A robust diagnostic approach for uncertain systems: an example for the jet engine sensor systems

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ABSTRACT

This paper presents a novel eigenstructure assignment approach for synthesizing robust fault detection and isolation (FDI) systems with known inputs. After formulating the FDI problem in eigenstructure assignment, we proceed to develop a parametric characterization of all allowable eigenspaces for disturbance decoupling to achieve robust fault detection. In addition to the structured uncertainties, the robustness of the diagnostic observer to unstructured modeling errors is discussed. A numerical algorithm is further proposed to suppress the effects due to the unstructured uncertainties. The overall robustness of the diagnostic strategy is verified through simulation studies on jet engine systems.

Keyword: fault detection and isolation, unknown inputs, eigenstructure assignment, unstructured uncertainties.

1. INTRODUCTION

The methodology of analytical redundancy is widely employed to economically perform fault detection and isolation (FDI) on controlled systems. The FDI concept based on analytical redundancy uses a mathematical model of the dynamic system to generate the redundancy required to detect malfunctions and isolate faulty components. Methods based on analytical redundancy have been surveyed by Frank,[1] Gertler,[2] and Patton and Chen.[3]

Approaches based on state observers are now the most favored and important methods of performing FDI. However, limitations regarding accuracy, reliability, and robustness lead to restrictions on the applicability of model-based FDI in practice. Inevitable mismatches between the actual process and its mathematical model frequently render FDI systems unable to tell faults from disturbances. The result is false alarms which adjudge the physical system abnormal even if no fault has occurred, and missing alarms when unexpected uncertainties are confused with effects due to faults. Robustness thus has become a key issue in model-based FDI. The FDI robustness problem has been studied by Lou et al.,[4] Viswanadham and Srichander,[5] Frank et al.,[6] Patton and Chen,[7] and Frank.[8] In this paper, our interest lies in the problem of structured type of uncertainties in which all uncertainties are summarized as unknown inputs[9] with the known distribution matrix acting on the nominal plant.

Eigenstructure assignment has gained special attention because of its flexibility in MIMO control system design, in which the eigenvalues determine the speed of response and the eigenvectors shape the transient response.[10,11] Patton and Chen[12] first decoupled structured uncertainties and achieved fault detection by means of eigenstructure assignment. However, the applicability of their method is limited because no complete solution and physical design criterion are provided making it difficult for engineers to implement on physical systems. In a similar way, Watanabe and Himmelblau and Wünnenberg and Frank proposed an unknown input observer (UIO) to achieve disturbance decoupling, in which the state estimate errors are decoupled from unknown input. Since the solution of an UIO is more complicated than the concept of disturbance decoupling with eigenstructure assignment, and the purpose of a FDI system is to detect fault occurrence instead of state estimate, we prefer the former way to achieve robust FDI design.

In this paper, we propose a robust FDI design based on disturbance decoupling using eigenstructure assignment. Complete solutions through disturbance decoupling with simple matrix computation are provided. In addition to structured uncertainties, the degree of robustness to unstructured type of uncertainties whose characteristics and distribution are unknown is also discussed. Based on time-domain analysis, we obtain robustness measure to bounded unstructured uncertainties that helps engineers determine which FDI system best satisfies their requirement. A reduced 5th-order model from a 17th-order jet engine model is presented to demonstrate this approach.

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2. ROBUST FAULT DETECTION OBSERVER USING EIGENSTRUCTURE ASSIGNMENT

2.1. Problem statement

Consider the following uncertain system with sensor faults present

$$x(k+1) = Gx(k) + Hu(k) + Ed(k) \quad (1)$$
$$y(k) = Cx(k) + Du(k) + Kf(k), \quad (2)$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ the known input vector, $y(k) \in R^r$ the measured signal vector, $d(k) \in R^q$ formulates the unknown input vector, and $f(k) \in R^p$ denotes a fault vector. G, H, C, D, E and K are known matrices of proper dimensions and $r > q$. Without loss of generality we may assume that C, E, and K have full rank, and the pair (C, G) is completely observable. The term $Kf(k)$ models the sensor fault effects. The FDI observer is constructed as:

$$\hat{x}(k+1) = (G - LC)\hat{x}(k) + (H - LD)u(k) + Ly(k) \quad (3)$$
$$\hat{y}(k) = C\hat{x}(k) + Du(k) \quad (4)$$
$$r(k) = W[y(k) - \hat{y}(k)], \quad (5)$$

where $r(k)$ is the residual vector for fault monitoring, $L \in R^{n \times r}$ is the observer feedback gain matrix, and $W \in R^{p \times r}$ is a constant weighting matrix to determine the dimension of residual vector. The corresponding FDI block diagram is illustrated as in Fig. 1.

