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Description of the Even Tin Isotopes in a Mixing of Shell Model and IBA Model

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The positive parity energy levels of even $^{108-124}$Sn isotopes are studied in terms of a mixing of shell model and interacting boson approximation model. In the calculation, the basis states consist of a pure boson configuration and N_B-1 bosons plus a fermion pair configuration, where N_B is the valence boson number outside the closed shell. The fermions are allowed to excite to the $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$ and $h_{11/2}$ single particle orbitals. The interaction Hamiltonian contains three parts: pure boson Hamiltonian, pure fermion Hamiltonian and boson-fermion mixing Hamiltonian. The calculated energy levels are compared with the observed data. The wavefunctions are used to calculate $B(E2)$ values. It is found that the calculated energy spectra and $B(E2)$ values agree reasonably well with the experimental values.

§ 1. Introduction

The nuclear properties of doubly-even Sn nuclei have been investigated recently by a number of experimental and theoretical works.[1)~34)] This is because Sn isotopes have been a testing ground for models and methods in nuclear structure calculations.[35)] The low-lying structures of such nuclei were found to be dominated by a variety of complicated excitations. Different band structures of Sn isotopes have been observed[8),12)] contrary to the expectation from simple shell model. On the basis of the shell model,[4),22)] various approximation methods have been developed and a quantitative understanding of level energies and transition rates in these semimagic isotopes has been obtained. It was found that shell effects might be very pronounced in the neutron-deficient Sn isotopes which have a closed proton shell and a small number of valence nucleons. Clement and Baranger[4)] performed a shell model calculation of even-mass Sn isotopes with mass number $A=108-124$. In their work, the structures of the collective 2^+ and 3^- states of even Sn isotopes were studied and the importance of core excitation was investigated. Since the late seventies, deformed states have been observed in the heavier Sn isotopes.[6)~8),12),15)] These states form a rotational-like band and can be distinguished from spherical states of quasiparticle excitation. These states have been interpreted as a consequence of two-particle two-hole excitations across the proton closed shell in terms of the spherical model. The level structures of even Sn isotopes for mass number around $A=108-124$ have also been studied in terms of the broken-pair model.[8),10),22),23),26)] The zero- and one-broken-pair states with two-broken-pair components were mixed[10)] to investigate the spectroscopy of even Sn nuclei for mass number $A=112-120$. Neutron two-quasiparticle calculations in BCS approach[36)] were used to investigate the experimental data of tin isotopes. Recently, odd and even Sn isotopes were also studied[34)]

within a quasiparticle multistep shell-model method.

It is known that the traditional interacting boson approximation[37] can describe the nuclear collective motion for nuclei which are far away from the closed shell. Therefore, it is not suitable to study tin isotopes which are close to the doubly magic core by using the traditional IBA model. It is also not suitable to describe the level structure of tin isotopes by the shell model because the model space is too large to be manageable for tin isotopes considered in the present work. Therefore, we report here a study on even Sn isotopes by means of the mixing of shell model and the interacting boson approximation model with one boson being allowed to break into a fermion pair. This model has been applied to study the high spin states of even Pt-isotopes.[38] Because of the spin-coupling of bosons and fermions, high spin states of Pt-isotopes are not allowed in the traditional IBA model but they are able to be well described by the extension of the IBA model as far as the level energies are concerned. The purpose of this investigation is twofold: in the first place we want to present a systematical study of the structure of positive parity energy spectra of even-mass Sn isotopes. Second and the most important, we desire to investigate to what extent the extension model of IBA can be applied to interpret the observed low lying energy states of even-mass tin isotopes. These isotopes have closed proton shell $Z=50$ and small boson numbers. They cannot be described satisfactorily by the shell model and IBA model. The rest part of this paper is organized as follows: In § 2 we describe the model. In § 3 we present results of this work. The fourth section presents our discussion and conclusions. A brief summary will be presented in the final section.

§ 2. The model

The even-mass Sn isotopes with $Z=50$ and $124\geq A\geq 108$ will be studied systematically. Taking $Z=50$, $N=50$ as a closed shell, the boson numbers for isotopes $^{108-116}$Sn are $N_B=4, 5, 6, 7$ and 8, respectively. For the other isotopes which pass the neutron midshell, $Z=50$, $N=82$ is taken as a closed shell, the neutron boson numbers are counted as one-half of the number of neutron holes. Therefore, isotopes $^{118-124}$Sn have valence boson (hole) numbers 7, 6, 5 and 4, respectively. In this work it is assumed that one of the bosons can be broken to form a fermion pair. The fermions are allowed to occupy the $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$ and $h_{11/2}$ single particle orbitals.

Our model space includes an IBA subspace with N_B bosons and a subspace with N_B-1 bosons plus two fermions. Explicitly, it can be expressed as:
$|n_s n_d \nu I\rangle \oplus |\tilde{n}_s \tilde{n}_d \tilde{\nu} L, (j_1 j_2) J; I\rangle$, where $n_s+n_d=\tilde{n}_s+\tilde{n}_d+1$ and $J=0, 2, 4, 6, \cdots$.

The model Hamiltonian can be expressed as[38]

$$H=H_B+H_F+V_{BF}\,. \tag{1}$$

H_B is the IBA boson Hamiltonian

$$H_B=\varepsilon_d \hat{n}_d+a_1 P^{\dagger}\cdot P+a_2 L\cdot L+a_3 Q\cdot Q\,, \tag{2}$$

where ε_d is the single d-boson energy and $P^{\dagger}\cdot P$, $L\cdot L$, and $Q\cdot Q$ are the pairing interboson, the spin-dependent and quadrupole interactions respectively and can be

expressed as:

$$\hat{n}_d = d^\dagger \cdot \tilde{d}\,, \tag{3}$$

$$P = (1/2)(\tilde{d}\cdot\tilde{d}) - (1/2)(\tilde{s}\cdot\tilde{s})\,, \tag{4}$$

$$L = \sqrt{10}\,(d^\dagger \times \tilde{d})^{(1)} \tag{5}$$

and

$$Q = Q^B + \alpha \sum_{j_1,j_2} (a^\dagger_{j_1} \times \tilde{a}_{j_2})^{(2)} + \beta \sum_{j_1,j_2} [(a^\dagger_{j_1} \times a^\dagger_{j_2})^{(4)} \times \tilde{d} - d^\dagger \times (\tilde{a}_{j_1} \times \tilde{a}_{j_2})^{(4)}]^{(2)}\,, \tag{6}$$

where $a^\dagger_j$ is the nucleon creation operator and

$$Q^B = (d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d})^{(2)} - \frac{\sqrt{7}}{2}(d^\dagger \times \tilde{d})^{(2)}\,. \tag{7}$$

