# Energy-Bandwidth Efficiency Tradeoff in MIMO Multi-Hop Wireless Networks

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Abstract—This paper considers a MIMO multi-hop network and analyzes the relationship between its energy consumption and bandwidth efficiency. Its minimum energy consumption is formulated as an optimization problem. By taking both transmit antennas (TAs) and receive antennas (RAs) into consideration, the energy-bandwidth efficiency tradeoff in the networks is investigated. Moreover, the minimum energy of an equally-spaced relaying strategy is investigated for various numbers of antennas. In addition, the minimum energy over all possible antenna pairs is derived. Finally, the effect of the number of hops on the energy-bandwidth efficiency tradeoff is considered. For a fixed antenna pair, the minimum energy over all possible rates and hop numbers are obtained. Generally, the routes with more hops minimize the energy consumption in the low effective rate region. On the other hand, in the high effective rate region, the routes with fewer hops minimize the energy consumption.

Index Terms—Communication networks, MIMO, Communication systems

## I. Introduction

THE DEMAND for portable devices with wireless real-time and high-rate multimedia services has been growing rapidly in this decade. Recent research shows that multipleinput and multiple-output (MIMO) communication systems are one of many solutions to achieving high data rate for these applications. Besides, more efficient bandwidth usage than before is also constantly an important factor to consider in order to fulfill the required transmission rate. However, compared to a single-input and single output (SISO) system, a major disadvantage of the MIMO communication system is that it needs more radio-frequency (RF) circuitry, digitalto-analog converters (DACs) and analog-to-digital converters (ADCs) which accordingly consume more power. Therefore, for portable devices, it is crucial to understand the tradeoff between the efficient bandwidth usage and the available battery energy, in MIMO multi-hop networks. As such, in this work, the relationship between bandwidth efficiency and energy is investigated.

For a long-range wireless communication system, the communication between a cellular phone and its base station, the power consumption at the receiver is usually much less than the energy at the transmitter, because the power amplifier (PA) at the transmitter needs to output very high power to send

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the signal to a distant receiver. This is a very high portion of the consumed power in such a system. Therefore, the energy consumed at the receiver is usually ignored for long-range communication systems. However, for wireless ad hoc networks, the distance from the source to the destination via relay nodes is relatively short. As such, the power consumption of the transmitter might not be as dominant as that in the single-hop case. The energy consumed at the receiver then becomes an important factor in the bandwidth-energy efficiency optimization problem. Thus, it is important to explore the tradeoff between power consumption and the data rate when the multi-hop transmissions occur and receiver processing energy is incorporated in the total energy consumption.

Aspects of the energy-bandwidth efficiency tradeoff problem have been investigated previously. In [1], the energybandwidth tradeoff under some optimal signaling methods is considered. Minimizing the transmission energy with different packet intervals is described in [2]. In [3], [4] and [5], the authors take the transmission and signal processing energy into consideration but the performance measures do not include the bandwidth efficiency or the end-to-end data rate. In [6] and [7], the authors discuss the energy-bandwidth efficiency and end-to-end throughput in linear multi-hop networks. The case of equally-spaced multi-hop networks is explored in [8] and [9]. Subsequently, works, [10] and [11], show the energybandwidth efficiency tradeoff by considering transmission energy, signal processing, and end-to-end throughput at the same time. However, those mentioned works mainly discuss these issues in multi-hop single-antenna networks. For MIMO systems, the works in [12], [13] and [14] characterize the system capacity, and the work in [15] provides the system throughput in a Gaussian broadcast channel. Moreover, the relationship between the transmission power and bandwidth in the MIMO channel is investigated in [16] without considering the signal processing energy.

In this paper, we optimize the overall system performance by taking the variables, the end-to-end throughput and energy consumption in a MIMO multi-hop network as well as the transmission and receiving signal processing energy, into consideration. The energy-bandwidth efficiency optimization problem also includes the factor of different numbers of transmit and receive antennas. Because of the downside of the increased processing energy for multiple receive antennas, we also derive analytical results for the optimization of antenna numbers for minimum energy consumption under the capacity-achieving end-to-end rate. By considering the number of hops and the end-to-end rate jointly, the optimal number of hops as well as the end-to-end data rate can be obtained.

The rest of this paper is organized as follows. The system model is described in Section II. In Section III the performance metrics of energy and bandwidth efficiency are described and optimized with respect to the transmitted energy on different hops. We also derive the energy-bandwidth efficiency tradeoff as well. The minimum total energy consumption for varying number of antennas is derived in Section IV. In this section, we also derive the optimal number of hops and the optimal end-to-end rate for the minimum energy consumption. By presenting the numerical results, the effects of the number of antennas on the tradeoff are discussed and summarized in Section V. Finally, we conclude in Section VI.

#### II. SYSTEM MODEL

In this paper, a network which has k-1 relay nodes between a source node and a destination node is considered. All nodes operate in half-duplex mode. Fig. 1 shows the system architecture for the MIMO multi-hop wireless network. Node i has  $M_{i+1}$  transmit antennas (TAs) and  $N_i$  receiver antennas (RAs). That is, there is a  $M_i \times N_i$  multiple-input and multiple-output (MIMO) channel between node i-1 and node i. The input-output relationship of the i-th MIMO channel can be represented as

$$\mathbf{Y}_i = \mathbf{H}_i \mathbf{X}_i + \mathbf{n}_i, \tag{1}$$

where  $\mathbf{H}_i$  is the channel matrix,  $\mathbf{X}_i$  the transmitted data on TAs,  $\mathbf{Y}_i$  the received data on RAs, and  $\mathbf{n}_i$  is the additive white Gaussian noise (AWGN). The distance between node i-1 and node i is  $d_i$ , while the end-to-end distance is  $d_e$ . The distance ratio  $\alpha_i$  of each node is defined as the proportion of its hop node distance to end-to-end distance,  $\alpha_i = \frac{d_i}{d_e}$ . Each relay node is assumed to be in the transmitter's far-field region. Hence, the relationship between the received power  $P_r$  and the transmitted power  $P_t$  is given by

$$P_r = \frac{\beta}{d^{\eta}} P_t, \ d > 1, \tag{2}$$

where  $\beta$  is related to the antenna properties,  $\eta$  is the path-loss parameter, and d is the distance between the transmitter and receiver. In this paper,  $\beta$  is assumed to be one for simplicity.

For each MIMO channel, there is a transmission rate limitation for reliable communication, namely the channel capacity. The effective end-to-end rate can be represented in the form of a combination of all hops' transmission rates. A higher effective end-to-end rate is possible with higher energy consumption. This tradeoff between the effective transmission rate and the total energy consumption will be investigated in the following section.

