by

$$\mathbf{s}=\mathbf{G}\mathbf{Q}_{xx}^{-1}\mathbf{a}_d=\mathbf{G}\mathbf{Q}_{nn}^{-1}\mathbf{a}_d. \tag{31}$$

Using (1) and (15), the sum beamformer output desired signal power is given by

$$P_d=\sigma_d^2\mathbf{s}^H\mathbf{a}(u_d,v_d)\mathbf{a}^H(u_d,v_d)\mathbf{s}=16\sigma_d^2(\mathbf{a}_d^H\mathbf{Q}_{nn}^{-1}\mathbf{a}_d)^2. \tag{32}$$

Similarly, the output interference-plus-noise power is given by

$$P_{in}=\mathbf{s}^H\mathbf{R}_{nn}\mathbf{s}=\mathbf{a}_d^H\mathbf{Q}_{nn}^{-1}\mathbf{G}^H\mathbf{R}_{nn}\mathbf{G}\mathbf{Q}_{nn}^{-1}\mathbf{a}_d, \tag{33}$$

such that

$$\mathrm{SINR}_o=\frac{P_d}{P_{in}}=\frac{16\sigma_d^2(\mathbf{a}_d^H\mathbf{Q}_{nn}^{-1}\mathbf{a}_d)^2}{\mathbf{a}_d^H\mathbf{Q}_{nn}^{-1}\mathbf{G}^H\mathbf{R}_{nn}\mathbf{G}\mathbf{Q}_{nn}^{-1}\mathbf{a}_d}. \tag{34}$$

1. Case 1: No interference

In the absence of interference, we have $\mathbf{R}_{nn}=\mathbf{R}_{ww}$ and $\mathbf{Q}_{nn}=\mathbf{Q}_{ww}$. The output SNR reduces to

$$\mathrm{SNR}_o=\frac{\gamma_d\tau_{dd}^2}{\sigma_n^2\mathbf{a}_d^H\mathbf{Q}_{ww}^{-1}\mathbf{G}^H\mathbf{R}_{ww}\mathbf{G}\mathbf{Q}_{ww}^{-1}\mathbf{a}_d}. \tag{35}$$

For nonoverlapping subarrays, we have $\mathbf{R}_{ww}=\sigma_n^2\mathbf{I}_{4L}$. In this case, τ_{xy} reduces to ρ_{xy} due to (14), and the output SNR simplifies accordingly to

$$\mathrm{SNR}_o=4\gamma_d\rho_{dd}=4\gamma_d\|\mathbf{a}_d\|^2, \tag{36}$$

where $\|\ \|$ denotes the 2-norm. This is also the maximum output SNR produced by the MVDR beamformer.[5]

2. Case 2: Single interferer

In the presence of a single interferer, we have

$$\mathbf{R}_{nn}=\sigma_1^2\mathbf{a}(u_1,v_1)\mathbf{a}^H(u_1,v_1)+\mathbf{R}_{ww} \tag{37}$$

and

$$Q_{nn}^{-1}=[\sigma_1^2\mathbf{H}^H\mathbf{a}(u_1,v_1)\mathbf{a}^H(u_1,v_1)\mathbf{H}+\mathbf{Q}_{ww}]^{-1}$$
$$=[\sigma_1^2\kappa_1\mathbf{a}_1\mathbf{a}_1^H+\mathbf{Q}_{ww}]^{-1}$$
$$=\mathbf{Q}_{ww}^{-1}-\frac{\sigma_1^2\kappa_1\mathbf{Q}_{ww}^{-1}\mathbf{a}_1\mathbf{a}_1^H\mathbf{Q}_{ww}^{-1}}{1+\frac{1}{4}\gamma_1\kappa_1\tau_{11}}. \tag{38}$$

The third equality in (38) holds due to the matrix inversion lemma. Substituting (37) and (38) in (34) leads to

$$\mathrm{SINR}_o=\left[\frac{1}{\sigma_n^2}\gamma_d\left(\tau_{dd}-\frac{\gamma_1\kappa_1|\tau_{d1}|^2}{4+\gamma_1\kappa_1\tau_{11}}\right)^2\right]\times\left(\frac{\gamma_1\mu_1|\tau_{d1}|^2}{\sigma_n^2(4+\gamma_1\kappa_1\tau_{11})^2}+\mathbf{a}_d^H\mathbf{Q}_{nn}^{-1}\mathbf{G}^H\mathbf{R}_{ww}\mathbf{G}\mathbf{Q}_{nn}^{-1}\mathbf{a}_d\right)^{-1}. \tag{39}$$

For nonoverlapping subarrays, we have the simplified expression for the output noise-only power

$$P_n=4\sigma_n^2\mathbf{a}_d^H\mathbf{Q}_{nn}^{-1}\mathbf{Q}_{nn}^{-1}\mathbf{a}_d$$
$$=\frac{1}{4\sigma_n^2}\left(\mathbf{a}_d^H-\frac{\gamma_1\kappa_1\rho_{d1}}{4+\gamma_1\kappa_1\rho_{11}}\mathbf{a}_1^H\right)\left(\mathbf{a}_d-\frac{\gamma_1\kappa_1\rho_{1d}}{4+\gamma_1\kappa_1\rho_{11}}\mathbf{a}_1\right)$$
$$=\frac{1}{4\sigma_n^2}\left[\rho_{dd}-2\frac{\gamma_1\kappa_1|\rho_{d1}|^2}{4+\gamma_1\kappa_1\rho_{11}}+\left(\frac{\gamma_1\kappa_1|\rho_{d1}|}{4+\gamma_1\kappa_1\rho_{11}}\right)^2\rho_{11}\right]. \tag{40}$$

