

# Optical Birefringence of Volume Photopolymer Holograms

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## ABSTRACT

Optical birefringence of volume photopolymer holograms could be applied as polarization elements and an important compensated device for display. In this paper, an analysis of the optical birefringence in volume photopolymer hologram is given. This characteristic analysis is mainly based on the Kogelnik's coupled wave theory. These characteristic properties for index modulation and optical birefringence in hologram formation are theoretically analyzed with respect to the parameter of diffraction efficiency, film thickness, angular and wavelength selectivity. This paper is also considered the holograms in slanted and non-slanted structures. Results of the characteristics of optical birefringence is provided an approach to realize as an optical compensation. This study shows that the property of negative birefringence in volume photopolymer will give a newer application in liquid crystal display technology.

**Keywords :**Holographic Optical Elements, Optical Birefringence, Diffraction Optics, Display Optics, Optical compensator.

## 1. INTRODUCTION

One practical application of holograms is their use as optical components. It is there of interest to investigate the optical birefringence of volume holographic structure in considering the actual application. In general, the holograms was recorded by two plane waves with the same or different incident angles to the recording holographic plane, *i.e.* called non-slanted and slanted holograms. The interference of two plane waves will form a periodic grating structure with sinusoidal index distribution in the emulsion. The periodic grating structure with any index variation function can act as many layers of dielectric material. Grating of dielectric material can consider as homogeneous birefringence material if the wavelength of the incident light is much larger than the period of the grating, *i.e.* called form birefringence<sup>1</sup>. The phase retardation of optical elements is dependent on the optical birefringence and thickness of holograms. Some polarization characteristics, utilized a hologram, can be illustrated in view of phase retardation.

There are many novel devices that take advantage of the form birefringence in surface-relief grating and volume holograms, including polarizing element<sup>2</sup>, wave plate<sup>3</sup>, antireflection coating<sup>4,5</sup>, Fresnel lenses, prism, diffuser<sup>6</sup>, and optical compensator<sup>7</sup>. For the reason of high quality volume holograms are inexpensively and conveniently realized in holographic recording films such as

photopolymer and dichromated gelatin. In general, the high-spatial-frequency grating more realized with the volume hologram than with the surface relief grating. This paper considers the characteristics of volume holograms in optical birefringence. The volume holograms is employed the photopolymer material<sup>8</sup> for the good reason of easy dry processing, good photospeed, wide spectral sensitivity, excellent holographic properties, and long shelf-life commercial products.

This paper is considered electric and dielectric fields is applied to a sinusoidally modulated volume hologram with infinitesimal-layered approximation. The optical birefringence of volume holograms could be described as a function of index modulation of volume hologram. The characteristics of optical birefringence in volume holograms is investigated by the method of the behavior of volume hologram formation, which holographic properties and performance of various formulation are discussed. The angular and wavelength selectivity are presented with a view to evaluate the performance of index modulation and optical birefringence when the volume hologram is to be formed. This crystal axis of volume hologram is also illustrated when the hologram is developed. We found the hologram formation is the best way to present the direction of crystal axis when it is needed and expectable.

## 2. OPTICAL BIREFRINGENCE OF VOLUME HOLOGRAMS

The simple way to understand the optical birefringence of volume holograms can concern the continuity of the electric- and the dielectric-wave vectors at the boundary between two media. A sinusoidal modulated volume hologram can be considered as a infinite stack of infinitesimally thin dielectric layers of varying index modulation.

