Wireless Pers Commun (2011) 60:171–190
DOI 10.1007/s11277-010-9937-3

A Robustness and Low-Complexity Selection Criterion for Switching between Multiplexing and Diversity in MIMO Transmission

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Published online: 23 February 2010

Abstract In this paper, we propose a new QR-based selection criterion with reduced complexity for the development of switching between spatial multiplexing (SM) and spatial diversity (SD) transmission. Then, we investigate an effect of the QR-decomposition processes by exploiting orthogonal structure of channel matrix. Additionally, to solve the problem of the error transmission mode in transmitter, we claim that the proposed QR-based detection at receiver can be used to form the successive interference cancellation (SIC) detection and orthogonal space-time block code (O-STBC) decoding for the SM transmission and the SD transmission, respectively. Simulation results show that the proposed scheme is capable of achieving optimum performance, but at a low-complexity level.

Keywords Multiple-input multiple-output · Switching · Minimum Euclidean distance · QR-decomposition

1 Introduction

In multiple-input multiple-output (MIMO) channel (**H**) system [1–3], spatial multiplexing (SM) [4,5] and spatial diversity (SD) [6–8] can achieve high transmission data rate and high reliability of the wireless communication link, respectively. In SM transmission, different data streams are transmitted from different antennas simultaneously and detected based on their unique spatial signature at the receiver. This implies the creation of parallel spatial channels for maximizing the data rate [4,5]. In SD transmission, the orthogonal space-time block coding (O-STBC) technique is designed for exploiting multiple transmitting antennas. O-STBC introduces temporal and spatial correlation into signals by sending the same information

Part of this work was presented at the 2007 IEEE Conference on Intelligent Transportation Systems (ITS), Seattle, WA, USA, Sept. 2007. This work is sponsored jointly by the National Science Council of in Taiwan.

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through different paths. Thus, multiple, independently faded replicas of the data symbols can be coherently combined to average the fading gains. From this point of view, fading can in fact be beneficial through increasing the degrees of freedom available for communication [6–8]. It is clear that there is a tradeoff between SD and SM in MIMO transmission [3,8].

To improve the transmission performance, the switching technique, where either SM or SD is chosen based on channel state information (CSI), was proposed in [9]. For the switching transmission, Heath in [9] proposed the minimum square Euclidean distance selection criterion and Demmel-based selection criterion to choose an optimum transmission mode employed in the feedback system [10–14]. However, at the receiver, this selection criterion involves high computational complexity for the minimum square Euclidean distance selection criterion because of the exhaustive search of each codeword operating on the channel for the codebook construction [12]. Then, the Demmel-based selection criterion may involve complicated computation due to the SVD processes.

By using QR-decomposition process ($\mathbf{H} = \mathbf{QR}$), the QR-based detection with reduced complexity has been devised to realize the successive interference cancellation (SIC) detection [15–19]. We note that the sorted QR detection (SQRD) for enhancing detection performance is depicted in [15], and the sphere decoding via QR scheme to realize the maximum likelihood (ML) detection performance is described in [16]. Then, considering low computational complexity, the performance of the QR-based genetic algorithm to achieve the performance of the near-ML algorithm is developed in [17–19]. Therefore, in this paper, we propose a low-complexity QR-based selection criterion in switching transmission to realize an optimum transmission. To achieve this, for SM, we claim that the minimum diagonal entry of $\mathbf{R}$ operating on the minimum distance of each codeword difference is less than or equal to the minimum square Euclidean distance in SM. For SD, we claim that the minimum square Euclidean distance in SD is equal to a diagonal entry of $\mathbf{R}$ operating on the minimum distance of each codeword difference. This is because $\mathbf{R}$'s diagonal entries are identical for the space-time signature matrix channel in O-STBC transmission. Based on these two conditions, the condition number obtained by using the diagonal entry of $\mathbf{R}$ is proposed to find a useful measure for characterizing matrix channels in the switching system [9,20]. Therefore, the proposed QR-based selection criterion can realize the minimum square Euclidean distance criterion, but without an exhaustive search of each codeword operating on the channel. Especially, the proposed QR-based selection criterion complexity is zero because $\mathbf{R}$'s entries can be obtained from the QR-based detection process, and it is less complex than the SVD complexity of Demmel-based selection criterion. Then, to reduce the QR-decomposition processes via the Gram–Schmidt scheme in the real system, we propose that the real and imaginary parts of the channel elements are rearranged to achieve a column-wise orthogonal structure. This column-wise orthogonal structure means that the neighboring columns of $\mathbf{H}$ are orthogonal. By using this orthogonal structure, the proposed QR-decomposition processes are less complex than conventional QR-decomposition processes [16] because elements in the neighboring columns of the upper triangular matrix ($\mathbf{R}$) are repeated, and thus only half of the elements are computed.

Additionally, the error transmission mode may occur in the transmitter via imperfect feedback channel [21]. For example, the receiver informs the transmitter about the SD transmission mode but transmitter chooses the SM transmission mode via imperfect feedback channel. Thus, the detection performance is degraded because the match filter matrix ($\mathbf{H}^H\mathbf{H}$) is not diagonal at the orthogonal space-time block code (O-STBC) decoding in receiver. To solve this error transmission problem in the transmitter, we propose a QR-based detection at receiver to realize the SIC detection and the O-STBC decoding for the SM transmission and the SD transmission, respectively. First, for the SD transmission, we claim that the performance of QR-based detection is equal to the performance of the O-STBC decoding, where $\mathbf{H}$ is the space-time signature channel [2,6]. This is because QR-based detection in the O-STBC transmission is error-propagation-free. That is, in O-STBC transmission, $\mathbf{R}$ is a diagonal matrix after the QR-decomposition of an orthogonal channel. Second, for the SM transmission, the QR-based detection has developed in realizing the SIC detection [17–19]. Hence, based on these two realized schemes, the error transmission problem can be solved because a robust QR-based detection is single at the receiver employed in the switching transmission. Simulation results show that the proposed scheme is capable of achieving optimum performance, but at a low-complexity level.

This paper is organized as follows. In Sect. 2, the MIMO detection for SM and SD are described. In Sect. 3, a low-complexity selection criterion employed in switching transmission is depicted. In Sect. 4, analytic symbol-error-rate (SER) performance and computational complexity are analyzed. In Sect. 5, we conduct the computer simulation to confirm the effectiveness of the proposed algorithm. In Sect. 6, we give a conclusion and suggest future work. Finally, we collect all proofs in the Appendix in order to enhance the flow of the paper.

2 MIMO Detection for SM and SD

We consider the switching system with N transmitting antennas and M receiving antennas in the MIMO channel systems ($M \geq N$ is assumed). To achieve a robust MIMO detection in receiver, we describe the QR-based detection in realizing the SIC detection and the O-STBC decoding for A) SM and B) SD as follows.

2.1 SM

For SM in this paper, we consider the QR-based detection with reduced complexity to realize the SIC performance as follows. The equivalent system model can be denoted as [1–5]

$$\mathbf{Y} = \mathbf{Hx} + \mathbf{v}, \tag{1}$$

where $\mathbf{Y} \in C^{M\times 1}$ is the received signal vector, $\mathbf{H} \in C^{M\times N}$ is the MIMO channel, $\mathbf{x} \in C^{N\times 1}$ is the transmitted signal vector, $\mathbf{v} \in C^{M\times 1}$ has *i.i.d.* complex Gaussian entries with noise power σ_v^2. For the QR-based detection, the QR-decomposition of $\mathbf{H}$ can be expressed as

$$\mathbf{H} = [\mathbf{Q}_1\mathbf{Q}_2]\begin{bmatrix}\mathbf{R}\\ \mathbf{0}\end{bmatrix} = \mathbf{QR}, \tag{2}$$

where $\mathbf{Q} = [\mathbf{Q}_1 \in C^{M\times N}\mathbf{Q}_2 \in C^{M\times(M-N)}]$ is an $M\times M$ unitary matrix so $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$ and $\mathbf{R}$ is an $N \times N$ upper triangular matrix. Additionally, the QR-based detection can be depicted as follows [17–19]

$$\tilde{\mathbf{y}} = \begin{bmatrix}\tilde{y}_1\\ \tilde{y}_2\\ \vdots\\ \tilde{y}_N\end{bmatrix} = \mathbf{Q}^H\mathbf{Y} = \underbrace{\begin{bmatrix} r_{1,1} & r_{1,2} & \cdots & r_{1,N}\\ 0 & r_{2,2} & \cdots & r_{2,N}\\ \vdots & \vdots & \cdots & \vdots\\ 0 & \cdots & 0 & r_{N,N}\end{bmatrix}}_{\mathbf{Rx}} + \tilde{\mathbf{v}} = \mathbf{Rx} + \tilde{\mathbf{v}}, \tag{3}$$

where $\mathbf{R} = \mathbf{Q}^H \mathbf{H}$, $\tilde{\mathbf{v}} = \mathbf{Q}^H \mathbf{v}$, and $r_{i,j}$, $1 \le i, j \le N$, is the (i, j)th element of $\mathbf{R}$. The ith element of modified received signals is detected as

$$\hat{y}_i = \tilde{y}_i - \sum_{j=i+1}^{N} r_{i,j}\hat{x}_j = r_{i,i}x_i + \sum_{j=i+1}^{N} r_{i,j}(x_j - \hat{x}_j) + \tilde{v}_i, \tag{4}$$

where $\hat{x}_j$ is the jth element of detected transmit signals. Assuming that there is no error in the previous symbol detection, we can obtain [17–19]

$$\hat{x}_i = \text{Decision}\left(\frac{\hat{y}_i}{r_i, i}\right), \text{ when } \hat{y}_i = r_{i,i}x_i + \tilde{v}_i. \tag{5}$$

