PHYSICAL REVIEW E

STATISTICAL PHYSICS, PLASMAS, FLUIDS, AND RELATED INTERDISCIPLINARY TOPICS

THIRD SERIES, VOLUME 51, NUMBER 4 PART B APRIL 1995

ARTICLES

Catalytic interface erosion

Hsin-Fei Meng* and E. G. D. Cohen
The Rockefeller University, New York, New York 10021
(Received 1 August 1994)

We study interface erosion processes: catalytic erosions. We present two cases. (1) The erosion of a completely occupied lattice by one single moving particle starting from somewhere inside the lattice, considering deterministic as well as probabilistic erosion rules. In the latter case, the eroded regions appear to have interfaces with continuously tunable fractal dimensions. (2) The kinetic roughening of an initially flat surface, where ballistic or diffusion-limited particles, which remain intact themselves, erode the surface coming from the outside, using the same erosion rules as in (1). Many features resembling realistic interfaces, for example, islands and inlets, are generated. The dependence of the surface width on the system size is due to both the erosion mechanism and the way particles move before reaching the surface.

PACS number(s): 82.20.Wt, 05.40.+j

I. INTRODUCTION

Surface growth and erosion have been modeled by diffusion-limited processes [1], since diffusion-limited aggregation was first proposed as a model for dendrite growth in solidification [2]. In all of these processes, the incoming particle moves as a random walker until it reaches the surface. For the case of aggregation, the particle sticks to the aggregate and becomes a part of it. For the case of diffusion-limited erosion, as first discussed by Krug and Meakin, the moving particle annihilates with the surface particle it encounters and the surface site is eroded [3]. In a three-dimensional model, Nagatani [4] studied numerically the pitting corrosion of metal surfaces by diffusion-limited motion of particles causing depassivation and corrosion of the surface. The scaling behavior of the pit-size distribution of the corrosion patterns was obtained. In all diffusion-limited cases, the moving velocity of the surface is proportional to the gradient of a Laplacian field [5]. The relaxation of surface fluctuations can thus be treated by a linear analysis based on the Laplacian field [6]. In a more complicated model, Meakin, Jøssang and Feder [7] simulated pitting corrosion in a two-dimensional model where the corrosion is controlled by the diffusion of moving corrosive sites as well as by the passivation and depassivation of surface sites. The pit growth and the scaling behavior of the corrosion current were given. Although these corrosion models have similarities with those considered in this paper, the problem they address and the results they yield are quite different. This seems to be related to the more chemical nature of the corrosion problem as compared to the more physical nature of the erosion problem considered here, so that models with different features are employed for the two problems.

In this paper, we study an alternative mechanism of surface erosion—catalytic erosion, where unlike the surface erosion in the previous models, the eroding particle moves through the solid over potentially extended distances, until it reemerges and is eliminated. Consider a square lattice with each lattice site assigned not only a permanent (probabilistic or deterministic) scattering rule, but also an "erodable property," which is removed (eroded) by the visit of a moving (eroding) particle. The trajectory of the particle is exclusively determined by the scattering rule at each lattice site independent of whether the site is or has been visited (eroded) or not. Thus, the

*Permanent address: Institute of Physics, National Chiao Tung University, Hsinchu, Taiwan 30050.

sites can be classified into two kinds: not visited or occupied and visited or empty. Occupied and empty sites are separated spatially into two phases with an interface in between, which we call solid and liquid phases, respectively, for convenience. The erosion process is taking place by a "catalytic particle" starting in the liquid region; when the particle hits an occupied site in the solid region, this site becomes empty, i.e., part of the liquid. In other words, the liquid region grows at the expense of the solid region. We use this to model the erosion at the interface between two physically or chemically different phases. The eroding particles are called catalytic because they mediate the erosion of the solid while remaining intact themselves and behave, therefore, like catalysts till they return to the liquid. Alternatively, one can consider this a model for the damage done by a moving particle to a surface. A possible example of an "erodable" property is the occupation of a lattice site by a particle, which is eliminated when the moving particle hits the lattice site, i.e., creates an empty site.

The dynamic rules we use for catalytic particles to move are either those of a biased random walk or a deterministic rule, to be specified below. We first discuss the biased random walk. In that case, in each step, the probability for the particle to move from its present site to an occupied neighbor is smaller than the probability of moving to an empty neighbor. Thus, we introduce a bias between motion to occupied or empty neighbors from the current position of the particle. As the simplest case, we start one single particle from a chosen position in a completely occupied lattice, i.e., a solid. Then a wide variety of interfaces between the eroded and not (yet) eroded part of the lattice can be generated by tuning the bias, so that a tunable fractal dimension of the surface of the eroded region, i.e., of the liquid, can be obtained. We applied this same rule also to study the erosion of an initially flat surface, where the particles are launched one by one from a large distance above it.

The two dynamical rules both imply that when the particle coming from the liquid hits the solid, it feels a resistance of the solid and can only penetrate into it within a certain depth. Thus, the catalytic particle will eventually move back to the liquid, leaving a number of solid sites eroded. We eliminate the particle after it returns to the liquid and start another one and so on. In the $1+1$ flat surface case that we will consider in this paper, it is found that the surface eroded in this way is qualitatively different from that in Krug and Meakin. Many islands and inlets formed along the surface and the surface appears to be rougher than that of Krug and Meakin for small system sizes as discussed below. Furthermore, as we increase the initial surface length, the surface roughness approaches a finite saturation value, in contrast to the case of Krug and Meakin where the surface fluctuations diverge as the square root of the logarithm of the system size. As in Krug and Meakin, our catalytic particle moves like an unbiased random walker before it hits the surface, so that the present phenomenon is due to the dynamics of the erosion process, rather than to the way the particles approach the surface.

In order to study the effect on the erosion process of the way the particles approach the surface, we also consider ballistic erosion processes, where an incoming particle moves in a straight line with a random direction before hitting the first occupied lattice site. After that, the particle moves as a biased random walker and erodes the sites along its path just like in the diffusion-limited processes described above. We can vary the range of the random angles of the straight line along which the particle moves. As expected, the appearances of the surfaces generated by various ranges are quite different from one another. For very wide ranges, when the particle can hit the solid with almost any angle, the surfaces look like the ones obtained in the diffusion-limited process. For narrow ranges where the particles come in almost parallel to each other, the surfaces generated have sharp stalagmite-like spikes, which are absent in all other models considered here. However, interestingly, even for the wide range angle cases where the surface appears to be similar to the ones obtained in diffusion-limited processes, the surface width has a different asymptotic behavior than that in diffusion-limited processes when the system size goes to infinity. Thus, it becomes clear that the nature of the eroded surfaces are determined by both the erosion mechanism and the way the particles move before reaching the surfaces.