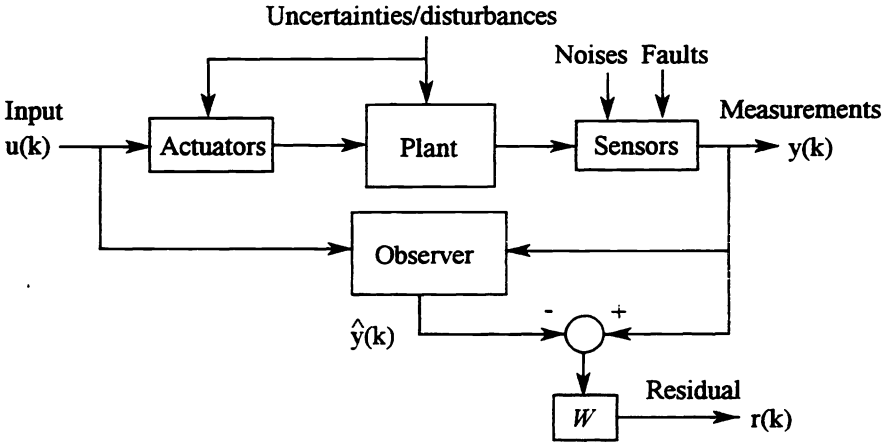

Fig. 1 The block diagram of a FDI observer

To obtain the residual vector dynamics, let $e(k) = x(k) - \hat{x}(k)$ be the state estimation error. By Eqs. (1-5), we obtain

$$e(k+1) = G_L e(k) + Ed(k) - LKf(k) \quad (6)$$
$$r(k) = WCe(k) + WKf(k), \quad (7)$$

where $G_L = (G - LC)$. Therefore, the complete response of the residual vector is

$$r(z) = G_{rd}(z)d(z) + G_{rf}(z)f(z), \quad (8)$$

where

$$G_{rd}(z) = WC(zI - G_L)^{-1}E \quad (9)$$
$$G_{rf}(z) = -WC(zI - G_L)^{-1}LK + WK$$
$$= -W[C(zI - G_L)^{-1}L - I]K. \quad (10)$$

To achieve robust fault detection, the following conditions must hold true:

(i) $G_{rd}(z) = 0$, which decouples disturbances from residual vector $r(k)$;

(ii) $[G_{rf}(z)]_i \neq 0$ and $[G_{rf}(1)]_i \neq 0$, where the subscript i represent the i-th column of $G_{rf}(z)$, which makes the i-th fault detectable by the residual vector $r(z)$.

Expanding Eq. (9) in eigenstructure, we have

$$G_{rd}(z) = WC\sum_{i=1}^{n}\frac{v_1 p_i^T}{(z-\lambda_i)}E. \tag{11}$$

There are two different ways to make $G_{rd}(z) = 0$ with eigenstructure assignment. One is to choose a $W \in R^{p\times r}$ that satisfies $WCE = 0$ first and, secondly, assign the rows of WC to be the first p left eigenvectors of $(G - LC)$.[10] This assignment gives that $WCv_i = 0$ for $i = p+1,\ldots,n$ and $p_i^T E = \langle WC \rangle_i\ E = 0$ for $i = 1,2,\ldots,p$, and yields that

$$G_{rd}(z) = WC\sum_{i=1}^{n}\frac{v_1 p_i^T}{(z-\lambda_i)}E = WC\sum_{i=1}^{p}\frac{v_1 p_i^T}{(z-\lambda_i)}E = 0$$

However, it is seen that the selection of weighting matrix W determines the assignability of eigenvectors. In other words, the rows of WC may be unassignable to the first p left eigenvectors of $(G - LC)$ even if $WCE = 0$. An alternative method is to first determine the last $(n-q)$ left eigenvectors which satisfy $p_i^T E = 0$ for $i = q+1,\ldots,n$ from the assignable eigenspaces of pair (C, G), then, arbitrarily assign the remaining eigenvectors from allowable eigenspaces and determine $W \in R^{p\times r}$, which makes $WCv_i = 0$ for $i = 1,2,\ldots,q$. The unknown inputs are decoupled from the residual vector. In this approach, there is no eigenstructure assignability problem. Therefore, we propose this idea to achieve robust FDI.

2.2. Robust fault detection using the Eigenstructure Assignment Approach

The following Theorem 1 is used to formulate the FDI robustness problem from unknown inputs in eigenstructure.

Theorem 1: Suppose that v_i and p_i^T are respectively the right and left eigenvectors of G_L corresponding to the eigenvalue λ_i. If

$$\text{(i) } p_i^T E = 0 \text{ for } i = q+1,\ldots,n \tag{12}$$

and

$$\text{(ii) } WCv_i = 0 \text{ for } i = 1,2,\ldots,q, \tag{13}$$

then $G_{rd}(z) = 0$.

Proof:

Eq. (9) can be rewritten as

$$G_{rd}(z) = WC[\sum_{i=1}^{n}\frac{v_i p_i^T}{(z-\lambda_i)}]E$$

$$= [\sum_{i=1}^{q}\frac{(WCv_i)p_i^T}{(z-\lambda_i)}]E + WC[\sum_{i=q+1}^{n}\frac{v_i(p_i^T E)}{(z-\lambda_i)}].$$

It entails that $G_{rd}(z) = 0$. ■

By Theorem 1, the residual vector is completely decoupled from the unknown inputs so that the robust fault detection is achieved. Since it is obviously complicated to obtain the assignable eigenvectors fulfilling Eqs. (12-13), efficient numerical algorithm for obtaining the allowable eigenvector subspaces is desirable.