The octupole term $T_3 \cdot T_3$ and the hexadecapole term $T_4 \cdot T_4$ have been omitted in H_B since they are generally believed to be less important. The fermion Hamiltonian H_F is

$$H_F = \sum_j \varepsilon_j \sqrt{2j+1}\,(a^\dagger_j \times \tilde{a}_j)^{(0)} + (1/2) \sum_{J,j_1,j_2,j_3,j_4} V_0 \sqrt{2J+1}\,[a^\dagger_{j_1} \times a^\dagger_{j_2})^J \times (\tilde{a}_{j_3} \times \tilde{a}_{j_4})^J]^{(0)}\,. \tag{8}$$

The mixing Hamiltonian V_{BF} is

$$V_{BF} = Q^B \cdot Q - Q^B \cdot Q^B\,, \tag{9}$$

which can be rewritten as

$$V_{BF} = \alpha \sum_{j_1,j_2} Q_B \cdot (a^\dagger_{j_1} \times \tilde{a}_{j_2})^{(2)} + \beta \sum_{j_1,j_2} Q_B \cdot [(a^\dagger_{j_1} \times a^\dagger_{j_2})^{(4)} \times \tilde{d} - d^\dagger \times (\tilde{a}_{j_1} \times \tilde{a}_{j_2})^{(4)}]^{(2)}\,, \tag{10}$$

where the j's run over five single fermion orbits: $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$ and $h_{11/2}$. In the above expressions, as usual the boson-fermion exchange interaction is not included.[38)~40)] The constants α and β in principle depend on j_1 and j_2. For a quadrupole force one may expect that α is roughly proportional to the reduced matrix elements of Y_2 or $r^2 Y_2$. For partially filled orbits one expects in addition a modification along the line predicted by BCS. Similar arguments may apply to β. However, in this calculation we assume that the parameters α and β are independent of the quantum numbers of the single particle state energies because tin isotopes considered in our work are in the vicinity of a closed shell, and therefore, the variation of single particle energies is not large. In the calculation of two-body matrix elements, the form of fermion potential is assumed to be the Yukawa type with Rosenfeld mixture and the harmonic oscillator wavefunction is adopted. The oscillator constant $\nu = 0.96A^{-1/3}\,\mathrm{fm}^{-2}$ with $A=100$ is assumed. The parameter V_0 in Eq. (8) is the interaction strength of fermion-fermion interaction which is assumed to be -60 MeV. The other interaction strength parameters contained in H_B and single particle energies contained in the total Hamiltonian are chosen to reproduce the energy level spectra of even Sn isotopes with mass number between 108 and 124. In the shell-model calculation, energy spacings between $s_{1/2}$ and $d_{3/2}$ orbitals, and between $d_{5/2}$ and $g_{7/2}$ orbitals are found to be very

small. To reduce the number of parameters, we thus assume that the single particle energies of the $s_{1/2}$ and $d_{3/2}$ orbitals in all tin isotopes have the same value, and the energies of the single particle orbits $d_{5/2}$ and $g_{7/2}$ are also assumed to be the same value. The energy bands with the above-mentioned two kinds of configurations are mixed through the diagonalization of the energy matrix in the whole model space.

§ 3. Results

The interaction strengths and single-particle energies for Sn isotopes are allowed to be mass number dependent. Table I shows the best fitted interaction strengths, single particle energies, the number of levels included in the least squares fitting (n) and the root mean square (rms) deviations of the calculated level energies from the observed data for all isotopes in the present calculation. The interaction strength a_3 of the quadrupole term $Q\cdot Q$, and the mixing parameters α and β are found to be able to be unified as (in MeV) $a_3=0.033$, $\alpha=0.0$ and $\beta=0.025$. One can note from Table I that the interaction strength for the d-boson energy ε_d decreases monotonically from isotope ^{108}Sn to ^{116}Sn and then increases monotonically from the nucleus ^{118}Sn to ^{124}Sn; the parameter a_1 of the pairing term $P^\dagger\cdot P$ decreases from ^{108}Sn to ^{110}Sn and then changes sign from ^{110}Sn to ^{112}Sn. It decreases again from ^{112}Sn to ^{116}Sn. Finally a significant change of a_1 happens from ^{116}Sn to ^{118}Sn and then becomes a constant from ^{118}Sn to ^{124}Sn; the parameter a_2 of the $L\cdot L$ term increases monotonically from isotope ^{108}Sn to ^{116}Sn and then decreases monotonically from ^{118}Sn to ^{124}Sn. These parameters have significant changes around the isotopes ^{116}Sn and ^{118}Sn. This is because in our work we assume the $Z=50$, $N=50$ nucleus as a closed shell for $^{108-116}$Sn isotopes, and the boson number for these nuclei is counted as one-half of the number of nucleons outside the closed shell, while for the other Sn isotopes ($^{118-124}$Sn) that pass the neutron midshell, the nucleus $Z=50$, $N=82$ is assumed as a closed shell and the neutron boson numbers are counted as one half of the number of neutron holes. Therefore, we have particle-particle to hole-hole transition from ^{116}Sn nucleus to ^{118}Sn nucleus. It is not surprising that we have a significant change of the interaction parameters of ε_d, $P^\dagger\cdot P$ and $L\cdot L$ around the $A\sim116$ region. The boson pairing interaction $P^\dagger\cdot P$ is normally

Table I. The interaction parameters and single particle energies (in MeV) adopted in this work.

Nucleus	ε_d	a_1	a_2	$\epsilon_{1/2}$ ($\epsilon_{3/2}$)	$\epsilon_{5/2}$ ($\epsilon_{9/2}$)	$\epsilon_{11/2}$	n	rms
			particle-particle					
^{108}Sn	1.2750	0.1500	−0.0054	1.60	1.43	2.310	12	0.100
^{110}Sn	1.1167	0.0616	−0.0048	1.60	1.43	2.310	14	0.094
^{112}Sn	0.9020	−0.0180	0.0008	1.60	1.43	2.310	15	0.097
^{114}Sn	0.7167	−0.0568	0.0074	1.60	1.43	2.003	18	0.113
^{116}Sn	0.5121	−0.0687	0.0200	1.40	1.22	1.583	17	0.112
			hole-hole					
^{118}Sn	0.7220	−0.0100	0.0318	1.75	1.55	1.562	12	0.118
^{120}Sn	0.7529	−0.0100	0.0315	1.57	1.55	1.454	11	0.082
^{122}Sn	0.9215	−0.0100	0.0048	1.57	1.60	1.365	13	0.096
^{124}Sn	1.0196	−0.0100	−0.0032	1.57	1.60	1.338	10	0.085