Substituting this back into (39) yields

$$\mathrm{SINR}_o=\left[\gamma_d\left(\rho_{dd}-\frac{\gamma_1\kappa_1|\rho_{d1}|^2}{4+\gamma_1\kappa_1\rho_{11}}\right)^2\right]\left\{\frac{\gamma_1\mu_1|\rho_{d1}|^2}{(4+\gamma_1\kappa_1\rho_{11})^2}+\frac{1}{4}\left[\rho_{dd}-2\frac{\gamma_1\kappa_1|\rho_{d1}|^2}{4+\gamma_1\kappa_1\rho_{11}}+\left(\frac{\gamma_1\kappa_1|\rho_{d1}|}{4+\gamma_1\kappa_1\rho_{11}}\right)^2\rho_{11}\right]\right\}^{-1}$$
$$=\frac{4\gamma_d[4\rho_{dd}+\gamma_1\kappa_1(\rho_{11}\rho_{dd}-|\rho_{d1}|^2)]^2}{16\rho_{dd}+4\gamma_1\mu_1|\rho_{d1}|^2+\gamma_1\kappa_1(\rho_{dd}\rho_{11}-|\rho_{d1}|^2)(8+\gamma_1\kappa_1\rho_{11})}. \tag{41}$$

It is straightforward to verify that (41) reduces to

$$\mathrm{SINR}_o=4\gamma_d\rho_{dd},\quad \text{for } \gamma_1\ll 1,$$
$$\mathrm{SINR}_o=4\gamma_d\rho_{dd}\left(1-\frac{|\rho_{d1}|^2}{\rho_{dd}\rho_{11}}\right),\quad \text{for } \gamma_1\gg 1. \tag{42}$$

As expected, the first result is identical to that given in (36). The second result indicates that the effect of interference suppression degrades as the interferer approaches the desired source.

3. Case 3: Multiple interferers

There is no simple expression for the output SINR with multiple interferers present. Nevertheless, the behavior of the sum beamformer follows the same trend as in the case of a single interferer, so long as the interferers are mutually uncorrelated. The beamformer can handle correlated interference, but fails to cancel multiple coherent (100% correlated) interferers.[13] Some remedies, such as virtual spatial dither[6] and spatial smoothing[19] can be applied to decorrelate the interference.

B. Effect of pointing errors

With pointing errors present, the desired signal is not removed by the difference preprocessor. The sum weight vector is given as in (18),

$$\mathbf{s}=\mathbf{G}\mathbf{Q}_{xx}^{-1}\mathbf{a}_o, \tag{43}$$

and the corresponding output SINR is given by

$$\mathrm{SINR}_o=\frac{\sigma_d^2\mu_d|\mathbf{a}_o^H\mathbf{Q}_{xx}^{-1}\mathbf{a}_d|^2}{\mathbf{a}_o^H\mathbf{Q}_{xx}^{-1}\mathbf{G}^H\mathbf{R}_{nn}\mathbf{G}\mathbf{Q}_{xx}^{-1}\mathbf{a}_o}. \tag{44}$$

1. Case 1: No interference

In this case, we have $\mathbf{R}_{nn}=\mathbf{R}_{ww}$ such that

$$\mathbf{R}_{xx}=\sigma_d^2\mathbf{a}(u_d,v_d)\mathbf{a}^H(u_d,v_d)+\mathbf{R}_{ww} \tag{45}$$

and

$$\mathbf{Q}_{xx}^{-1}=[\sigma_d^2\mathbf{H}^H\mathbf{a}(u_d,v_d)\mathbf{a}^H(u_d,v_d)\mathbf{H}+\mathbf{Q}_{ww}]^{-1}$$
$$=[\sigma_d^2\kappa_d\mathbf{a}_d\mathbf{a}_d^H+\mathbf{Q}_{ww}]^{-1}$$
$$=\mathbf{Q}_{ww}^{-1}-\frac{\sigma_d^2\kappa_d\mathbf{Q}_{ww}^{-1}\mathbf{a}_d\mathbf{a}_d^H\mathbf{Q}_{ww}^{-1}}{1+\frac{1}{4}\gamma_d\kappa_d\tau_{dd}}. \tag{46}$$

Substituting (46) in (44) yields

$$\mathrm{SNR}_o=\left[\frac{1}{\sigma_n^2}\gamma_d\mu_d\left(\frac{|\tau_{do}|}{4+\gamma_d\kappa_d\tau_{dd}}\right)^2\right]\times(\mathbf{a}_o^H\mathbf{Q}_{xx}^{-1}\mathbf{G}^H\mathbf{R}_{ww}\mathbf{G}\mathbf{Q}_{xx}^{-1}\mathbf{a}_o)^{-1}. \tag{47}$$

For nonoverlapping subarrays, the output noise-only power can be simplified as

$$P_n=4\sigma_n^2\mathbf{a}_o^H\mathbf{Q}_{xx}^{-1}\mathbf{Q}_{xx}^{-1}\mathbf{a}_o$$
$$=\frac{1}{4\sigma_n^2}\left(\mathbf{a}_o^H-\frac{\gamma_d\kappa_d\rho_{od}}{4+\gamma_d\kappa_d\rho_{dd}}\mathbf{a}_d^H\right)\left(\mathbf{a}_o-\frac{\gamma_d\kappa_d\rho_{do}}{4+\gamma_d\kappa_d\rho_{dd}}\mathbf{a}_d\right)$$
$$=\frac{1}{4\sigma_n^2}\left[\rho_{oo}-2\frac{\gamma_d\kappa_d|\rho_{do}|^2}{4+\gamma_d\kappa_d\rho_{dd}}+\left(\frac{\gamma_d\kappa_d|\rho_{do}|}{4+\gamma_d\kappa_d\rho_{dd}}\right)^2\rho_{dd}\right]. \tag{48}$$