2.3. Computation of allowable subspaces: the numerical solution

To systematically develop the numerical procedure for allowable eigenvector subspaces, we introduce a partitioned matrix,

$$\begin{bmatrix} N_{\lambda_i} \\ M_{\lambda_i} \end{bmatrix}, \tag{14}$$

whose columns constitute the basis of the augmented vector

$$\begin{bmatrix} p_i \\ \xi_i \end{bmatrix}, \tag{15}$$

where $i = 1,2,\ldots,n$ with respect to eigenvalue λ_i. Therefore, the eigenvector p_i and the vector ξ_i are respectively parametrized by a column vector α_i, i.e., $p_i = N_{\lambda_i}\alpha_i$ and $\xi_i = M_{\lambda_i}\alpha_i$. Before computing the allowable subspaces that fulfill Eq. (12-13), we firstly examine the key idea of observer design using the eigenstructure assignment approach by the following Theorem.

Theorem 2. Consider the nominal system representation $R = \{G, H, C, D\}$ as in Eqs. (1-2). Suppose that $\mathcal{L} = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ is a distinct self-conjugate set including the unobservable eigenvalues of the pair (*C*,*G*) and vectors $p_i \in R^n$ and $\xi_i \in R^r$. If $P = [p_1 \quad p_2 \quad \cdots \quad p_n]$ being linearly independent and $\Xi = [\xi_1 \quad \xi_2 \quad \cdots \quad \xi_n]$ satisfies

$$\begin{bmatrix} p_i \\ \xi_i \end{bmatrix} \in N([\lambda_i I - G^T \quad C^T]) \text{ for } i = 1,2,\ldots,n, \tag{16}$$

then, $p_i^T[\lambda_i I - (G - LC)] = 0$ iff $L = P^{-T}\Xi^T$.

Proof:

($\Leftarrow$)

By Eq. (16), $p_i^T(\lambda_i I - G) - \xi_i^T C = 0$. Since $L = P^{-T}\Xi^T$, we have that $p_i^T L = \xi_i^T$ for $i = 1,2,\ldots,n$. They yield

$$p_i^T[\lambda_i I - (G - LC)] = 0.$$

($\Rightarrow$)

If $p_i^T[\lambda_i I - (G - LC)] = 0$, we have that

$$p_i^T(\lambda_i I - G) - p_i^T LC = p_i^T(\lambda_i I - G) - \xi_i^T C,$$

or, equivalently, $p_i^T LC = \xi_i^T C$ for $i = 1,2,\ldots,n$. Since $C \in R^{r\times n}$ is full row-rank and $r \le n$, it yields that

$$p_i^T L = \xi_i^T \text{ for } i = 1,2,\ldots,n.$$

Also, since P is linearly independent, then $L = P^{-T}\Xi^T$. ■

The bases N_i and M_i can be obtained as follows:

(i) $p_i^T E = 0$ for $i = q+1,\ldots,n$:

Since the assignable eigenvector p_i of the FDI observer with respect to the eigenvalue λ_i must satisfy Eq. (16), we have the augmented equation

$$\begin{bmatrix} \lambda_i - G^T & C^T \\ E^T & 0 \end{bmatrix} \begin{bmatrix} p_i \\ \xi_i \end{bmatrix} = 0. \tag{17}$$

Since $r > q$, Eq. (17) is always solvable so that we obtain the bases N_i and M_i directly with linear algebra. For arbitrary α_i, $p_i = N_{\lambda_i}\alpha_i$ is assignable in eigenstructure assignment.

(ii) $WCv_i = 0$ for $i = 1,\ldots,q$:

The bases N_i and M_i can be directly obtained from the relationship stated in Eq. (16), i.e., the columns of the partitioned matrix

$$\begin{bmatrix} N_{\lambda_i} \\ M_{\lambda_i} \end{bmatrix}$$

constitute the basis of $N(\begin{bmatrix} \lambda_i I - G^T & C^T \end{bmatrix})$. Arbitrarily choose α_i for $i = 1,2,\ldots,n$ to determine the linearly independent eigenvector set $P = \begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix}$ and obtain $V = P^{-T}$. Consequently, the weighting matrix W is determined by

$$w_i^T \in N(CV_1), \tag{18}$$

where w_i^T is the i-*th* row of W and $V_1 = \begin{bmatrix} v_1 & v_2 & \cdots & v_q \end{bmatrix}$.

Remark 1. If the effect of a fault or the fault itself is soft, the steady-state gain from fault to residual vector has to be scaled by W to avoid missing alarm.

2.4. Summarized numerical algorithm

Step 1. Given $R = \{G, H, C, 0\}$, disturbance distribution matrix E, and sensor fault distribution matrix K. Determine the eigenvalue set $\mathcal{L} = \{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ which is distinct self-conjugate including the unobservable eigenvalues of the pair (C,G).