To achieve low-complexity, we propose a column-wise orthogonal structure for QR-decomposition. This column-wise orthogonal structure means that the neighboring columns of $\mathbf{H}$ are orthogonal. To achieve this, the real and imaginary parts in the elements of $\mathbf{H}$ are rearranged as

$$\mathbf{H}_r := \begin{bmatrix} \text{Re}\{\mathbf{h}_1\} & -\text{Im}\{\mathbf{h}_1\} & \text{Re}\{\mathbf{h}_2\} & -\text{Im}\{\mathbf{h}_2\}, \ldots, \text{Re}\{\mathbf{h}_N\} & -\text{Im}\{\mathbf{h}_N\} \\ \text{Im}\{\mathbf{h}_1\} & \text{Re}\{\mathbf{h}_1\} & \text{Im}\{\mathbf{h}_2\} & \text{Re}\{\mathbf{h}_2\}, \ldots, \text{Im}\{\mathbf{h}_N\} & \text{Re}\{\mathbf{h}_N\} \end{bmatrix} \in \Re^{2(M \times N)}, \tag{6}$$

where $\mathbf{h}_i$, $i \in \{1, \ldots, N\}$, is the ith column of $\mathbf{H}$ and thus (1) can be rewritten as

$$\mathbf{Y}_r = \mathbf{H}_r \mathbf{x}_r + \mathbf{v}_r, \tag{7}$$

where $\mathbf{Y}_r = [\text{Re}\{y_1\}\text{Im}\{y_1\}\text{Re}\{y_2\}\text{Im}\{y_2\} \ldots \text{Re}\{y_M\}\text{Im}\{y_M\}]^T \in \mathbf{R}^{2M\times 1}$ is the equivalent received signal vector, $\mathbf{x}_r = [\text{Re}\{x_1\}\text{Im}\{x_1\}\text{Re}\{x_2\}\text{Im}\{x_2\} \ldots \text{Re}\{x_N\}\text{Im}\{x_N\}]^T \in \mathbf{R}^{2N\times 1}$ is the equivalent transmitted signal vector, and $v_r = [\text{Re}\{v_1\}\text{Im}\{v_1\}\text{Re}\{v_2\}\text{Im}\{v_2\} \ldots \text{Re}\{v_M\}\text{Im}\{v_M\}]^T \in \mathbf{R}^{2M\times 1}$ is the equivalent noise vector. By exploiting this column-wise orthogonal structure, the QR-decomposition (QRD) of $\mathbf{H}_r$ can be formulated as follows.

Property 1 *Assume $\mathbf{H}_r(:, c)$ is the cth column of H_r, $\mathbf{H}_r \in \Re^{2(M\times N)}$, where $(\mathbf{H}_r(:, c))^H \mathbf{H}_r(:, c+1) = \mathbf{0}$ for $c \in \{1, 3, 5, \ldots, 2N-1\}$ and let $\mathbf{Q}$ and $\mathbf{R}$ be QRD of $\mathbf{H}$ ($\mathbf{H} = \mathbf{QR}$). Then, we have $\mathbf{R} \in \mathbf{B}_{2^n\times 2^n}$, where*

$$\begin{aligned} \mathbf{B}_{2^n\times 2^n} \equiv \{&\mathbf{B} \in \Re^{2^n\times 2^n};\ \forall i,\ j \in \left\{1,\ 3,\ 5, \ldots,\ 2^{n-2}+1\right\}, \mathbf{B}(i, i+1; j, j+1) \\ &= \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \text{ or } \begin{pmatrix} a & b \\ b & -a \end{pmatrix} \text{ for some } a, b \in \Re\}. \end{aligned} \tag{8}$$

Proof See appendix I. □

Therefore, due to the involvement of the column-wise orthogonal structure in (5), the proposed QR-decomposition processes with $O(\sum_{i=1}^{2N-3} 2Mi + 12M(N-1))$ have about half of the computational complexity compared to conventional QR-decomposition processes with $O(2(N-1)(4MN + 2N))$ (see [16] Eq. 3) via Gram–Schmidt scheme in the real system. That is, by using (6) and (8), the proposed QR-decomposition processes are less complex than conventional QR-decomposition processes because elements in the neighboring columns of $\mathbf{R}$ are repeated, and thus only half of the elements are computed, depicted in Appendix I.

2.2 SD

To achieve a robust detection at the receiver for the SD transmission, we consider the QR-based detection to realize the O-STBC detection [6–8] as follows. Considering the O-STBC transmission for SD, there are K complex information symbols prior to space-time encoding which are denoted as $\bar{\mathbf{x}} = [x_1 x_2 \ldots x_K]$, and $\mathbf{A}(\mathbf{H}) \in C^{MT \times K}$ is the space-time signature matrix with transmitting time $T(1 \le t \le T)$ satisfying [6–8]

$$\mathbf{A}(\mathbf{H})\mathbf{A}^H(\mathbf{H}) = ||\mathbf{A}(\mathbf{H})||^2 \, \mathbf{I}_{MT}, \tag{9}$$

where $\mathbf{I}_{MT}$ is MT x MT identity matrix and thus the equivalent system of (1) can be rewritten as

$$\bar{\mathbf{Y}} = \mathbf{A}(\mathbf{H})\mathbf{x} + \bar{\mathbf{v}}, \tag{10}$$

where $\bar{\mathbf{Y}} = [\mathbf{y}(1)\mathbf{y}(2)\ldots\mathbf{y}(T)]^T$ with $\mathbf{y}(t) = [y_1(t)\ldots y_M(t)]$ and $\bar{\mathbf{v}} = [\mathbf{v}(1)\mathbf{v}(2)\ldots\mathbf{v}(T)]^T$ with $\mathbf{v}(t) = [\mathrm{v}_1(t)\ldots\mathrm{v}_M(t)]$. To realize the performance of maximum-likelihood receiver, the QR-decomposition of the space-time signature matrix in (10) can be given as

$$\mathbf{A}(\mathbf{H}) = \mathbf{QR}, \tag{11}$$

where $\mathbf{R} = \bar{r}_{i,i}\mathbf{I}_M$, $i \in \{1, 2, \ldots, N\}$, has $\bar{r}_{1,1} = \bar{r}_{2,2} = \cdots = \bar{r}_{N,N}$ because $\mathbf{A}(\mathbf{H})$ is orthogonal (depicted at (A.1) in appendix I). By using QR-decomposition, (10) can be rewritten as [17–19]

$$\tilde{\mathbf{y}} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_N \end{bmatrix} = \mathbf{Q}^H\bar{\mathbf{Y}} = \mathbf{Rx} + \mathbf{Q}^H\bar{\mathbf{v}} = \begin{bmatrix} \bar{r}_{1,1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \bar{r}_{N,N} \end{bmatrix} \mathbf{x} + \underline{\mathbf{v}}, \tag{12}$$

where $\underline{\mathbf{v}} = \mathbf{Q}^H\bar{\mathbf{v}}$ is the AWGN which has the same statistical distribution as that of $\bar{\mathbf{v}}$. For QR-based detection in O-STBC, to realize the maximum-likelihood decoding performance, the proposed QR-based detection via (12) is given as follows

$$\begin{aligned} \hat{\mathbf{X}} &= \min_{\mathbf{x}\in S} \arg \left\|\bar{\mathbf{Y}} - \mathbf{A}(\mathbf{H})\mathbf{x}\right\|^2 = \min_{\mathbf{x}\in S} \arg \left\|\mathbf{Q}^H\bar{\mathbf{Y}} - \mathbf{Q}^H\mathbf{A}(\mathbf{H})\mathbf{x}\right\|^2 = \min_{\mathbf{x}\in S} \arg \|\tilde{\mathbf{y}} - \mathbf{Rx}\|^2 \\ &\Rightarrow \hat{\mathbf{X}} = \mathbf{R}^\dagger\tilde{\mathbf{y}}, \end{aligned} \tag{13}$$

where $(\cdot)^\dagger$ is a pseudo-inverse [2] and the right-hand side of (13) holds because $\mathbf{R}$ is a diagonal matrix depicted in (12). Furthermore, (13) can be rewritten as the symbol-by-symbol detection is

$$\hat{x}_i = Decision\left(\frac{\tilde{y}_i}{r_{i,i}}\right), \text{ for } 1 \le i \le N, \tag{14}$$

where $\hat{x}_i$ the symbol decision at the ith layer. For O-STBC transmission, the conventional match filter decoding ($\mathbf{H}^H\mathbf{H}$) was developed in receiver to realize the maximum-likelihood decoding performance [6]. However, this match filter decoding may degrade the detection performance in the SM transmission because $\mathbf{H}^H\mathbf{H}$ is not a diagonal.

Remark Based on (3)–(5) and (12)–(14), we claim the QR-based detection can realize the SIC detection and the O-STBC decoding for the SM transmission and the SD transmission, respectively. Hence, for switching transmission, a robust QR-based detection is proposed in the receiver no matter it's in the SM transmission or in the SD transmission.