In Sec. II, we present our results for the erosion done by a single particle starting inside a completely occupied lattice. Next, in Sec. III, we discuss the roughening of a flat one dimensional interface by particles from the outside moving according to a biased random walk. We consider in particular the dependence of the surface height fluctuations on time and system size and also compare the results of a probabilistic erosion rule with those of a deterministic rule. In Sec. IV, we consider the same erosion problem as in Sec. III, if the particles approach the surface ballistically rather than in a random walk. Finally, in Sec. V, we discuss some possible reasons for the various erosion results found in the preceding sections.

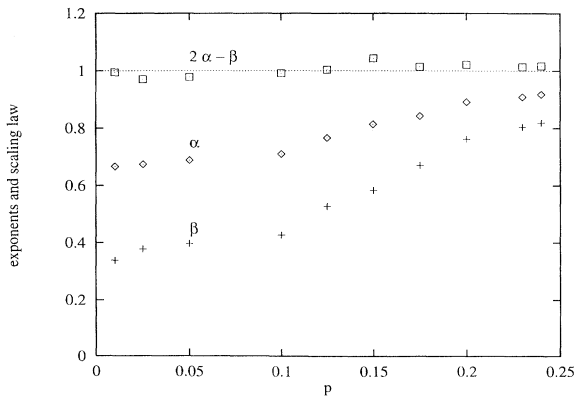

FIG. 1. The growth of the eroded region by one single eroding particle inside a solid can be described by two exponents α and β, defined by $A \sim t^{\alpha}$ and $B \sim t^{\beta}$, where A and B are the size of the area and boundary of the region, respectively. The p dependence of α and β on the bias p is shown here. For all p, these two exponents obey a scaling law $2\alpha-\beta=1$ as predicted in Ref. [8].

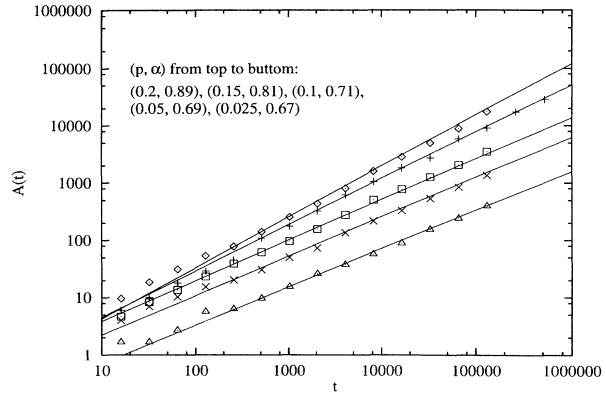

FIG. 2. The p dependence of the asymptotic behavior of the area $A(t)$ is illustrated by plotting A against t in log-log scales for various p's. Different slopes for different p's are clearly seen. When p goes from 0 to $\frac{1}{4}$, the slope α goes from $\frac{2}{3}$ to 1. The data points have been shifted to avoid overlap. For $t > 1000$ the error bars (not shown) are within the data symbols.

Depending on the context, the terms "surface" and "interface" will be used interchangeably.

II. INSIDE EROSION BY ONE SINGLE PARTICLE

A. Introduction

As a simple model for the catalytic erosion process, we define a biased random walk as follows. Consider a particle on a square lattice moving at every time step from its present site to one of its nearest neighbors. As discussed in the Introduction, the lattice sites are classified into two kinds: occupied and empty. We assign a probability p for the particle to move to a nearest occupied neighbor site. Then, assuming that there are n occupied neighbors around the current position of the particle, the probability for the particle to move to any of the other $4-n$ empty neighbors is $(1-np)/(4-n)$, so that the total probability to move is one. After the particle moves, the site at the new particle position is considered as empty or eroded no matter whether it was empty or occupied before the move. Then the particle makes the next move with a probability based on the nature of its new neighbors and so on. We choose p always smaller than $\frac{1}{4}$, so that the particle is more likely to move to an empty site than to an occupied site. The difference between $\frac{1}{4}$ and p measures the bias. When $p=\frac{1}{4}$, we have an unbiased random walk. This biased random walk can be interpreted directly as an erosion process if we identify the transformation of occupied sites to empty sites as the process through which matter (occupied sites) becomes eroded (empty sites) by the particle.

B. Erosion from the inside

Here, we consider the case that the particle starts from somewhere inside a perfectly occupied lattice. Then empty sites are defined as those sites that have been visited by the particle in its walk through the lattice. For each p, we measure the time step t dependence of the area A and boundary B of the eroded (empty) region, where A is defined as the number of eroded sites, and B as the number of sites on the interface, i.e., the sites in A that have at least one occupied nearest neighbor. In general, we found

$$A(t)\sim t^{\alpha(p)}, B(t)\sim t^{\beta(p)} , \tag{1}$$

where $\alpha(p)$ and $\beta(p)$ are the asymptotic critical exponents. Interestingly, there appears to be a continuous, i.e., tunable p dependence of α and β. Moreover, α and β are related for all p by the equation $2\alpha-\beta=1$, which is