responsible for the corresponding fermion interaction and its strength parameter in general should not change from a microscopic point of view. One can note from Table I that our parameter a_1 keeps constant for isotopes $^{118-124}$Sn and thus it is consistent with a microscopic point of view. However, the parameter a_1 decreases monotonically from ^{108}Sn to ^{116}Sn. The reason for this inconsistency with a microscopic point of view can be explained as follows. It is known that for a nucleus in the $O(6)$ limit, only $P^\dagger \cdot P$ and $L \cdot L$ terms appear in the Hamiltonian H_B; and for a nucleus in the $SU(3)$ limit, only $L \cdot L$ and $Q \cdot Q$ terms appear in H_B.[41] From the correlation of the variation of interaction parameters to the limiting symmetries, the decreasing of the parameter a_1 manifests that tin isotopes have the tendency of deviation from $O(6)$ symmetry to become the $SU(3)$ symmetry as the mass number A changes from $A=108$ to 116. One can note from Table I that the single particle energies $\epsilon_{1/2}$, $\epsilon_{3/2}$, $\epsilon_{5/2}$, $\epsilon_{9/2}$ and $\epsilon_{11/2}$ vary smoothly with respect to the mass number A. This is due to the following reasons: the nuclei under considering are near the closed shell and the mass region of the nuclei considered in this work is not large. In addition to these, according to our model tin isotopes have closed proton shells, thus only $n-n$ (neutron-neutron) interactions are involved. This makes the single particle energies vary smoothly with respect to the mass number. One can also note from Table I that the interaction parameters and single particle energies can most likely be unified for the isotopes which are close to the closed shell. For example, the parameters of the isotopes ^{108}Sn, ^{110}Sn and ^{112}Sn are almost the same. A similar situation can be found also in the isotopes ^{122}Sn and ^{124}Sn.

The calculated and observed energy spectra of the Sn isotope chain are shown in Figs. 1~4. In these figures, we present all states which are included in the least squares fitting. For those states which are not included in the least squares fitting, we present only the lowest energy level of each angular momentum. The states with too high excitation energy are also omitted to make the figure as clear as possible. The states marked with asterisks are not included in the fitting. These states are beyond our model space because they contain core excitation configuration or the configuration of breaking two bosons into two fermion pairs. In Fig. 1 we compare the calculated energy levels with observed data for isotopes ^{108}Sn and ^{110}Sn. There have been abundant experimental data for the isotopes ^{108}Sn[1)~3)] and ^{110}Sn[3),5)~8)] observed in recent years. In the ^{108}Sn nucleus, the experimental data reveal that there exists a distinct positive-parity band up to the (22^+) state, while in the ^{110}Sn nucleus, the positive-parity band is found up to the (20^+) state. It was found[2)] that a deformed band above the $J=14\hbar$ level can be identified in the nucleus ^{108}Sn and a collective behavior exists in the yrast states of ^{110}Sn above the 10_3^+ state. The even-mass Sn isotopes have been studied by using shell-model calculation.[4)] Fairly good results have been obtained. In our calculation, one can note from Fig. 1 that energy levels for the ^{108}Sn nuclei can be reproduced quite satisfactorily. For the nucleus ^{110}Sn, the large deviation of 4^+ states may be due to the inclusion of 0_4^+, 2_4^+ and 2_5^+ states in the nucleus with a small boson number (the boson number of ^{110}Sn is only 5). In recent years, intruder states in nearly single-closed shell nuclei have been investigated both experimentally and theoretically.[6)~8),12),15)] In even-even nuclei near $Z=50$ and 82, these states are known as the 0_2^+ states at very low excitation energies and several

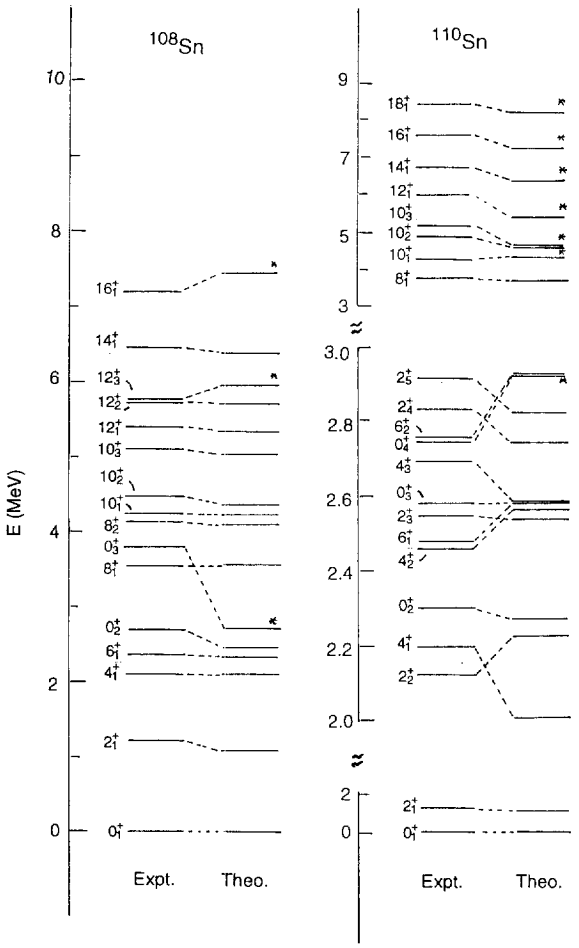

Fig. 1. Calculated and observed energy spectra for the nuclei ^{108}Sn and ^{110}Sn.

collective states which have been found to be built on them.[12] They are understood as proton excitations through the closed shell to the next major shell.[7] In the isotope ^{110}Sn, the levels such as 0_2^+, 12_1^+, 14_1^+, 16_1^+ and 18_1^+ are assigned to the members of the intruder states.[7] Since in our model we do not consider the core excitation, thus in the beginning of our calculation these states were not included in the least squares fitting. However, large rms deviation was found in our calculation unless the 0_2^+ state was included in the fitting. This may be ascribed to the reason that the 0_2^+ state is not a pure intruder state. In fact, the wave function analysis, as presented in Table II, manifests the 0_2^+ consists mainly of the pure boson configuration and thus cannot be regarded as a pure intruder state. One can see from Fig. 1 that surprisingly, our calculated results for other intruder states still agree reasonably well with observed values although they are excluded from the least squares fitting. The satisfactory reproduction of intruder states in the present calculation may be ascribed to the reason that these spurious states may not be pure intruder states, instead, they may be mixings of the core excitation configuration and the core inert configuration. In

Table II. The relative intensities of wave functions for energy levels of isotopes ^{108}Sn, ^{110}Sn and ^{112}Sn. The total intensity is normalized to 1.0. j_1 (or j_2)$=s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$ or $h_{11/2}$, and $j_1 \neq j_2$.