Step 2. In the case of $i = q+1,\ldots,n$, define

$$S_{\lambda_i} \equiv \begin{bmatrix} \lambda_i - G^T & C^T \\ E^T & 0 \end{bmatrix} \tag{19}$$

and a compatibly partitioned matrix,

$$\Sigma_{\lambda_i} = \begin{bmatrix} N_{\lambda_i} \\ M_{\lambda_i} \end{bmatrix}, \tag{20}$$

whose columns constitute the bases of $N(S_{\lambda_i})$.

Step 3. In the case of $i = 1,2,\ldots,q$, define $S_{\lambda_i} \equiv \begin{bmatrix} \lambda_i I - G^T & C^T \end{bmatrix}$ and a compatibly partitioned matrix Σ_{λ_i}, as defined in Eq. (20), whose columns constitute the bases of $N(S_{\lambda_i})$.

Step 4. Determine the linear independent eigenvectors $p_i = N_{\lambda_i}\alpha_i$ and $\xi_i = M_{\lambda_i}\alpha_i$ for $i = 1,2,\ldots,n$. Obtain the observer feedback gain $L = P^{-T}\Xi^T$.

Step 5. Compute $V = P^{-T}$. Determine the weighting matrix W from WCV_1 that meets the specific steady-state gain from faults to residual vector.

3. ROBUSTNESS TO UNSTRUCTURED UNCERTAINTIES

In addition to so-called unknown inputs, another robustness problem arises when uncertainties affecting on system are unstructured for which both characteristics and distribution are unknown. Consider the fault-free and the unknown input-free system in Eqs. (1-2) involving the unstructured uncertainty

$$x(k+1) = [G + \Delta G(x,u)]x(k) + [H + \Delta H(x,u)]u(k) \tag{21}$$
$$y(k) = Cx(k) + n(k), \tag{22}$$

where $n(k)$ represents output noises, and $\Delta G(x,u)$ and $\Delta H(x,u)$ formulate the unstructured uncertainty. It is noted that $\Delta G(x,u)$ and $\Delta H(x,u)$ are non-constant; if $\Delta G(x,u)$ and $\Delta H(x,u)$ are constant, they can be characterized as unknown inputs. The state estimate error and residual dynamics of the diagnostic observer in Eq. (6-7) then becomes

$$e(k+1)=G_L e(k)+v(k)-Ln(k) \quad (23)$$
$$r(k)=WCe(k)+Wn(k), \quad (24)$$

where $v(k)=\Delta G(x,u)x(k)+\Delta H(x,u)u(k)$. One can see that the robust FDI observer of Eqs. (3)-(5) is no longer reliable under such circumstances even if the unknown inputs are decoupled. To achieve robust FDI subject to system uncertainties, the observer feedback gain L must be appropriately specified from the allowable solution. Eqs. (23-24) lead to the following time response equation for fault-free residuals:

$$r(k)=WC\{G_L{}^k e(0)+\sum_{i=1}^{k} G_L{}^{i-1}s(k)\}+Wn(k), \quad (25)$$

where $s(k)=Ln(k-i)+v(k-i)$, $e(0)=x(0)-\hat{x}(0)$ is the initial error condition, and $\Lambda_L = diag(\lambda_1,\lambda_2,...,\lambda_n)$. By taking the 2-norm of both sides of Eq. (25), we obtain the i-th residual output $r_i(k)$ as

$$\begin{aligned}|r_i(k)| &= \left| w_i^T C[G_L{}^k e(0)+\sum_{i=1}^{k} G_L{}^{i-1}s(k-i)]+w_i^T n(k)\right| \\ &\le \left\|w_i^T C\right\|_2 \cdot \left\| V\Lambda_L{}^k P^T e(0)+\sum_{i=1}^{k} V\Lambda_L{}^{i-1}P^T s(k-i)]\right\|_2 + \left|w_i^T n(k)\right| \\ &\le \left\|P^T\right\|_2 \cdot \|V\|_2 \cdot \left\|w_i^T C\right\|_2 \cdot \Delta(k)+\left\|w_i^T\right\|_2 \|n(k)\|_2 \\ &= \kappa(P)\cdot\left\|w_i^T C\right\|_2 \cdot \Delta(k)+\left\|w_i^T\right\|_2 \|n(k)\|_2, \end{aligned} \quad (26)$$

where $\Delta(k)=\left\|\Lambda_L{}^k\right\|_2 \cdot \|e(0)\|_2 + \sum_{i=1}^{k}\left\|\Lambda_L{}^{i-1}\right\|_2 \cdot \|s(k-i)\|_2$. If $\Delta(k)$ is bounded, $\kappa(P)$, $\left\|w_i^T C\right\|_2$, and $\left\|w_i^T\right\|_2$ can be treated as measures of robustness to unstructured uncertainties and measurement noises, i.e., they should be designed to be as small as possible. Note that to reflect the magnitude of a fault and detect a soft fault, a specific steady state gain is desired. Consequently, $\kappa(P)$, $\left\|w_i^T C\right\|_2$, and $\left\|w_i^T\right\|_2$ are minimized in specific steady-state gain from certain fault to its corresponding residual. To minimize $\kappa(P)$, the eigenvectors must be determined as orthogonal as possible (reference). Unfortunately, the design of W and P is coupled. By Eq. (10), the steady-state gain from i-th faults to i-th residuals is

$$-w_i^T[C(I-G_L)^{-1}L-I]k_i, \quad (27)$$

where k_i is the i-th column of K. It is seen that in a specific DC gain of Eq. (27) the selection of w_i^T is dependent on $[C(I-G_L)^{-1}L-I]k_i$ and L depends on P. The most orthonormal modal matrix P does not guarantee small $\left\|w_i^T\right\|_2$. No optimal solution of Eq. (26) is provided here, while, the solution of w_i^T for minimal 2-norm in specific eigenvectors P is developed by the following Lemma.