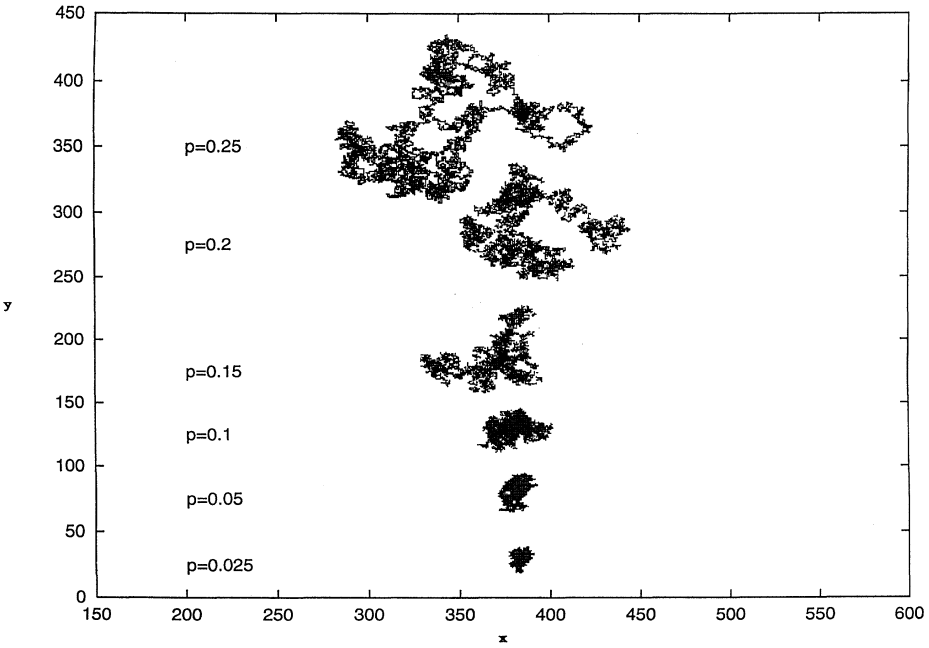

FIG. 3. Typical single particle trajectories for various p's are shown; as p decreases, they become more and more compact and the fractal dimension of the boundary of the trajectories approaches 1 when p approaches 0. All the trajectories have 2^{14} time steps.

exactly the same scaling relation derived before for the growth of the liquid region in the deterministic Lorentz lattice gas [8] described below, which can also be considered as an erosion process. However, in the biased random walk, the fractal dimension of the boundary can be tuned by adjusting p, while there is no such adjustable parameter in the Lorentz lattice model. We plot $\alpha(p)$ and $\beta(p)$ as well as $2\alpha-\beta$ in Fig. 1. Note that the dimension of the surface is $2\beta/\alpha$, since $B\sim(\sqrt{A})^{2\beta/\alpha}$. It is in general larger than one, i.e., fractal. To illustrate the dependence of the asymptotic behavior of $A(t)$ on t, we plot both on a log-log scale in Fig. 2. The behavior of $B(t)$ looks similar to that of $A(t)$. A continuous variation of the slopes can clearly be seen. Six typical trajectories of the particle in the solid for different p are shown in Fig. 3. As expected, the trajectory becomes more and more compact as p is reduced from $\frac{1}{4}$. In the extreme cases, the trajectory is like a circular disk for $p<0.05$, or has a loose random-walk-like shape for $p>0.20$. Equation (1) differs from the usual result that the asymptotic exponents have a set of discrete values and do not change continuously. Here, by varying a single parameter p, we can generate eroded regions with extremely diversified fractal properties, at least for the up to a million time steps we considered. This suggests that many more realistic and complicated erosion processes, deterministic or stochastic, can be approximated and simulated by this model with an effective p. For example, the deterministic Lorentz lattice gas model studied before [8] corresponds effectively to $p\simeq 0.125$. This is due to the fact that the motion of the eroding particle will be determined by the current configuration of occupied and empty sites in its neighborhood. The response of the particle motion to the various configurations can then be summarized, on average, by an effective resistance exercised by the occupied side of the interface on the particle, which can be expressed by a particular value of the parameter p.

III. OUTSIDE ROUGHENING BY DIFFUSION-LIMITED EROSION

A. Introduction and definition of model

Now we turn to the surface erosion of an initially flat one dimensional interface by particles hitting the surface from the outside and then carrying out the biased random walk described above. We use the same strip geometry as Krug and Meakin [3] [cf. Fig. 4(a)]. Let L be the horizontal size of the system, i.e., the initial length of the interface. At first, beneath the surface all sites are occupied (solid) and above the surface all sites are empty (liquid). A particle is launched from a point in the liquid far above the surface and moves in the liquid as a (nor-

