

# Multiresolution interpolative DPCM for data compression

Wen-Jen Ho & Wen-Thong Chang  
Department of Communication Engineering  
National Chiao Tung University, Hsinchu, Taiwan, ROC

## ABSTRACT

A family of multirate representation of a given signal is defined for data compression. This family of multirate signals is constructed by polynomial interpolation of these direct decimated versions of a given signal. Interpolated signal from decimated signal is used to predict the higher resolution signal. The prediction error is the difference between the interpolated signal from lower resolution and the higher resolution one. This kind of signal representation can be called as interpolation compensated signal prediction. A multiresolution interpolative DPCM is then proposed to represent the prediction errors with a hierarchical multirate structure. This structure possesses the advantages of both the pyramid structure and the DPCM structure.

**Keywords:** multiresolution, DPCM, iterative polynomial interpolation

## 1 INTRODUCTION

A family of multiresolution signals is constructed by polynomial interpolation of the direct decimated version of a given signal. Beginning from the lowest resolution, interpolated signal from the decimated signal with polynomial interpolation is used to predict the higher resolution signal. The prediction error is the difference between the interpolated signal from lower resolution and the original higher resolution one. Based on this interpolation, a scheme called as interpolation compensated signal prediction is developed. A multiresolution interpolative DPCM is then proposed to represent the prediction errors with a hierarchical multirate structure. To derive the discrete prediction algorithm, we begin by showing how polynomial interpolation can be implemented with discrete iterative computation structure. From that, a new iterative polynomial interpolation is derived.

The characteristic of iterative interpolation is very suitable for rate conversion in multiresolution representation. Iterative interpolation was first proposed by Dubuc<sup>8</sup> and Dyn et al.<sup>9</sup> In our approach, a timelimited and symmetric self-similar cardinal function is chosen as the interpolation basis. Since the interpolation basis is constrained to be symmetric, cardinal and timelimited, the corresponding interpolation filter is a symmetric halfband FIR filter. With this filter used in the iterative interpolation, the interpolated sequence will converge to the continuous function interpolated with the corresponding interpolation basis.

This multiresolution interpolative DPCM has the following characteristics. The first is the use of interpolator as the predictor. Similar to the Laplacian pyramid, the prediction is noncausal due to the nature of multiresolution representation. The performance of such predictor is better than that of the traditional causal predictor commonly used with DPCM.<sup>13</sup> The second is the representation forms a multiresolution structure. It is possible to access a given signal at different resolutions and support a progressive rendition structure for devices having different resolutions. The third is the effect of error propagation is different from the traditional DPCM and depends on

the resolution level where the error occurs. Due to the nature of multiresolution permutation and subsequent interpolation, the error signal possesses unequal importance. The importance decreases from low resolution prediction level to high resolution prediction level. Since high resolution prediction depend on the result of low resolution prediction.

The organization of this paper is as follows. In section 2, iterative polynomial interpolation is defined. In section 3, interpolation compensated signal prediction is introduced. The generation and reconstruction of the multiresolution interpolative DPCM based on iterative polynomial interpolation is then described. In section 4, numerical experiments are used to show the efficiency of the multiresolution interpolative DPCM for data compression. In section 5, a conclusion is made.

## 2 ITERATIVE POLYNOMIAL INTERPOLATION

Iterative polynomial interpolation is used to successively interpolate a polynomial from a set of data points. In this section, the choice of timelimited and symmetric self-similar cardinal basis for iterative polynomial interpolation is discussed. Based on the self-similar equation which defines the interpolation basis, interpolation filter is derived. The corresponding characteristic of the interpolation filter used in iterative interpolation is then discussed.

Consider the interpolated function  $I(t; f(\frac{k}{W})) = \sum_{k=-\infty}^{\infty} f(\frac{k}{W})\Phi(Wt - k)$  defined on the real line  $R$ , where  $f(\frac{k}{W})$  denotes the sampled value of the function  $f(t)$  and  $W > 0$  denotes the sampling rate. If the interpolation basis  $\Phi(t)$  is timelimited with timelimit  $T$ , i.e. it vanishes outside the interval  $[-T, T]$ , then the interpolated function  $I(t; f(\frac{k}{W}))$  can also be written as  $I(t; f(\frac{k}{W})) = \sum_{k=\lfloor Wt-T \rfloor+1}^{\lfloor Wt+T \rfloor} f(\frac{k}{W})\Phi(Wt - k)$  where  $\lfloor x \rfloor$  denotes the largest integer which is smaller than or equal to  $x$ . The number of samples needed for the computation of the interpolated function  $I(t; f(\frac{k}{W}))$  at an arbitrary point is  $\lceil -2T \rceil$ , i.e. the smallest integer which is larger or equal to  $2T$ .