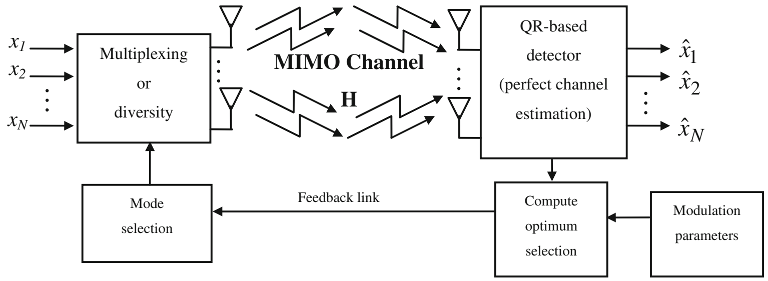


Fig. 1 The transmitter switches between SM and SD in MIMO system based on feedback from receiver

3 Low-Complexity Selection Criterion Employed in Switching Transmission

Considering various selection criteria are employed in the receiver, either SM or SD is selected according to the instantaneous channel state information (CSI), and then the receiver informs the transmitter about the transmission decision via a feedback channel, as depicted in Fig. 1. In this section, we propose a low-complexity selection criterion with QR-based scheme to achieve an optimum transmission in switching between SM and SD. To achieve this, A) minimum distance selection criterion, B) Demmel-based selection criterion, and C) the proposed QR-based selection criterion are depicted as follows.

3.1 Minimum Distance Selection Criterion

To achieve an optimal transmission by switching between SM and SD, the squared minimum Euclidean distance computation is described as follows. For SM, the minimum squared Euclidean distance is achieved by minimizing each codeword difference operating on the channel as [9]

$$d^2_{\min,\mathrm{SM}}(\mathbf{H}) := \min_{\mathbf{s},\mathbf{c}\in S,\mathbf{s}\neq\mathbf{c}} \|\mathbf{H}(\mathbf{s}-\mathbf{c})\|^2 . \tag{15}$$

where $\mathbf{s}$, $\mathbf{c} \in S$ are two different transmitted codewords and S denotes the feasible set containing the possible symbol vector. However, $d^2_{\min,\mathrm{sm}}(\mathbf{H})$ leads to more complicated computation due to the involvement of an exhaustive search of each codeword operating on the channel for the codebook construction.

3.2 Demmel-Based Selection Criterion

To reduce an exhaustive search complexity, the Demmel-based selection criterion is described as follows. By applying the Rayleigh-Ritz theorem to (15), the upper and lower bounds on the minimum square Euclidean distance [9] can be given as

$$\lambda^2_{\min}(\mathbf{H})\frac{d^2_{\min,\mathrm{sm}}}{N} \le d^2_{\min,\mathrm{SM}}(\mathbf{H}) \le \lambda^2_{\max}(\mathbf{H})\frac{d^2_{\min,\mathrm{sm}}}{N}. \tag{16}$$

where λ_{min} is the minimum singular value of $\mathbf{H}$, λ_{max} is the maximum singular value of $\mathbf{H}$ and $\frac{d^2_{\min,\mathrm{sm}}}{N}$ is the minimum Euclidean distance of each codeword difference [9] given by

$$\frac{d^2_{\min,\mathrm{sm}}}{N} = \min_{\mathbf{s},\mathbf{c}\in S,\mathbf{s}\neq\mathbf{c}} \|\mathbf{s}-\mathbf{c}\|^2 . \tag{17}$$

For SD, the minimum squared Euclidean distance is achieved by minimizing each codeword difference operating on the space-time signature channel as

$$d^2_{\min,\mathrm{SD}}(\mathbf{H}) := \min_{\mathbf{s},\mathbf{c}\in S,\mathbf{s}\neq\mathbf{c}} \|\mathbf{A}(\mathbf{H})(\mathbf{s}-\mathbf{c})\|^2 , \tag{18}$$

Then, the minimum square Euclidean distance property [9] can be shown as

$$d^2_{\min,\mathrm{SD}}(\mathbf{H}) \leq \frac{1}{N}\|\mathbf{A}(\mathbf{H})\|_F^2\, d^2_{\min,\mathrm{md}} = \frac{1}{N} d^2_{\min,\mathrm{sd}} \sum_{k=1}^{L} \lambda_{\mathrm{k}}^2(\mathbf{A}(\mathbf{H})), \tag{19}$$

where $\|\cdot\|_F^2$ is the squared Frobenius norm and $L = \min(N, M)$. Considering the selection criterion via (16)–(19), SM is preferred when

$$\|\mathbf{A}(\mathbf{H})\|_F^2\, d^2_{\min,\mathrm{sd}} \leq \lambda^2_{\min}(\mathbf{H}) d^2_{\min,\mathrm{sm}}. \tag{20}$$

Rewriting (20) by taking the Demmel condition number κ_D [9] is given as

$$\kappa_D := \frac{\|\mathbf{A}(\mathbf{H})\|_F}{\lambda_{\min}(\mathbf{H})} \leq \frac{d_{\min,\mathrm{sm}}}{d_{\min,\mathrm{sd}}}, \text{ where} \frac{d^2_{\min,\mathrm{sd}}}{N} = \min_{\mathbf{s},\mathbf{c}\in S,\mathbf{s}\neq\mathbf{c}} \|\mathbf{s}-\mathbf{c}\|^2 , \tag{21}$$

In many measurement papers [9], SM is preferred when $d^2_{\min,\mathrm{sd}} \leq \lambda^2_{\min}(\mathbf{H}) d^2_{\min,\mathrm{sm}}$ via $\|\mathbf{A}(\mathbf{H})\|_F = 1$.

3.3 The Proposed QR-Based Selection Criterion

In this section, we propose the QR-based selection criterion with reduced complexity to achieve an optimum transmission as follows. To achieve this, the following theorem shows that the minimum square Euclidean distance can be bounded in terms of the diagonal entries of **R** in the QR decomposition of a channel matrix.

Theorem 1 *Let* $\mathbf{H} \in C^{M\times N}$ *and* **R** *be the upper triangular matrix in the QR decomposition of* **H** *(*$\mathbf{H}=\mathbf{QR}$*). Denoting* $r_{1,1}, r_{2,2}, \ldots, r_{N,N}$ *as the diagonal elements of* **R***, we have*

$$\min_{i\in 1,2,\ldots,N} r^2_{i,i} \cdot \frac{d^2_{\min,\mathrm{SM}}}{N} \leq d^2_{\min,\mathrm{SM}}(\mathbf{H}) \leq r^2_{1,1} \cdot \frac{d^2_{\min,\mathrm{sm}}}{N}. \tag{22}$$

Proof See Appendix II. □

Furthermore, by applying Theorem 1, we can obtain the following property.

Property 2 *Let* $\mathbf{H} \in C^{M\times N}$*, and then the upper triangular matrix* **R** *has the equal diagonal elements as* $r_{1,1} = r_{2,2} = \ldots = r_{N,N}$*. Based on* (22)*, we have*

$$d^2_{\min,\mathrm{SM}}(\mathbf{H}) = r^2_{k,k} \cdot \frac{d^2_{\min,\mathrm{sm}}}{N}, \forall 1 \leq k \leq N. \tag{23}$$

Based on (22) *and* (23)*, to achieve low-complexity selection criterion without the singular value decomposition (SVD) processes, by using* (22)*, we propose the minimum Euclidean distance property by using* [20] *as follows*

$$\lambda^2_{\mathrm{in}}(\mathbf{H}) \frac{d^2_{\min,\mathrm{sm}}}{N} \leq \min_{1\leq k\leq N} r^2_{k,k} \frac{d^2_{\min,\mathrm{sm}}}{N} \leq d^2_{\min,\mathrm{SM}}(\mathbf{H}). \tag{24}$$

To claim the lower bound in (24) *(the left-hand side of* (24)*), the following theorem shows that the minimum singular value of* **H** *is less than the minimum eigenvalue of* **H**.

Theorem 2 *Let* $\mathbf{H} \in C^{M\times N}$, *and let* $\mathbf{H}=\mathbf{QR}$ *be QR-decomposition of the channel matrix* $\mathbf{H}$, *where* $\mathbf{Q} \in C^{M\times N}$ *is an unitary matrix and* $\mathbf{R} \in C^{N\times N}$ *is an upper triangular matrix. Then, the minimum singular value of the matrix* $\mathbf{H}$ *is bounded above by* $\min_{1\le i\le N} \left|\mathbf{R}_{i,i}\right|$. *That is,* $\lambda_{min}(\mathbf{H}) \le \min_{1\le i\le N} \left|\mathbf{R}_{i,i}\right|$.