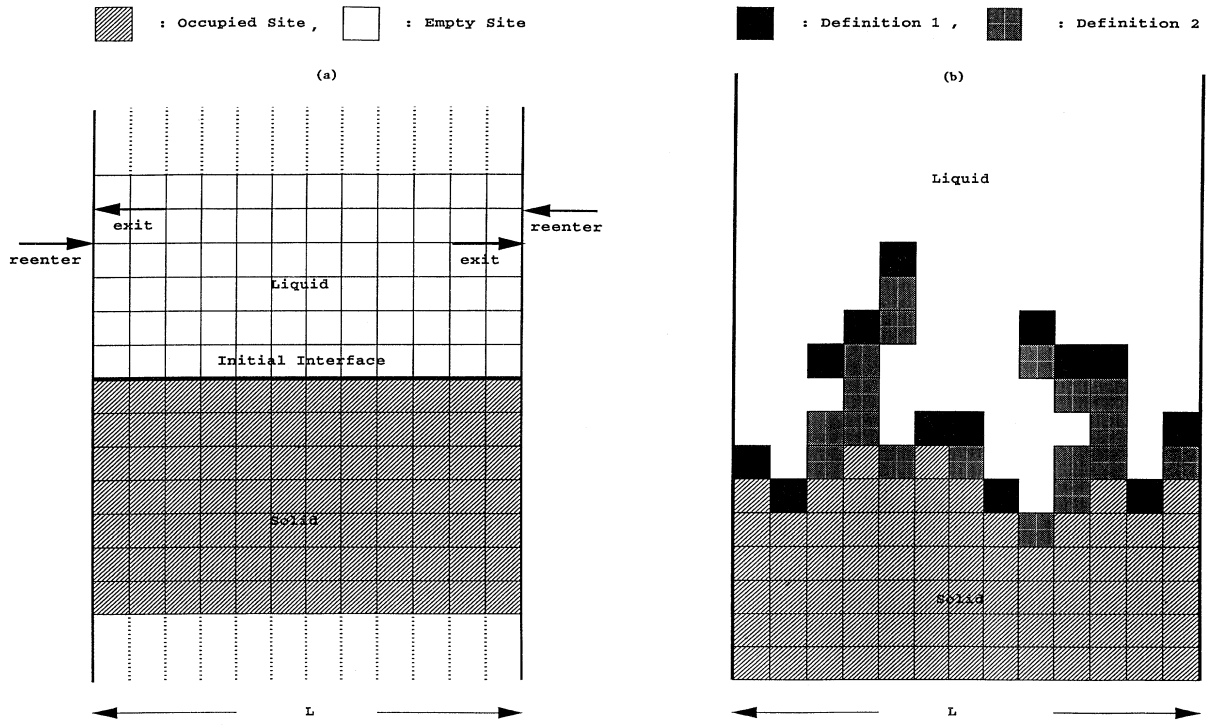

FIG. 4. (a) The initial arrangement of the lattice in the case of flat surface roughening. Below the initially flat interface all the sites are occupied (solid), above the interface empty (liquid). The horizontal size of the lattice is L, while the lattice extends to infinity in the vertical directions. The eroding particles are launched far above the interface one by one. When hitting the wall, an incoming eroding particle reenters from the opposite wall, a periodic boundary condition, as indicated in the figure. (b) Two definitions of surface sites are illustrated. Black squares are the top of each column (definition 1). Gray squares are the sites that have at least one empty neighbor (definition 2). Note that many overhangs are present in the catalytic erosion surfaces. The grid in the liquid region is omitted for clarity.

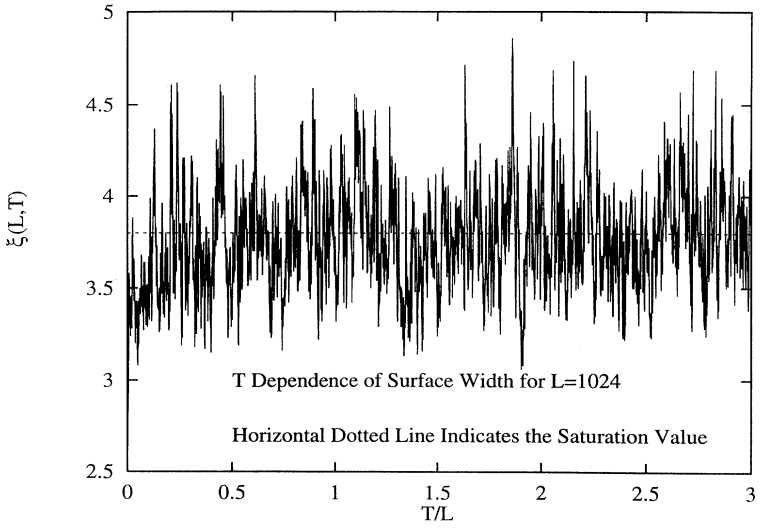

FIG. 5. Behavior of the surface width $\xi(L,T)$ as a function of reduced time T/L for a fixed L. Starting from zero, ξ saturates quickly at $T/L \approx 1$ and begins to fluctuate around a certain value, $\xi(L)$. This particular example is for a biased random walk with $p=0.125$, $L=1024$. The saturation value 3.799 is indicated by the dashed horizontal line.

mal) random walker on a square lattice according to the biased probability rule. However, after the particle hits the surface and erodes along its path, it feels the effective repulsion caused by the bias coming from the solid (occupied) neighbor sites. As a consequence, the particle cannot penetrate into the solid indefinitely, since the probability to go into the solid is smaller than to come back to the liquid. So, eventually the particle will leave the solid and return to the liquid, where it is then eliminated and a new particle is launched randomly from another remote point above the liquid-solid interface. We impose periodic side boundary conditions to reduce boundary effects near the end points of the interface as much as possible. So the particle reenters through the left-hand-side wall of the strip if it exits through the right-hand-side wall of the strip, and vice versa.

B. Surface height fluctuation

The surface height fluctuation is, in general, characterized by its width ξ, which is defined as the root mean square deviation of the heights of the surface sites from their average. We consider two kinds of definitions of the surface sites, which we call 1 and 2. For 1, surface sites are the highest occupied sites in each column. For 2, a surface site is an occupied site that has at least one empty nearest neighbor [Fig. 4(b)]. Let h_i be the height of the ith surface site, then ξ is expressed by

$$\xi(L,T)=\left[\frac{\sum_i [h_i(T)-\bar{h}_L(T)]^2}{\sum_i 1}\right]^{1/2}, \tag{2}$$

where $\bar{h}_L$ is the arithmetic mean of h_i's and the sums are

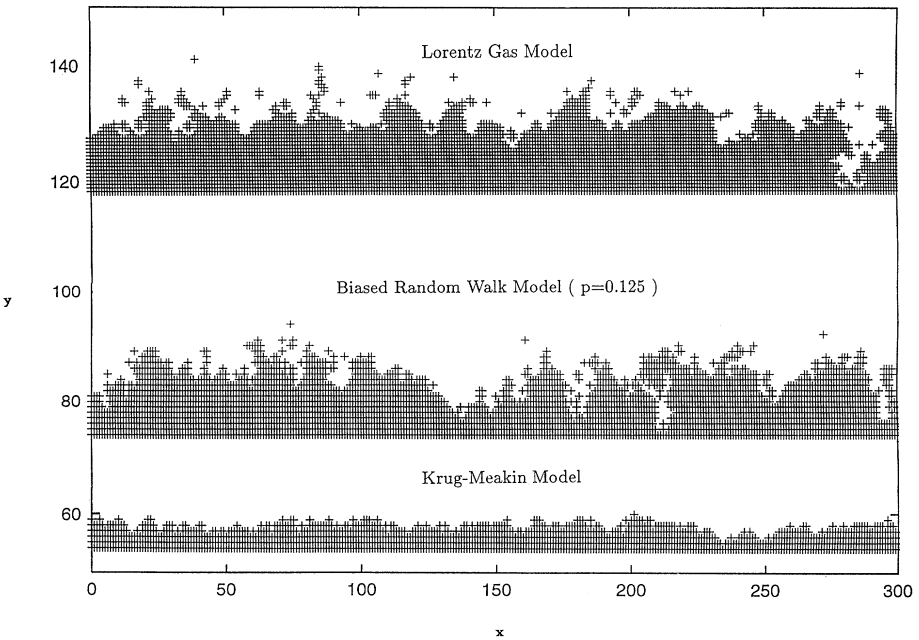

FIG. 6. Typical surfaces after saturation of $\xi(L,T)$ for the Lorentz gas, biased random-walk model and Krug-Meakin model are shown. Many islands and inlets are seen in the first two cases, while they are absent in the third. However, the function $\xi(L)$ has a different asymptotic behavior even for the first two models (see text).