In this paper, the case of dyadic sampling rate with  $W = 2^{-j}, j \in Z$ , is considered. Let  $c_k^j$  denote the sampled value of the function  $f(t)$ , i.e.  $c_k^j = f(2^j k)$ , then the interpolated function  $I(t; c_k^j)$  is described as:  $I(t; c_k^j) = \sum_k c_k^j \Phi(2^{-j}t - k)$ . In order to satisfy the requirement that the interpolated function at points  $2^j k$  will coincide with the sequence  $c_k^j$ , i.e.  $I(2^j k; c_k^j) = c_k^j$ , cardinal functions are chosen as the interpolation bases. Besides the timelimited property mentioned above, symmetric property is also considered. Timelimited and symmetric properties will make the subsequent filter design easier. Further, since the interpolation  $I(t; c_k^j)$  depends on the resolution of the sampled data  $c_k^j$ , vector space  $V_j$  as the interpolation  $V_j = \{I(t; c_k^j) | I(t; c_k^j) = \sum_k c_k^j \Phi_k^j(t), \text{ where } \Phi_k^j(t) := \Phi(2^{-j}t - k)\}$  is defined. In order to establish a hierarchical vector space relationship, that is  $V_j \subset V_{j-1}, j \in Z$ , the basis  $\Phi(t)$  is also constrained to be self-similar, that is  $\Phi(t) = \sum_k p_k \Phi(2t - k), k \in Z$ .<sup>5</sup> The sequence  $p_k$  will be shown later to be the interpolation filter for discrete interpolation.

Now let us consider the computation of the interpolation  $I(t; c_k^j)$  by iterative interpolation. Iterative interpolation is a discrete implementation of the polynomial interpolation. Each time, new point is interpolated between the original points. The same process can be repeatedly applied until the final interpolated sequence converges to the function  $I(t; c_k^j)$ .

To derive the iterative interpolation, consider the interpolation

$$I(t; c_k^j) = \sum_k c_k^j \Phi(2^{-j}t - k). \tag{1}$$

From  $I(t; c_k^j)$ , the new sequence  $I(2^{j-1}k; c_k^j)$  can be obtained with sampling interval  $2^{j-1}$ . With  $I(2^{j-1}k; c_k^j)$ , a

new interpolation  $I'(t; c_k^j)$  can be written as

$$I'(t; c_k^j) = \sum_k I(2^{j-1}k; c_k^j)\Phi(2^{-j+1}t - k). \quad (2)$$

The purpose of iterative interpolation is to derive a computational formula such that  $I'(t; c_k^j)$  can be made to be equivalent to  $I(t; c_k^j)$ . From Eq.(1) and Eq.(2), we have

$$\sum_k c_k^j \Phi(2^{-j}t - k) = \sum_k I(2^{j-1}k; c_k^j)\Phi(2^{-j+1}t - k). \quad (3)$$

As can be seen from Eq.(3), the relationship between  $I(2^{j-1}k; c_k^j)$  and  $c_k^j$  is constrained by the interpolation basis. From the self-similar property defined above, we have

$$I(2^{j-1}k; c_k^j) = \sum_n c_n^j p_{k-2n}. \quad (4)$$

This means that if  $c_k^j$  is given, according to the interpolation filter  $p_k$ , the new sequence  $I(2^{j-1}k; c_k^j)$  can be obtained by Eq.(4). If the process is repeated for  $l$  times, the sequence  $I(2^{j-l}k; c_k^j)$  can be derived. The interpolation  $I(t; c_k^j)$  can then be written as

$$I(t; c_k^j) = \sum_k I(2^{j-l}k; c_k^j)\Phi(2^{-j+l}t - k). \quad (5)$$

When  $l$  approaches to infinity, the kernel  $\Phi(2^{-j+l}t)$  will converge to delta function  $\delta(t)$  and  $\lim_{l \rightarrow \infty} I(2^{j-l}k; c_k^j) = I(t; c_k^j)$ .