# III. PROBLEM FORMULATION

This section will provide the end-to-end rate and the total energy consumption per information bit for evaluating the system performance. The minimization of the total energy consumption for a given effective end-to-end rate will be found. One can obtain the energy-bandwidth efficiency trade-off through the solution of an optimization problem.

A. Channel Capacity of a MIMO Fading Channel and Endto-end Rate

The channel capacity of each MIMO channel will depend on the knowledge of the channel condition at the transmitter, receiver or both. Channel information knowledge at the transmitter will affect the adopted power allocation strategy. Indirectly, it will affect the channel capacity as well. If each transmitter can obtain the full channel state information (CSI) of the MIMO channel, an optimal power allocation strategy, namely water-filling strategy, can be adopted among the transmitter antennas. However, obtaining full CSI at either the transmitter or receiver in practice is not possible due to estimation and quantization noises. On the other hand, if the receiver is required to feedback the estimated CSI to the transmitter, then the bandwidth efficiency will be reduced accordingly. Moreover, since the feedback CSI is generally noisy, it may cause error propagation problems in practice, even when optimized water-filling power allocation strategies in the transmitter are adopted. Due to these concerns, this work only considers the system model in which transmitter does not know CSI. We assume that the channel is sufficiently random which means that the elements of channel matrix  $H_i$ are independent naturally. In this scenario, the optimal power allocation strategy for the i-th hop is to use equal amounts of energy on all TAs [17]. Therefore, for the i-th  $M_i$ -by- $N_i$  MIMO fading channel with full rank, the upper bound of channel capacity for the real dimension (coming from combining equation 8.15 and 8.17 in [17]) when transmitters do not know the CSI is given by

$$C(\gamma_i) \le \frac{n_{min}}{2} \log_2(1 + \frac{2\gamma_i}{M_i n_{min}} \sum_{j=1}^{n_{min}} \hat{\lambda}_j^2),$$
 (3)

where  $n_{min} = \min(M_i, N_i)$  represents the channel rank,  $\gamma_i$  is the ratio of received energy per channel use to noise power spectral density, and  $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq ... \geq \hat{\lambda}_{n_{min}}$  are the ordered singular values of the channel matrix  $\mathbf{H}_i$ . Furthermore, the expectation value of  $\sum_{j=1}^{n_{min}} \hat{\lambda}_j^2$  can be represented as

$$E\{\sum_{j=1}^{n_{min}} \hat{\lambda}_j^2\} = E\{Tr\{\mathbf{H}_i \mathbf{H}_i^T\}\} = E\{\sum_{m_i, n_i} h_{m_i, n_i}^2\}.$$
 (4)

Note that  $\mathbf{H}_i^T$  denotes the transpose matrix of  $\mathbf{H}_i$ . For further simplifying (3), we assume that the term of  $\sum_{j=1}^{n_{min}} \hat{\lambda}_j^2$  in (3) will be close to its expectation value as shown in (4).

As mentioned, we consider that the channel matrix is sufficiently random, and assume the normalized  $E\{h_{m_i,n_i}^2\}=1$ , where  $h_{m_i,n_i}$  is the element of channel matrix  $\mathbf{H}_i$ . Equality holds in (3) when the channel matrix  $\mathbf{H}_i$  is sufficiently random and statistically well-conditioned [17]. As such, with the assumption of sufficiently random and statistically well-conditioned channel matrix, the channel capacity becomes

$$C(\gamma_i) \approx \frac{n_{min}}{2} \log_2(1 + \frac{2\gamma_i N_i}{n_{min}}).$$
 (5)

The rate of the *i*-th MIMO channel within nodes i-1 and i is given by  $R_i$ . Then, the end-to-end rate [8][9] (which is named as the effective rate in [11]) is given by

$$R_e = \frac{1}{\sum_{i=1}^k R_i^{-1}}. (6)$$

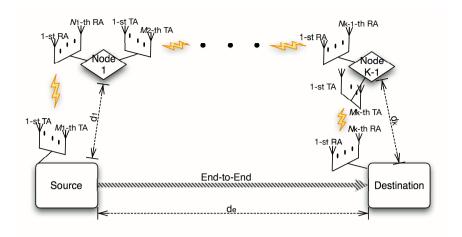


Fig. 1. The system architecture for the MIMO multi-hop wireless network

For reliable communications, the transmission rate  $R_i$  for the i-th MIMO channel should not be larger than the capacity  $C(\gamma_i)$ . We assume that the capacity-achieving codes are applied at each transmitter. As such, the maximum end-to-end rate corresponding to a set of received energy-to-noise ratios (i.e.,  $\gamma_i$ , for i=1,...,k) will be

$$R_e = \frac{1}{\sum_{i=1}^k (C(\gamma_i))^{-1}}.$$
 (7)

# B. Total Energy Consumption

The energy consumption  $E_i$  for the i-th MIMO channel consists of transmission energy consumption ( $E_{t,i}$  per coded symbol on each TA) and receiver processing energy consumption ( $E_{p,i}$  per coded symbol on each RA). Therefore, the energy consumption per information bit for the i-th  $M_i$ -by- $N_i$  MIMO channel is

$$E_{i} = \frac{M_{i}E_{t,i} + N_{i}E_{p,i}}{R_{i}}.$$
 (8)

Further, let the total energy consumption be represented as

$$E_{tot} = \sum_{i=1}^{k} E_i. (9)$$

The total energy consumption will be normalized by the noise power spectral density  $N_0$ . Moreover, in order to compare the energy consumption with the results of single-link reliable communications, the total energy consumption will be normalized by the end-to-end propagation loss as well. Hence, the metric for evaluating the energy consumption is given by

$$\Gamma = \frac{E_{tot}}{N_0 d_o^{\eta}}.$$
(10)

By substituting (8) and (9) into (10), the energy metric can be rewritten as

$$\Gamma = \sum_{i=1}^{k} \frac{M_i E_{t,i} + N_i E_{p,i}}{N_0 d_e^{\eta} R_i}$$
 (11)

$$=\sum_{i=1}^{k}\frac{M_{i}\alpha_{i}^{\eta}\gamma_{i}+N_{i}\gamma_{p}}{R_{i}},$$
(12)

where  $\gamma_i \equiv \frac{E_{t,i}}{N_0 d_i^{\eta}}$  and  $\gamma_p \equiv \frac{E_{p,i}}{N_0 d_e^{\eta}}$ . Note that  $\gamma_i$  is the ratio of received energy per channel use to noise power spectral density.