carried out over all surface sites either according to definition 1 or definition 2. It is convenient [1] to replace the actual "running" time t of the erosion process by another quantity T, the decrease of $\bar{h}$ due to erosion since the beginning of the process. For large t, T is roughly proportional to the total number of the particles launched so far. Obviously, ξ is a function of L and T. It is found numerically that for both surface definitions and for all L, $\xi(L,T)$ approaches a saturation value, around which it fluctuates, as $T/L \gg 1$. The fluctuations around the saturation value are smaller for larger L, since ξ is a summation over all the surface sites and the statistical error reduces as the number of surface sites increases. One typical behavior of the approach of ξ to its saturation value is plotted in Fig. 5. In the following, we will use $\xi(L)$ to denote $\sqrt{\langle \xi^2(L,T)\rangle_L} = \sqrt{\langle [h(0)-\bar{h}_L]^2\rangle_L}$, where the average is carried out over many T's, i.e., over many interfaces, after saturation has been reached. $h(0)$ simply means the height of the column at position 0, which in view of the periodic boundary conditions, could be any position from 0 to L. In Fig. 6, we show a typical saturated surface generated by this process, together with that for the Krug-Meakin's process [3], where the incoming particles erode only the very first solid sites they hit, as well as that for the Lorentz gas model discussed below. It is seen that many new features show up in the surface properties when compared with those obtained by Krug and Meakin. For example, there are many "islands" and "inlets," not present before, caused by the eating away of the solid by the particle. In fact, it seems that the present erosion mechanism is able to generate many features that also exist in real systems. The effect of the occurrence of islands and inlets on $\xi(L,T)$ can be studied by comparing the $\xi(L,T)$ obtained from the two different surface definitions. In general, 1 gives a smaller $\xi(L,T)$ than 2 does. This is so because 1 only takes into account the effective surface when one looks from above down onto the surface, so that all the vacancies below the highest site in each column are blocked and the underlying surface structure is ignored. However, 2 properly takes into account all the solid sites that are exposed to the liquid and thus includes all the islands and inlets. We note that the order of magnitude for these two definitions in the Krug-Meakin model as compared to ours is simply the reverse. This is so because in the Krug-Meakin model there are almost no vacancies below the highest sites in each column and 2 only adds sites that are between adjacent highest sites when the height difference is more than one (see Fig. 4).

C. Surface correlation functions

Now we discuss the properties of the surfaces quantitatively. Before doing this, we point out that, instead of finite surfaces with periodic boundary conditions, the systems we really want to study are infinite surfaces free from the unnatural constraints coming from the boundary conditions. In light of this, we will first define a set of correlation functions suitable for an ensemble of infinite surfaces, which characterize the fluctuation properties of the surfaces as a function of the erosion processes. Then we will discuss how we approximate the correlation functions of the infinite surfaces by computations on finite surfaces with various system sizes L's. It turns out that for the infinite system the correlation functions are not independent of each other, and only one of them is needed to be determined in order to obtain all the others.

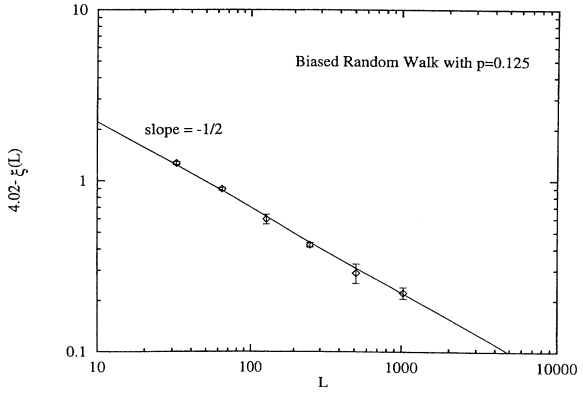

FIG. 7. The general behavior of $\xi(L)$ for the biased random-walk model with p not too close to 0 or $\frac{1}{4}$ is found to be $\xi(L)=C-D/\sqrt{L}$, where C and D depend on p. $C-\xi(L)$ for $p=0.125$ is shown here in log-log scale with $C=4.02$. A line with slope $-\frac{1}{2}$ is also shown.

It is natural to consider three correlation functions, $\xi^{\infty}(L)$, $C^{\infty}(r)$, and $F^{\infty}(r,L)$, to characterize the properties of the height fluctuations for an ensemble of infinite surfaces. They are defined as follows:

$$
\begin{aligned}
&[\xi^{\infty}(L)]^2 \equiv \langle [h(0)-\bar{h}_L]^2 \rangle_{[L]} , \\
&C^{\infty}(r) \equiv \langle [h(0)-h(r)]^2 \rangle , \\
&F^{\infty}(r,L) \equiv \langle [h(0)-\bar{h}_L][h(r)-\bar{h}_L] \rangle_{[L]} .
\end{aligned}
\tag{3}
$$

Here, $\langle\ \rangle$ means an average over many realizations (many T's in practice) of the surface after saturation is reached. The subscript $[L]$ indicates that the average is over a segment of finite size L of the infinite surface and

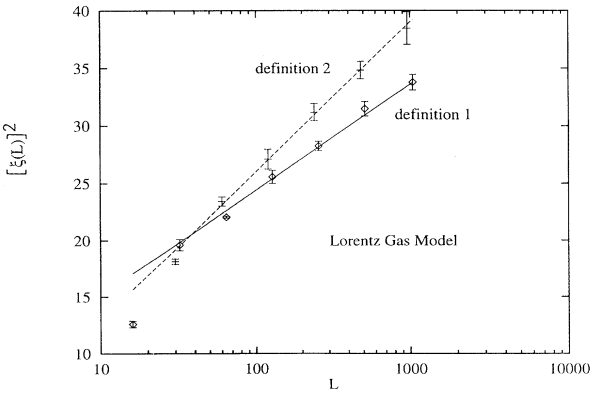

FIG. 8. The behavior of $\xi(L)$ for the Lorentz gas model is found to be $\xi(L)=C\sqrt{\log_{10}(L/a)}$, where C and a are certain constants. $[\xi(L)]^2$ is shown here for two definitions of the surface sites. In definition 1 only the top of each column is included, so that inlets are ignored. In definition 2 all the sites exposed to the liquid are included so that a larger $\xi(L)$ results.