Now we consider the corresponding constraints of the interpolation filter if timelimited and symmetric self-similar cardinal basis  $\Phi(t)$  is chosen. Since  $\Phi(t)$  is symmetric and timelimited so that the filter  $p_k$  will be symmetric and finite in length. Let the odd term and the even term of the filter  $p_k$  be denoted as  $e_k^0 = p_{2k}$  and  $e_k^1 = p_{2k+1}$ . Hence,  $e_k^0$  and  $e_k^1$  are called as the 0th and the 1st polyphase component of  $p_k$ , respectively.<sup>15</sup> Due to the cardinal property of  $\Phi(t)$ , the sequence  $p_k = \Phi(k/2), k \in Z$ , then  $p_{2k} = e_k^0 = \delta_{k,0}$ . The  $z$  transform of the  $e_k^0$ ,  $E^0(z)$ , is equal to 1, and the  $z$  transform of  $p_k$ ,  $P(z)$ , can be represented with its polyphase components as

$$P(z) = 1 + z^{-1}E^1(z^2), \quad (6)$$

where  $E^1(z)$  is the  $z$  transform of the  $e_k^1$ . From the above equation, we have the following result:

$$P(z) + P(-z) = 2, \quad (7)$$

or in the frequency domain,

$$P(e^{jw}) + P(-e^{jw}) = 2. \quad (8)$$

This result means that the frequency response of the filter and its frequency translation version with amount  $\pi$  are mirror image symmetric about the frequency  $w = \pi/2$ . This condition is called as the halfband condition. A filter satisfies this condition is called as the halfband filter. From the above discussion, we can see that the filter needed for iterative polynomial interpolation is a halfband filter.

### 3 MULTIREOLUTION INTERPOLATIVE DPCM

In this section, based on iterative polynomial interpolation, the structure of multiresolution interpolative DPCM is introduced. In the first part, we introduce a family of multiresolution hierarchical representation of a signal based on iterative polynomial interpolation. Then based on that representation, multiresolution interpolative DPCM is constructed. Multiresolution interpolative DPCM is a compact structure for multiresolution

representation of a given signal. It is composed of the difference between two consecutive resolution versions of a signal.

As shown in section 2,  $I(t; c_k^j)$  represents the interpolated version of  $f(t)$  with  $c_k^j$ . A multirate representation of  $f(t)$  is the family of functions  $I(t; c_k^j)$ , where  $j = 0, \dots, J$ , with  $I(t; c_k^0) = f(t)$ . Multiresolution interpolative DPCM is used to represent this family of functions with a very compact hierarchical structure.

Assume  $f(t) \in V_0$ , the signal  $f(t)$  can be represented as

$$\begin{aligned} f(t) &= I(t; c_k^0) \\ &= I(t; c_k^1) + Q_1 f(t) \end{aligned} \quad (9)$$

where  $Q_1 f(t) := I(t; c_k^0) - I(t; c_k^1)$ .  $Q_1 f(t)$  represents the prediction error in predicting  $I(t; c_k^0)$  from the signal  $I(t; c_k^1)$ . This scheme is called as interpolation compensated signal prediction. This representation shows that the signal  $f(t)$  can be regarded as the combination of the coarser scale structure  $I(t; c_k^1)$  and an error structure  $Q_1 f(t)$ . In fact, the coarser scale of the signal can be repeatedly decomposed with the same way until the coarsest resolution  $J$  is reached, then we have

$$f(t) = I(t; c_k^J) + \sum_{j=1}^J Q_j f(t), \quad (10)$$

where  $Q_j f(t) = I(t; c_k^{j-1}) - I(t; c_k^j)$  representing the prediction error in predicting  $f(t)$  from resolution  $j$  to  $j-1$ .

For the discrete domain description, we define two dyadic grids to represent the sampling positions. A set of the dyadic nested grids  $\Omega^j(k)$  is defined as  $\Omega^j(k) := \{2^j k; j, k \in Z\}$  where  $j$  denotes the resolution level and  $k$  denotes the coordinate index of the grid  $\Omega^j(k)$ . For each of the grids  $\Omega^j(k)$ , a sampled version of the signal  $f(t)$ , i.e.  $c_k^j$ , can be obtained. Another set of the dyadic nested grids  $\Delta^j(k)$  is defined as  $\Delta^j(k) := \{2^{j-1}(2k+1); j, k \in Z\}$ . It can be regarded as the complementary grid of  $\Omega^j(k)$ . We can see that the union of the grids  $\Delta^j(k)$  and  $\Omega^j(k)$  is the grid  $\Omega^{j-1}(k)$ .