# C. Optimization Problem

Before formulating the optimization problem, a set of received energy-to-noise ratios  $(\gamma_1, \gamma_2, ..., \gamma_k)$  can be one-to-one mapped to a set of capacity-achieving transmission rates  $(C(\gamma_1), ..., C(\gamma_k))$ . Therefore, a set of rates which minimizes the energy metric under a given end-to-end transmission rate can be formulated as follows,

$$\begin{cases} \min_{(C(\gamma_1),\dots,C(\gamma_k))} \sum_{i=1}^k \frac{M_i \alpha_i^{\eta} \gamma_i + N_i \gamma_p}{C(\gamma_i)} \\ \text{subject to: } R_e^{-1} = \sum_{i=1}^k (C(\gamma_i))^{-1}. \end{cases}$$
(13)

One can apply the method of Lagrange multiplier to find the optimal solution of the above problem. The corresponding Lagrange function is defined as

$$L(\lambda) = \sum_{i=1}^{k} \frac{M_i \alpha_i^{\eta} \gamma_i + N_i \gamma_p}{C(\gamma_i)} - \lambda (R_e^{-1} - \sum_{i=1}^{k} (C(\gamma_i))^{-1}).$$
(14)

where  $\lambda$  is Lagrange multiplier. By setting  $\frac{\nabla L(\lambda)}{\nabla C(\gamma_i)}$  to zero (i.e., the Karush-Kuhn-Tucker (KKT) condition), one can obtain

$$\lambda = M_i \alpha_i^{\eta} (C(\gamma_i) \frac{\partial \gamma_i}{\partial C(\gamma_i)} - \gamma_i) - N_i \gamma_p.$$
 (15)

By substituting (5) into (15), (15) can be expressed as

$$\lambda = M_i \alpha_i^{\eta} \left\{ \frac{n_{min}}{2N_i} \left( 1 - 2^{\frac{2C(\gamma_i)}{n_{min}}} \right) + \frac{C(\gamma_i) \ln 2}{N_i} 2^{\frac{2C(\gamma_i)}{n_{min}}} \right) \right\} - N_i \gamma_p. \tag{16}$$

Further, (16) can be rewritten as

$$\frac{2N_i}{M_i \alpha_i^{\eta} n_{min} e} (\lambda + N_i \gamma_p) - \frac{1}{e} = (\frac{2C(\gamma_i) \ln 2}{n_{min}} - 1) e^{(\frac{2C(\gamma_i) \ln 2}{n_{min}} - 1)}.$$
(17)

Then, an useful lemma is introduced for solving (17).

• Lemma A: Lambert  $\omega$ -function [18] The Lambert  $\omega$ -function is the inverse function of

$$f(X) = Xe^X,$$

where X is any complex number. Moreover, if there is another equation in the form of (namely Lambert's transcendental equation)

$$\ln X = \rho X^{\theta}$$

the solution is given by

$$X = e^{-\frac{\omega\{-\theta\rho\}}{\theta}}.$$

By using the Lambert  $\omega$ -function in (17), the channel capacity  $C(\gamma_i)$  in the form of  $\lambda$  is given by

$$C(\gamma_i) = \frac{n_{min}}{2 \ln 2} \left(1 + \omega \left\{ \frac{2N_i}{M_i \alpha_i^{\eta} n_{min} e} (\lambda + N_i \gamma_p) - \frac{1}{e} \right\} \right), (18)$$

where  $\omega\{.\}$  is the principal branch of the Lambert  $\omega$ -function. Hence, all the performance metrics in the form of  $\lambda$  are listed as in (19), where  $S_{i,\lambda} = \frac{2N_i}{M_i\alpha_i^{\eta}n_{min}e}(\lambda+N_i\gamma_p)-\frac{1}{e}$ . Finally, by varying  $\lambda$ , one can obtain the tradeoff between the energy metric  $\Gamma$  and the effective rate  $R_e$ .

#### IV. PERFORMANCE EVALUATION

The performances of MIMO multi-hop wireless networks under the equally-spaced hops will be investigated. We can obtain an analytical expression for the performance as a function of the number of antennas. More number of antennas generally leads to higher bandwidth efficiency, while increasing the number of antennas also increases the energy consumption. Therefore, we derive the optimal number of antennas (both TAs and RAs) for the lowest energy consumption for the MIMO channel. The analyses provide the tradeoff between energy and bandwidth efficiency. In fact, the number of antennas affects the rank of the MIMO channel and the channel capacity depends on the channel rank. Hence, the following discussion will be divided the energy metric which relates with the channel capacity into two parts for each hop-allocation condition,  $N_i \leq M_i$  and  $M_i \leq N_i$ . Then,  $n_{min}$  of (5) is  $N_i$ when  $N_i \leq M_i$ , while  $n_{min}$  is  $M_i$  when  $M_i \leq N_i$ .

First, the equally-spaced hops are given by  $\alpha_1=\ldots=\alpha_k=\alpha$ . We assume that all hops are the same which means they all have the same number of TAs and RAs. As such, it will result in  $C(\gamma_1)=\ldots=C(\gamma_k)=C(\gamma)$ . Note that the following discussion will ignore the suffix i on  $M_i$ ,  $N_i$ , and  $\alpha_i$ . Therefore, the end-to-end rate can be represented as

$$R_e = \frac{C(\gamma)}{k}. (20)$$

By substituting (5) and (20) into (12), the energy metric can be rewritten as

$$\Gamma(M,N) = \frac{M\alpha^{\eta}(\frac{n_{min}}{2N})(e^{\frac{2kR_e \ln 2}{n_{min}}} - 1) + N\gamma_p}{R_e}.$$
 (21)

In this scenario, (21) represents the energy consumption of the capacity-achieving effective rate. In the following discussion, we will optimize numbers of TAs M and RAs N to find the minimum energy consumption with the maximum effective rate.

1) When  $N \leq M$ : When  $n_{min} = N$ , the energy metric in (21) will become

$$\Gamma(M,N)|_{N \le M} = \frac{M\alpha^{\eta} (e^{\frac{2kR_e \ln 2}{N}} - 1) + 2N\gamma_p}{2R_e}.$$
 (22)

According to (22), in the low  $R_e$  region, the term of  $2N\gamma_p$  dominates the energy metric and this results in that increasing number of RAs N is expected to increase the energy consumption. On the other hand, in the high  $R_e$  region, the term of  $e^{\frac{2kR_e\ln 2}{N}}$  dominates the energy metric. As such, increasing N will decrease the energy consumption. The simulation results in Section V will exhibit this phenomenon. As for the number of TAs M, increasing M will always increase the energy consumption.