Substituting this back into (47), we get

$$\mathrm{SNR}_o=\left[4\gamma_d\mu_d\left(\frac{|\rho_{do}|}{4+\gamma_d\kappa_d\rho_{dd}}\right)^2\right]\times\left[\rho_{oo}-2\,\frac{\gamma_d\kappa_d|\rho_{do}|^2}{4+\gamma_d\kappa_d\rho_{dd}}+\left(\frac{\gamma_d\kappa_d|\rho_{do}|}{4+\gamma_d\kappa_d\rho_{dd}}\right)^2\rho_{dd}\right]^{-1}=\frac{4\gamma_d(\mu_d/16)|\rho_{do}|^2}{\rho_{oo}+\frac{1}{16}\gamma_d\kappa_d(8+\gamma_d\kappa_d\rho_{dd})(\rho_{oo}\rho_{dd}-|\rho_{do}|^2)}. \tag{49}$$

Some interesting points are observed regarding (49). First, SNR_o is inversely proportional to γ_d for a large γ_d and a nonzero pointing error. This is reasonable since the desired signal remaining after difference preprocessing is in fact treated as interference. Second, SNR_o reduces to $4\gamma_d\rho_{dd}$ as the pointing error approaches zero such that $\kappa_d\approx 0$, $\mu_d\approx 16$, and $\rho_{do}\approx\rho_{dd}$. This coincides with the results in (36). Third, when either u_o or v_o is correct, the desired signal will be eliminated after difference preprocessing. As a result, κ_d equals zero and SNR_o reduces to $\gamma_d\mu_d|\rho_{do}|^2/(4\rho_{oo})$. In this case, the drop in SNR_o is caused by the drop in the subarray beamformer gain, which is much less significant than the effect of desired signal cancellation. This is an advantage of using separable quadruplet weighting for the difference preprocessor.

For comparison, the output SNR for the MVDR beamformer using the same nonoverlapping quadruplet array is given by[2]

$$\mathrm{SNR}_o=\frac{4\gamma_d|\rho_{do}|^2}{\rho_{oo}+8\gamma_d(1+2\gamma_d\rho_{dd})(\rho_{oo}\rho_{dd}-|\rho_{do}|^2)}. \tag{50}$$

We note that the major distinction between (49) and (50) lies in the second term of the denominator. Clearly, both beamformers give the same optimum value of SNR_o without pointing errors. For a moderately large range of angle, the term $\rho_{oo}\rho_{dd}-|\rho_{do}|^2$ increases as the pointing error increases. The increasing rate is much smaller with the proposed beamformer than with the MVDR beamformer, due to the presence of κ_d in (49). Consequently, SNR_o in (50) decreases much more dramatically with pointing errors than that in (49). In particular, both expressions decrease faster with a large input SNR γ_d. However, this effect is greatly lessened in the proposed beamformer since a large γ_d is most likely offset by κ_d in (49). These observations conclude that the proposed beamformer is much more robust to pointing errors than the MVDR beamformer.

2. Case 2: Multiple interferers

Treating the desired signal as interference after difference preprocessing, the sum beamformer behaves exactly in the same way as described in Sec. II A. In the presence of interference correlated with the desired signal, the effect of mutual cancellation will complicate the behavior of the beamformer. Nevertheless, mutual cancellation will not cause the desired signal to be eliminated even when the interference is coherent with it. The reason is that mutual cancellation is only effective for the difference beamformer. The gain/phase relationship between the desired and interfering signals which causes their mutual cancellation is in fact destroyed in the sum beamformer. Moreover, since the subarray beamformer does not actually put a null in the desired signal direction, the sum beamformer output SINR will not degrade much with pointing errors as in the case of uncorrelated interference. These observations will be confirmed by simulations shortly.

III. DESIRED SOURCE LOCALIZATION

Although the sum-and-difference based beamformer is more robust than the MVDR beamformer, it still exhibits performance degradation with a large pointing error. In this section, a DOA estimator for the desired source using the beamspace MUSIC technique is derived based on the sum-and-difference data. We demonstrate that with the sum-and-difference beamformers constructed according to (5), a closed-form DOA estimator can be obtained. The DOA estimator provides an accurate look direction, alleviating the effect of desired signal cancellation.

A. Beamspace MUSIC based on sum-and-difference data

A class of computationally efficient DOA estimators that exhibit excellent asymptotic performance are those based on the method of beamspace MUSIC.[11] The beamspace MUSIC procedure involves a transformation of the array data into beamspace data via a suitably chosen matrix beamformer, an execution of the generalized eigenvalue decomposition (GEVD) of the beamspace data/noise correlation matrix pencil, and a spectral search over the spatial spectrum to determine the DOAs. In 2-D DOA estimation, the spectral search is a time consuming task and should be avoided. A more efficient implementation would be to decompose the problem into two 1-D problems in which u_d and v_d are estimated individually. According to the conventional 2-D sum-and-difference source localization, a sum beam and two difference beams, one for the u angle and the other for the v angle, are formed with weight vectors $\mathbf{s}$, $\mathbf{d}_u$, and $\mathbf{d}_v$, respectively. These weight vectors are selected so as to provide a reliable and unambiguous estimate of (u_d,v_d). In order to obtain a reliable DOA estimate, it is necessary to suppress the interference in the beamspace data. To this end, the sum-and-difference beams should both exhibit nulls in the interfering directions. The commonality of interference nulls can be achieved with the structures