$\bar{h}_L$ is the mean height of the columns in the segments. We use the superscript ∞ to emphasize that the definitions refer to infinite surfaces, in contrast to the surface width function $\xi(L)$ defined for an ensemble of finite surfaces with periodic boundary conditions. The argument of h refers to the horizontal position of the surface sites. $\xi^{\infty}(L)$ and $\sqrt{C^{\infty}(r)}$ are two different ways to represent the magnitude of the vertical excursions of the surface heights within the horizontal displacement of L, while $F^{\infty}(r,L)$ measure the height correlation between two points separated by a distance $r \leq L$ within a segment of size L of the infinite surface. $[\xi^{\infty}(L)]^2$ and $C^{\infty}(r)$ are expected to be monotonically increasing functions of L and r, respectively, while $F^{\infty}(r,L)$ is a monotonically decreasing function of r. As stated before, all we need is $[\xi^{\infty}(L)]^2$, since the other two functions can be derived from it in a simple way. First, it is easy to see that $C^{\infty}(r)=\{2[\xi^{\infty}(L)]^2-F^{\infty}(r,L)\}$. Note that the explicit L dependence on the right-hand side of the above equation is canceled, so $C^{\infty}(r)$ is a function of r only. Integrating both sides of the equation and noting that $\int_0^L F^{\infty}(r,L)dr=0$, we have $\int_0^L C^{\infty}(r)dr=2L[\xi^{\infty}(L)]^2$. The integration is short hand for a discrete sum over the interface. However, we do assume that all the functions are sufficiently smooth, since they are ensemble averages, that they can be differentiated. Thus, $C^{\infty}(L)$ and $F^{\infty}(r,L)$ can be expressed in terms of $[\xi^{\infty}(L)]^2$ through

$$C^{\infty}(L)=2([\xi^{\infty}(L)]^2+L\{[\xi^{\infty}(L)]^2\}'),$$
$$F^{\infty}(r,L)=[\xi^{\infty}(L)]^2-[\xi^{\infty}(r)]^2-r\{[\xi^{\infty}(r)]^2\}', \quad (4)$$

where the primes mean differentiations with respect to the indicated arguments. Note that the correlation between the heights of the surface sites r lattice spaces apart are measured by the correlation function $C^{\infty}(r)$. The smaller $C^{\infty}(r)$ is for a given r, the more the heights are correlated. The asymptotic behavior of $C^{\infty}(r)$ characterizes, therefore, the long-range height correlations of the surface. In this paper, we will give only the results for $\xi(L)$, which is an approximation to $\xi^{\infty}(L)$. One can easily see from Eq. (4) that C^{∞} and $\xi^{\infty 2}$ share the same asymptotic behavior.

Needless to say, we are not able to compute the correlation functions of the infinite surfaces. Therefore, we proceed by making the crucial assumption that $\xi^{\infty}(L)$ is well approximated by $\xi(L)$, as computed from an ensemble of surfaces of size L with periodic boundary conditions. This assumption is justified since $\xi^{\infty}(L)$ measures the mean deviation of surface heights of all columns from 0 to L, so that the effect of the boundary conditions should concentrate for sufficiently large L near 0 and L and have a small influence on $\xi(L)$. Thus, while $C^{\infty}(r)$ and $F^{\infty}(r,L)$ can be rigorously obtained only from $\xi^{\infty}(L)$ by Eq. (4), they can be obtained approximately from $\xi(L)$ which can be actually determined in practice.

In the following, we will therefore only study the behavior of $\xi(L)$. For a biased random walk it is found that $\xi(L)$ itself approaches a constant value as L goes to infinity, where the difference with that constant decreases as $L^{-1/2}$ (see Fig. 7). So the surface is asymptotically very smooth. Only the results for $p=0.125$ are shown here, but we found that as long as p is not too close to 0.25 or 0, there are no qualitative differences. Even though the surface width ξ depends on the system size L in a nontrivial way, the fractal dimension of the surface, as measured by the standard ϵ method [9], remains one for all T. In other words, the eroded surface is not fractal. This holds, in fact, for all the cases considered in this paper.

D. Lorentz lattice gas model

Finally, for a comparison, we also consider a more complicated deterministic mechanism of erosion than that considered by Krug and Meakin: the Lorentz lattice

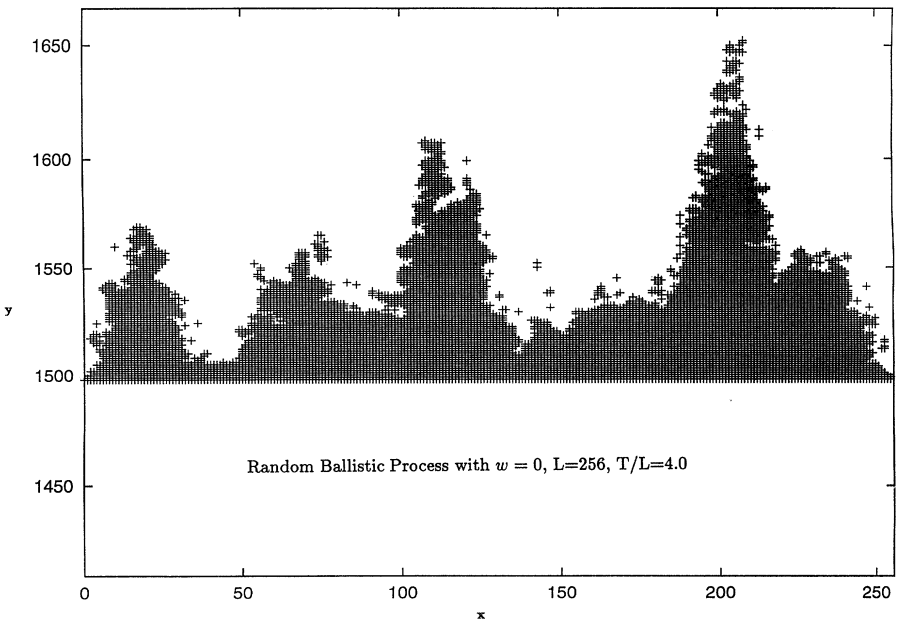

FIG. 9. A snapshot of the surface of a ballistic erosion process with perfectly parallel incoming particles ($\omega=0$) at some T. The surface width $\xi(T,L)$ will eventually diverge with T because of the lack of correlations between remote columns. Large peaks are seen in this case only.