For finding the minimum energy consumption, by treating M and N as continuous variables, the derivatives of  $\Gamma(M,N)|_{N\leq M}$  concerning M and N, respectively, are

$$\frac{\partial \Gamma(M,N)}{\partial M} = \frac{\alpha^{\eta} \left(e^{\frac{2kR_e \ln 2}{N}} - 1\right)}{2R_e}$$

$$\frac{\partial \Gamma(M,N)}{\partial N} = -\frac{M\alpha^{\eta}k \ln 2}{N^2} e^{\frac{2kR_e \ln 2}{N}} + \frac{\gamma_p}{R_e}.$$
(23)

Note that there are two natural restrictions of  $M \geq 1$  and  $N \geq 1$ . As such, for M, since the energy metric (22) is a linear increasing function and  $M \geq N \geq 1$ , the minimum value of energy metric is located at  $M^* = N$ . Then, setting (23) to zero for N will yield  $N^*$ . Hence, the optimal  $M^*$  and  $N^*$  (when the other variable is fixed) is

$$\begin{cases} M^* = N \\ N^* = \left(\frac{M\alpha^{\eta} k R_e \ln 2}{\gamma_p} e^{2\omega \left\{ \left(\frac{\gamma_p k R_e \ln 2}{M\alpha^{\eta}}\right)^{\frac{1}{2}} \right\} \right)^{\frac{1}{2}}. \end{cases}$$
(24)

By substituting (24) into (22), the energy metric with individual optimal  $N^*$  and  $M^*$  for given M and N can be respectively represented as

$$\Gamma(M, N^*)|_{N^* \le M} = \frac{M\alpha^{\eta}}{2R_e} (e^{2\omega\{Q\}} - 1) + \frac{\gamma_p k \ln 2}{\omega\{Q\}}$$

$$\Gamma(M^*, N)|_{N \le M^*} = \frac{N\alpha^{\eta} (e^{\frac{2kR_e \ln 2}{N}} - 1) + 2N\gamma_p}{2R_e},$$
(25)

where  $Q=(\frac{\gamma_p k R_e \ln 2}{M \alpha^\eta})^{\frac{1}{2}}$ . The above energy metrics  $\Gamma(M,N^*)|_{N^* \leq M}$  and  $\Gamma(M^*,N)|_{N \leq M^*}$  are the minimum energy consumptions with the maximum effective rates for given numbers of TAs and RAs, respectively. That is, for a given antenna number either TA or RA, the energy-bandwidth efficiency tradeoff is bounded by (25).

Moreover, for an arbitrary antenna pair when  $N \leq M$ , setting the derivatives of  $\Gamma(M^*,N)$  concerning N to zero yields

$$M^* = N^* = \frac{2kR_e \ln 2}{\omega(\frac{2\gamma_p}{\alpha^{\eta_e}} - \frac{1}{e}) + 1}.$$
 (26)

Then, by substituting (26) into (22), the energy metric with the joint optimal  $M^*$  and  $N^*$  can be obtained by

$$\Gamma(M^*,N^*)|_{N^* \le M^*} = \frac{\alpha^{\eta} k \ln 2(e^{\omega \{\frac{2\gamma_p}{\alpha^{\eta}_e} - \frac{1}{e}\} + 1} - 1) + 2k\gamma_p \ln 2}{\omega \{\frac{2\gamma_p}{\alpha^{\eta}_e} - \frac{1}{e}\} + 1} (27)$$

$$\begin{cases} \text{Channel Capacity: } C(\gamma_{i}) = \frac{n_{min}}{2 \ln 2} (1 + \omega \{S_{i,\lambda}\}) \\ \text{End-to-End Rate: } R_{e} = \frac{n_{min}}{2 \ln 2} (\sum_{i=1}^{k} \frac{1}{1 + \omega \{S_{i,\lambda}\}})^{-1} \\ \text{Energy Metric: } \Gamma = \ln 2 \sum_{i=1}^{k} \frac{M_{i} \alpha_{i}^{\eta} n_{min} (e^{1 + \omega \{S_{i,\lambda}\}} - 1) + 2N_{i}^{2} \gamma_{p}}{N_{i} n_{min} (1 + \omega \{S_{i,\lambda}\})}, \end{cases}$$
(19)

2) When  $M \leq N$ : In this condition,  $n_{min}$  is equivalent to M. Therefore, the energy metric with capacity-achieving effective rate in (21) becomes

$$\Gamma(M,N)|_{M \le N} = \frac{M^2 \alpha^{\eta} \left(e^{\frac{2kR_e \ln 2}{M}} - 1\right) + 2N^2 \gamma_p}{2NR_e}.$$
 (28)

Under the condition of  $N \leq M$ , increasing the number of RAs N will increase the energy consumption in the low  $R_e$  region, while increasing N in the high  $R_e$  region results in decreased energy consumption. Furthermore, for obtaining the minimum energy consumption, the derivatives of  $\Gamma(M,N)|_{M\leq N}$  concerning M and N are

$$\frac{\partial\Gamma(M,N)}{\partial M} = \frac{\left[(2M\alpha^{\eta} - 2\alpha^{\eta}kR_e \ln 2)e^{\frac{2kR_e \ln 2}{M}} - 2M\alpha^{\eta}\right]}{2NR_e} 
\frac{\partial\Gamma(M,N)}{\partial N} = \frac{4R_e N^2 \gamma_p - 2R_e M^2 \alpha^{\eta}(e^{\frac{2kR_e \ln 2}{M}} - 1)}{(2NR_e)^2}.$$
(29)

Hence, by setting (29) to zero, one can obtain the optimal  $M^*$  and  $N^*$  based on Lemma A as

$$\begin{cases}
M^* = \frac{2kR_e \ln 2}{2+\omega\{\frac{-2}{e^2}\}} \\
N^* = \left[\frac{M^2\alpha^{\eta}(e^{\frac{2kR_e \ln 2}{M}} - 1)}{2\gamma_p}\right]^{\frac{1}{2}}.
\end{cases}$$
(30)

Further, the energy metric with individual optimal  $M^*$  and  $N^*$ , respectively, are

$$\Gamma(M^*, N)|_{M^* \le N} = \frac{(\frac{2kR_e \ln 2}{2+\omega\{\frac{-2}{e^2}\}})^2 \alpha^{\eta} (e^{2+\omega\{\frac{-2}{e^2}\}} - 1) + 2N^2 \gamma_p}{2NR_e}$$
(31)

$$\Gamma(M,N^*)|_{M \leq N^*} = \frac{M[2\alpha^{\eta}\gamma_p(e^{\frac{2kR_e \ln 2}{M}}-1)]^{\frac{1}{2}}}{R_e},$$

where  $M^*$  and  $N^*$  are defined in (30). Hence, (31) is the minimum energy consumption which is also a performance bound of the energy-bandwidth efficiency tradeoff for an arbitrary antenna pair when  $M \leq N$ .