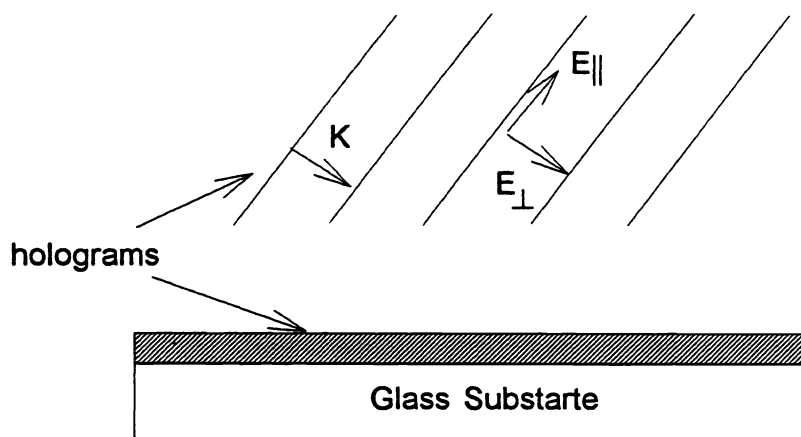


Fig. 1. Slanted volume hologram with sinusoidal index modulation, and the orientation of the normal and tangential electric field directions to the fringe plane.

Consider the interference structure of planar volume hologram, as shown in Fig. 1. It has a sinusoidal modulated permittivity distribution in accordance with the equation

$$\varepsilon = \varepsilon_0 + \varepsilon_1 \cdot \cos(\mathbf{K} \cdot \mathbf{x}) \quad (1)$$

where  $\Lambda$  is the period of the grating,  $\varepsilon_0$  is the average permittivity of hologram,  $\varepsilon_1$  is the modulated permittivity of hologram,  $x$  is the perpendicular direction to the lines of the grating, and  $K$  is the grating vector. The grating vector is normally directed to the fringe plane and is defined as,

$$K = \frac{2\pi}{\Lambda} \quad (2)$$

The electric vector of the incident plane wave is perpendicular to the lines. Then the normal component of the electric displacement  $D$  should be constant along  $x$ . The averaged electric field over a period of the grating is,

$$E_{\perp} = \frac{1}{\Lambda} \int_0^{\Lambda} \frac{D_{\perp}}{\varepsilon} dx = \frac{D_{\perp}}{\sqrt{\varepsilon_0^2 - \varepsilon_1^2}} \quad (3)$$

Then,

$$\varepsilon_{\perp} = \frac{D_{\perp}}{E_{\perp}} = \sqrt{\varepsilon_0^2 - \varepsilon_1^2} \approx \varepsilon_0 - \frac{\varepsilon_1^2}{2\varepsilon_0} \quad (4)$$

The electric vector of the incident wave is parallel to the lines. The electric field  $E$  must keep constant along the direction  $x$ , and the average electrical displacement is

$$\overline{D}_{\parallel} = \frac{1}{\Lambda} \int_0^{\Lambda} D_{\parallel}(x) dx = \frac{E_{\parallel}}{\Lambda} \int_0^{\Lambda} \varepsilon(x) dx = E_{\parallel} \cdot \varepsilon_0 \quad (5)$$

$$\varepsilon_{\parallel} = \frac{\overline{D}_{\parallel}}{E_{\parallel}} = \varepsilon_0 \quad (6)$$

Thus,

$$\Delta\varepsilon = \varepsilon_{\perp} - \varepsilon_{\parallel} = -\frac{(\varepsilon_1)^2}{2\varepsilon_0} \leq 0 \quad (7)$$

That is, the birefringence property in volume holograms can be considered as a negative uniaxial crystal. In terms of the refractive index, Eq. (7) can be described as following form,

$$\Delta n = n_{\perp} - n_{\parallel} = -\frac{n_1^2}{n_0} \quad (8)$$

where  $n_0$  is the average index of hologram. and  $n_1$  is the index modulation during hologram formation.

It is apparent that the birefringence effect in volume holograms is dependent on their index modulation, of which the value will decide many efficiency properties of holograms. It is significant to understand the behavior of optical birefringence in holograms through the investigation of index modulation during hologram formation. So, it is also necessary to investigate the optical birefringence from this point of view in angular and wavelength sensitivity, and other related parameters. The phase changes of hologram is then given by,

$$\Gamma = 2\pi(n_e - n_o) \frac{d}{\lambda} \quad (9)$$

where  $d$  represented the thickness of hologram. It could be given an important approach to realize an application from understanding the characteristics of the phase retardation in volume holograms, such as an application in wave plate, polarization optical element, or optical compensation.