Proof From [20], we know that

$$\lambda_{\min}(\mathbf{H}) = \sigma_{\min}(\mathbf{H}^H\mathbf{H}). \tag{25}$$

where $\sigma_{min}(\mathbf{H}^H\mathbf{H})$ is the minimum eigenvalue of $\mathbf{H}^H\mathbf{H}$. From the definition at (25), it is easy to show that

$$\sigma_{\min}(\mathbf{H}^H\mathbf{H}) = \sigma_{\min}(\mathbf{R}^H\mathbf{R}). \tag{26}$$

The equality at (26) holds, since $\mathbf{H}^H\mathbf{H} = \mathbf{R}^H\mathbf{Q}^H\mathbf{Q}\mathbf{R} = \mathbf{R}^H\mathbf{R}$, for $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$. Then, we have (depicted in [20] at Theorem 3.3.14, p176])

$$\sigma_{\min}(\mathbf{R}^H\mathbf{R}) \le |\sigma_{\min}(\mathrm{R})| = \min_{1\le i\le N} \left|\mathbf{R}_{i,i}\right|. \tag{27}$$

Therefore, the proof is complete. □

In (24), we show that the minimum diagonal entry of $\mathbf{R}$ operating on the minimum distance of each codeword difference is great than or equal to the minimum singular value of $\mathbf{H}$ operating on the minimum distance of each codeword difference (depicted in Demmel selection criterion [20]). Hence, the minimum diagonal entry of $\mathbf{R}$ operating on the minimum distance of each codeword difference is close to the minimum square Euclidean distance. Based on these two conditions, to achieve a better transmission, considering low-complexity, we propose the QR-based selection criterion via (12) and (18) as follows.

$$\begin{aligned} d^2_{\min,\mathrm{SD}}(\mathbf{H}) :&= \min_{\mathbf{s},\mathbf{c}\in S,\mathbf{s}\ne\mathbf{c}} \|\mathbf{H}(\mathbf{s}-\mathbf{c})\|^2 = \min_{\mathbf{s},\mathbf{c}\in S,\mathbf{s}\ne\mathbf{c}} \|\mathbf{R}(\mathbf{s}-\mathbf{c})\|^2 = \left\|\bar{r}_{k,k}\right\|^2 \|(\mathbf{s}-\mathbf{c})\|^2 \\ &= \left\|\bar{r}_{k,k}\right\|^2 d^2_{\min,\mathrm{sd}}(\mathbf{H}), \end{aligned} \tag{28}$$

where $\mathbf{R} = \bar{r}_{k,k}\mathbf{I}_N$ from (12) and $\bar{r}_{1,1} = \ldots = \bar{r}_{k,k} = \ldots = \bar{r}_{N,N}$. Without an exhaustive search of each codeword operating on the channel in (28), we can achieve low-complexity due to only the computation of the minimum of each codeword difference. By applying (24) in the selection criterion, we propose a low-complexity QR-based selection criterion which is employed in SM when

$$\kappa_{\mathrm{QR}} := \frac{\bar{r}_{k,k}}{\min_{1\le k\le N} r_{k,k}} \le \frac{d_{\min,\mathrm{sm}}}{d_{\min,\mathrm{sd}}}, \tag{29}$$

where κ_{QR} is the proposed condition number. Without involving computational complexity, $r_{k,k}$ and $\bar{r}_{k,k}$ can be obtained from the MIMO detection of (3) and (12), respectively.

4 Performance and Complexity Analysis

In this section, the switching mode will be analyzed via 1) SER performance and 2) computational complexity. For 1), the closed-form SER for switching between SM and SD transmission is introduced as follows. For SM, considering the $N=M=2$ case, the condition symbol error probability of the stream 2 is

$$P_2(e|r_{2,2}) = Q\left(\sqrt{\frac{|r_{2,2}|^2 E_s}{2N_0}}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{E_s|r_{2,2}|^2}{4N_0\sin^2\theta}\right)d\theta, \tag{30}$$

where $Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta$, E_s is the energy per symbol, and $N_0/2$ is the power spectral density. $|r_{2,2}|^2$ follows a chi-square distribution with two degrees of freedom (depicted in appendix III) [21]. By averaging (30) with respect to $|r_{2,2}|^2$, we have (see [22] p. 117)

$$\begin{aligned}\bar{P}_{\mathrm{SM},2}(e) &= E\left(\frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{|r_{2,2}|^2 E_s}{4N_0\sin^2\theta}\right)d\theta\right) = \frac{1}{\pi}\int_0^{\pi/2} -\frac{\sin^2\theta}{\frac{E_s}{4N_0}+\sin^2\theta}d\theta \\ &= \frac{1}{\pi}\int_0^{\pi/2} 1 - \frac{\frac{E_s}{4N_0}}{\frac{E_s}{4N_0}+\sin^2\theta}d\theta = \frac{1}{2} - \frac{\frac{E_s}{4N_0}}{\sqrt{\left(\frac{E_s}{2N_0}+1\right)^2 - 1}},\end{aligned} \tag{31}$$

where $E(\cdot)$ is an expectation operation. Then, the conditional symbol error probability of stream 1 is

$$P_1(e|r_1, 1) = Q\left(\sqrt{\frac{|r_{1,1}|^2 E_s}{2N_0}}\right) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp\left(-\frac{E_s|r_{1,1}|^2}{4N_0\sin^2\theta}\right)d\theta, \tag{32}$$

where $|r_{1,1}|^2$ follows a chi-square distribution with four degrees of freedom and S_j follows a chi-square distribution with two degrees of freedom. By averaging (32) with respect to $|r_{1,1}|^2$, we have (see [2] Eq. (3.26))

$$\begin{aligned}\bar{P}_{\mathrm{SM},1}(e) &= E\left(Q\left(\sqrt{\frac{2\left|r_{1,1}\right|^2 E_s}{4N_0}}\right)\right) = E\left(\frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{\left|r_{1,1}\right|^2 E_s}{4N_0\sin^2\theta}\right)d\theta\right) \\ &= E\left(\frac{1}{\pi}\prod_{j=1}^{2}\int_0^{\pi/2} \exp\left(-\frac{S_j E_s}{4N_0\sin^2\theta}\right)d\theta\right) = E \\ &\times \left(\frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{E_s/4N_0+\sin^2\theta}\right)^2 d\theta\right) \\ &= \left(\frac{1-\sqrt{\frac{E_s/4N_0}{(E_s/4N_0)+1}}}{2}\right)^2 \sum_{k=0}^{1}\binom{k+1}{k}\left(\frac{1+\sqrt{\frac{E_s/4N_0}{(E_s/4N_0)+1}}}{2}\right)^k.\end{aligned} \tag{33}$$

Based on (31) and (33), the average SER over two streams is given as

$$\bar{P}_{\mathrm{SM}}(e) = \left(\bar{P}_{\mathrm{SM},1}(e) + \bar{P}_{\mathrm{SM},2}(e)\right)/2. \tag{34}$$

For SD, considering the Alamouti transmission at the QR-based receiver, the pairwise error probability is given by (see [2] Eq. (3.27))

$$P_{\mathrm{SD}}(\mathbf{X},\hat{\mathbf{X}}) = \frac{1}{2}\left(1-\sqrt{\frac{d_E^2(\mathbf{X},\hat{\mathbf{X}})E_s/4N_0}{1+d_E^2(\mathbf{X},\hat{\mathbf{X}})E_s/4N_0}}\sum_{k=0}^{3}\binom{2k}{k}\right. \quad (35)$$

$$\left.\times\left(\frac{1}{4(1+d_E^2(\mathbf{X},\hat{\mathbf{X}})E_s/4N_0)}\right)^k\right),$$

where $d_E^2(\mathbf{X},\hat{\mathbf{X}})$ is the squared Euclidean distance between $\mathbf{X}$ and $\hat{\mathbf{X}}$ denoted as [2]

$$d_E^2(\mathbf{X},\hat{\mathbf{X}}) = \left|x_1-\hat{x}_1\right|^2 + \left|x_2-\hat{x}_2\right|^2. \quad (36)$$

The equality at (35) holds because the symbol-by-symbol detection is error-propagation-free via (12)–(14). This symbol-by-symbol detection has the same performance as maximum-likelihood decoding in the SD transmission [2]. In following simulation, for Alamouti transmission, we will show the SER performance of the proposed QR-based detection that is equal to (35). Therefore, the average SER of switching model is given as

$$\bar{P}_{SW}(e) = (N_M\cdot\bar{P}_{\mathrm{SM}}(e) + N_D\cdot P_{\mathrm{SD}}(\mathbf{X},\hat{\mathbf{X}}))/(N_M+N_D). \quad (37)$$

where N_M and N_D are the number of iterations employed in SM and SD, respectively. For 2), considering the real multiplication processes, the computational complexity of the proposed QR-based detection for SM and SD are $O(\sum_{i=1}^{2N-3}2Mi+12M(N-1)+2NM^2+(2N+1)N)$ and $O(4M)$, respectively. For selection computational complexity, the computational complexity of minimum square Euclidean distance selection criterion is $O(2N\cdot 2^{\bar{m}N}MN+2N\cdot 2^{mN}MN) where$ m and $\bar{m}$ are the modulation order for SM and SD, respectively. Without the exhaustive search of each codeword operating on the channel, the computational complexity of SVD selection criterion is $O(8N^3/3+2N^2)$. The computational complexity of the QR-based selection criterion is zero. This is because the computational complexity of $\mathbf{R}$'s entries can be obtained from the QR-based detection processes in (3) and (12).

5 Simulation Results

This section uses several numerical examples to illustrate the performance of the proposed scheme. When $N=M=2$, we consider SM (an uncode system is assumed), SD (Alamouti's code is assumed), and the switching mode under the assumption that CSI is given to the receiver but not to the transmitter. The proposed QR-based scheme evaluates the effects of A) SM and SD, B) switching between SM and SD C) complexity analysis, and D) error transmitted transmission as follows.