On the other hand, substituting  $M^*$  into M of  $N^*$  yields the joint optimal  $M^*$  and  $N^*$  for the energy metric. The energy metric with the joint optimal solution is represented as

$$\Gamma(M^*, N^*)|_{M^* \le N^*} = \frac{2\sqrt{2}k \ln 2}{2 + \omega\{\frac{-2}{e^2}\}} \sqrt{\alpha^{\eta} \gamma_p(e^{2 + \omega\{\frac{-2}{e^2}\}} - 1)}.$$

Besides, for satisfying the initial condition of  $M \leq N$ , the optimal  $M^*$  should be less or equal to the optimal  $N^*$ . Therefore, by substituting (30) into  $M^* \leq N^*$ , one can obtain that the energy metric with joint optimal solution in (32) is subject to

$$\gamma_p \le \frac{\alpha^{\eta} \left(e^{2+\omega\left\{\frac{-2}{e^2}\right\}} - 1\right)}{2}.$$
 (33)

Finally, by comparing (27) with (32), the energy metric with joint optimization whenever the relationship of M and N is

$$\Gamma(M^*,N^*) = min\{\Gamma(M^*,N^*)|_{N^* < M^*},\Gamma(M^*,N^*)|_{M^* < N^*}\}$$
 (34)

Consequently, according to the detailed derivation in Appendix A, the energy metric with joint optimization is

$$\Gamma(M^*,N^*) = \begin{cases} \Gamma(M^*,N^*)|_{N^* \leq M^*}, \text{ as shown in (27),} \\ \text{when } \gamma_p > \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2} \\ \Gamma(M^*,N^*)|_{N^* \leq M^*} = \Gamma(M^*,N^*)|_{M^* \leq N^*}, \\ \text{as shown in (27) and (32),} \\ \text{when } \gamma_p = \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2} \\ \Gamma(M^*,N^*)|_{M^* \leq N^*}, \text{ as shown in (32),} \\ \text{when } \gamma_p < \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}, \end{cases}$$

3) Optimization of the number of hops: So far, previous discussions are under the given number of hops. Further, this section explores the energy-bandwidth efficiency tradeoff with the optimal number of hops and the optimal end-to-end rate. By setting the derivative of (21) concerning the number of hops k to zero, the optimal number of hops can be obtained

$$k^* = \frac{n_{min}}{2R_e \ln 2} (\omega \{ \frac{-\eta}{e^{\eta}} \} + \eta).$$
 (36)

By substituting (36) into (21), the energy metric with the optimal  $k^*$  is

$$\Gamma(k^*, R_e) = \frac{PMR_e^{\eta - 1}}{Nn_{min}^{\eta - 1}} + \frac{\gamma_p N}{R_e},\tag{37}$$

where 
$$P \equiv [\frac{e^{\omega\{\frac{-\eta}{e^\eta}\} + \eta} - 1}{2}][(\frac{2\ln 2}{\omega\{\frac{-\eta}{e^\eta}\} + \eta})^\eta].$$

Next, setting the derivate of (37) respect to  $R_e$  equal to zero yields the optimal end-to-end rate  $R_e^*$  as

$$R_e^* = \left(\frac{\gamma_p N^2 n_{min}^{\eta - 1}}{P(\eta - 1)M}\right)^{\frac{1}{\eta}}.$$
 (38)

Hence, by substituting the optimal  $R_e^*$  in (38) into the optimal  $k^*$  in (36), the optimal number of hops  $k^*$  can be rewritten

$$k^* = \left(\frac{M(e^{\omega \left\{\frac{-\eta}{e^{\eta}}\right\} + \eta} - 1)}{2N^2 n_{min}^{\eta - 1}}\right)^{\frac{1}{\eta}} \left(\frac{\gamma_p}{\eta - 1}\right)^{\frac{-1}{\eta}}$$
(39)

$$= \left(\frac{M(e^{\omega\{\frac{-\eta}{e^{\eta}}\}+\eta}-1)}{2N^2 n_{min}^{\eta-1}}\right)^{\frac{1}{\eta}} \left(\frac{E_{p,i}}{N_0(\eta-1)}\right)^{\frac{-1}{\eta}} d_e. \tag{40}$$

Then, the energy metric in (21) with the optimal  $k^*$  and  $R_e^*$ 

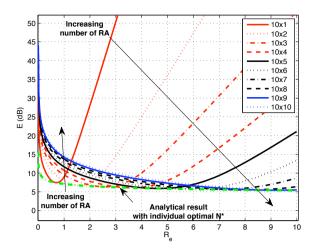


Fig. 2. The energy-bandwidth characteristic of 4 equally-spaced hops with a fixed number of TA and different numbers of RAs when  $N \leq M$ , and the analytical result in (25).

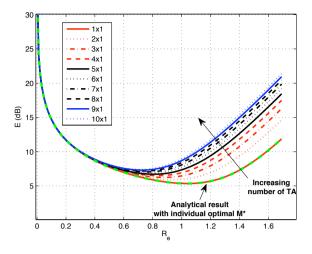


Fig. 3. The energy-bandwidth characteristic of 4 equally-spaced hops with different numbers of TAs and a fixed number of RA when  $N \leq M$ , and the analytical result in (25).

can be represented as

$$\begin{split} &\Gamma(k^*,R_e^*)\\ &=\frac{M\alpha^{\eta}(\frac{n_{min}}{2N})(e^{\frac{2k^*R_e^*\ln 2}{n_{min}}}-1)+N\gamma_p}{R_e^*}\\ &=\{\frac{MP}{Nn_{min}^{\eta-1}}(\frac{N^2n_{min}^{\eta-1}}{P(\eta-1)M})^{\frac{\eta-1}{\eta}}+N(\frac{N^2n_{min}^{\eta-1}}{P(\eta-1)M})^{\frac{-1}{\eta}}\}\\ &\times\gamma_p^{\frac{\eta-1}{\eta}}\\ &=\{\frac{MP}{Nn_{min}^{\eta-1}}(\frac{N^2n_{min}^{\eta-1}}{P(\eta-1)M})^{\frac{\eta-1}{\eta}}+N(\frac{N^2n_{min}^{\eta-1}}{P(\eta-1)M})^{\frac{-1}{\eta}}\}\\ &=\{\frac{MP}{Nn_{min}^{\eta-1}}(\frac{N^2n_{min}^{\eta-1}}{P(\eta-1)M})^{\frac{\eta-1}{\eta}}+N(\frac{N^2n_{min}^{\eta-1}}{P(\eta-1)M})^{\frac{-1}{\eta}}\}\\ &\times(\frac{E_{p,i}}{N_0})^{\frac{\eta-1}{\eta}}(d_e)^{1-\eta}. \end{split} \tag{43}$$