nucleus	states	0	$s^2_{1/2}$	$d^2_{3/2}$	$d^2_{5/2}$	$g^2_{9/2}$	$h^2_{11/2}$	$j_1 j_2$
^{108}Sn	0_1	1.000	0.000	0.000	0.000	0.000	0.000	0.000
	0_2	0.000	0.000	0.000	0.500	0.500	0.000	0.000
	0_3	0.997	0.000	0.000	0.001	0.001	0.000	0.001
	0_4	0.000	0.500	0.500	0.000	0.000	0.000	0.000
	2_1	0.998	0.000	0.000	0.000	0.000	0.000	0.002
	4_1	0.080	0.000	0.000	0.262	0.177	0.000	0.481
	6_1	0.000	0.000	0.000	0.000	0.259	0.001	0.740
	8_1	0.000	—	0.000	0.000	0.500	0.000	0.500
	8_2	0.942	—	0.000	0.012	0.012	0.001	0.033
	10_1	—	—	—	0.000	0.000	0.000	1.000
	10_2	—	—	—	0.000	0.500	0.000	0.500
	10_3	—	—	—	0.003	0.028	0.920	0.049
	12_1	—	—	—	—	0.000	0.000	1.000
	12_2	—	—	—	—	0.500	0.000	0.500
	12_3	—	—	—	—	0.135	0.735	0.130
	12_4	—	—	—	—	0.025	0.941	0.034
	14_1	—	—	—	—	0.000	0.000	1.000
	14_2	—	—	—	—	0.000	0.004	0.996
	16_1	—	—	—	—	—	0.000	1.000
^{110}Sn	0_1	1.000	0.000	0.000	0.000	0.000	0.000	0.000
	0_2	0.995	0.000	0.000	0.001	0.001	0.000	0.003
	0_3	0.000	0.000	0.000	0.500	0.500	0.000	0.000
	0_4	0.000	0.500	0.500	0.000	0.000	0.000	0.000
	0_5	0.990	0.000	0.000	0.002	0.002	0.000	0.006
	2_1	0.998	0.000	0.000	0.000	0.000	0.000	0.002
	2_2	0.996	0.000	0.000	0.001	0.001	0.000	0.002
	2_3	0.000	0.000	0.001	0.318	0.318	0.026	0.337
	4_1	0.946	0.000	0.000	0.010	0.010	0.002	0.032
	4_2	0.014	0.000	0.000	0.001	0.001	0.003	0.981
	4_3	0.030	0.000	0.000	0.319	0.319	0.008	0.324
	6_1	0.000	0.000	0.000	0.000	0.500	0.000	0.500
	6_2	0.949	0.000	0.000	0.009	0.009	0.001	0.032
	8_1	0.961	0.000	0.000	0.007	0.004	0.001	0.027
	10_1	0.000	—	0.000	0.000	0.000	1.000	0.000
	10_2	0.000	—	0.000	0.000	0.500	0.000	0.500
	10_3	0.954	—	0.000	0.008	0.008	0.001	0.029
	12_1	—	—	—	0.000	0.000	1.000	0.000
	14_1	—	—	—	—	0.000	1.000	0.000
	16_1	—	—	—	—	0.000	1.000	0.000
	18_1	—	—	—	—	—	1.000	0.000
^{112}Sn	0_1	1.000	0.000	0.000	0.000	0.000	0.000	0.000
	0_2	0.996	0.000	0.000	0.001	0.001	0.000	0.002
	2_1	0.998	0.000	0.000	0.000	0.000	0.000	0.002
	2_2	0.997	0.000	0.000	0.001	0.001	0.000	0.001
	2_3	0.000	0.000	0.002	0.318	0.318	0.021	0.341
	4_1	0.962	0.000	0.000	0.005	0.003	0.000	0.030
	4_2	0.014	0.000	0.000	0.002	0.000	0.006	0.978

(continued)

4_3	0.000	0.000	0.044	0.062	0.044	0.190	0.660
6_1	0.000	0.000	0.000	0.000	0.244	0.001	0.755
6_2	0.000	0.000	0.000	0.000	0.743	0.025	0.232
6_3	0.933	0.000	0.000	0.013	0.018	0.004	0.032
8_1	0.000	0.000	0.000	0.000	0.489	0.021	0.490
8_2	0.938	0.000	0.000	0.012	0.010	0.005	0.035
10_1	0.000	0.000	0.000	0.000	0.000	1.000	0.000
10_2	0.942	0.000	0.000	0.010	0.006	0.006	0.036
12_1	0.000	—	0.000	0.000	0.000	1.000	0.000
12_2	0.000	—	0.000	0.000	0.500	0.000	0.500
14_1	—	—	—	0.000	0.500	0.000	0.500
16_1	—	—	—	—	0.000	1.000	0.000
18_1	—	—	—	—	0.000	1.000	0.000
20_1	—	—	—	—	—	1.000	0.000

Sn isotopes the intruder states were usually interpreted as a consequence of two-particle two-hole excitations across the proton closed shell in terms of the spherical model. Within the framework of IBA model these two-particle two-hole fermion configurations can be mapped onto a two-boson state making no distinction between the particle boson or hole boson.[42] Therefore, it is expected that such intruder states should contain the admixtures of N_B and N_B+2 configurations for these intruder states. Hence the reproduction of these states in this work seems to support the above interpretation.

Figure 2 shows the calculated and observed energy levels for the nuclei ^{112}Sn and ^{114}Sn. These two nuclei have been intensively investigated both theoretically[8),10)~12)] and experimentally.[7)~9),12)~16)] It was found that for the nucleus ^{112}Sn a distinct positive-parity band up to the 22^+ state exists, and an intruder band which starts from the 6^+ state was observed. The collective bands existing in these two nuclei can be in general reproduced by the proton excitation model. Wenes et al.[11] studied the nuclear structures of doubly-even Sn nuclei $^{112-118}$Sn by constructing a model taking into account both pure quadrupole vibrational excitations as well as proton $2p-2h$ configurations coupled with the quadrupole vibrational excitations. Satisfactory results were obtained by them. Recently, Harada et al.[15] used a model based on the spin-projected Hartree-Bogoliubov model to study high spin energy levels and back-bending phenomena of the nucleus ^{114}Sn. The backbending phenomenon in the intruder band of ^{114}Sn was reproduced satisfactorily. One can note from Fig. 2 that our calculation can also reproduce the energy spectra of ^{112}Sn reasonably well, especially the high spin states. In ^{114}Sn, states 0_2^+, 6_3^+, 8_1^+, 10_2^+, 12_1^+, 12_2^+, 14_1^+, 16_1^+, 18_1^+ and 20_1^+ were assigned to the intruder bands.[7] In our calculation, we include 0_2^+, 6_3^+, 8_1^+, 10_2^+, 12_1^+, 12_2^+ states in our least squares fitting in order to yield smaller rms deviation. Furthermore, the calculated energy values for 14_1^+, 16_1^+, 18_1^+ and 20_1^+ states agree also reasonably with their experimental counter parts although in our model we did not include the core excitation configuration. The reason of the coincidence may be ascribed to the reason that these states may not be pure intruder states, instead, they may include the core excitation configuration and the core inert configuration altogether as mentioned above. For nucleus ^{114}Sn, the calculated energy levels agree quite reasonably well with the experimental values. Most of the