Consider the sampled data  $c_k^{j-1}$  of a signal at grid  $\Omega^{j-1}(k)$ , we have

$$c_k^{j-1} = I(2^{j-1}k; c_k^j) + (c_k^{j-1} - I(2^{j-1}k; c_k^j)). \quad (11)$$

$I(2^{j-1}k; c_k^j)$  is the interpolated data at the grid  $\Omega^{j-1}(k)$  given the data  $c_k^j$  at the grid  $\Omega^j(k)$  with discrete interpolation. The interpolated data  $I(2^{j-1}k; c_k^j)$  can be seen as the prediction of the original data  $c_k^{j-1}$  from  $c_k^j$ . The term  $c_k^{j-1} - I(2^{j-1}k; c_k^j)$  is the prediction error and is defined as  $d_{2k+1}^{j-1}$ . Since  $c_{2k}^{j-1} = c_k^j$ , we have  $d_{2k}^{j-1} = 0$ . Therefore, a new error sequence can be defined as  $\bar{d}_k^j := d_{2k+1}^{j-1}$ . Note that the length of the sequence  $\bar{d}_k^j$  is half of the length of the sequence  $d_k^{j-1}$  and is also defined as the sampled value of the error signal  $Q_j f(t)$  at the grid  $\Delta^j(k)$ .  $\bar{d}_k^j$  is the prediction error when using the interpolation procedure  $I$  to predict  $c_k^{j-1}$  from the knowledge of  $c_k^j$ .

In discrete implementation, the prediction process can be viewed as the prediction of points at the complementary grid  $\Delta^j(k)$  from the points at the grid  $\Omega^j(k)$  with a discrete interpolator. Let us define the discrete interpolator  $B$  in terms of the interpolation  $I$  as

$$B : \Omega^j \rightarrow \Delta^j, \quad B(c_k^j) = I(2^{j-1}(2k+1); c_k^j). \quad (12)$$

As shown in Eq.(4),  $I(2^{j-1}k; c_k^j)$  at the grid  $\Omega^{j-1}(k)$  can be derived from  $c_k^j$  at the grid  $\Omega^j(k)$  with the filter  $p_k$ . Since  $c_{2k}^{j-1} = c_k^j$ , only  $c_{2k+1}^{j-1}$  at the grid  $\Delta^j(k)$  needed to be predicted. Therefore, the prediction can be described as

$$B(c_k^j) = \sum_n c_n^j p_{2(k-n)+1} = \sum_n c_n^j e_{k-n}^1, \quad (13)$$

where  $e_k^1$  is the 1st polyphase component of the filter  $p_k$ . The purpose of  $B$  is to predict the sampled values of function  $f$  at the grid  $\Delta^j(k)$  when the sampled values of function  $f$  at the grid  $\Omega^j(k)$  are given. This polyphase algorithm provides a fast and convenient way to predict the points at the grid  $\Delta^j(k)$  with  $e_k^1$ . This multirate signal prediction procedure serves as the fundamental structure for the proposed multiresolution interpolative DPCM.

With the above prediction procedure, the original sampled sequence  $c_k^0$  can be decomposed into the sequences of prediction errors at various resolutions,  $(\bar{d}_k^J, \bar{d}_k^{J-1}, \dots, \bar{d}_k^1)$ . The decomposition procedure is, for  $j = 1 \sim J$ ,

$$c_k^j = c_{2k}^{j-1}, \quad (14)$$

$$\bar{d}_k^j = c_{2k+1}^{j-1} - B(c_k^j). \quad (15)$$

For reconstruction,  $c_k^0$  can be successively predicted from its low resolution version with the knowledge of prediction error. Exact reconstruction can be obtained as the sum of the predicted sequences and the prediction errors at various resolution levels,  $(\bar{d}_k^J, \bar{d}_k^{J-1}, \dots, \bar{d}_k^1)$ . The reconstruction procedure is, for  $j = J \sim 1$ ,

$$c_{2k}^{j-1} = c_k^j, \quad (16)$$

$$c_{2k+1}^{j-1} = B(c_k^j) + \bar{d}_k^j. \quad (17)$$

For implementation, a direct form consisting of decimator, expander and interpolation filter is shown in Fig.1. From the original sequence  $c_k^0$  as shown in Fig.1(a), the coarser sequence  $c_k^1$  is obtained by downsampling the sequence  $c_k^0$ . The process to interpolate the sequence  $I(k; c_k^1)$  consists of upsampling and filtering with the halfband interpolation filter  $p_k$  as shown in Fig.1(a). The error sequence  $d_k^0$  is then obtained by subtracting  $I(k; c_k^1)$  from the original sequence  $c_k^0$ . Since  $d_{2k}^0 = 0$ , the prediction error  $\bar{d}_k^1$  is then obtained by downsampling the sequence  $d_k^0$  as shown in Fig.1(a). This structure can be cascaded to calculate the prediction errors  $(\bar{d}^2, \dots, \bar{d}^J)$ . For reconstruction, a direct form implementation structure is shown in Fig.1(b).