From the above results, one can obtain

The optimal number of hops  $k^* \propto d_e$  The optimal energy metric  $\Gamma(k^*,R_e^*) \propto {d_e}^{1-\eta}$  (44) The optimal energy metric  $\Gamma(k^*,R_e^*) \propto E_{p,i}^{\frac{\eta-1}{\eta}}$ .

Note that  $d_e$  is the end-to-end distance and  $E_{p,i}$  is the processing energy. Furthermore, according to (10), the total energy consumption  $E_{total}$  (which is the un-normalized energy metric) increases linearly with  $d_e$ .

Besides, if the number of TAs is equivalent to the number of RAs, namely M=N, the energy metric with optimal  $k^*$  and  $R_e^*$  in (43) can be reduced to

$$\Gamma(k^*, R_e^*)|_{M=N}$$

$$= \{P(\frac{1}{P(\eta - 1)})^{\frac{\eta - 1}{\eta}} + (\frac{1}{P(\eta - 1)})^{\frac{-1}{\eta}}\}$$

$$\times (\frac{E_{p,i}}{N_0})^{\frac{\eta - 1}{\eta}} (d_e)^{1-\eta}$$

$$= \frac{\eta}{\eta - 1} (\frac{1}{P(\eta - 1)})^{\frac{-1}{\eta}} (\frac{E_{p,i}}{N_0})^{\frac{\eta - 1}{\eta}} (d_e)^{1-\eta}.$$
(45)

The above result shows that the minimum energy consumptions with the optimal k and  $R_e$ , in the scenario of M=N,

are independent of the antenna numbers. Surely, the proportional relationships in (44) still hold in this scenario.

#### V. NUMERICAL RESULTS

This section provides the numerical results of the tradeoff between energy and bandwidth for various configurations in the number of transmitting and receiving antennas. The adopted system parameters are as follows,

- Path-loss exponent:  $\eta = 4$
- End-to-end distance:  $d_e = 3000 \text{m}$
- Signal processing energy:  $E_{p,i}=0.95\mu\mathrm{J/symbol}$  unless otherwise specified

In the figures, each curve represents the energy-bandwidth efficiency tradeoff for a specific antenna pair. For instance, 1x10 denotes that there are one TA and ten RAs in each MIMO channel.

Fig. 2 shows the tradeoff of energy and bandwidth with a fixed number of TAs and different numbers of RAs when  $N \leq M$ , for the case of *four equally-spaced hops*. As shown in Fig. 2, for low end-to-end transmission rates, increasing number of RAs will result in increasing energy consumption. However, increasing number of RAs will lead to decreasing energy consumption in the high end-to-end rate region. As explained in Section IV-1, this phenomenon comes from that the term of  $2N\gamma_p$  in (22) dominates the energy metric (i.e., (22)) in the low  $R_e$  region, while the term of  $e^{\frac{2kR_e\ln 2}{N}}$  dominates the energy metric in the high  $R_e$  region.

Fig. 3 shows the tradeoff in different numbers of TAs and a fixed number of RAs when  $N \leq M$ . With a fixed energy consumption, Fig. 2 shows that increasing the number of RAs can lead to higher end-to-end transmission rate, while Fig. 3 shows that decreasing the number of TAs can result in higher end-to-end transmission rate. For explaining why more TAs lead to smaller rates as shown in Fig. 3, we can start from (22). In (22), with a fixed energy consumption  $\Gamma(M,N)|_{N\leq M}$  and a given number N of RAs, increasing M will result in decreasing  $R_e$  naturally. The analytical results  $\Gamma(M,N^*)|_{N^*< M}$ 

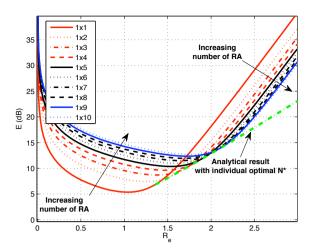


Fig. 4. The energy-bandwidth characteristic of 4 equally-spaced hops with a fixed number of TAs and different numbers of RAs when  $M \leq N$ , and the analytical result in (31).

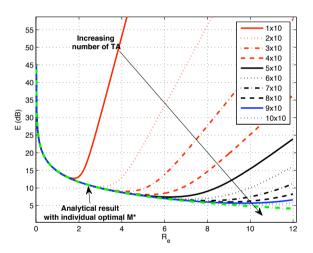


Fig. 5. The energy-bandwidth characteristic of 4 equally-spaced hops with different numbers of TAs and a fixed number of RAs when  $M \leq N$ , and the analytical result in (31).

and  $\Gamma(M^*,N)|_{N\leq M^*}$  in (25) are also shown in Fig. 2 and Fig. 3, respectively. Note that M in (25) for Fig. 2 and N in (25) for Fig. 3 are set to the fixed number of TAs and RAs, namely 10 and 1, respectively. The analytical results  $\Gamma(M,N^*)|_{N^*\leq M}$  and  $\Gamma(M^*,N)|_{N\leq M^*}$  in (25) provide a very good approximation for the energy-bandwidth efficiency tradeoff for Fig. 2 and Fig. 3, respectively.

The following discussion focuses on the numerical results corresponding to  $M \leq N$ . For this scenario, Fig. 4 shows the energy-bandwidth efficiency tradeoff of four equally-spaced hops with a single transmitting antenna and different numbers of RAs. For the tradeoff between energy and rate, Fig. 4 exhibits the same phenomenon as Fig. 2. Besides, for plotting the analytical result in Fig. 4, the term M of the analytical result  $\Gamma(M, N^*)|_{M \leq N^*}$  with individual optimal  $N^*$  (i.e., (31)) is set to a fixed number of TAs which is one here.