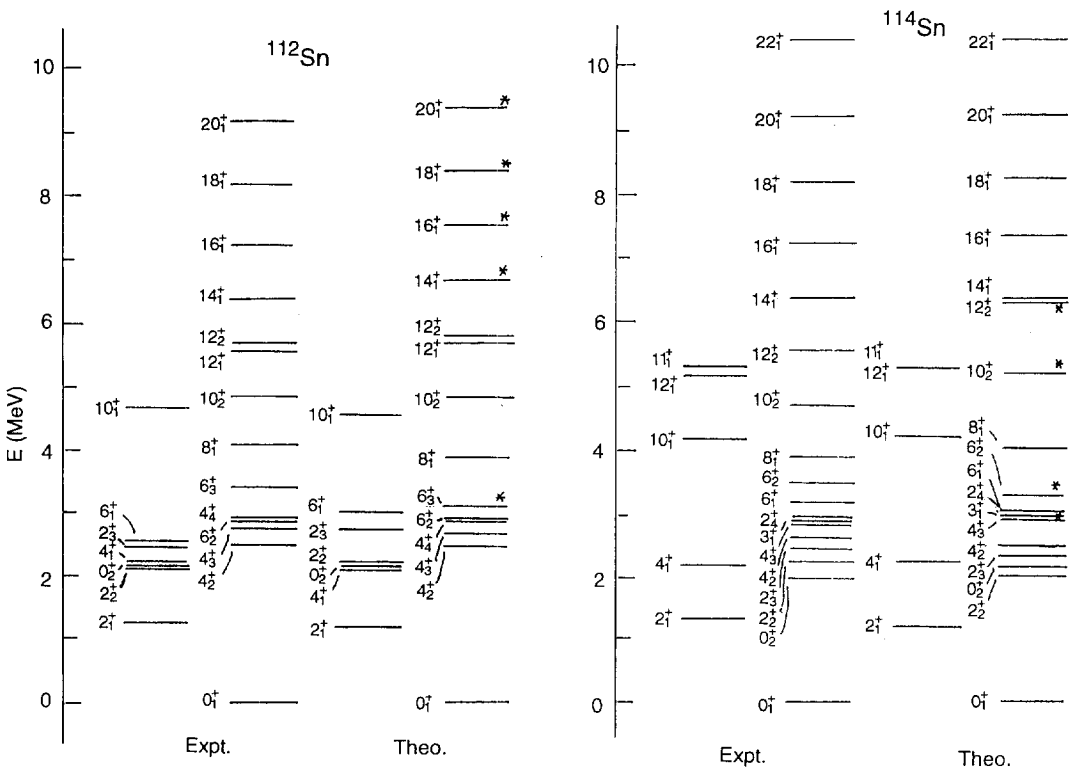

Fig. 2. Calculated and observed energy spectra for the nuclei ^{112}Sn and ^{114}Sn.

calculated and observed levels agree within a few tens of KeVs. There are only two levels, 0_2^+ and 2_2^+ which are calculated in reversed order.

The calculated and observed energy levels for the nuclei $^{116-118}$Sn are shown in Fig. 3. Many works have investigated the level structures of the nuclei $^{116-118}$Sn experimentally and theoretically.[18)~26)] Raman and his co-workers[20),21)] studied the level scheme of ^{116}Sn by combining the results of ^{115}Sn$(n, \gamma)^{116}$Sn and ^{116}Sn$(n, n'\gamma)^{116}$Sn experiments. They constructed a nearly complete level scheme of the ^{116}Sn nucleus by comparing the observed data to the combined predictions of the two-broken-pair model, the interacting boson model and the deformed collective model. From a neutron pickup reaction,[19)] it can be found that the neutron shell occupations in the nucleus ^{116}Sn are not restricted to $2d_{5/2}$ and $1g_{7/2}$ orbitals only, but other orbitals, $1h_{11/2}$, $2d_{3/2}$ and $3s_{1/2}$, have also sizable strengths. The broken-pair model was usually adopted to study the energy levels for these two nuclei and reasonable results were obtained for the lower collective states. Our calculation for the nuclei ^{116}Sn yields satisfactory agreement with the observed data. Most of the calculated levels are in correct order except 2_2^+ and 3_1^+ states. For nucleus ^{118}Sn, our theoretical results show that reasonable agreement exists between the theoretical and experimental data except for 0_2^+, 2_2^+, 2_3^+, 2_4^+, 2_5^+ and 6_1^+ states.

Figure 4 shows the calculated and observed energy spectra for the doubly even nuclei $^{120-124}$Sn. There are abundant observed data for these three nuclei.[24),26)~33)] The experimental level schemes of these nuclei have been compared with the level

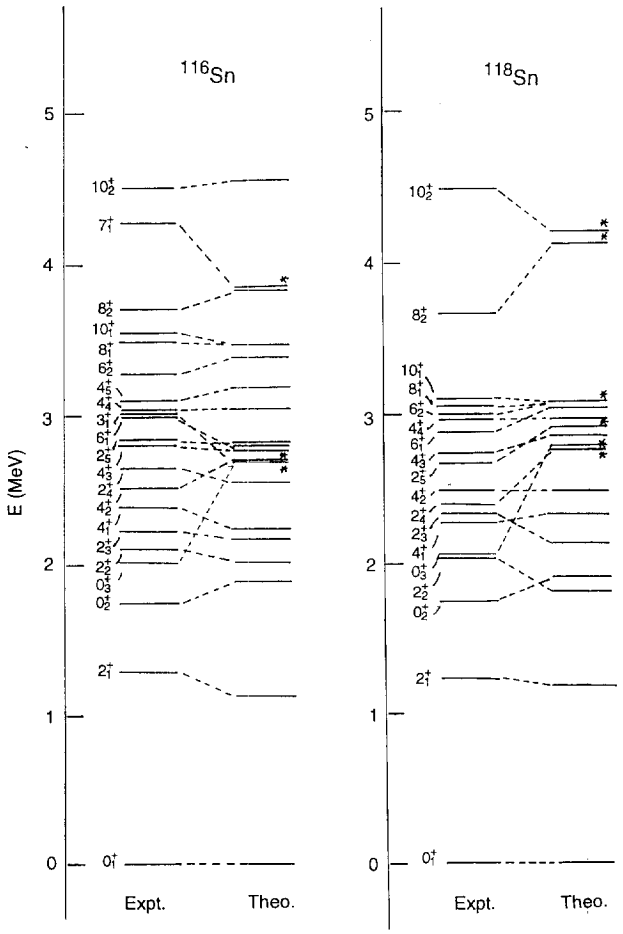

Fig. 3. Calculated and observed energy spectra for the nuclei ^{116}Sn and ^{118}Sn.

schemes calculated on the basis of a broken-pair model which included up to two broken pairs,[26] and reasonable agreement has been obtained. Our calculation on these three nuclei shows that satisfactory agreement can be obtained for the lower lying levels. However, some calculated levels almost shrink together in the level spectra of nuclei ^{122}Sn and ^{124}Sn.