The direct form implementation is conceptually very easy to understand. However, many operations can be combined to make the implementation more compact. First, the shifter  $z$  and the decimator after the subtracter in Fig.1(a) can be moved inside the loop. Consider the structure on the left side of Fig.2(a). The structure is a cascade of an expander followed by the filter  $zP(z)$  and a decimator. This structure on the right side of Fig.2(a) is the discrete interpolator  $B$  where  $E^1(z)$  is the 1th polyphase component of  $P(z)$ . The polyphase implementation structure for the generation and reconstruction of the multiresolution DPCM is shown in Fig.2(b) and 2(c), respectively. In fact, it is easy to check that the structure shown in Fig.2(b) actually realizes the generation algorithm of Eq.(14)(15) and the structure shown in Fig.2(c) actually realizes the reconstruction algorithm of Eq.(16)(17). Note that in the direct form and polyphase implementations, sequences  $c_k^j, j = 1 \sim J$ , can be obtained simultaneously when sequence  $c_k^0$  is given, so all the prediction errors  $\bar{d}_k^j, j = 1 \sim J$ , can be computed parallelly.

Now, if we view the generation of the error signal as a separate process, a filter bank structure can be derived. Consider the generation of  $\bar{d}_k^{j+1}$  from  $c_k^j$  as shown in Fig.3(a). This is done by moving the downsamplers in Fig.2(b) to after the subtracter. According to the noble identity of multirate system,<sup>15</sup> the structure as shown in Fig.3(a) can be derived. Since  $z - E^1(z^2) = zP(-z)$ , the upper and the lower branches of Fig.3(a) can be combined to form a single filter as shown in Fig.3(b). With this, the filter bank implementation structure for the generation and reconstruction is shown in Fig.3(c). This structure provides a more clear way to inspect the generation of the error signals. From this structure, it is easy to see that the values of the prediction error  $\bar{d}_k^{j+1}$  are controlled by the operation  $C^j(z)P(-z)$  where  $C^j(z)$  is the  $z$  transform of  $c_k^j$ . Since  $P(z)$  is usually a lowpass filter, the prediction error  $\bar{d}_k^{j+1}$  can be viewed as the high frequency component of the input signal  $c_k^j$  shaped by  $P(-z)$ .

## 4 NUMERICAL EXPERIMENT

In the first part of the numerical experiment, a polynomial signal is used. The test signal used is the following cubic polynomial:

$$f(t) = 10t^3 - 20t^2 + 10t, \quad 0 \leq t \leq 1.$$

In the second part of the numerical experiment, the more complicated image signal is used. The test image used is the  $256 \times 256$  standard image "Lena".

Let us investigate the effect of different interpolation filters on the prediction errors of multiresolution DPCM. The interpolation filters used are the maximally flat filters with length  $L = 3, 7$  and  $11$ .<sup>11</sup> The test signal is the uniform sampled sequence  $c_k^0$  of the cubic polynomial  $f(t)$  and is shown in Table.1. In the second column,  $c_k^0$  is represented with multirate structure. The multiresolution DPCM of  $c_k^0$  generated with these maximally flat filters are shown in the 3th, 4th and 5th columns of Table.1. Initially, the first level of prediction errors at the grid  $\Delta^4(k)$  are relatively large. From the second level, the prediction errors at the more finer grids  $\Delta^j(k)$ ,  $j = 3 \sim 1$  become small. For the case of  $L = 7$ , the errors are almost zero except these boundary points at the grids  $\Delta^j(k)$ ,  $j = 3 \sim 1$ . The errors at the boundary points are mainly due to the discontinuity at the boundary of signal. For the case of  $L = 11$  the result is similar. For cubic polynomial, it can be found that the maximally flat filters with length 7 and 11 can predict the sampled signal very well. The prediction errors at resolution  $j + 1$  depend on the product  $C^j(z)P(-z)$  as discussed in section 2.  $C^j(z)$  is the multirate spectrum of the signal  $c_k^0$  and  $P(z)$  is the spectrum of filter  $p_k$ . Therefore, the errors depend on the resolution level and the frequency response of the filter. In the beginning, when only a few samples are used for prediction, the product of  $C^j(z)$  and  $P(-z)$  is large, so the initial prediction errors are large. When sufficient data is used for prediction, the product of  $C^j(z)$  and  $P(-z)$  becomes small and the errors will reduce. This fact can be seen in Table.1, the prediction errors decrease from top to bottom with increasing resolution.