5.1 SM and SD

For a complex Gaussian channel, the channel coefficient is obtained from transmitting antenna $n(n=1,2,\ldots,N)$ to receiving antenna $m(m=1,2,\ldots,M)$ as

$$h_{m,n} = h_{\mathrm{R}} + jh_{\mathrm{I}}, \quad (38)$$

where h_R and h_I are *i.i.d.* complex Gaussian random variables with zero mean for the real part and the imaginary part, respectively. These simulations demonstrate the predicted SER

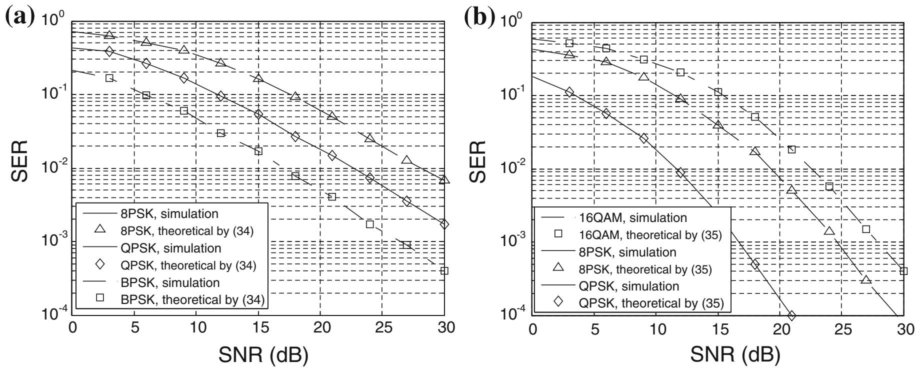


Fig. 2 Theoretical and simulation average SER for different constellations at $N = M = 2$ employing in **a** multiplexing (SM) and **b** diversity (SD)

performance in Sect. 3 and the corresponding simulated outcomes. For SM, Fig. 2a shows the average SER for three different symbol constellations: BPSK, QPSK and 8-PSK; the theoretical values are computed by (34). For SD, Fig. 2b shows the average SER for three different symbol constellations: QPSK, 8-PSK and 16QAM; the theoretical values are computed by (35) [2]. As we can see from Fig. 2a and b, the simulated results closely match the theoretical solutions. This implies that the proposed QR-based detection has same the performance as the maximum-likelihood detection. Based on these simulation results, Fig. 2 shows that the proposed QR-based detection can realize the SIC detection and maximum-likelihood detection for SM and SD, respectively.

5.2 Switching between SM and SD

In this subsection, we use the Demmel-based selection criterion (Demmel selection) via (20), QR-based selection criterion (QR-selection) via (29) and the square minimum Euclidean distance selection criterion (min-selection) to exhibit the switching between SM and SD performance. In min-selection criterion, SM is preferred when

$$d^2_{\min,\mathrm{MD}}(\mathbf{H}) \le d^2_{\min,\mathrm{SM}}(\mathbf{H}). \tag{39}$$

For simulation setting, the symbol constellations used are 8PSK for SM and 64QAM for SD so that the bit data rates are the same ($R = 6$ bits/symbol). We assume that the transmitter receives the perfect transmission and modulation order decision from the receiver via the perfect feedback channel. In a Ricean channel, $\mathbf{H}_{\mathrm{sp}}$ denotes the specular component that illuminates the entire array and is spatially deterministic from antenna to antenna. $\mathbf{H}_{\mathrm{sc}}$ denotes the scattered component that varies randomly from antenna to antenna (Rayleigh-distributed) and thus the channel response can be given as [24]

$$\mathbf{H} = \sqrt{\frac{\kappa}{\kappa+1}}\mathbf{H}_{\mathrm{sp}} + \sqrt{\frac{1}{\kappa+1}}\mathbf{H}_{\mathrm{sc}}, \tag{40}$$

where Ricean factor κ is defined as the ratio of deterministic-to-scattered power; when $\kappa \to \infty$, the channel is full correlated, but as $\kappa \to 0$, the channel is rich-scattered. The large κ implies severe correlations; the extreme selections $\kappa = 0$ and $\kappa = 100$, respectively, render the channel to independently fading and being almost light-of-sight.

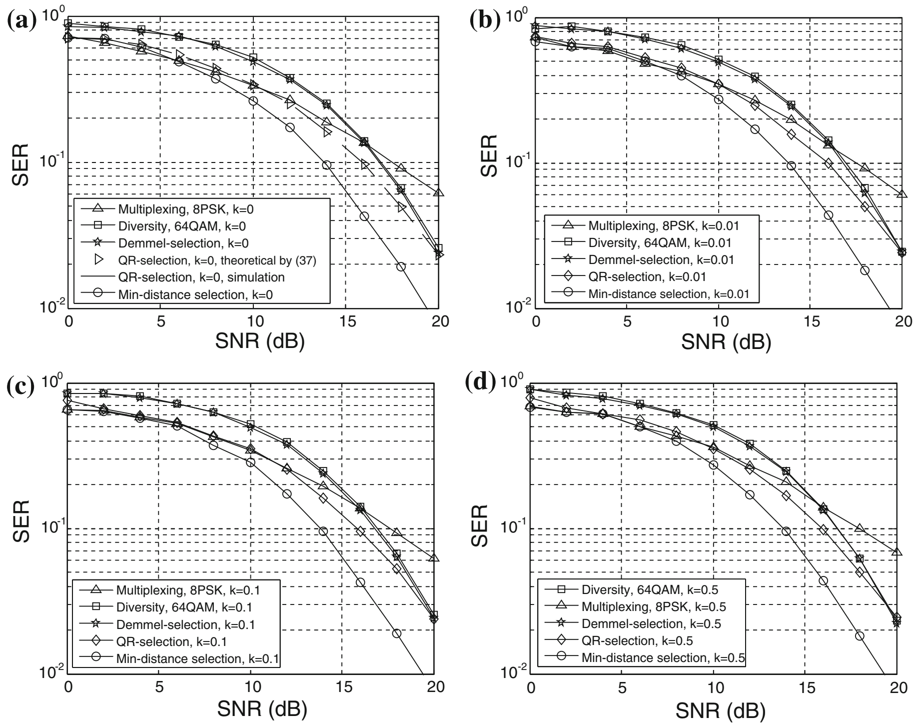


Fig. 3 SER versus SNR for $R = 6$ bits/symbol at $N = M = 2$ with different channel factors employing in **a** $k = 0$, **b** $k = 0.01$, **c** $k = 0.1$, and **d** $k = 0.5$

By using a medium eigenvalue spread = 5.3 in (40), first, we consider four Ricean κ-factors: 0, 0.01, 0.1, and 0.5. In this rich-scattering channel, in Fig. 3a–d, to select one modulation scheme for all channel realizations, SM is preferred at SNR < 16 dB while SD is preferred at SNR ≥ 16 dB. Figure 3a–d show that the proposed QR-selection criterion has better performance than Demmel-selection criterion. This is because the lower bound of the squared minimum Euclidean distance can be bounded tightly by the minimum diagonal entry of the **R** operating on the minimum distance of each codeword difference (depicted in (23)). When SNR is high (SNR = 20 dB), the performance of the proposed QR-selection criterion is close to the performance of the Demmel-selection criterion because SD is usually preferred. Especially, in Fig. 3a–d, the performance of the proposed QR-selection criterion can achieve the performance of the min-distance selection criterion incurring about a 2.5 dB loss when SNR is high. However, the proposed QR-selection criterion has less computational complexity than others. By using the switching mode, when the number of SM's transmission and that of SD's transmission are known, in Fig. 3a, the simulated results closely match the theoretical value in QR-selection criterion via (37).

Second, considering spatial correlation in MIMO channel, the probability of ill-conditioned matrix increases and thus it results in a higher error rate [25]. Hence, with enlarging κ ($\kappa = 10$, 100) to realize this ill-conditioned channel, Fig. 4a–b show that SVD-selection, QR-selection, and min-selection criteria have worse performance due to incensement of the error selection effect in this rank deficient channel. Figure 4a–b demonstrate that the performance of the proposed QR-selection criteria can achieve the performance of the min-

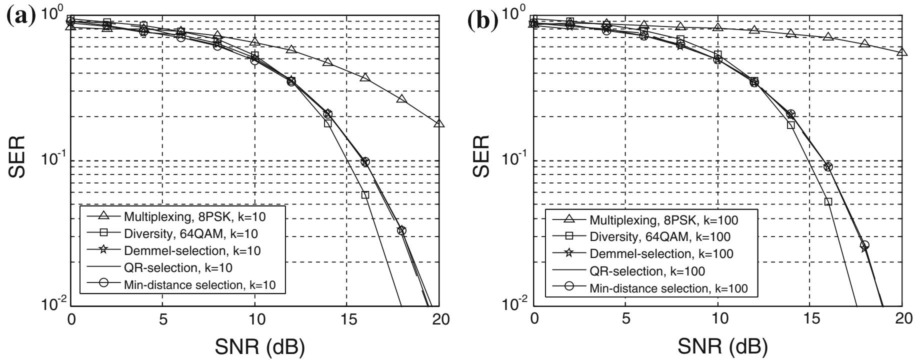


Fig. 4 SER versus SNR for $R = 6$ bits/symbol at $N = M = 2$ with different channel factors employing in **a** $k = 10$, and **b** $k = 100$

selection criteria, but at a low-complexity level. Particularly, in ill-conditioned channel, the proposed QR-selection criterion has high probability of error selection because upper triangular matrix ($\mathbf{R}$) is nearly singular, which can degrade the detection performance. Similarly, the SVD-selection criterion (Demmel selection) in the ill-conditioned channel involves degraded detection performance since large condition number causes high probability of error selection. Additionally, as shown in Fig. 4a–b, to select one modulation scheme for all channel realizations in the low-rank channel, SD is preferred.