$$\mathbf{s}=\mathbf{G}\mathbf{c},\quad \mathbf{d}_u=\mathbf{H}_u\mathbf{c},\quad \mathbf{d}_v=\mathbf{H}_v\mathbf{c}, \tag{51}$$

where $\mathbf{G}$ and $\mathbf{c}$ are as defined before. $\mathbf{H}_u$ and $\mathbf{H}_v$ have the same structure as $\mathbf{H}$, except that the quadruplet weight vector is replaced by

$$\begin{aligned}\mathbf{h}_u&=[1,\quad -e^{jk_uu_o},\quad e^{jk_vv_o},\quad -e^{j(k_uu_o+k_vv_o)}]^T,\\ \mathbf{h}_v&=[1,\quad e^{jk_uu_o},\quad -e^{jk_vv_o},\quad -e^{j(k_uu_o+k_vv_o)}]^T,\end{aligned} \tag{52}$$

respectively. As will be shown shortly, choosing $\mathbf{h}_u$ and $\mathbf{h}_v$ by (52) provides unambiguous estimates of u_d and v_d. The patterns associated with **s**, $\mathbf{d}_u$, and $\mathbf{d}_v$ are given accordingly by

$$\begin{aligned} s(u,v) &= \mathbf{s}^H\mathbf{a}(u,v) = g(u,v)c(u,v) \\ &= g_u(u)g_v(v)c(u,v), \\ d_u(u,v) &= \mathbf{d}_u^H\mathbf{a}(u,v) = h_u(u,v)c(u,v) \\ &= h_u(u)g_v(v)c(u,v), \\ d_v(u,v) &= \mathbf{d}_v^H\mathbf{a}(u,v) = h_v(u,v)c(u,v) \\ &= g_u(u)h_v(v)c(u,v). \end{aligned} \tag{53}$$

It is noteworthy that $s(u,v)$ shares the factor $g_v(v)c(u,v)$ with $d_u(u,v)$. Thus the only distinction between them is the pattern factor associated with the u angle. A similar observation is made regarding $s(u,v)$ and $d_v(u,v)$.

Employing beamspace MUSIC based on the sum-and-difference technique dictates that u_d and v_d be determined by the following two beamspace data vectors:

$$\mathbf{x}_u = \begin{bmatrix} \mathbf{s}^H\mathbf{x} \\ \mathbf{d}_u^H\mathbf{x} \end{bmatrix} = \mathbf{W}_u^H\mathbf{x}, \quad \mathbf{x}_v = \begin{bmatrix} \mathbf{s}^H\mathbf{x} \\ \mathbf{d}_v^H\mathbf{x} \end{bmatrix} = \mathbf{W}_v^H\mathbf{x}, \tag{54}$$

respectively, where $\mathbf{W}_u = [\mathbf{s},\mathbf{d}_u]$ and $\mathbf{W}_v = [\mathbf{s},\mathbf{d}_v]$ are the beamforming matrices. The corresponding beamspace data correlation matrices are given by

$$\begin{aligned} \mathbf{R}_{uu} &= E\{\mathbf{x}_u\mathbf{x}_u^H\} = \mathbf{W}_u^H\mathbf{R}_{xx}\mathbf{W}_u \\ &\approx \sigma_d^2\mathbf{W}_u^H\mathbf{a}(u_d,v_d)\mathbf{a}^H(u_d,v_d)\mathbf{W}_u^H + \mathbf{W}_u^H\mathbf{R}_{ww}\mathbf{W}_u, \\ \mathbf{R}_{vv} &= E\{\mathbf{x}_v\mathbf{x}_v^H\} = \mathbf{W}_v^H\mathbf{R}_{xx}\mathbf{W}_v \\ &\approx \sigma_d^2\mathbf{W}_v^H\mathbf{a}(u_d,v_d)\mathbf{a}^H(u_d,v_d)\mathbf{W}_v^H + \mathbf{W}_v^H\mathbf{R}_{ww}\mathbf{W}_v, \end{aligned} \tag{55}$$

where we have assumed that the subarray beamformer suppresses the interference such that $\mathbf{W}_u^H\mathbf{a}(u_k,v_k) \approx \mathbf{W}_v^H\mathbf{a}(u_k,v_k) \approx \mathbf{0}_2$, $k=1,\ldots,K$. In practice, $\mathbf{R}_{uu}$ and $\mathbf{R}_{vv}$ are replaced by $\hat{\mathbf{R}}_{uu} = \mathbf{W}_u^H\hat{\mathbf{R}}_{xx}\mathbf{W}_u$ and $\hat{\mathbf{R}}_{vv} = \mathbf{W}_v^H\hat{\mathbf{R}}_{xx}\mathbf{W}_v$, respectively.