Now, let us examine the application of this multiresolution DPCM for image signal. For the image "Lena" with size  $256 \times 256$ , the histogram of this image is shown in Fig.4(a) and the histogram of the prediction errors with the 7-tap maximally flat filter is shown in Fig.4(b). As can be seen from the histogram, the values of the prediction errors are small and are close to zero. The mean of this histogram is 0.751 and the standard deviation is 16.763. The histograms of the prediction errors with different filters are all very similar. The means and standard deviations with filters of different length are shown in Table.2. The length of the filters used are 3, 7, 11 and 15, respectively. From these data, we can see that multiresolution DPCM is quite robust with respect to the filter used. Although the prediction errors depend on the filter used, the dependency is not strong. For comparison, the distribution of prediction errors of the traditional DPCM is also shown. The predictor used is  $\hat{c}_{i,j} = \rho \cdot c_{i,j-1} + \rho \cdot c_{i-1,j} - \rho^2 \cdot c_{i-1,j-1}$ , where  $c$  and  $\hat{c}$  are the original and the predicted image, respectively, and  $\rho$  is the parameter of the predictor. The means and the standard deviations of the prediction errors with different  $\rho$  values are shown in Table.3. As can be seen from Table.3, the performance of the proposed multiresolution interpolative DPCM is comparable with that of the traditional DPCM. Therefore, for data representation, the efficiency is about the same. The advantage of multiresolution interpolative DPCM is its space scalability. The advantage can be very useful in recent video transmission application.

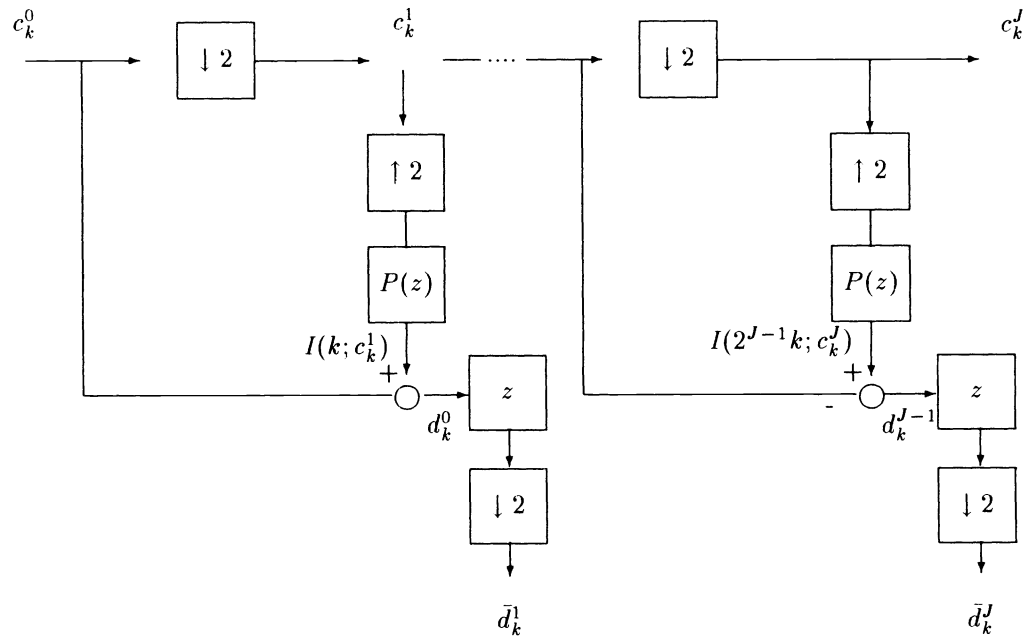
## 5 CONCLUSION

A multiresolution interpolative DPCM has been proposed for compact multiresolution signal representation. In this representation, iterative polynomial interpolation is used for signal prediction. To derive the prediction algorithm, we begin by showing how polynomial interpolation can be implemented with discrete iterative polynomial interpolation in discrete domain. This iterative computation structure is then applied for multiresolution signal prediction. Based on the multirate signal prediction, the multiresolution DPCM is then built to represent the multirate prediction error. The advantage of such representation is that the prediction error can be computed