5.3 Complexity Analysis

In this subsection, considering Gram–Schmidt scheme in the real multiplication system, to compare the complexity of the proposed QR-decomposition via (8) and the conventional QR-decomposition (see [16] Eq. 3), the computational effectiveness (CE) ratio is given as

$$\text{CE}_{\text{decmp}} := \frac{\text{PQRD complexity}}{\text{CQRD complexity}}, \tag{41}$$

where PQRD complexity denotes the proposed QR-decomposition complexity, and CQRD complexity denotes the conventional QR-decomposition complexity. Especially, in the conventional QR-decomposition processes, the real channel matrix (see [16] Eq. 3) is given as

$$\mathbf{H}_r := \begin{bmatrix} \text{Re}\{\mathbf{H}\} & -\text{Im}\{\mathbf{H}\} \\ \text{Im}\{\mathbf{H}\} & \text{Re}\{\mathbf{H}\} \end{bmatrix}. \tag{42}$$

Then, based on (41), with $(N, M) = (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8)$, and $(9, 9)$, Fig. 5 shows that the proposed QR-decomposition has about half of the computational complexity compared to conventional QR-decomposition. This is because elements in the neighboring columns of upper triangular matrix ($\mathbf{R}$) are repeated, and thus only half the elements are computed. For detection complexity, the complexity of the signals decision in (5) and the QR-decomposition complexity is considered in the SM transmission. Thus, to compare the complexity of the proposed QR-detection and the conventional QR-detection, CE ratio is given as

Fig. 5 CE ratio versus various antennas for comparing the proposed QR-decomposition complexity with conventional QR-decomposition complexity

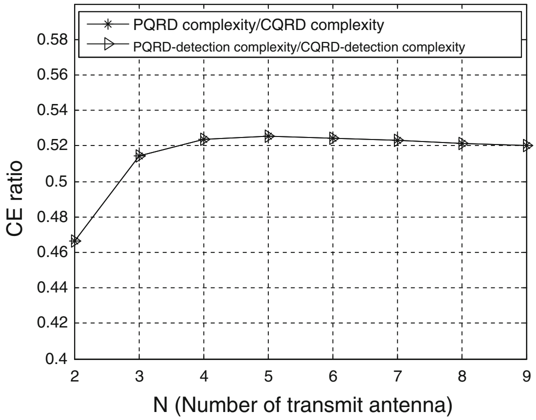
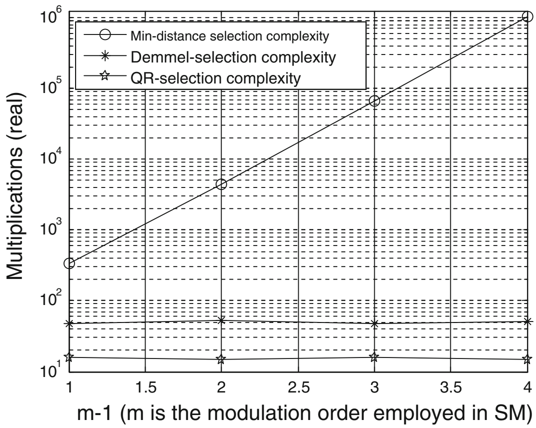


Fig. 6 Number of real multiplications versus various modulation orders employing in various selection criteria complexity



$$\mathrm{CE}_{\mathrm{detection}} := \frac{\text{PQRD-detection complexity}}{\text{CQRD-detection complexity}}, \tag{43}$$

where the PQRD-detection complexity denotes the proposed QR-detection complexity, and CQRD-detection complexity denotes the conventional QR-detection complexity. Figure 5 shows that the proposed QR-detection complexity has about half of the computational complexity compared to conventional QR-detection. This is because the proposed QR-detection involves a low-complexity QR-decomposition of (8), where the complexity of the signals decision is less than the QR-decomposition complexity in the detection processes.

Then, to compare the QR-based detection complexity with various selection criteria, we consider $(m, \bar{m}) = (1, 2), (2, 4), (3, 6), (4, 8)$ and a complex Gaussian channel in simulations at Fig. 6. Figure 6 shows that the proposed QR selection (QR-selection) criterion has less complexity than the minimum distance (min-distance) selection criterion or the Demmel selection (Demmel-selection) criterion. This is because the computational complexity of $\mathbf{R}$'s entries of (29) can be obtained from the QR-based detection processes in (3) and (12), and it does not involve exhaustive searching processes of (15) or the SVD processes of (21).

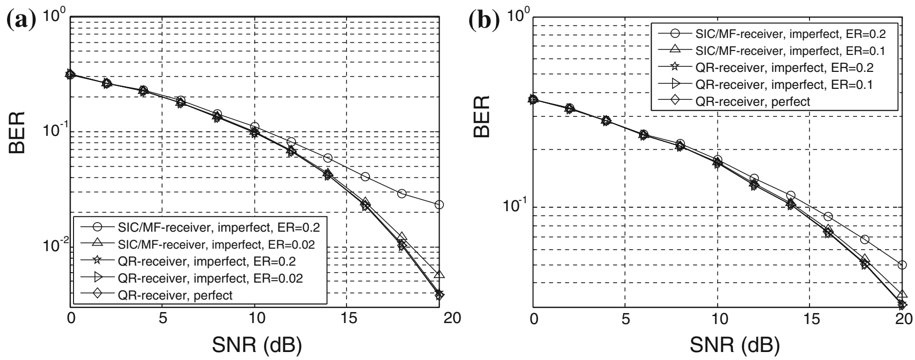


Fig. 7 BER versus SNR with different error transmitted transmission mode rates (ERs) at $N = M = 2$ employing in **a** $R = 6$ bits/symbol (8PSK for SM, 64QAM for SD), and **b** $R = 8$ bits/symbol (16QAM for SM, 256QAM for SD). (In Fig. 7, imperfect denotes imperfect transmitted transmission decision, and perfect denotes ER = 0)

5.4 Error Transmitted Transmission

In this subsection, we assume that the error transmission decision occurs at the transmitter in a complex Gaussian channel. Additionally, the perfect channel knowledge is given in receiver, and the modulation order decision is perfectly known in transmitter and receiver. To compare the detection performance in the proposed QR-based detection and the conventional detection, we derive the error rate of the transmitted transmission mode in the switching transmission as

$$\text{ER} := \frac{\text{Total error transmitted transmissions}}{\text{Total transmissions}}. \tag{44}$$

For conventional detection in the receiver, the SIC detection (QR-based detection is used) is employed in the SM transmission, and the match filter (MF) decoding is employed in the SD transmission for switching transmission. This MF scheme is given as

$$\mathbf{HY} = \mathbf{H}^H\mathbf{Hx} + \mathbf{Hv}. \tag{45}$$

This conventional receiver with the SIC detection and the MF detection is called SIC/MF-receiver in this paper. Then, the proposed QR-based receiver (QR-receiver) uses a single QR-detection employed in the switching transmission. With $N = M = 2$, 8PSK for SM, and 64QAM for SD in a complex Gaussian channel, Fig. 7a show that the proposed QR-receiver has a better performance than the conventional SIC/MF-receiver. This is better because the proposed QR-receiver can realize the SIC detection performance for SM and the O-STBC decoding performance for SD, respectively. That is, the proposed QR-receiver is single in the receiver that has no the transmission mismatch problem between the transmitter and receiver for degrading detection performance. Based on these two realized schemes, with increasing ER in imperfect transmitted transmission mode, the proposed QR-receiver has a robust detection performance for achieving the detection performance of the perfect transmitted transmission mode (ER = 0). In this perfect case (ER = 0), the proposed QR-receiver is considered for the detection scheme at the switching transmission in Fig. 7. Similarly, considering 16QAM for the SM transmission and 256QAM for the SD transmission, Fig. 7b shows that the proposed QR-receiver has a better performance than the conventional SIC/MF-receiver, and it can achieve the performance of the perfect transmitted transmission mode. Especially, for SIC/MF receiver in these simulations, the detection performance is degraded because the match filter matrix ($\mathbf{H}^H\mathbf{H}$) of (45) is not diagonal at the orthogonal space-time block code (O-STBC) decoding for the SM transmission in Fig. 7.

6 Conclusions

In this paper, to achieve high data transmission rate and high link reliability simultaneously, switching between SM and SD schemes are exhibited in MIMO communications. Considering a robust detection at the receiver, we proposed a QR-based detection to realize SIC detection and O-STBC decoding, respectively. Considering low-complexity implementation, QR-based selection criterion is proposed to achieve an optimum transmission in switching between SM and SD. In simulation results, the proposed QR-based selection criterion with reduced complexity has a better performance than SVD-based selection criterion, and can achieve the performance of the minimum Euclidean distance selection criterion in various channel condition factors. Our future work will investigate MIMO techniques in accordance with precoder and STBC for mobile broadband wireless access applications.

Acknowledgments The study is supported by the National Science Council (NSC), Taiwan under Contract Nos. NSC 95-2221-E-009-057-MY3 and NSC 96-2221-E-009-030-MY3. Additionally, I would like to express my deepest gratitude to my advisor, Professor Sin-Horng Chen, for his enthusiastic guidance and great patience.