Beamspace MUSIC applied here for estimating u_d requires the following procedure. First, the GEVD of $\{\hat{\mathbf{R}}_{uu}, \mathbf{W}_u^H\mathbf{R}_{ww}\mathbf{W}_u\}$ is computed. Second, a noise eigenvector $\mathbf{e} = [e_1,e_2]^T$ is determined as that generalized eigenvector (GEV) associated with the smaller generalized eigenvalue (Gev). It is well known that this noise eigenvector is ideally orthogonal to span$\{\mathbf{W}_u^H\mathbf{a}(u_d,v_d)\}$ such that we may determine u_d by solving the following nonlinear equation in u:

$$\begin{aligned} \Phi(u,v) &= \mathbf{e}^H\mathbf{W}_u^H\mathbf{a}(u,v) \\ &= [e_1^*g_u(u) + e_2^*h_u(u)]g_v(v)c(u,v) = 0, \end{aligned} \tag{56}$$

where we have invoked (53). The equation is equivalent to

$$\Psi(u) = e_1^*g_u(u) + e_2^*h_u(u) = 0 \tag{57}$$

so long as the desired source is not in the direction of any of the nulls of $g_v(v)c(u,v)$. This is in general true since the desired source DOA should be close to the look direction where $g_v(v)c(u,v)$ has a sufficiently large gain. Substituting (13) into (57) and solving for u, we get the estimator for u_d,

$$\hat{u}_d = u_o - \frac{1}{k_u}\arg\left\{\frac{e_2+e_1}{e_2-e_1}\right\}, \tag{58}$$

where $\arg\{z\}$ extracts the phase angle of z. A closed-form expression for $\hat{u}_d$ can be obtained by expressing e_1 and e_2 in terms of the components of $\hat{\mathbf{R}}_{uu}$. We can develop a procedure similar to that outlined above for the estimation of v_d based on $\{\mathbf{R}_{vv}, \mathbf{W}_v^H\mathbf{R}_{ww}\mathbf{W}_v\}$. The detail is omitted for brevity.

B. Preliminary desired source localization

In practical amplitude-comparison schemes, a preliminary estimate of the desired source DOA is usually determined as the angle where the sum beam receives the maximum response. The preliminary DOA estimate can be used as the look direction for the proposed estimator to obtain a "fine tuned" secondary DOA estimate. The secondary estimate may still be inaccurate such that the procedure should be repeated several times to gain further fine tuning. That is, at each iteration, the beamformer is formed, using the previous DOA estimate as the look direction. The major issue in this procedure is that severe performance degradation may occur due to a large initial pointing error. With a large pointing error, the desired signal is eliminated by the subarray beamformer. We now present remedies for two cases.

1. Case 1: Weak interference

If the interferers are relatively weak compared to the desired source, then the sidelobes of the sum-and-difference beamformers are sufficiently low to suppress them. In this case, it is adequate to use the quiescent beamformer for DOA estimation. By quiescent, we mean that the beamformer is constructed with $\mathbf{R}_{xx}$ replaced by $\mathbf{R}_{ww}$, assuming that the environment is free of any directional sources. Since the construction of the quiescent beamformer does not involve any sources, the desired signal will not be suppressed even in the presence of pointing errors (of course, the error cannot be larger than a half-mainbeam width).

2. Case 2: Strong interference

If strong interferers exist, the sidelobes of the quiescent beamformer may not be low enough to provide effective suppression of these undesired sources. This necessitates performing simultaneous nulling in the interfering directions for the sum-and-difference beamformers. To avoid desired signal cancellation due to pointing errors, we propose the use of subspace techniques to separate the strong signals (interference) from the weak signals (desired signal and noise).[8] Note that there are possibly weak interferers present as well, but they are suppressed by the sidelobes of the beamformer and can thus be ignored.

Assume that the interferers are mutually incoherent. The separation of the interference and noninterference subspaces is accomplished via the GEVD of the difference preprocessed data/noise correlation matrix pencil $\{\mathbf{Q}_{xx}, \mathbf{Q}_{ww}\}$:

$$\mathbf{Q}_{xx}\mathbf{u}_i = \xi_i\mathbf{Q}_{ww}\mathbf{u}_i, \quad i=1,\ldots,L, \tag{59}$$

where $\mathbf{u}_i$, $i=1,\ldots,L$ are the GEVs and $\xi_1 \geqslant \xi_2 \geqslant \ldots \xi_L$ are the corresponding Gevs arranged in descending order. According to the theory of subspace decomposition, the $L-K$ "smallest" GEVs $\mathbf{u}_i$, $i=K+1,\ldots,L$ are nearly orthogonal to the interfering subspace spanned by $\mathbf{H}^H\mathbf{a}(u_k,v_k)$, $i=1,\ldots,K$. As a result, the matrix

$$\bar{\mathbf{U}}_I=[\mathbf{u}_{K+1}, \quad \mathbf{u}_{K+2},\ldots,\mathbf{u}_L] \tag{60}$$

has a range space nearly orthogonal to the interference subspace. Since $\mathbf{H}^H\mathbf{a}(u,v)$ is proportional to $\mathbf{a}_s(u,v)$, as seen from (15), choosing $\mathbf{c}=\bar{\mathbf{U}}_I\mathbf{y}$ as a linear combination of the columns of $\bar{\mathbf{U}}_I$ leads to $\mathbf{c}^H\mathbf{a}_s(u_k,v_k)\approx 0$, $k=1,\ldots,K$. Consequently, the sum-and-difference beamformers constructed in accordance with (51) exhibit deep nulls in the directions of the strong interferers. In the presence of pointing errors, a portion of the desired signal remains in $\mathbf{Q}_{xx}$ and is contained in the subspace spanned by $\mathbf{u}_i$, $i=1,\ldots,K+1$. Since the remaining desired signal is much weaker than the interfering signals, the K "largest" GEVs contains very little desired signal component. As a result, the subarray beamformer will not cancel the desired signal as effectively as with the regular beamformer described in Sec. I B. This makes the modified beamformer very robust to pointing errors.