We also analyze our wave function for each state of these even-even Sn isotopes to study the relative intensities of the pure N_B bosons configuration and the N_B-1 bosons plus two fermions configuration. For illustration, we list the relative intensities of the wave functions for isotopes ^{108}Sn, ^{110}Sn and ^{112}Sn in Table II. For the nucleus ^{108}Sn only 0_1^+, 0_3^+, 2_1^+ and 8_2^+ states are dominated by the pure boson configuration. All other states contain significant components of the configuration of N_B-1 bosons coupled with two fermions. Most of the states with N_B-1 bosons coupled with two fermions occur with a fermion pair in the same orbit. Table II also shows that states 0_1^+, 0_2^+, 0_3^+, 2_1^+, 2_2^+, 3_3^+, 4_1^+, 4_4^+ and 6_2^+ in ^{116}Sn; states 0_1^+, 0_2^+, 0_3^+, 2_1^+, 2_3^+ and 4_2^+ in ^{118}Sn; states 0_1^+, 0_2^+, 0_3^+, 2_1^+, 2_2^+ and 4_2^+ in ^{120}Sn; states 0_1^+, 0_2^+, 2_1^+,

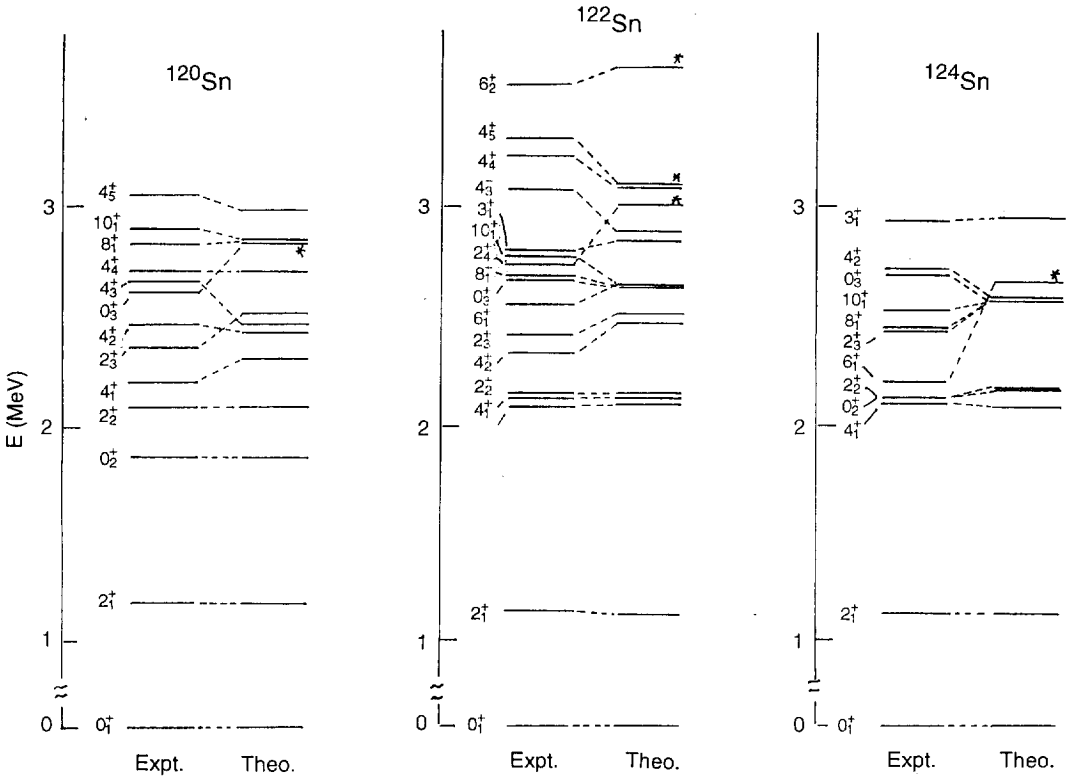

Fig. 4. Calculated and observed energy spectra for the nuclei ^{120}Sn, ^{122}Sn and ^{124}Sn.

2_2^+, 2_4^+ and 4_1^+ in ^{122}Sn; and states 0_1^+, 0_2^+, 2_1^+, 2_2^+ and 4_1^+ in ^{124}Sn are all dominated by pure boson configurations. One can note from Table II that the 0_2^+ state in ^{108}Sn consists mainly of N_B-1 bosons configuration, while in other nuclei the 0_2^+ state consists of pure boson configuration. This can be attributed to the reason below. The isotope ^{108}Sn lies closer to the doubly closed shell and its 0_2^+ state is higher than the 6_1^+ state. However, in isotopes ^{110}Sn and ^{112}Sn, their 0_2^+ states are all lower than the corresponding 6_1^+ states. Hence, the 0_2^+ state in ^{108}Sn possesses relatively higher excitation energy than those 0_2^+ states in other two isotopes. And this is why it consists mainly of N_B-1 bosons configuration. One can see, from Table II, that, in those states which have both N_B boson and N_B-1 bosons plus two fermions configurations, the intensities of the former component are in general close to either 1 or 0. This means the mixing between the two configurations is usually small. However, the inclusion of the configuration of boson breaking is still necessary in some cases although the mixing is small. Because of the spin-coupling of bosons and fermions, high-spin states are not allowed in the traditional IBA model but they can be well described by the extension of the IBA model as far as the level energies are concerned.[38] In Sn isotopes, the proton shell is closed, the level structure thus cannot be well described by the traditional IBA model. However, as we have seen from Tables II and III, our extension of the IBA model with a small mixing between the configurations of the pure boson and N_B-1 bosons plus two fermions can well

describe the level energies and the $B(E2)$ values of the Sn isotopes.

Some experimental $B(E2)$ values for tin isotopes[1),3),5),9),13),18),25),27),28),30)~33)] have been observed. A study of $B(E2)$ values will give us a good test of the model wave functions. The electric quadrupole operator can be written as

$$T(E2)=e^B Q^B+e^F\alpha\sum_{j_1,j_2}(a^\dagger_{j_1}\tilde{a}_{j_2})^{(2)}+\beta e^B\sum_{j_1,j_2}[(a^\dagger_{j_1}a^\dagger_{j_2})^{(4)}\tilde{d}-d^\dagger(\tilde{a}_{j_1}\tilde{a}_{j_2})^{(4)}]^{(2)}, \qquad (11)$$

where Q^B is taken as

$$Q^B=(d^\dagger\tilde{s}+s^\dagger d)^{(2)}-\kappa(d^\dagger\tilde{d})^{(2)}. \qquad (12)$$

In our calculation the fermion effective charge e^F is assumed to be 0.5 eb and the boson effective charge e^B is assumed to be 0.1 eb. It was found that a different fermion effective charge cannot yield a significant change in $B(E2)$ values. The parameters α and β are assumed to be the same as those used in the mixing Hamiltonian. Two different values of κ are chosen in our calculation; one is chosen as 0.0 (calculation I) and the other is chosen as $-\sqrt{7}/2$ (calculation II) which is one of the generators of the $SU(3)$ group. It is found that the different values of κ can only yield some change in the $B(E2)$ values of the inter-band transitions. For illustration, we plot the calculated $B(E2)$ values for the yrast band transitions for the nucleus ^{114}Sn by using two different values of κ.