### 3. CHARACTERISTICS ANALYSIS

#### 3.1 Non-slanted holograms

Diffraction characteristics of volume hologram is analyzed and based on the Kogelnik's coupled wave theory<sup>1</sup>. For the simple non-slanted reflection holograms, the index modulation  $n_1$  may be written as following form,

$$\eta = \tanh^2\left(\frac{\pi \cdot \Delta n \cdot d}{\lambda_0 \cdot \cos\theta}\right)$$

$$n_1 = \frac{\lambda_0 \tanh^{-1}(\sqrt{\eta})}{\pi d} \cdot \cos\theta \quad (10)$$

where  $\eta$  is the diffraction efficiency of hologram,  $\lambda_0$  is the wavelength at Bragg condition,  $\theta$  is the incident Bragg angle, and  $d$  is the grating thickness. The performance of reflection holograms, as exhibited by their diffraction efficiency and bandwidth, depends on their index modulation recorded in the films. However, the high index modulation in hologram will achieve a better performance in volume hologram, that is, the better performance in optical birefringence. From equation of index modulation, it is evident that the index modulation of a volume hologram depends on the parameters of wavelength, total efficiency, and incident angle at Bragg condition when a volume phase hologram is characterized as a device of optical birefringence. In other word, the phase retardation of a volume hologram also depends on these above parameters.

#### 3.2 Slanted holograms

For slanted hologram, it is constructed with different angles in reference and object beam. The fringe plane of slanted hologram incline to a angle with surface plane of volume hologram, and the grating vector of hologram is no more directed to surface normal. According to Kogelnik's theory,

the reflection efficiency  $\eta$ , of a slanted volume reflection hologram utilizing a lossless phase grating, may be expressed as

$$\eta = \left[ 1 + \frac{1 - \xi^2 / v^2}{\sinh^2 \sqrt{v^2 - \xi^2}} \right]^{-1}$$

$$v = \frac{\pi \cdot n_1 \cdot d}{\lambda \sqrt{c_r \cdot -c_s}}$$

$$\xi = -\frac{d}{2} \cdot \frac{K \cos(\phi - \theta) - K^2 \lambda / 4\pi n}{c_s}$$

$$K = \frac{2\pi}{\Lambda} = \frac{4\pi n \cos(\phi - \theta_0)}{\lambda_0} \quad (11)$$

where  $c_r = \cos \theta_r$ , and  $c_s = \cos \theta_r - 2 \cos(\phi - \theta_0) \cos \phi$ . The parameters  $c_r, c_s$  is a measure of the exposure angle from the surface normal,  $\lambda$  is the playback wavelength, and  $\theta$  is the playback angle,  $d$  is the film thickness, and  $\xi$  is a detuning factor of the deviation from the Bragg condition ( $\lambda_0, \theta_0$ ). The index modulation of slanted hologram could be obtained by utilizing numerical methods in term of controlling the related parameters ( $\lambda, \theta, \phi$  etc.), i.e. the index modulation of slanted hologram may be illustrated by the characteristics of wavelength and angular selectivity. Therefore, the optical birefringence of a slanted hologram also could be expressed in this way.

### 3.3 The c-axis of the equivalent uniaxial medium<sup>11</sup>

From the equations 1 to 8, the subscripts " $\perp$ " and " $\parallel$ " indicate that the direction of the electric field vector is parallel or perpendicular to the absorbing medium, respectively. The crystal axis of the equivalent uniaxial medium is perpendicular to the slabs for platelet structures, as shown in Fig. 2. Thus the ordinary and extraordinary refractive indices of the composite medium are given by,

$$\begin{aligned} n_o &= n_{\parallel}, \\ n_e &= n_{\perp} \end{aligned} \quad (12)$$

To be evident, an expectable formation in crystal axis can be obtained in slanted volume hologram, which the hologram is formed by a predictable constructed structure. This merit characteristics of volume hologram will provide an important approach to some specific application, as different in conventional hologram application.