With the subarray weight vector so obtained, the difference beamformer output is essentially free of any directional signals. The beamformer is thus constructed as in the quiescent case. Substitution of $\mathbf{c}=\bar{\mathbf{U}}_I\mathbf{y}$ in (16) along with the replacement of $\mathbf{R}_{xx}$ by $\mathbf{R}_{ww}$ leads to

$$\min_{\mathbf{y}} \ \mathbf{y}^H\bar{\mathbf{U}}_I^H\mathbf{H}^H\mathbf{R}_{ww}\mathbf{H}\bar{\mathbf{U}}_I\mathbf{y}, \quad \text{subject to} \ \ \mathbf{y}^H\bar{\mathbf{U}}_I^H\mathbf{a}_s(u_o,v_o)=\alpha. \tag{61}$$

Solving for $\mathbf{y}$ yields the subarray weight vector

$$\mathbf{c}=\bar{\mathbf{U}}_I(\bar{\mathbf{U}}_I^H\mathbf{H}^H\mathbf{R}_{ww}\mathbf{H}\bar{\mathbf{U}}_I)^{-1}\bar{\mathbf{U}}_I^H\mathbf{a}_s(u_o,v_o). \tag{62}$$

Although the GEVD based beamformer is effective for a large initial pointing error, it is much more computationally burdensome than the regular one. Some efficient implementations of adaptive subspace tracking have been proposed,[20] and can be used here to speed up the computation of the weight vector. Once the desired source is in lock, the beamformer should be switched back into its regular mode to save computations.

IV. COMPUTER SIMULATIONS

Computer simulations were conducted to evaluate the performance of the proposed sum-and-difference based beamforming schemes. The quadruplet array used was rectangular, consisting of 8×8 identical omnidirectional elements with λ/2 spacing between adjacent elements, as shown in Fig. 2. Also shown in Fig. 2 are some possible ways to choose the four subarrays. In general, choosing a small subarray size L leads to a lower computational complexity in updating the weight vector, but reduces the effectiveness of interference cancellation. The opposite is true regarding a large subarray size. As we are more concerned with the performance of the beamformer, the largest subarray size [option (a)] will be used in all of the following simulations. It was assumed that the desired signal arrived from the broadside of the array, i.e., $(\theta_d,\phi_d)=(0°,0°)$, and an interfering signal arrived at $(\theta_1,\phi_1)=(20°,-25°)$. Finally, the SNR and SIR (signal-to-interference ratio) in decibels were defined as $10\log_{10}\gamma_d$ and $10\log_{10}(\sigma_d^2/\sigma_1^2)$, respectively.

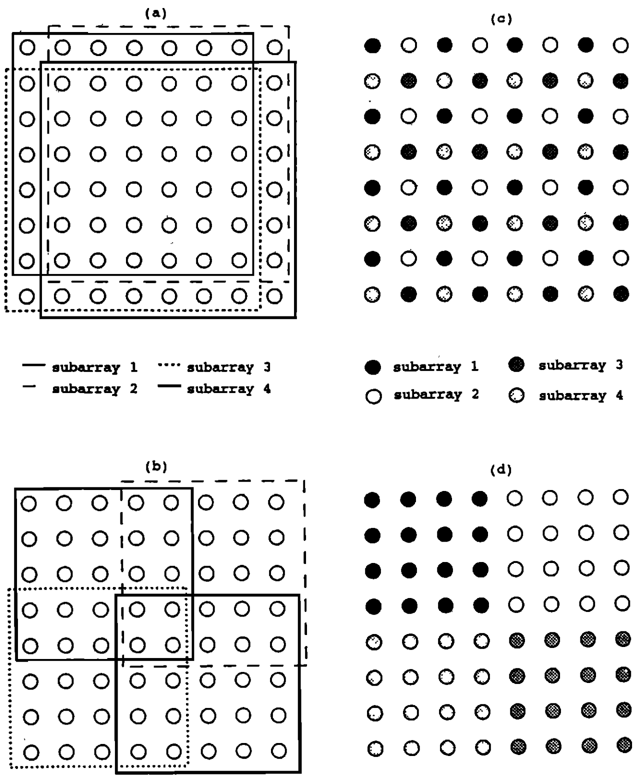

FIG. 2. Some options of the subarray structures associated with an 8×8 rectangular array: (a) and (b) overlapping subarrays; (c) and (d) nonoverlapping subarrays.

The first set of simulations examines the performance of the proposed beamformer against spatially white noise. In this case, the interference power was set to zero. Figure 3 shows the beamformer output SNR versus pointing errors for the proposed and MVDR beamformers, with SNR=0 and 20

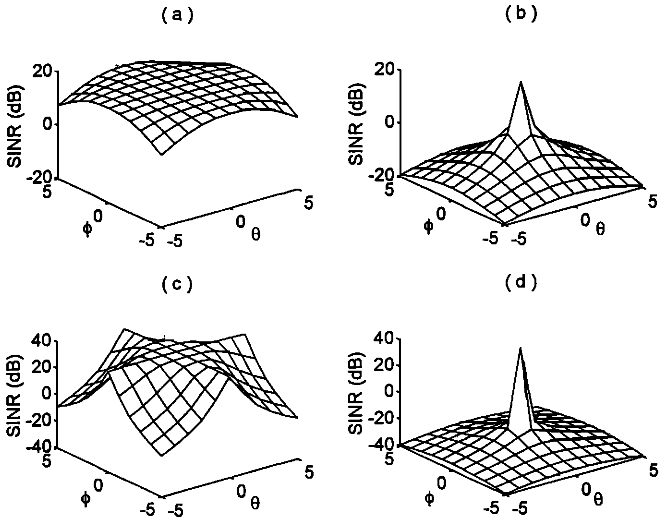

FIG. 3. Output SINR as a function of pointing errors (in degrees) under spatially white noise only. $(\theta_d,\phi_d)=(0°,0°)$. (a) Proposed; SNR=0 dB. (b) MVDR; SNR=0 dB. (c) Proposed; SNR=20 dB. (d) MVDR; SNR=20 dB.