Lemma 1. Suppose that N_w is an orthonormal basis whose columns span the space of w_i, then
(i) if w_i lies on the direction $\eta_i = N_w\delta_i$ where $\delta_i = N_w^T[C(I-G_L)^{-1}L-I]k_i$, we have minimal $\|w_i\|_2$ in constant $\left|w_i^T[C(I-G_L)^{-1}L-I]k_i\right|$;
(ii) following (i), if the steady state gain in Eq. (27) is specified as a_i, then

$$w_i = -\frac{a_i}{\|\delta_i\|_2^2}\eta_i. \quad (28)$$

Proof:

(i) Suppose that w_i lies in the direction $\eta_i = N_w\beta_i$ where β_i is a column vector. It is known that if β_i has minimal angle to δ_i, then Eq. (27) is maximized in all $w_i \in N_w$ as $\|w_i\|_2$ is constant, i.e., we have minimal $\|w_i\|_2$ in specific magnitude in Eq. (27). In other words, the column vector β_i is the least square solution of

$$N_w\beta_i = -[C(I-G_L)^{-1}L-I]k_i. \tag{29}$$

This yields that

$$N_w^TN_w\beta_i = \beta_i = \delta_i,$$

then,

$$\eta_i = N_w\beta_i = N_w\delta_i.$$

(ii) By part (i), we have that

$$\begin{aligned} w_i &= \eta_i \times \frac{a_i}{\eta_i^T\cdot\{-[C(I-G_L)^{-1}L-I]k_i\}} \\ &= \eta_i \times \frac{a_i}{\delta_i^T\cdot\{-N_w^T[C(I-G_L)^{-1}L-I]k_i\}} \\ &= -\frac{a_i}{\|\delta_i\|_2^2}\eta_i. \end{aligned}$$

■

4. A DESIGN EXAMPLE: JET ENGINE SYSTEM

To demonstrate the numerical algorithm, a jet engine model are used. The complete system model is originally 17th-order, for practical reasons and convenience of design, Patton and Chen[7] approximated the behavior of the engine with a reduced 5th-order model and a disturbance distribution matrix to cover the remaining nonlinearity and modeling error. Consider the following linearized 5th-order discrete time model with 0.026 sec. sampling time

$$G = \begin{bmatrix} -0.9813 & 7.5320 & -0.5983 & 0.48575 & -0.6979 \\ 0.2838 & -0.0826 & 0.0779 & -0.0617 & 0.0928 \\ -6.8588 & 28.9161 & -2.0561 & 1.6083 & -2.2612 \\ 1.2235 & -5.6607 & 0.4020 & -0.3192 & 0.4141 \\ 13.2662 & -53.4047 & 4.7390 & -3.7710 & 5.3669 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.000139 & 0.000195 \\ 0.000067 & -0.000005 \\ 0.003188 & 0.000601 \\ 0.007840 & -0.000273 \\ 0.003123 & -0.001516 \end{bmatrix}, C = I_{5\times5}, \text{ and } D = 0_{5\times2}.$$

The disturbances were summarized in the dominant distribution matrix

$$E = \begin{bmatrix} 0.4126 & 1.0511 \\ -0.0617 & -0.1545 \\ 1.5659 & 4.3087 \\ -0.2776 & -0.9646 \\ -2.9231 & -7.8282 \end{bmatrix}$$

and another minor part

$$E' = \begin{bmatrix} 0.5334 & 1.1580 \\ -0.0768 & -0.1644 \\ 1.9658 & 4.3874 \\ -0.3698 & -0.8722 \\ -3.7068 & -8.2010 \end{bmatrix}.$$

The physical meaning of the sensors and states is described in Patton and Chen's reports.[7] In the following discussion and simulation, we employ the distribution of E as the unknown inputs, and, the sensor noises and effects from E' represent the unstructured uncertainties. Based on the nominal model $R = \{G, H, C, 0, E, K\}$, we can design a FDI observer by Algorithm 1:

Step 1. Determine the eigenvalue set $\mathcal{L} = \{0.6000, 0.7000, 0.8000, 0.9000, 0.9500\}$.

Step 2 and Step 3. Obtain the bases N_{λ_i} and M_{λ_i} for p_i and ξ_i, $i = 1, 2, \ldots, n$.