On the other hand, Fig. 5 shows the effect of the number of TAs on the energy-bandwidth efficiency tradeoff of four

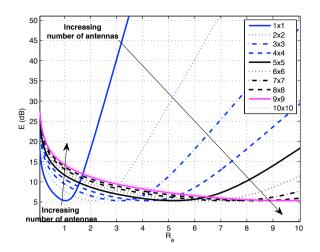


Fig. 6. The energy-bandwidth characteristic of 4 equally-spaced hops with different numbers of TAs when M=N.

TABLE I EFFECTS OF ANTENNA NUMBERS ON THE EFFECTIVE RATES FOR THE FIXED ENERGY CONSUMPTION AND FOR THE FIXED EFFECTIVE RATE

Fixed E	N < M	N = M	N > M
Increasing	$R_e \uparrow$	$R_e \uparrow$	$R_e \uparrow$
number of RA $(N\uparrow)$			
Increasing	$R_e \downarrow$	$R_e \uparrow$	$R_e \uparrow$
number of TA $(M\uparrow)$			
Fixed R <sub>o</sub>	N < M	A7 A4	37 36
riacu Ite	IV < IVI	N = M	N > M
Increasing	$R < M$ Low $R_e$ : $E \uparrow$	N = M Low $R_e$ : $E \uparrow$	N > M Low $R_e : E \uparrow$
-0	1		
Increasing	Low $R_e$ : $E \uparrow$	Low $R_e$ : $E \uparrow$	Low $R_e$ : $E \uparrow$

equally-spaced hops. In Fig. 5, increasing the number of TAs can decrease the energy consumption. Besides, with a fixed energy consumptions, more TAs lead to higher transmission rates. Also, as described in Section IV-2, the analytical result  $\Gamma(M^*,N)|_{M^*\leq N}$  in (31) forms a good approximation to the energy-bandwidth efficiency tradeoff in Fig. 5, when N is set to a fixed number of RAs.

Fig. 6 shows the energy-bandwidth efficiency tradeoff when M=N. For M=N, increasing the number of antennas results in increasing end-to-end rate with a fixed energy consumption. With a fixed effective rate, increasing the number of antennas leads to increasing energy consumption in the low effective rate region and decreasing energy consumption in the high effective rate region. Further, the effects of different numbers of antennas are summarized in Table I for the fixed energy consumption and for the fixed effective rate. According to this table, one can choose a suitable antenna pair for effective transmissions.

Then, according to (35), since the lower bound of the energy metric for an arbitrary antenna pair relates to quantity of the normalized signal processing energy  $\gamma_p$ , the following three figures (i.e., Fig. 7, Fig. 8, and Fig. 9) show all possible antenna pairs for up to four antenna numbers and

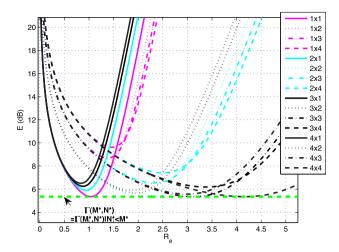


Fig. 7. The energy-bandwidth characteristic of 4 equally-spaced hops with all possible antenna pairs for up to four antenna numbers, and the analytical results in (27) and (35) when  $\gamma_p > \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ .

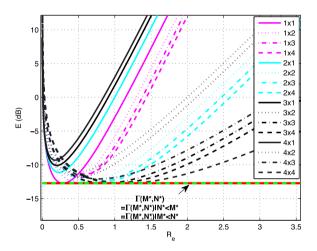


Fig. 8. The energy-bandwidth characteristic of 4 equally-spaced hops with all possible antenna pairs for up to four antenna numbers, and the analytical results in (27), (32) and (35) when  $\gamma_p = \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ .

the analytical results with joint optimal  $M^*$  and  $N^*$ . Fig. 7, Fig. 8, and Fig. 9 correspond to  $\gamma_p=2.946$  (which is larger than  $\frac{\alpha^n(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ ),  $\gamma_p=0.0077$  (which is equivalent to  $\frac{\alpha^n(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ ), and  $\gamma_p=7.6593\times 10^{-4}$  (which is less than  $\frac{\alpha^n(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ ), respectively. In the high end-to-end rate region, the curves fall into four groups and this phenomenon is more obvious in the high  $\gamma_p$  condition especially. Because the energy metric in (19) relates to  $n_{min}$ , each group corresponds to a value of  $n_{min}$ . For instance,  $n_{min}$  value corresponding to the curve of 2x3 is two. This curve will be in the curve group of 2x2, 2x4, 3x2, and 4x2 because of the same  $n_{min}$ . Moreover, the relationships between  $\Gamma(M^*,N^*)|_{N^*\leq M^*}$  and  $\Gamma(M^*,N^*)|_{M^*\leq N^*}$  in (35) for different conditions of  $\gamma_p$  can be verified in those figures.

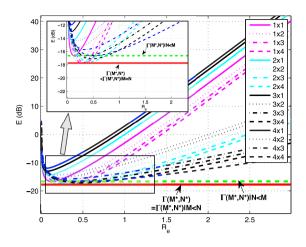


Fig. 9. The energy-bandwidth characteristic of 4 equally-spaced hops with all possible antenna pairs for up to four antenna numbers, and the analytical results in (27), (32) and (35) when  $\gamma_p < \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ .

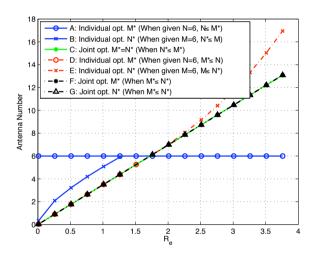


Fig. 10. The optimized antenna numbers versus the effective rate when  $\gamma_n = \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{e^{2}}.$ 

Fig. 10 shows the results of the optimized antenna numbers in Section IV versus the effective rate, when  $\gamma_p = \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ . Because of  $\gamma_p = \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ , as shown in (35), the results of joint optimization corresponding to  $N^* \leq M^*$  and  $M^* \leq N^*$  will be the same. Hence, in Fig. 10, the curves of C, D, F, and G overlap with each other. Curve A is a constant line and its value is equal to the given N. Curve B will not exceed the given M due to the restriction of  $N^* \leq M$ , while curve E will tower above the given M because of  $M \leq N^*$ .