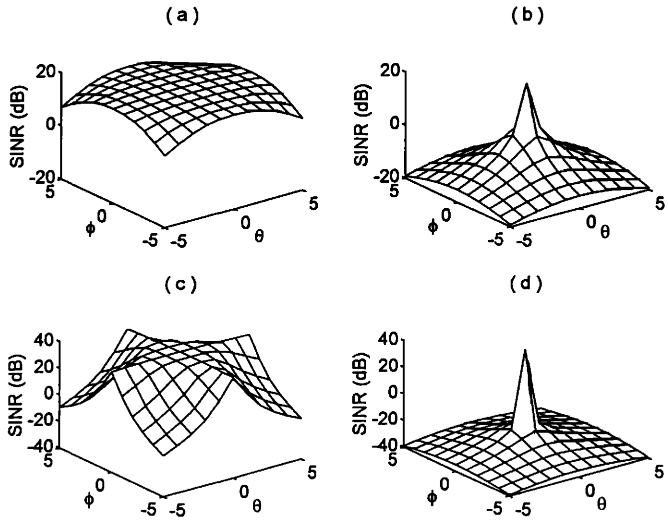

FIG. 4. Output SINR as a function of pointing errors (in degrees) under spatially white noise plus an uncorrelated interferer with SIR = −10 dB. $(\theta_d, \phi_d) = (0°, 0°)$. $(\theta_1, \phi_1) = (20°, -25°)$. (a) Proposed; SNR = 0 dB. (b) MVDR; SNR = 0 dB. (c) Proposed; SNR = 20 dB. (d) MVDR; SNR = 20 dB.

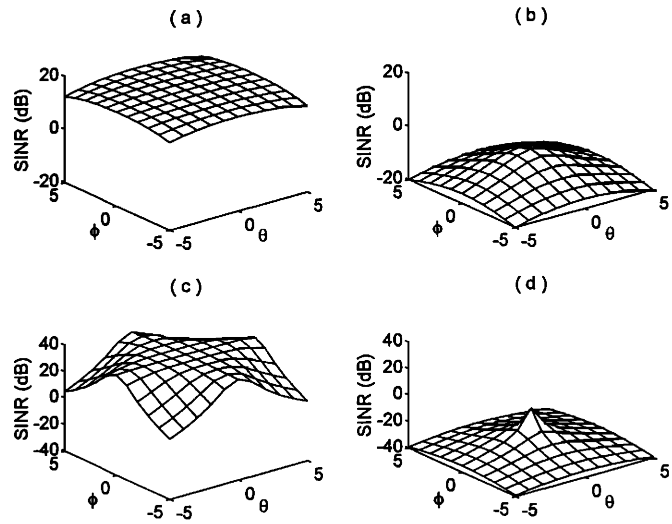

FIG. 5. Output SINR as a function of pointing errors (in degrees) under spatially white noise plus a coherent interferer with SIR = −10 dB. $(\theta_d, \phi_d) = (0°, 0°)$. $(\theta_1, \phi_1) = (20°, -25°)$. (a) Proposed; SNR = 0 dB. (b) MVDR; SNR = 0 dB. (c) Proposed; SNR = 20 dB. (d) MVDR; SNR = 20 dB.

dB. We observe that for both cases, the SNR plot associated with the proposed beamformer is much flatter than that associated with the MVDR beamformer, confirming our earlier statement that the proposed beamformer is robust against pointing errors. As predicted by the analysis results, the output SNR drops more dramatically with pointing errors for a higher SNR due to the effect of desired signal cancellation.

The second set of simulations investigates the interference rejection capability of the proposed beamformer. The interferers were assumed uncorrelated with the desired signal, with an SIR of −10 dB. The output SINR versus pointing errors for the proposed and MVDR beamformers, with SNR = 0 and 20 dB, are depicted in Fig. 4. The results are quite similar to those shown in Fig. 3. This is an indication that the interference has been sufficiently suppressed so that the beamformer performs as if no interference is present.

The third set of simulations demonstrates the effect of correlated interference. The parameter setting was the same as in the previous case, except that the interferer was assumed correlated with the desired source, with the correlation coefficient equal to 0.9. Figure 5 shows the output SINR versus pointing errors for the proposed and MVDR beamformers. In this case, the effective desired signal power and interference power are defined as

$$P_d = |\sigma_d \mathbf{s}^H \mathbf{a}(u_d, v_d) + 0.9\sigma_1 \mathbf{s}^H \mathbf{a}(u_1, v_1)|^2, \quad (63)$$

$$P_i = 0.19\sigma_1^2 |\mathbf{s}^H \mathbf{a}(u_1, v_1)|^2, \quad (64)$$

respectively, to account for the partial contribution of the interferer to the output desired signal. Compared with Fig. 4, we find that the MVDR beamformer suffered serious degradation due to the mutual cancellation between the desired and interfering signals. This did not happen with the proposed beamformer, as already mentioned in Sec. II B 2. Interestingly, the beamformer seems to be more robust to pointing errors than it was in the previous case. A plausible reason for this is that the desired signal remaining after difference preprocessing was significantly weaker than the interfering signal. The subarray beamformer, in an attempt to perform mutual cancellation by combining the two signals, put a large gain at (u_d, v_d) and a small gain at (u_1, v_1). The gain pattern was then translated to the sum beamformer, producing a high output SINR even with a large pointing error.