Step 4. Obtain an eigenvector set

$$P = \begin{bmatrix} -0.1243 & -0.2501 & 0.1668 & -0.1426 & 0.9348 \\ 0.0186 & 0.0483 & -0.6830 & -0.7281 & 0.0262 \\ -0.4651 & 0.3453 & -0.5641 & 0.5478 & 0.2148 \\ 0.0783 & -0.8746 & -0.3673 & 0.2845 & -0.1147 \\ 0.8728 & 0.2258 & -0.2293 & 0.2617 & 0.2573 \end{bmatrix}$$

whose $\kappa(P) = 1$.

Step 5. Compute the observer feedback gain matrix

$$L = \begin{bmatrix} -1.9051 & 7.5163 & -0.6177 & 0.5258 & -0.7576 \\ 0.2681 & -0.9354 & 0.1169 & -0.0445 & 0.1152 \\ -6.8782 & 28.9551 & -2.8378 & 1.5589 & -2.3572 \\ 1.2636 & -5.6435 & 0.3526 & -1.0516 & 0.4050 \\ 13.2065 & -53.3823 & 4.6430 & -3.7801 & 4.7076 \end{bmatrix}.$$

Arbitrarily choose weighting matrix w_1 from the null space of CV_1 such that the steady state gain from the 1st sensor fault to residual output is 100 as

$$W = [54.6 \quad -408.6 \quad -152.7 \quad -112.6 \quad -54.8].$$

5. SIMULATION RESULTS

Fig. 2 shows the fault detection result from the first sensor. In this case, sensor noises are not present and a soft fault, whose pick value is 0.0001, is detected. To observe the detection performance in the presence of measurement noise, we add bounded-random artificial noise with a standard deviation of 0.0001 at each measurement. Fig. 3 shows the system outputs, and Fig. 4 indicates the detection results under the same operating conditions as those in Fig. 2 except that sensor noise is included. It can be seen that the detection performance degrades in the presence of noise. According to the discussion in Sec. 3 on robustness to unstructured uncertainties, we apply Lemma 1 to determine an alternative weighting matrix

$$W = [35.4426 \quad 16.3019 \quad -30.4057 \quad -23.6654 \quad -9.3822].$$

The corresponding diagnostic result is shown in Fig. 5. Clearly, the effects of noise on residual are suppressed without loss of detectionability. To verify our work, diagnostic results from the third sensor in the jet engine system are demonstrated in Figs. 6 and 7, with step-type and sinusoidal-type faults, respectively. Fig. 6(a) and Fig. 7(a) show the results in which the weighting matrix W is arbitrarily chosen to be

$$W = [-21.8 \quad 163.4 \quad 61.0 \quad 45.0 \quad 21.9],$$

and Fig. 6(b) and Fig. 7(b) are the diagnostic results of a robust weighting design by Lemma 1:

$$W = \begin{bmatrix} 0.4662 & -7.9472 & 22.7318 & 11.3300 & 11.3350 \end{bmatrix}.$$

Apparently, the proposed robust FDI design significantly improves the diagnostic performance when system uncertainty is present. The simulation results in Figs. 6 and 7 provide a direct verification of our work.

Remark 2. Most published papers about eigenstructure assignment claim that the eigenvectors should be chosen as mutually orthogonal as possible to obtain a well-conditioned modal matrix. While, in Sec. 3, the robustness of detection performance to unstructured uncertainties is found to be dependent on the weighting matrix W and $\kappa(P)$ from which the effects are coupled. In other words, well-conditioned eigenvector assignment may result in high 2-norm of W. The conditioning of the observer modal matrix should be properly determined. Take the case in Fig. 3 above as an example, the eigenvectors are alternatively chosen as

$$P = \begin{bmatrix} -0.1226 & -0.2510 & 0.6294 & -0.4671 & -0.4645 \\ 0.0183 & 0.0484 & 0.7040 & 0.0824 & 0.0790 \\ -0.4674 & 0.3422 & -0.2267 & -0.7231 & -0.7245 \\ 0.0841 & -0.8740 & -0.2371 & -0.2585 & -0.2601 \\ 0.8712 & 0.2316 & -0.0250 & -0.4305 & -0.4307 \end{bmatrix}$$

whose condition number is about 421. Consequently, the output feedback gain matrix

$$L = \begin{bmatrix} 0.8892 & 7.1113 & 3.6208 & 2.0311 & 1.7732 \\ -3.1170 & -0.3467 & -5.3067 & -2.0136 & -3.0935 \\ -8.3418 & 29.1509 & -5.1531 & 0.7197 & -3.7288 \\ -0.3431 & -5.4144 & -2.1485 & -1.9504 & -1.0804 \\ 13.0411 & -53.3688 & 4.3527 & -3.8896 & 4.5386 \end{bmatrix}$$

is obtained and the W determined by Lemma 1 is

$$W = \begin{bmatrix} 0.2192 & -0.2777 & -0.1204 & -0.1295 & -0.0154 \end{bmatrix}.$$

The diagnostic result is shown in Fig. 8. It is seen that the noise rejection is improved but the detection time gets slower. Readers should note the trade-off between detection time and robustness to measurement noise.