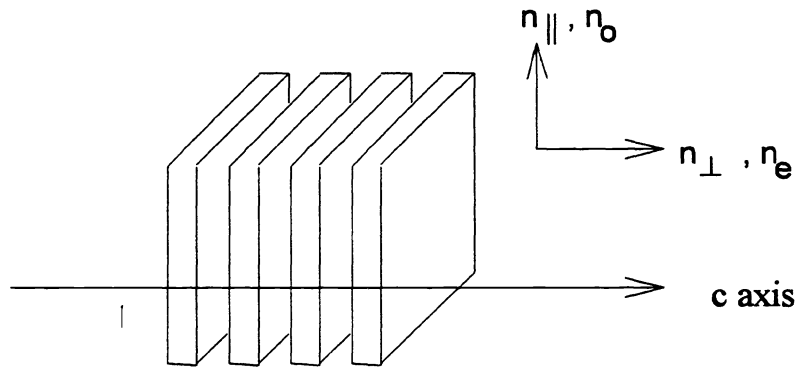


Fig. 2. Schematic drawing of the slab structure,  $n_e$  is presented the orientation of normal to the fringe plane and  $n_o$  is the tangential direction of fringe plane.

#### 4. RESULTS AND DISCUSSION

Simple nonslanted data were analyzed according to Equations (10). Data for reflection photopolymer are shown in Fig. 3-7. The characteristics of index modulation in volume hologram formation is analyzed in term of the parameters of total diffraction efficiency, film thickness, wavelength, and angular deviation. The analysis in Fig. 3A is based on the condition of 10  $\mu\text{m}$  film thickness, and normal incidence. Fig. 3B shows the lower curve distribution, which the film thickness is 20  $\mu\text{m}$ , and the incident Bragg angle is 20 degree. The simple holographic mirror at normal incidence will get higher index modulation when the total diffraction efficiency is high.

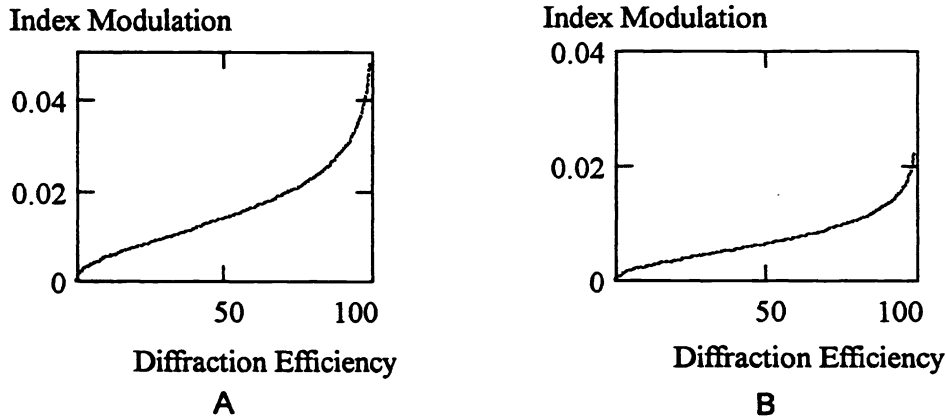


Fig. 3. Index modulation of volume hologram relates with total diffraction efficiency of hologram.

As shown in Fig. 4., the index modulation and optical birefringence is illustrated in term of the film thickness. It is evident to explain that the thicker film could not obtain high index modulation during volume hologram formation. This data is obtained at 20 degree of incident Bragg angle and 100% diffraction efficiency. Therefore, the lower index modulation is low in optical birefringence.