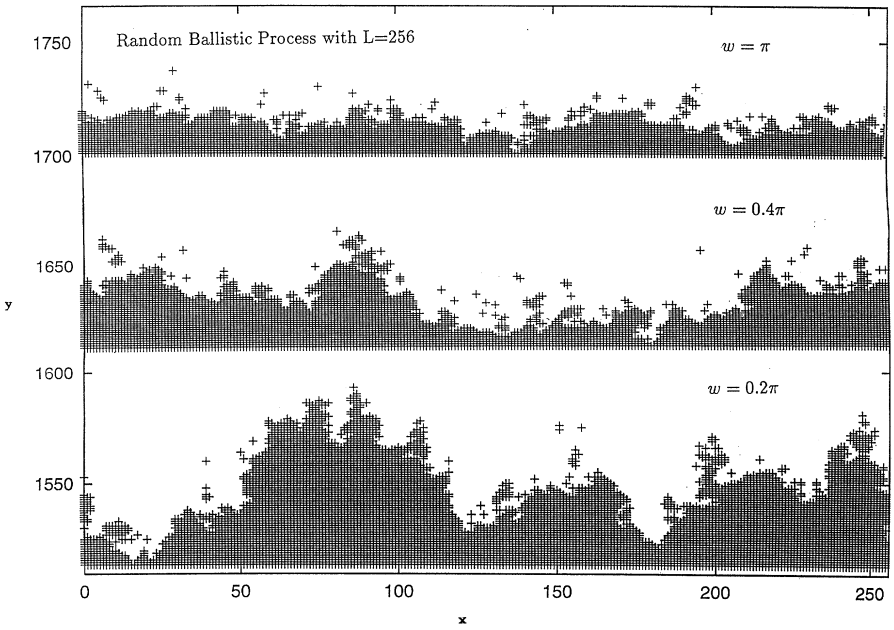

FIG. 10. Typical surfaces after saturation of $\xi(L)$ for a random ballistic process with various ranges ω of the angles of the incoming particles. The surfaces become smoother with increasing ω.

gas with flipping rotators of two types [8]. In this case, the particle moves inside a lattice whose sites scatter the particle according to a deterministic scattering rule: that of right or left rotators, which rotate, after collision, the velocity of the moving eroding particle either to the right or to the left, respectively. After a scattering, the type of a rotator flips to the other one, i.e., left to right and right to left. As shown in the previous paper [8], the particle can then destroy an initially periodic array of right and left rotators (a solid) and make it into a completely random distribution of right and left rotators (a liquid). Moreover, the particle motion in such a self-generated random configuration of rotators is very similar to a random walk. So we can replace the liquid by a random distribution of right and left rotators and the solid by a periodic array of rotators initially. In practice, we use the array composed of a 3×3 basic cell, which has four right rotators in the upper left 2×2 block and five left rotators in the rest of the cell. However, our result does not depend on the choice of the initial array for the solid, as long as the array can be completely randomized by the moving particle (cf. [8]). Independent of its rotator orientation, each site can be either occupied or empty. Similar to the case of the biased random walk considered above, initially we assign all the sites in the solid phase to be occupied and in the liquid phase to be empty. Then, we launch the particle in the liquid region far above the interface and let it move deterministically. After reaching the liquid-solid interface, the particle erodes all the occupied sites along its path, i.e., makes them empty and flips the orientations of the rotators associated with them. It was shown before [8] that the particle cannot penetrate into the solid too deeply. So, like the biased random walk, the particle will eventually return to the liquid after eroding several solid sites. Then, we eliminate it and start another particle from a remote point in the liquid. The surface generated this way is shown in Fig. 6. It looks rather similar to the one generated by the biased random walk. However, it turns out that the surface fluctuations have a different asymptotic behavior. For the (deterministic) flipping rotator model, $\xi(L)$ is proportional to $\sqrt{\log_{10}(L/a)}$ just like in the (deterministic) Krug-Meakin process [3], where a is a characteristic short-range cutoff length of the order of the lattice constant (see Fig. 8), while for the biased random walk $\xi(L)$ approached a constant.

IV. OUTSIDE EROSION BY BALLISTIC EROSION

A. Introduction and definition of model

Let us now turn to the case of the biased random-walk erosion process, when particles hit the surface ballistical-

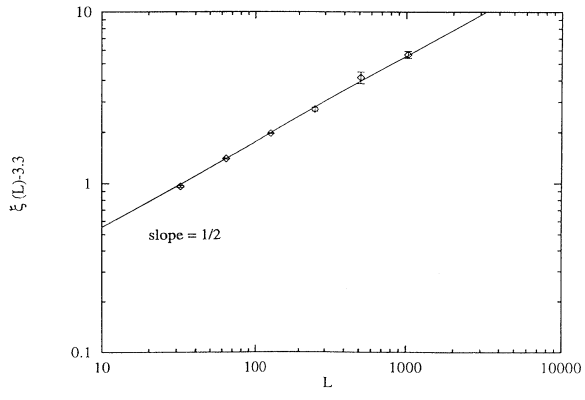

FIG. 11. The general behavior of the surface width $\xi(L)$ for a random ballistic process is $\xi(L)=C+D\sqrt{L}$, where C,D depend on p. For $p=0.125$, $\xi(L)-C$ is plotted against L on a log-log scale for $\omega=\pi$, for which $C=3.3$. A line with slope $\frac{1}{2}$ is also shown.

TABLE I. Here we list the behavior of all the models of surface roughening considered in this paper. $\xi(L)$ is the surface width, after saturation has been reached. C, D, and a are constants whose values depend on the specific models. In the case of random ballistic erosion with $\omega=0$, $\xi(L)$ cannot be defined because $\xi(L,T)$ diverges with T. One can see clearly that both the erosion mechanism (biased random walk, Lorentz lattice gas or Krug-Meakin) and the way the particles approach the surface (diffusion limited or ballistic) are important to determine the behavior of the surface width.