6. CONCLUSION

In this paper, we proposed a FDI observer robust to unknown inputs through disturbance decoupling with eigenstructure assignment. Other than the concept of disturbance decoupling proposed by Patton and Chen, we formulate the robustness problem starting from allowable eigenspaces decoupling unknown inputs so that the desired eigenstructure is always assignable. Our numerical treatment provides complete solutions for allowable FDI which decouples unknown inputs from failures. In addition to structured uncertainties, unstructured uncertainties, for which neither characteristics nor distribution are known in advance, were addressed in this paper since a physical system can not be completely described by analytical models. In our simulation results, the presence of measurement noise and modeling errors brought up the problem of robustness, even though the unknown inputs were decoupled. Also, the weighting matrix and the condition number of eigenvectors were treated as two important indices of the FDI system's robustness to unstructured uncertainties. In real applications, it would be more appropriate to use eigenvector assignment to decouple the dominant unknown inputs and then suppress the effects of unstructured uncertainties by minimizing the total quantity of $\kappa(P)$ and $\|w_i\|_2$.

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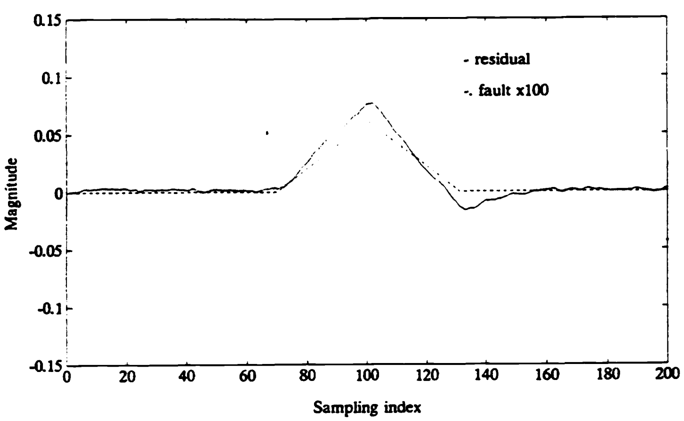