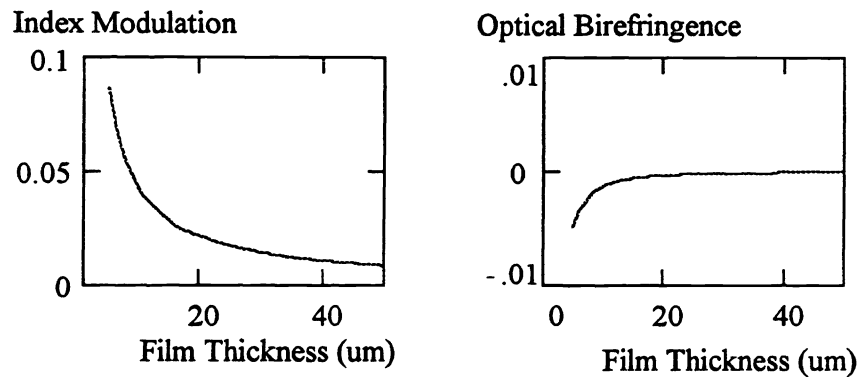


Fig. 4. Index modulation and optical birefringence of volume hologram in terms of film thickness.

The angular selectivity for index modulation is presented by a sinusoidal distribution. The higher index modulation will occur at normal incidence.

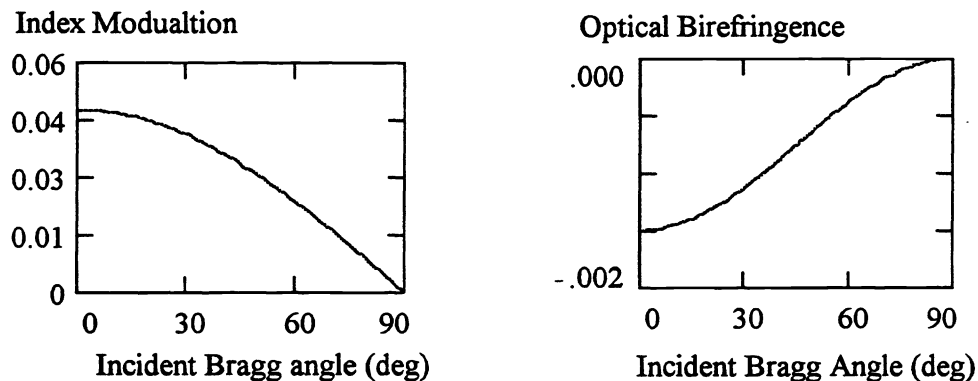


Fig. 5. Index modulation of volume hologram in term of incident Bragg angle.

For wavelength selectivity, the index modulation is a linear distribution with the wavelength of incident beam. The simple holographic mirror can be obtained a very higher index modulation up to 0.075. The data shows that the volume hologram is leading a significant application in longer wavelength region, especially on infrared ray application. As we known, the holographic optical element is also an important design and application in the region of infrared ray. There are many contribution papers have been investigated in the infrared field application.