Furthermore, for the case with the optimal number of hops  $k^*$ , the energy-bandwidth efficiency characteristic for the four equally-spaced hops are shown in Fig. 11. The analytical result with the optimal  $k^*$  forms a low bound of the energy consumption as a function of rate. As shown in Fig. 11, increasing number of hops leads to increasing energy consumption in the

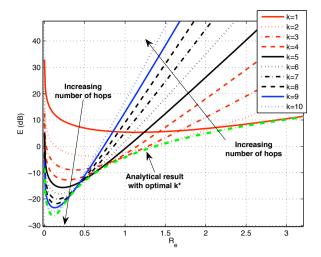


Fig. 11. The energy-bandwidth characteristics under the 1x1 antenna pair for the four equally-spaced hops with k=1,...,10, and the analytical result in (37).

high end-to-end rate region, while increasing number of hops results in decreasing energy consumption in the low end-toend rate region.

# VI. CONCLUSION

The energy-bandwidth efficiency tradeoff in MIMO multihop wireless networks is derived in this paper and the effects of different number of antennas on the energy-bandwidth efficiency tradeoff are investigated at the same time. Besides, we also optimize over the number of antennas to find the minimum energy consumption with the maximum effective rate. The joint optimization over the numbers of TAs and RAs are performed as well. In addition, by optimizing over the number of hops and the effective rate, the optimal number of hops and the optimal effective rate can be obtained. According to the results, one can choose suitable system parameters with considering the energy and bandwidth in the meantime.

#### APPENDIX A

### THE ENERGY METRIC WITH JOINT OPTIMIZATION

First of all, the following discussion is divided into three parts according to the relationship between  $\gamma_p$  and  $\frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{e^2}$ .

1) When  $\gamma_p > \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ : Since  $\Gamma(M^*,N^*)|_{M^* \leq N^*}$  holds only when  $\gamma_p \leq \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ , the  $\Gamma(M^*,N^*)$  in (34) become

$$\Gamma(M^*, N^*) = \Gamma(M^*, N^*)|_{N^* \le M^*},$$
 (47)

in this scenario.

2) When  $\gamma_p = \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ :: By letting  $A \equiv 2+\omega\{\frac{-2}{e^2}\}$  and substituting  $\gamma_p$  into (32),  $\gamma_p$  and  $\Gamma(M^*,N^*)|_{M^*\leq N^*}$  can be rewritten as

$$\gamma_p = \frac{\alpha^{\eta}(e^A - 1)}{2} \tag{48}$$

and

$$\Gamma(M^*, N^*)|_{M^* \le N^*} = \alpha^{\eta} k \ln 2 \times \frac{2(e^A - 1)}{A},$$
 (49)

respectively.

Besides, with A, one can represent  $B \equiv \omega \{ \frac{2\gamma_p}{\alpha^{\eta}e} - \frac{1}{e} \} + 1$  in (27) as

$$B = \omega \{ \frac{e^A - 2}{e} \} + 1. \tag{50}$$

Note that  $e^B=e^{\omega\{\frac{e^A-2}{e}\}+1}$  holds as well because of (50). Then,  $e^B=e^{\omega\{\frac{e^A-2}{e}\}+1}$  can be rewritten to match the form of  $X=e^{-\frac{\omega\{-\theta\rho\}}{\theta}}$  in Lemma A as

$$e^{B-1} = e^{\omega\left\{\frac{e^A - 2}{e}\right\}} \tag{51}$$

Hence, X,  $\theta$ , and  $\rho$  in Lemma A are equivalent to  $e^{B-1}$ , -1, and  $\frac{e^A-2}{e}$ , respectively. As such, by substituting  $X=e^{B-1}$ ,  $\theta=-1$ , and  $\rho=\frac{e^A-2}{e}$  into  $\ln X=\rho X^\theta$  in Lemma A, one can obtain

$$e^B = \frac{e^A + e^B - 2}{B}. (52)$$

Then,  $\Gamma(M^*,N^*)|_{N^*\leq M^*}$  in (27) can be represented as

$$\Gamma(M^*, N^*)|_{N^* \le M^*} = \alpha^{\eta} k \ln 2(\frac{e^B - 1}{B}) + k \ln 2(\frac{2\gamma_p}{B})$$

$$= \alpha^{\eta} k \ln 2(\frac{e^B - 1}{B}) + \alpha^{\eta} k \ln 2(\frac{e^A - 1}{B})$$

$$= \alpha^{\eta} k \ln 2 \times e^B$$

$$= \alpha^{\eta} k \ln 2 \times e^{\omega \{\frac{e^A - 2}{e}\} + 1}. \tag{53}$$

Finally, with (49) and (53), the difference between  $\Gamma(M^*, N^*)|_{M^* < N^*}$  and  $\Gamma(M^*, N^*)|_{N^* < M^*}$  is represented as

$$\Gamma(M^*, N^*)|_{N^* \le M^*} - \Gamma(M^*, N^*)|_{M^* \le N^*}$$

$$= \alpha^{\eta} k \ln 2 \times \left(e^{\omega \left\{\frac{e^A - 2}{e}\right\} + 1} - \frac{2(e^A - 1)}{A}\right). \tag{54}$$

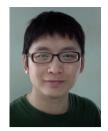
Since A is a constant, the result of  $(e^{\omega\{\frac{e^A-2}{e}\}+1}-\frac{2(e^A-1)}{A})$  can be proved as zero with the numerical methods. It means that  $\Gamma(M^*,N^*)|_{N^*\leq M^*}$  and  $\Gamma(M^*,N^*)|_{M^*\leq N^*}$  are the same when  $\gamma_p=\frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ . As such,  $\Gamma(M^*,N^*)$  in (34) is equivalent to  $\Gamma(M^*,N^*)|_{N^*\leq M^*}$  and  $\Gamma(M^*,N^*)|_{M^*\leq N^*}$  in this scenario.

3) When  $\gamma_p < \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ :: Due to (27) and (32),  $\Gamma(M^*,N^*)|_{N^*\leq M^*}$  decreases with  $\gamma_p$  approximately and  $\Gamma(M^*,N^*)|_{M^*\leq N^*}$  decreases with  $\sqrt{\gamma_p}$ , respectively. Moreover, since  $\Gamma(M^*,N^*)|_{N^*\leq M^*}$  and  $\Gamma(M^*,N^*)|_{M^*\leq N^*}$  are the same when  $\gamma_p = \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ ,  $\Gamma(M^*,N^*)|_{M^*\leq N^*}$  will be less than  $\Gamma(M^*,N^*)|_{N^*\leq M^*}$  when  $\gamma_p < \frac{\alpha^{\eta}(e^{2+\omega\{\frac{-2}{e^2}\}}-1)}{2}$ . Hence, the result of  $\Gamma(M^*,N^*)$  in (34) is equivalent to  $\Gamma(M^*,N^*)|_{M^*\leq N^*}$  in this scenario.

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