Table III. The calculated and observed $B(E2)$ values for Sn isotopes. The results listed in the column abbreviated as "cal. I" are obtained by using $\kappa=0.0$ and those listed in the column abbreviated as "cal. II" are obtained by using $\kappa=-1.322$.

A	J_i	J_f	$\Gamma_\nu(10^{-n}e^2b^2)$			n
			expt.	cal. I	cal. II	
112	2_1	0_1	4.87	4.02	4.23	2
	2_2	0_1	1.25	20.3	1.57	4
	4_1	2_1	1.79	6.82	6.64	2
	6_1	4_1	1.65	0.00	0.00	3
114	2_1	0_1	5.25	3.80	4.07	2
	0_2	2_1	7.22	9.25	8.20	2
	4_1	2_1	—	6.59	6.64	2
	8_1	6_1	—	2.07	2.18	3
116	2_1	0_1	3.90	3.47	3.80	2
	0_2	2_1	6.05	8.93	8.07	2
	0_3	2_1	1.65	9.05	0.91	3
	2_2	0_3	1.14	0.13	0.23	1
	2_2	0_2	7.39	0.72	0.20	2
	2_2	2_1	1.31	5.62	7.84	2
	2_2	0_1	2.00	2.92	0.25	3
	4_1	2_1	6.38	5.93	6.12	2
118	2_1	0_1	4.33	4.15	4.45	2
	0_2	2_1	6.53	9.77	8.41	2
120	2_1	0_1	5.94	3.80	4.06	2
	0_2	2_1	6.53	8.88	7.44	2
	2_2	2_1	2.11	5.90	8.22	2
	2_2	0_1	7.73	26.2	2.42	4
122	2_1	0_1	3.84	3.60	3.77	2
	0_2	2_1	<2.19	7.98	6.42	2
	4_1	2_1	3.59	5.71	5.45	2
	4_2	2_1	<2.16	0.32	0.31	2

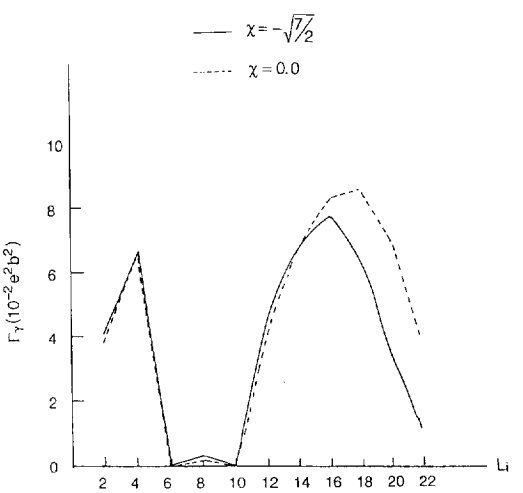

Fig. 5. Calculated and observed $B(E2)$ values of transitions from L_i state to L_i-2 state for the yrast band transitions of the nucleus ^{114}Sn.

As shown in Fig. 5, one can note that the different values of κ (the solid line is obtained by using $\kappa=-\sqrt{7}/2$ and the dotted line is obtained by using $\kappa=0.0$) cannot yield an important change in the calculated $B(E2)$ values for the yrast band transitions of the nucleus ^{114}Sn. The deviation becomes prominent only for J_i being equal to or larger than 16. Table III lists the calculated and available observed $B(E2)$ values for Sn isotopes. Two different values of κ are used to calculate the $B(E2)$ values. As shown in the table, the values listed in the fifth column (abbreviated as cal. I) were obtained by using the value of $\kappa=0.0$, while the sixth column (abbreviated as cal. II) are obtained by using the value $\kappa=-\sqrt{7}/2$. One can note from Table III that the calculated $B(E2)$ values for the yrast band transitions are almost insensitive to the different values of κ, while the calculated $B(E2)$ values for the inter-band transitions depend on the choice of the κ value. However, most of our calculated $B(E2)$ values agree with the experimental data within the same order of magnitude.

§ 4. Discussion and conclusions

As we mentioned above, we have studied the structure of the positive parity energy spectra and $B(E2)$ values of Sn isotopes. In a preliminary study of tin isotopes, we found that high spin states of tin isotopes were not able to be reproduced if only pure boson configurations were considered. For example, the highest value of the angular momenta for the states in nuclei ^{112}Sn and ^{114}Sn can be reproduced up to $L=12$ and 14, respectively if only pure boson configurations were taken into account. In the present work we find that accurate results for the energy spectra and $B(E2)$ values can be obtained by considering a mixing of the shell model with the extended IBA model which includes a pure boson configuration and a configuration of a boson being broken into a fermion pair. To compare our results with those obtained by previous works, we list the available calculated results of level energies of even mass tin isotopes in Table IV. Clement and Baranger[4] used shell model to calculate the energy levels of the first 2^+ and 3^- states in ^{116}Sn and ^{120}Sn with the Tabakin interaction as the residual interaction. In their calculation twelve single-particle proton and neutron levels and the BCS quasi-particle approximation were included. Although their calculated 2^+ level energies agree with the experiment within a few tens of KeVs, however, there is no evidence that the level energies for other higher spin states can also be reproduced well in their model. Van Poelgeest et al.[8] investigated high-spin ($J\leq 10$) neutron quasi-particle excitations in the even-mass $^{110-118}$Sn nuclei using Cd($\alpha, 2n\gamma$)Sn reactions. The experimental data were interpreted within the framework of the broken-pair model including proton 1p$-$1h excitations. Although the excitation energies calculated by them can fairly be accounted by their model, however, the experimental transition rates were not able to be reproduced well. Weenes et al.[11] investigated the collective bands in doubly-even Sn nuclei $^{114-118}$Sn. They treated proton two-particle two-hole excitations coupled with spherical quadrupole vibrations in interaction with the low-lying quadrupole vibrational states. The regular $\Delta J=2$ band structure on top of excited $J^\pi=0^+$ states were able to be accounted. One can note from Table IV that although the low lying states for these nuclei were accountable by their model, however, calculations of states in non-ground

Table IV. Comparison of available calculated level energies E_{cal} and observed data E_{expt}.