Appendix I

Detailed Proof of Property 1

Proof For $N = M = 1$, we assume $H := [a + bi \in C^{1x1}$ and hence the real-value channel is $\mathbf{H}_r := \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \in \Re^{2\times 2}$. We have $\mathbf{u}_1 = \mathbf{v}_1$ and $\mathbf{u}_2 = \mathbf{v}_2 - \frac{\langle \mathbf{v}_2, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 = \mathbf{v}_2$, where $\langle \mathbf{v}_2, \mathbf{u}_1\rangle = -ab + ba = 0$. Thus, the QR-decomposition of $\mathbf{H}$ is given as

$$\mathbf{H}_r = \underbrace{\begin{bmatrix} \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} & \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|} \end{bmatrix}}_{\mathbf{Q}} \underbrace{\begin{bmatrix} \|\mathbf{u}_1\| & 0 \\ 0 & |\mathbf{u}_2\| \end{bmatrix}}_{\mathbf{R}}, \tag{A.1}$$

where $\mathbf{R}_{11} = \mathbf{R}_{22}$, $\mathbf{R}_{12} = -\mathbf{R}_{21}$ and thus we have $\mathbf{R} \in \mathbf{B}_{2\times 2}$. For $N = 1$ and $M = 2$, we assume $H := \begin{bmatrix} a + bi \\ c + di \end{bmatrix} \in C^{2x1}$ and hence the real-value channel is $\mathbf{H}_r := \begin{bmatrix} a & -b \\ c & -d \\ b & a \\ d & c \end{bmatrix} \in \Re^{4\times 2}$. We have $\mathbf{u}_1 = \mathbf{v}_1$ and $\mathbf{u}_2 = \mathbf{v}_2 - \frac{\langle \mathbf{v}_2, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 = \mathbf{v}_2$, where $\langle \mathbf{v}_2, \mathbf{u}_1\rangle = -ab - cd + ba + cd = 0$. Thus, the QR-decomposition of $\mathbf{H}$ is given as

$$\mathbf{H}_r = \begin{bmatrix} \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} & \frac{\mathbf{u}_2}{\|\mathbf{u}_1\|} \end{bmatrix} \begin{bmatrix} \|\mathbf{u}_1\| & 0 \\ 0 & \|\mathbf{u}_1\| \end{bmatrix}, \tag{A.2}$$

For $N = 2$ and $M = 1$, we assume H $:= [a + bi \quad c + di] \in C^{1\times 2}$ and hence the real-value channel is $\mathbf{H}_r := \begin{bmatrix} a & -b & c & -d \\ b & a & d & c \end{bmatrix} \in \Re^{2\times 4}$, then $\mathbf{u}_1 = \mathbf{v}_1$ and $\mathbf{u}_2 = \mathbf{v}_2 - \frac{\langle \mathbf{v}_2, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 = \mathbf{v}_2$, where

$\langle \mathbf{v}_2, \mathbf{u}_1\rangle = -ab+ba = 0$. Then, we have $\mathbf{u}_3 = \mathbf{v}_3 - \frac{\langle \mathbf{v}_3, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 - \frac{\langle \mathbf{v}_3, \mathbf{u}_2\rangle}{\langle \mathbf{u}_2, \mathbf{u}_2\rangle}\mathbf{u}_2$, where $\langle \mathbf{v}_3, \mathbf{u}_1\rangle = ac+bd$ and $\langle \mathbf{v}_3, \mathbf{u}_2\rangle = -bc+ad$. To obtain $\mathbf{u}_4$, we have $\langle \mathbf{v}_4, \mathbf{u}_1\rangle = -ad-bc$ and $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = bd+ac$. Thus, we have $\langle \mathbf{v}_4, \mathbf{u}_3\rangle = \langle \mathbf{v}_4, \mathbf{v}_3\rangle_- \langle \mathbf{v}_4,\ \mathbf{u}_1\rangle \frac{\langle \mathbf{v}_3, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle} - \langle \mathbf{v}_4,\ \mathbf{u}_2\rangle \frac{\langle \mathbf{v}_3, \mathbf{u}_2\rangle}{\langle \mathbf{u}_2, \mathbf{u}_2\rangle} = 0$, where $\langle \mathbf{v}_4, \mathbf{u}_1\rangle = -\langle \mathbf{v}_3, \mathbf{u}_2\rangle$, $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = \langle \mathbf{v}_3, \mathbf{u}_1\rangle$ and $\langle \mathbf{v}_4, \mathbf{v}_3\rangle = 0$. Then, we have

$$\begin{aligned}\mathbf{u}_4 &= \mathbf{v}_4 - \frac{\langle \mathbf{v}_4,\ \mathbf{u}_1\rangle}{\langle \mathbf{u}_1,\ \mathbf{u}_1\rangle}\mathbf{u}_1 - \frac{\langle \mathbf{v}_4,\ \mathbf{u}_2\rangle}{\langle \mathbf{u}_2,\ \mathbf{u}_2\rangle}\mathbf{u}_2 - \frac{\langle \mathbf{v}_4,\ \mathbf{u}_3\rangle}{\langle \mathbf{u}_3,\ \mathbf{u}_3\rangle}\mathbf{u}_3 \\ &= \mathbf{v}_4 - \frac{-\langle \mathbf{v}_3,\ \mathbf{u}_2\rangle}{\langle \mathbf{u}_1,\ \mathbf{u}_1\rangle}\mathbf{u}_1 - \frac{\langle \mathbf{v}_3,\ \mathbf{u}_1\rangle}{\langle \mathbf{u}_2,\ \mathbf{u}_2\rangle}\mathbf{u}_2, \end{aligned} \tag{A.3}$$

where $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = \langle \mathbf{v}_3, \mathbf{u}_1\rangle$. Thus, the QR-decomposition of $\mathbf{H}$ is given as

$$\mathbf{H}_r = \underbrace{\begin{bmatrix} \frac{\mathbf{u_1}}{\|\mathbf{u_1}\|} & \frac{\mathbf{u_2}}{\|\mathbf{u_1}\|}\end{bmatrix}}_{\mathbf{Q}} \underbrace{\begin{bmatrix} \|\mathbf{u}_1\| & 0 & \langle \mathbf{v}_3, \mathbf{u}_1\rangle & -\langle \mathbf{v}_3, \mathbf{u}_2\rangle \\ 0 & \|\mathbf{u}_1\| & \langle \mathbf{v}_3, \mathbf{u}_2\rangle & \langle \mathbf{v}_3, \mathbf{u}_1\rangle \end{bmatrix}}_{\mathbf{R}}. \tag{A.4}$$

where $\mathbf{R}_{11} = \mathbf{R}_{22}, \mathbf{R}_{12} = -\mathbf{R}_{21}, \mathbf{R}_{13} = \mathbf{R}_{24}, \mathbf{R}_{14} = -\mathbf{R}_{23}$and thus we have $\mathbf{R} \in \mathbf{B}_{4\times 4}$. For $N = M = 2$, we assume $H := \begin{bmatrix} a+bi & e+fi \\ c+di & h+ji \end{bmatrix} \in C^{2x2}$ and hence the real-value channel is $\mathbf{H}_r := \begin{bmatrix} a & -b & e & -f \\ c & -d & h & -j \\ b & a & f & e \\ d & c & j & h \end{bmatrix} \in \Re^{4\times 4}$, then $\mathbf{u}_1 = \mathbf{v}_1$ and $\mathbf{u}_2 = \mathbf{v}_2 - \frac{\langle \mathbf{v}_2, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 = \mathbf{v}_2$, where $\langle \mathbf{v}_2, \mathbf{u}_1\rangle = -ab - cd + ba + dc = 0$. Then, we have $\mathbf{u}_3 = \mathbf{v}_3 - \frac{\langle \mathbf{v}_3, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle}\mathbf{u}_1 - \frac{\langle \mathbf{v}_3, \mathbf{u}_2\rangle}{\langle \mathbf{u}_2, \mathbf{u}_2\rangle}\mathbf{u}_2$, where $\langle \mathbf{v}_3, \mathbf{u}_1\rangle = ae + ch + bf + dj$ and $\langle \mathbf{v}_3, \mathbf{u}_2\rangle = -be - dh + af + cj$. To obtain $\mathbf{u}_4$, we have $\langle \mathbf{v}_4, \mathbf{u}_1\rangle = -af - cj + be + dh$ and $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = bf + di + ae + ch$. Thus, we have $\langle \mathbf{v}_4, \mathbf{u}_3\rangle = \langle \mathbf{v}_4, \mathbf{v}_3\rangle_- \langle \mathbf{v}_4,\ \mathbf{u}_1\rangle \frac{\langle \mathbf{v}_3, \mathbf{u}_1\rangle}{\langle \mathbf{u}_1, \mathbf{u}_1\rangle} - \langle \mathbf{v}_4,\ \mathbf{u}_2\rangle \frac{\langle \mathbf{v}_3, \mathbf{u}_2\rangle}{\langle \mathbf{u}_2, \mathbf{u}_2\rangle} = 0$, where $\langle \mathbf{v}_4, \mathbf{u}_1\rangle = -\langle \mathbf{v}_3, \mathbf{u}_2\rangle$, $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = \langle \mathbf{v}_3, \mathbf{u}_1\rangle$ and $\langle \mathbf{v}_4, \mathbf{v}_3\rangle = 0$. Then, we have