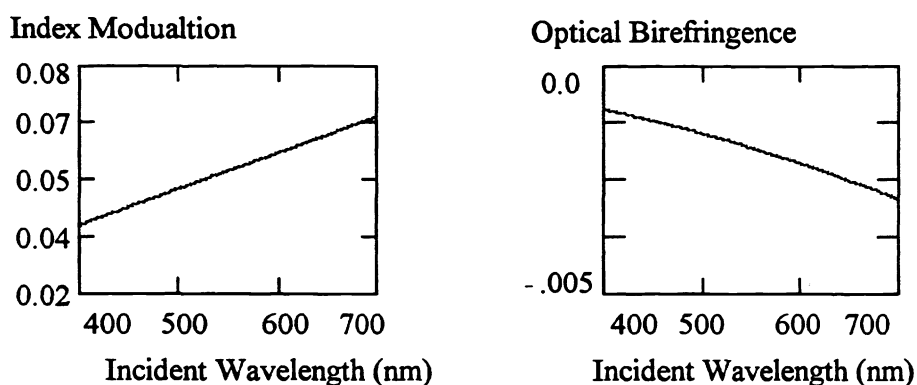


Fig. 6. Index modulation of volume hologram in term of incident wavelength

To achieve the best optical birefringence for volume hologram, the index modulation is dominant factor in the hologram formation. All of the analyzed data are based on the theoretical simulation. In practice, the processing of photopolymer will enhance the index modulation when grating-formed photopolymer is treated after the processing of UV curing and bake. However, there are still a lot of tricks in actual volume hologram formation which the recording film is utilized by the photopolymer. If the rectangular index distribution of volume photopolymer hologram could be obtained, the volume hologram can realize a platelet structure, and the optical birefringence will get better than sinusoidal index structure.

In application, the negative birefringence film compensators using holographic method have some merit over other compensators used in LCD, which the compensation is based on the equally phase retardation with opposite sign, *i.e.*  $\Delta n_{LC} \cdot L_{LC} = \Delta n_C \cdot L_C$ , where the subscript of "LC" means liquid crystal, and "C" is compensator. Since only the periodic structure of a hologram is used, the diffraction efficiency actually does not concern this compensator. As refractive index modulation is proportional to exposure intensity, by which birefringence is controlled, it is easy to mass produce by holograms copying. Over-exposure energy induces the high order of the sinusoidal grating that may affect the structure of holographic compensator.

## 5. SUMMARY

Optical birefringence occurs naturally in crystals or materials that consist of periodic arrays of long or nonspherical molecules. It is easy to create the negative or positive birefringence film using holographic method. It is also significant to emphasis that the holographic method will provide a convenient approach to develop a expectable crystal axis of birefringence film. Due to the important merit in hologram, there is an important application in LCD technology which the positive uniaxial structure of field on state in NW TN-LCD is required a negative uniaxial structure to compensate the display at large view angle. As we know, the volume hologram, a typical negative birefringence film, is formed by an artificially periodic layered medium, which it is easy to use holographic exposure to make these type of hologram. Not only a negative uniaxial structure can make by holographic methods, but also the biaxial structure with desirable crystal axis is convenient to be done. In this paper, we proposed a comprehensive analysis that the analyzed theory is based on the Kogelnik's



couple wave theory. It provides a valuable approach and application that based on the holographic method.

## 6. REFERENCES

1. M. Born and E. Wolf, *Principles of Optics*, Pergamon, New York, 1987, p. 706
2. F. A. Sattarov, "Polarizing properties of thick-film hologram grating", *Opt. Spectrosc.*, **47**, 422, 1979
3. L. H. Cescato, E. Gluch, and N. Streible, "Holographic quarter-wave plates", *Appl. Opt.* **29**, 3286, 1990
4. D. H. Raguin and G. M. Morris, "Antireflection structured surfaces for the infrared spectral region", *Appl. Opt.* **32**, 1154, 1993
5. D. L. Brundrett, E. N. Glytsis, and T. K. Gaylord, "Homogeneous-layer models for high-spatial-frequency dielectric surface-relief-grating: conical diffraction and antireflection designs", *Appl. Opt.* **33**, 2695, 1994
6. J. M. Tedesco, L. A. K. Brady and W. S. Colburn, "Holographic diffuser for LCD backlights and projection screens", *SID 93 DIGEST*, 29, 1993
7. J. Yoo and H. Shieh, "Novel compensator with grating structure for twisted nematic liquid crystal display application", *IDRC 94*, 217, 1994
8. A. M. Weber, W. K. Smothers, T. J. Trout and D. J. Mickish, "Hologram recording in Du Pont's new photopolymer material", *SPIE v.1212*, 14, 1990
9. H. Kogelnik, "Coupled wave theory for thick hologram grating", *Bell Syst. Tech. J.*, **48**, 2909, 1969
10. G. Campbell and R. K. Kostuk, "Effective-medium theory of sinusoidally modulated volume holograms", *J. Opt. Soc. Am. A* **12**, 1113, 1995
11. P. Yeh, "Generalized model for wire grid polarizers", *SPIE v. 307*, 13, 1981