Nucleus	states	E_{expt}	E_{cal}			
			This work	Ref. 11)	Ref. 20)	Ref. 26)
^{110}Sn	2_1	1.212	1.106			1.011[a)]
	6_1	2.477	2.582			2.248[a)]
^{112}Sn	2_1	1.257	1.159			1.082[a)]
	6_1	2.549	2.733			2.422[a)]
^{114}Sn	0_2	1.953	2.140	1.901		
	0_3	2.155	2.999	2.862		
	2_1	1.300	1.164	1.381		1.49[a)]
	2_2	2.239	2.016	2.324		
	2_3	2.454	2.266	2.693		
	4_1	2.188	2.262	2.581		
	6_1	3.189	3.049	2.970		
	8_1	3.871	4.001	3.582		
	10_1	4.140	4.145	3.983		
	12_1	5.183	5.263	4.901		
^{116}Sn	0_2	1.757	1.893	1.940	1.535	
	0_3	2.028	2.710	2.871	2.242	
	0_4	2.546	2.753		2.367	
	0_5	2.791	3.113		3.598	
	1_1	4.252	2.753		2.730	
	2_1	1.294	1.147	1.402	1.549	1.245[b)]
	2_2	2.112	2.019	2.403	2.200	
	2_3	2.225	2.174	2.702	2.742	
	2_4	2.650	2.560	3.734	2.846	
	2_5	2.844	2.813		3.176	
	3_1	2.996	2.711		2.920	
	3_2	3.180	2.753		3.190	
	3_3	3.315	2.969		3.264	
	3_4	3.371	3.059		3.697	
	4_1	2.391	2.238	2.651	2.501	
	4_2	2.529	2.709	3.065	2.977	
	4_3	2.801	2.786	4.058	3.071	
	4_4	3.046	3.076		3.247	
	4_5	3.212	3.186		3.642	
	6_1	3.032	2.803	3.142	3.345	
	6_2	3.277	3.391	3.970	3.697	
	8_1	3.493	3.478	3.437	3.535	
	8_2	3.714	3.838	4.712	5.380	
	10_1	3.547	3.478	4.552	3.443	3.750[a)]
	10_2	4.507	4.563		5.500	4.768[a)]
^{118}Sn	0_2	1.758	1.908	1.764		1.999
	0_3	2.057	2.847	2.694		2.375
	0_4	2.497	3.048			3.167
	0_5	3.137	3.072			3.622
	2_1	1.230	1.189	1.287	1.236[a)]	1.230
	2_2	2.043	1.819	2.247		2.045
	2_3	2.328	2.145	2.174		2.181

(continued)

	2_4	2.403	2.754			2.550
	2_5	2.677	2.897			2.835
	4_1	2.280	2.345	2.485		2.389
	4_2	2.489	2.477	2.883		2.749
	4_3	2.734	2.867			2.934
	4_4	2.963	2.969			3.053
	6_1	2.879	3.048	2.833		2.856
	6_2	2.999	3.072			3.798
	8_1	3.052	3.072	2.868		2.997
	8_2	3.692	4.134			3.675
	10_1	3.108	3.072	2.906	3.501[a]	3.034
	10_2	4.495	4.225			4.431
	12_1	5.379	4.225	5.242		4.415
^{120}Sn	0_2	1.875	1.863			2.066
	0_3	2.160	2.763			2.761
	0_4	2.587	2.834			3.324
	2_1	1.171	1.168		1.282[b]	1.171
	2_2	2.097	2.097			2.146
	2_3	2.355	2.507			2.364
	4_1	2.194	2.289			2.214
	4_2	2.466	2.438			2.817
	4_3	2.644	2.467			3.174
	4_4	2.698	2.692			3.416
	4_5	3.058	2.982			3.469
	8_1	2.836	2.834			2.823
	10_1	2.902	2.834			2.831
^{122}Sn	0_2	2.088	2.093			2.273
	0_3	2.674	2.629			3.023
	2_1	1.140	1.126			1.140
	2_2	2.153	2.155			2.248
	2_3	2.415	2.509			2.519
	2_4	2.735	3.009			2.826
	2_5	2.775	3.059			3.409
	3_1	2.797	2.847			3.080
	4_1	2.142	2.144			2.175
	4_2	2.331	2.474			2.735
	4_3	3.082	2.895			3.303
	4_4	3.233	3.080			3.536
	4_5	3.305	3.093			3.682
	6_1	2.555	2.629			2.645
	8_1	2.690	2.629			2.807
	10_1	2.765	2.629			2.809

a) Ref. 8).
b) Ref. 4).

state bands usually yielded prominent discrepancies between the theoretical values and observed ones. On the contrary, our calculation for the corresponding levels can yield more accurate results than theirs. Raman and his co-workers studied the nuclear excited states of even-even nuclei ^{116}Sn[20] and $^{118-122}$Sn[26] by combining the results of ^{115}Sn$(n, \gamma)^{116}$Sn and ^{116}Sn$(n, n'\gamma)^{116}$Sn experiments and by means of the decays of $^{118-122}$In isomers. Their observed data were compared with calculated level

energies obtained on the basis of two broken-pair model. One can note from Table IV that, in general, their calculations for the higher excitation states in non-ground state bands yielded larger deviations from the observed data when compared with our calculated results. Especially, the states 0_5, 2_3, 2_5, 4_3, 4_4, 4_5, 6_2, 8_2 and 10_2 of ^{116}Sn; 0_5, 4_3 and 6_2 of ^{118}Sn; 0_4, 4_2, 4_3, 4_4 and 4_5 of ^{120}Sn; 0_3, 2_5, 4_3, 4_4 and 4_5 of ^{122}Sn. All of these states were calculated by them with deviations around several hundred KeVs or even larger when compared with the experimental data. In our calculations, there are several levels, e.g., 1_1 and 3_3 of ^{116}Sn; 8_2 and 12_1 of ^{118}Sn; 2_3 of ^{120}Sn; and 2_4 of ^{122}Sn which are reproduced worse than the previous works. However, one can note from Table IV that our calculated results are superior to those of previous works for most of other levels. Therefore, one can conclude that positive parity states of even-even Sn isotopes with mass number $A=108-124$ can be well accounted systematically by the interacting boson approximation model provided the configuration of a boson being broken into a fermion pair is included in the traditional pure boson configuration.

§ 5. Summary

In summary, we have investigated the structure of the positive parity energy spectra of tin isotopes with mass number between 108 and 124. We mixed the shell model with the IBA model and include the configuration of a boson being broken to form two fermions which can occupy the $s_{1/2}$, $d_{3/2}$, $d_{5/2}$, $g_{7/2}$ and $h_{11/2}$ single particle orbitals. The calculated energy levels and $B(E2)$ values are in satisfactory agreement with the observed values for the whole chain of tin isotopes. The analysis of wave functions shows the mixing between the pure boson and N_B-1 bosons plus two fermions configurations is small. This is similar to the result obtained in the calculation of high spin states of Pt isotopes.[38] Because of spin-coupling of bosons and fermions, high-spin states are not allowed in the IBA model but they can be well described by the extension of the model. In the present study the level energies of Sn isotopes cannot be well described by the use of IBA model because these nuclei are close to doubly closed shell. However, our calculation shows a small mixing between the pure boson configuration and the N_B-1 bosons plus two fermions configuration can well describe the low-lying energy levels in Sn isotopes. The same philosophy is also able to apply to the coupling of three fermions to IBA bosons to explain odd-mass tin isotopes.

Acknowledgements

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