Fig. 2. Residual without measurement noise

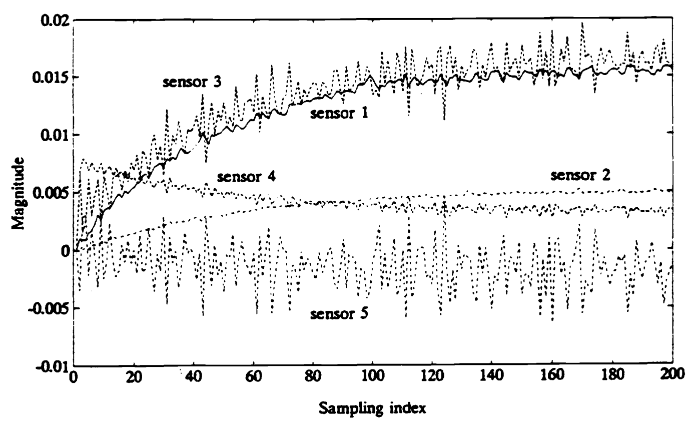

Fig. 3. Measurement with disturbance and noise

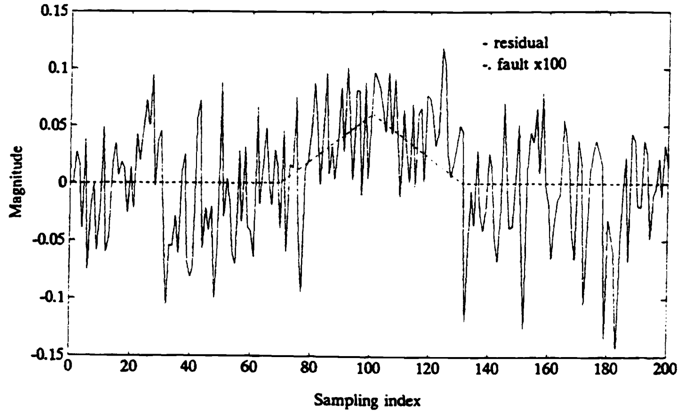

Fig. 4. Residual with measurement noise

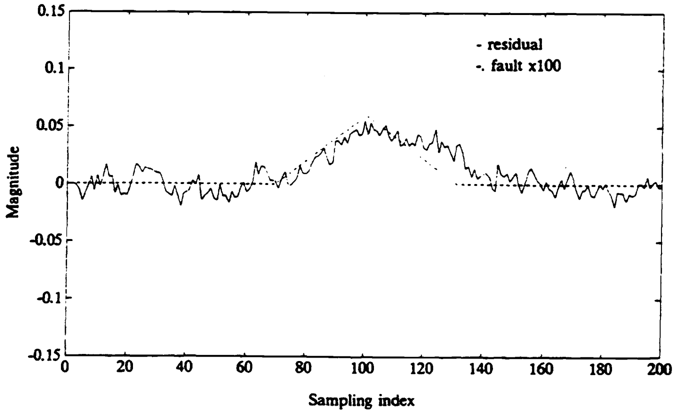

Fig. 5. Residual of robust FDI observer

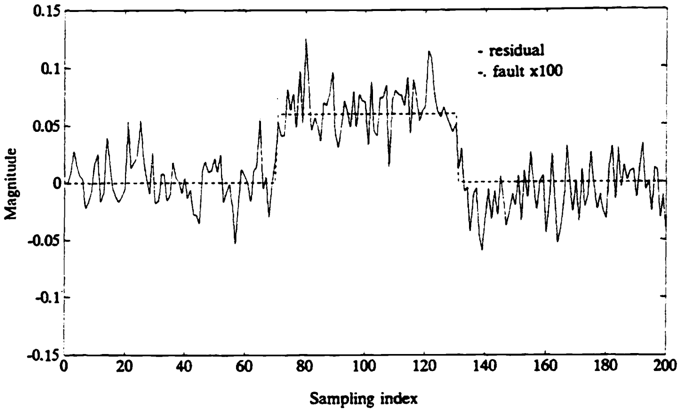

Fig. 6(a). Residual from 3-rd sensor in step-type fault

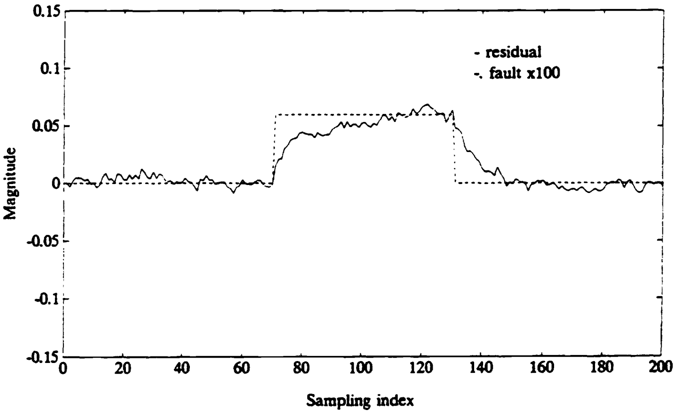

Fig. 6(b). Robust residual from 3-rd sensor in step-type fault

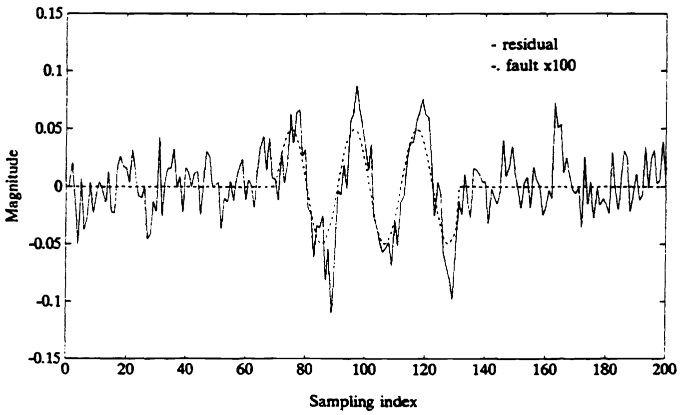

Fig. 7(a). Residual from 3-rd sensor in sin.-type fault

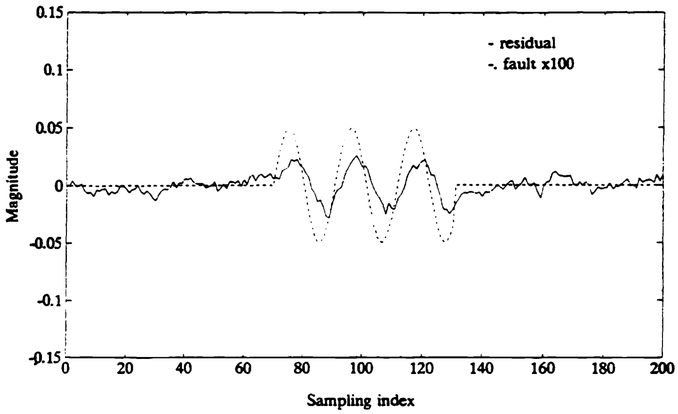

Fig. 7(b). Robust residual from 3-rd sensor in sin.-type fault

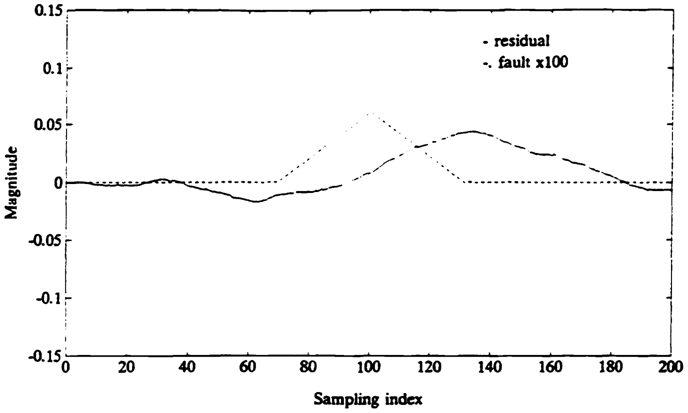

Fig. 8. Residual output with ill-conditioned eigenvector deisgn