Model	Behavior of $\xi(L)$
Diffusion limited, biased random walk	$C-DL^{-1/2}$
Diffusion limited, Lorentz lattice gas	$C[\log_{10}(L/a)]^{1/2}$
Krug-Meakin	$C[\log_{10}(L/a)]^{1/2}$
Ballistic, biased random walk with $\omega>0$	$C+DL^{1/2}$
Ballistic, biased random walk with $\omega=0$	$\xi(L,T)\sim\sqrt{T}$ as $T\to\infty$

ly. As stated before, the incoming particles move then from far above a flat liquid-solid interface along straight lines with random directions towards the surface. We consider the cases where the angles of the directions are uniformly distributed in an interval from $-\pi/2+\omega/2$ to $-\pi/2-\omega/2$. Here, we choose the axes such that the interface is initially along the horizontal x axis and the sites with $y\leq 0$ are occupied while those with $y>0$ are not. Therefore, $-\pi/2$ is in the direction of the erosion process and ω measures the range of the random directions. As before, we launch the particles from uniformly distributed random horizontal positions. We shall first consider two extreme cases $\omega=0$ and π.

B. $\omega=0$

In the first case $\omega=0$, the particles all drop down vertically. We find broad based, vertically thinning, stalagmitelike peaks, which differ from the thinner peaks and elevations of the previous models (see Fig. 9), where they would have been hit most and, consequently, eroded away in the diffusion-limited process. Moreover, it is found that the surface width $\xi(L,T)$ does not saturate at a finite value when T goes to infinity, as in all the cases discussed before; instead, $\xi(L,T)\sim\sqrt{T}$ as $T\to\infty$.

This can be understood as follows. When $\omega=0$, the horizontal positions where the particles touch the surface for the first time, are independent of the current shape of the surface because the x coordinates of the incoming particles remain the same before hitting the surface. If we further simplify the process so that the particles erode only the first site they hit, then the variation of the heights of the columns become independent of each other. Since we launch the particles with random x coordinates initially, the heights eroded for all the columns should then form a binomial distribution, with a mean equal to T. It is clear then that in this simplified example the mean deviation is proportional to $\sqrt{T}$, characteristic for binomial distributions. The fact that the same $\sqrt{T}$ behavior is also observed in the biased random walk with $\omega=0$ means that the coupling among the columns due to the horizontal motion of the particles after hitting the surface is not strong enough to establish long-range correlations between the surface heights to keep $\xi(L,T)$ bounded for all T.

C. $\omega=\pi$

For the second extreme case $\omega=\pi$, the particles move in straight lines with arbitrary directions before hitting the surface. We found that the surface eroded in this way is very similar to those obtained in the diffusion-limited processes (see Fig. 10), when the particles move as random walkers before hitting the surface. In both cases, a sharp or outstanding portion of the surface is not likely to last long, since it will be hit most often and erode away soon. So the resulting surface is relatively smooth compared with the $\omega=0$ case.

D. $0<\omega<\pi$

We can vary the appearance of the surfaces continuously between these two extremes by changing ω from π to 0. Larger and larger "stalagmites" emerge when we decrease ω. Interestingly, however, we found that $\xi(L,T)$ always approaches a finite saturation value $\xi(L)$ as long as ω is not strictly zero. In other words, the long-range correlations among the heights of the columns remain as long as the horizontal positions where the particles hit depend on the current shape of the surface, which is clearly true for all $\omega>0$. This is crucial since the mechanism to eliminate sharp portions of the surface is present for all $\omega>0$, however weak it might be. Furthermore, for all $\omega>0$, the surface width $\xi(L)$ exhibits the same behavior: $\xi(L)=C+D\sqrt{L}$. Here C,D are constants that depend on ω (see Fig. 11). Thus, $\xi(L)$ in the random ballistic model grows faster than in all the diffusion limited models considered before. In other words, the random ballistic trajectories do not eliminate the sharpness of the surface as effectively as the random walks.

V. SUMMARY AND DISCUSSION

From the above observations, we found that the surface appearance and the asymptotic behavior of its width, which are summarized in Table I, are two independent properties of interface erosion processes. Unlike the Laplacian field cases, the erosion mechanism discussed here is in principle difficult to treat analytically. This is mainly because the particles move around the surface eroding sites, after they reach the surface. Thus, the velocity with which the surface moves is not simply determined by the gradient of the Laplacian field, which gives the probabili-

ty to hit a surface position for the first time only. The fact that the surface height fluctuations generated by the diffusion-limited biased random-walk approach a constant means that there is a strong correlation among the heights of remote columns. We associate this long-range correlation with two mechanisms. First, the particles have the tendency to wander horizontally along the surface since the motion in the vertical direction is confined by the resistance from the solid. Thus, the positions of the sites that will be eroded are not very dependent on where the eroding particle first touches the surface and different portions of the surface move with more uniform probabilities, as in the Eden model [1]. Second, for a biased random walk, the particle can penetrate deeper when it hits a tip than when it hits a valley, again because the valley is surrounded by denser solid sites than the tip is. Thus, tips are going down faster than valleys. This has the effect of shrinking the surface fluctuations. We have confirmed this difference in the penetration depth numerically. Although these two effects certainly contribute to the strong height correlations found in an eroded interface, one should note that the same arguments can be made in the case of the flipping rotator model, which exhibits a logarithmically divergent surface fluctuation. In addition, also in the ballistic erosion process the eroding particles move as biased random walkers after they touch the surface for the first time and feel, therefore, the solid resistance, yet different asymptotic behaviors of the surface widths for the Lorentz gas and the ballistic models result. This shows that the interplay of the dynamics of the particle motion before reaching the surface and the erosion mechanism at the surface itself are both important in the determination of the surface properties. We have not yet been able to disentangle fully this complicated interplay between the erosion effects of the particle motions above and at the surface.

ACKNOWLEDGMENTS

We thank Professor H. van Beijeren and Professor R. Ziff for helpful suggestions and discussions. We are grateful to the Computing Services department at the Rockefeller University, as well as to Mr. F. Wang, for their help with the figures. We also acknowledge support from the Department of Energy under Contract No. DE-FG02-88-ER13847.

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