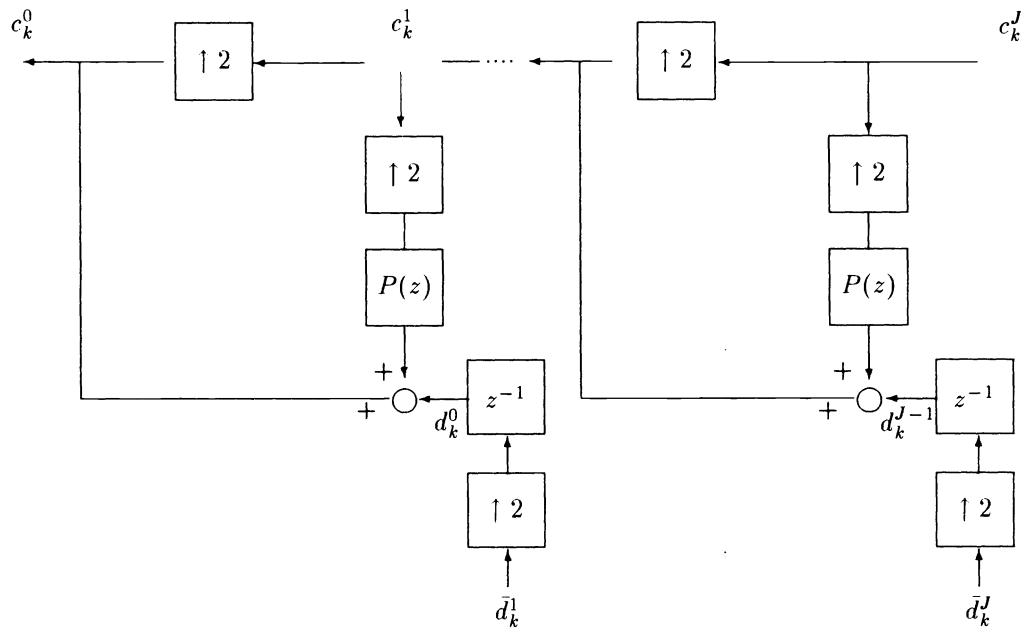
parallelly. The performance of such representation is shown to be as efficient as that of the traditional DPCM. Also, the representation allows for space scalability reconstruction which is very useful in recent progressive transmission application.

## 6 REFERENCES

- [1] A.Aldroubi and M.Unser," Families of wavelet transforms in connection with shannon sampling theorem and the gabor transform", in *Wavelets:A Tutorial in Theory and Applications*, C.K.Chui ed., Academic Press, NY, 1992, pp.509-528.
- [2] L.Arnold,"Interpolative coding of image with temporally increasing resolution", *Signal Processing*, 1989, pp.151-160.
- [3] P.J.Burt and E.H.Adelson,"The laplacian pyramid as a compact image code", *IEEE Trans. COM*, 1983, pp.532-540.
- [4] P.L.Buter, A.Fisher and K.Rückforth,"Scaling functions and wavelets with vanishing moments", to appear in *Computers and Mathematics with Applications*.
- [5] I.Daubechies, *Ten lectures on wavelets*, Philadelphia: SIAM, 1992.
- [6] G.Deslauriers and S.Dubuc,"Synmetric iterative interpolation process", *Constr. Approx.* 5, 1989, pp.49-68.
- [7] D.L.Donoho,"Interpolating wavelet transform", Preprint, Department of Statistics, Standford University, 1992.
- [8] S.Dubuc,"Interpolation through an iterative scheme", *J. Math. Anal. Appl.*, vol 24, 1986, pp.185-204.
- [9] N.Dyn, A. Gregory and D.Levin," A 4-point interpolatory subdivision scheme for curve design", *Comput. Aided. Geom. Des.*, 4, 1987, pp.257-268.
- [10] H.G.Musmann,"Predictive image coding", in *Image Transmission Technique*, W.K.Pratt ed., 1979, pp.73-112.
- [11] L.R.Rajagopal and S.C.Dutta-Roy,"Design of maximally-flat FIR filters using the bernstein polynomial", *IEEE Trans. CAS*, 1987, pp.1587-1590.
- [12] S.Ries,"On the reconstruction of signal by a finite number of samples", *Signal Processing*, 1991, pp.45-68.
- [13] M.L.Sethia and J.B.Anderson,"Interpolative DPCM", *IEEE Trans. COM*, 1984, pp.729-736.
- [14] N.Saito and G.Beylkin,"Multiresolution representations using the auto-correlation functions of compactly supported wavelets" *IEEE ICASSP*, 1992, pp.381-384.
- [15] P.P.Vaidyanathan, *Multirate Systems and Filter Banks*, Prentice Hall, 1993.