$$\begin{aligned}\mathbf{u}_4 &= \mathbf{v}_4 - \frac{\langle \mathbf{v}_4,\ \mathbf{u}_1\rangle}{\langle \mathbf{u}_1,\ \mathbf{u}_1\rangle}\mathbf{u}_1 - \frac{\langle \mathbf{v}_4,\ \mathbf{u}_2\rangle}{\langle \mathbf{u}_2,\ \mathbf{u}_2\rangle}\mathbf{u}_2 - \frac{\langle \mathbf{v}_4,\ \mathbf{u}_3\rangle}{\langle \mathbf{u}_3,\ \mathbf{u}_3\rangle}\mathbf{u}_3 = \mathbf{v}_4 - \frac{-\langle \mathbf{v}_3,\ \mathbf{u}_2\rangle}{\langle \mathbf{u}_1,\ \mathbf{u}_1\rangle}\mathbf{u}_1 \\ &\quad - \frac{\langle \mathbf{v}_3,\ \mathbf{u}_1\rangle}{\langle \mathbf{u}_2,\ \mathbf{u}_2\rangle}\mathbf{u}_2, \end{aligned} \tag{A.5}$$

where $\langle \mathbf{v}_4, \mathbf{u}_2\rangle = \langle \mathbf{v}_3, \mathbf{u}_1\rangle$. Thus, the QR-decomposition of $\mathbf{H}$ is given as

$$\mathbf{H}_r = \underbrace{\begin{bmatrix} \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} & \frac{\mathbf{u}_2}{\|\mathbf{u}_1\|} & \frac{\mathbf{u}_3}{\|\mathbf{u}_3\|} & \frac{\mathbf{u}_4}{\|\mathbf{u}_4\|}\end{bmatrix}}_{\mathbf{Q}} \underbrace{\begin{bmatrix} \|\mathbf{u}_1\| & 0 & \langle \mathbf{v}_3, \mathbf{u}_1\rangle & -\langle \mathbf{v}_3, \mathbf{u}_2\rangle \\ 0 & \|\mathbf{u}_1\| & \langle \mathbf{v}_3, \mathbf{u}_2\rangle & \langle \mathbf{v}_3, \mathbf{u}_1\rangle \\ 0 & 0 & \|\mathbf{u}_3\| & 0 \\ 0 & 0 & 0 & \|\mathbf{u}_3\| \end{bmatrix}}_{\mathbf{R}} = \mathbf{QR}, \tag{A.6}$$

where $\mathbf{R}_{11} = \mathbf{R}_{22}, \mathbf{R}_{12} = -\mathbf{R}_{21}, \mathbf{R}_{13} = \mathbf{R}_{24}, \mathbf{R}_{14} = -\mathbf{R}_{23}, \mathbf{R}_{31} = \mathbf{R}_{42}, \mathbf{R}_{32} = -\mathbf{R}_{41}, \mathbf{R}_{33} = \mathbf{R}_{44}, \mathbf{R}_{34} = -\mathbf{R}_{43}$ and thus we have $\mathbf{R} \in \mathbf{B}_{4\times 4}$.

$$\vdots$$

As it turns out, we have $\mathbf{u}_1 = \mathbf{v}_1$ and

$$\mathbf{u}_k = \mathbf{v}_k - \sum_{i=1}^{k-1} \frac{\langle \mathbf{v}_k, \mathbf{u}_i\rangle}{\langle \mathbf{u}_i, \mathbf{u}_i\rangle}\mathbf{u}_i, \text{for} 2 \le k \le 2N, \tag{A.7}$$

where $\mathbf{R}_{i,k} = \frac{\langle \mathbf{v}_k, \mathbf{u}_i \rangle}{\langle \mathbf{u}_i, \mathbf{u}_i \rangle}$ and it satisfies

$$\mathbf{R}_{m,n} = \mathbf{R}_{m+1,n+1}, \quad \text{and} \quad \mathbf{R}_{m,n+1} = -\mathbf{R}_{m+1,n}, \tag{A.8}$$

where $m \in 1, 3, \ldots, 2M-1$ and $n \in 1, 3, \ldots, 2N-1$. Hence, we have $\mathrm{R} \in \mathbf{B}_{2^n \times 2^n}$ for any N and M. □

Appendix II

Detailed Proof of Theorem 1

Proof Considering two different signal vectors $\mathbf{x} = [x_1, x_2, \ldots, x_N]^T$ and $\mathbf{c} = [c_1, c_2, \ldots, c_N]^T$. If $x_k = c_k$, $k = 2, 3, \ldots, N$, but $x_1 \neq c_1$, we have

$$\| |\mathbf{H}(\mathbf{x} - \mathbf{c})| \|^2 = r_{1,1}^2 \, \|x_1 - c_1\|^2 . \tag{A.9}$$

By taking the minima of both sides of (A.9), we have

$$d_{\text{min,SM}}^2(\mathbf{H}) \leq \min_{s_1, c_1 \in \chi, s_1 \neq c_1} r_{1,1}^2 \cdot |s_1 - c_1|^2 = r_{1,1}^2 \cdot \frac{d_{\text{min,sm}}^2}{N}, \tag{A.10}$$

where $\frac{d_{\text{min,sm}}^2}{N} = \min_{\mathbf{s},\mathbf{c} \in S, \mathbf{s} \neq \mathbf{c}} \|\mathbf{s} - \mathbf{c}\|^2$, S denotes the feasible set containing the possible symbol vector, and χ denotes the feasible containing the possible symbol. Equation (A.10) leads to $d_{\text{min,SM}}^2(\mathbf{H}) \leq r_{1,1}^2 \cdot \frac{d_{\text{min,SM}}^2}{N}$ for completing the proof of the right hand side of inequality (18). To prove the left-hand side of (18), we have

$$||\mathbf{H}(\mathbf{x} - \mathbf{c})||^2 = \sum_{i=1}^{N} \left| \sum_{j=i}^{N} r_{i,j} \cdot (x_j - c_j) \right|^2 . \tag{A.11}$$

Assuming $\mathbf{x} \neq \mathbf{c}$ and let k be an integer such that $x_i = c_i$, for $i > k$, but $x_k \neq c_k$. From (A.11), considering the upper triangularity of $\mathbf{R}$, we have

$$||\mathbf{H}(\mathbf{x} - \mathbf{c})||^2 = \sum_{i=1}^{N} \left| \sum_{i=1}^{N} r_{i,j} \cdot (x_j - c_j) \right|^2 \geq \left| r_{k,k} \right|^2 \cdot |s_k - c_k|^2 \geq \left| r_{k,k} \right|^2 \cdot \frac{d_{\text{min,sm}}^2}{N}. \tag{A.12}$$

Taking the minima of both sides of (A.12) yields

$$\min_{i \in 1,2,\ldots,N} r_{i,i}^2 \cdot \frac{d_{\text{min,sm}}^2}{N} \leq d_{\text{min,SM}}^2(\mathbf{H}), \tag{A.13}$$

and thus the proof is completed. □

Appendix III

Described Characterization of Degree of Freedom

To derive the SM performance, we describe a chi-square distribution with different degrees of freedom in $|r_{i,i}|^2$ when $1 \leq i \leq 2$. Considering $N = M = 2$, the QR-decomposition processes are given as follows:

$$r_{1,1} = \|\mathbf{h}_1\|_F, \mathbf{h}_1 = \left[h_{1,1} h_{2,1}\right]^T, \quad \text{and } \mathbf{q}_1 = \frac{\mathbf{h}_1}{r_{1,1}}, \tag{A.14}$$

$$r_{1,2} = \mathbf{q}_1^H \mathbf{h}_2 = \mathbf{q}_1^H \left[h_{1,2} h_{2,2}\right]^T, \tag{A.15}$$

$$r_{2,2} = \left\|\mathbf{h}_2 - r_{1,2}\mathbf{q}_1\right\|_F, \quad \text{and } \mathbf{q}_2 = \frac{\mathbf{h}_2 - r_{1,2}\mathbf{q}_1}{r_{2,2}}, \tag{A.16}$$

where $|r_{1,1}|^2$ has a chi-square distribution with four degrees of freedom because it is equal to $\|\mathbf{h}_1\|_F^2$. To the probability density distribution, we show $\mathbf{h}_2$ with QR-decomposition as

$$\mathbf{h}_2 = r_{1,2}\mathbf{q}_1 + r_{2,2}\mathbf{q}_2, \tag{A.17}$$

where $\|\mathbf{h}_2\|_F^2$ has a chi-square distribution with four degrees of freedom. To obtain the degree of freedom for $|r_{2,2}|^2$. Firstly, we represent that $r_{1,2} = \mathbf{q}_1^H \mathbf{h}_2$ is jointly Gaussian and it characterized by its covariance matrix as

$$E[r_{1,2} r_{1,2}^H] = \mathbf{q}_1^H E[\mathbf{h}_2 \mathbf{h}_2^H] \mathbf{q}_1 = \mathbf{I}_2 \mathbf{q}_1 = \mathbf{I}_1. \tag{A.18}$$

With (A.18), we describe that $|r_{1,2}|^2$ has a chi-square distribution with two degrees of freedom. Therefore, based on (A.17) and (A.18), $|r_{2,2}|^2$ has a chi-square distribution with two degrees of freedom.

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