The final set of simulations evaluates the performance of the proposed DOA estimator. The preliminary look direction was set to be (5°, −5°), corresponding to a pointing error of 5° for both angles. For all cases applicable, the beamformers were constructed with $\hat{\mathbf{R}}_{xx}$ based on $N = 250$ independent data samples. The first case examines the acquisition capability of the proposed iterative estimation procedure under spatially white noise only. The quiescent beamformer as described in Sec. III B 1 was employed. For each estimation, five beamspace data samples were used to form $\hat{\mathbf{R}}_{uu}$ and $\hat{\mathbf{R}}_{vv}$. The resulting trajectory of the estimate of θ_d over a period of 50 samples is shown in Fig. 6, with SNR = 0 and 20

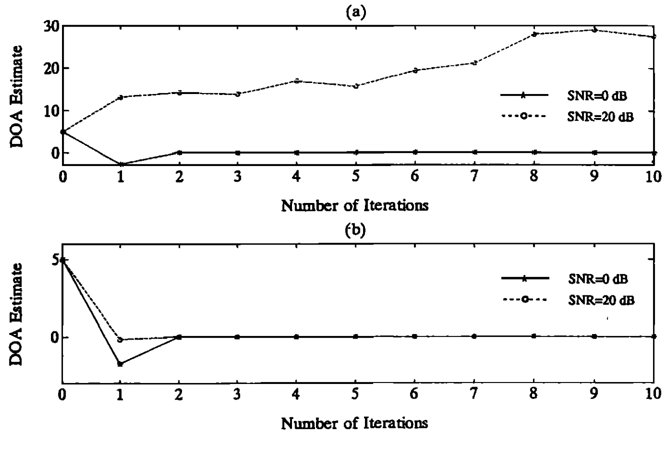

FIG. 6. Tracjectories of the estimate of θ_d (in degrees) obtained with the proposed estimator under spatially white noise only. $(\theta_d, \phi_d) = (0°, 0°)$. (a) Regular beamformer. (b) Quiescent beamformer.

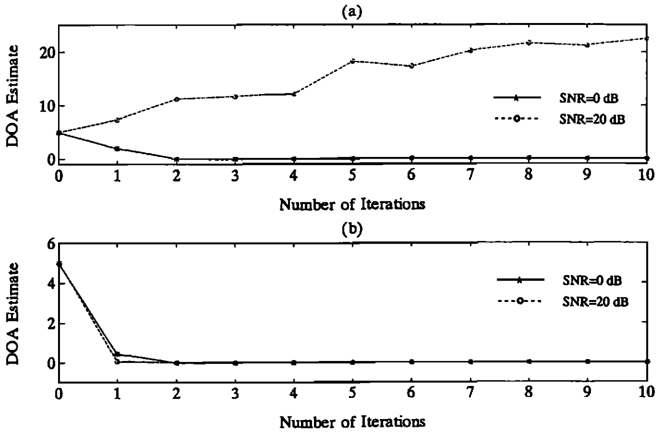

FIG. 7. Trajectories of the estimate of θ_d (in degrees) obtained with the proposed estimator under spatially white noise plus an uncorrelated interferer with SIR=−10 dB. $(\theta_d,\phi_d)=(0°,0°)$. $(\theta_1,\phi_1)=(20°,-25°)$. (a) Regular beamformer. (b) EVD based beamformer.

dB. For comparison, we also included the results obtained with the regular beamformer described in Sec. I B. It is observed that the regular beamformer was not able to keep track of the desired source with SNR=20 dB. With such a high SNR, a small pointing error can cause the desired signal to be canceled by the regular beamformer. On the other hand, the desired source was in lock with the quiescent beamformer in two iterations for both SNR values, and was kept in track over the entire test period. In the second case, an uncorrelated interferer with SIR=−10 dB was present at (20°,−25°). We repeated the previous simulation and plotted the DOA estimate trajectories in Fig. 7. The GEVD based beamformer described in Sec. III B 2 was employed for the first two iterations. After the desired source was in lock, the beamformer was switched back to its regular mode. Again, we find that the regular beamformer was not able to localize the desired source with a high SNR. The GEVD based beamformer performed reliably for both SNR values, confirming its robustness against large pointing errors.

V. CONCLUSIONS

This paper described a 2-D adaptive beamforming scheme based on the sum-and-difference technique for quadruplet arrays. The proposed beamformer was constructed so as to minimize the output power of the difference beamformer subject to a unit response constraint in the look direction on the sum beamformer. It was demonstrated analytically and numerically that the proposed beamformer exhibits a better SINR performance than the conventional MVDR beamformer in the presence of pointing errors and/or correlated interference. A 2-D DOA estimator for the desired source was developed using the beamspace MUSIC procedure operating on a set of judiciously constructed sum-and-difference data. The DOA estimator provides an accurate look direction for the beamformer, alleviating the effect of desired signal cancellation. To avoid performance breakdown due to beam squint in the initialization of the beamforming operation, a subspace separation technique was suggested to further improve the robustness of the beamformer. Simulation results confirmed the efficacy of the DOA estimator working in conjunction with the proposed beamformer. The proposed beamformer is similar in structure to the MVDR beamformer, rendering advanced hardware implementations, such as VLSI systolic arrays,[21] readily applicable.

ACKNOWLEDGMENT

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