(a)



(b)

Figure 1: Direct form implementation of multiresolution interpolative DPCM. (a) Generation structure. (b) Reconstruction structure.



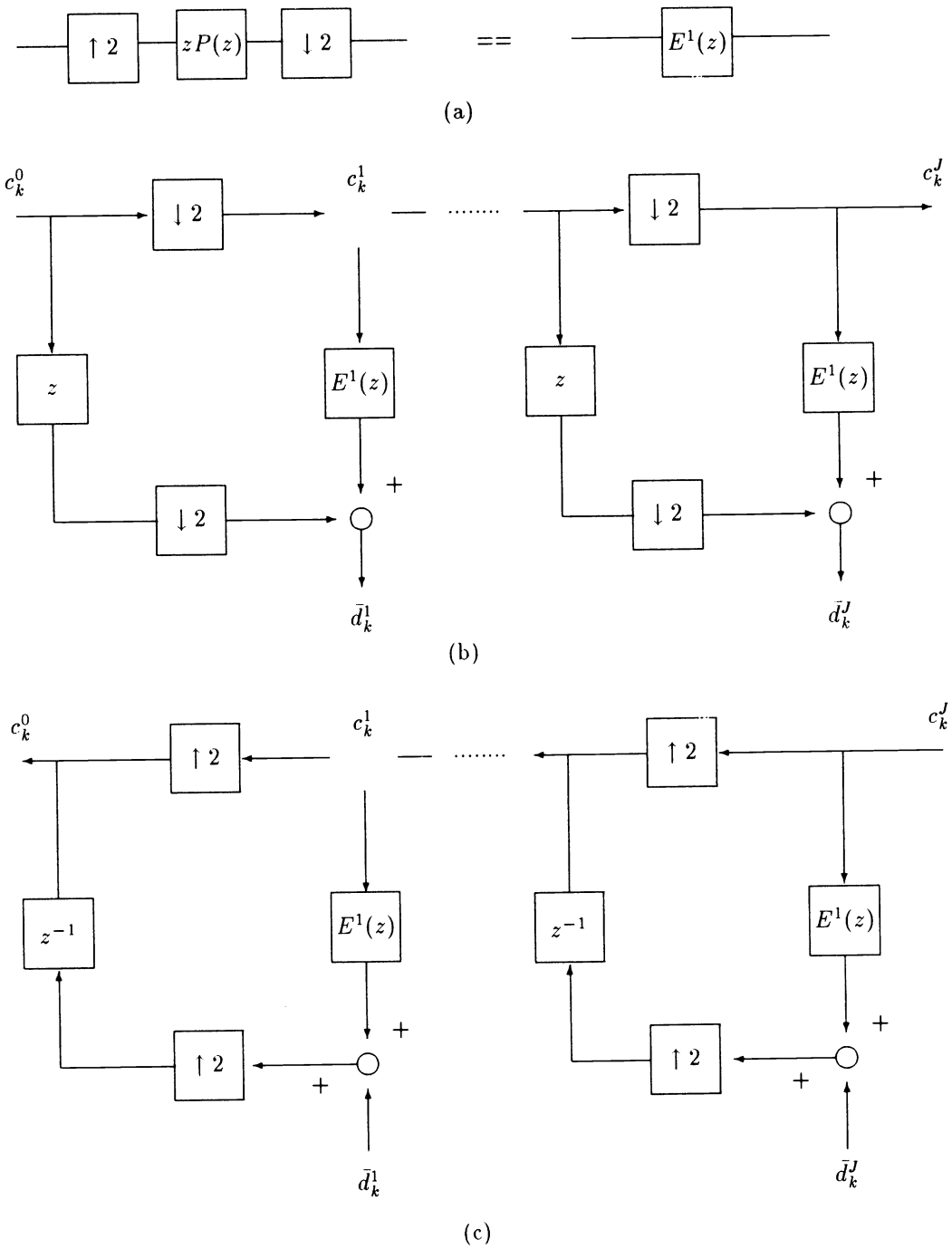


Figure 2: Polyphase implementation of multiresolution interpolative DPCM. (a) Polyphase implementation of predictor. (b) Generation structure. (c) Reconstruction structure.

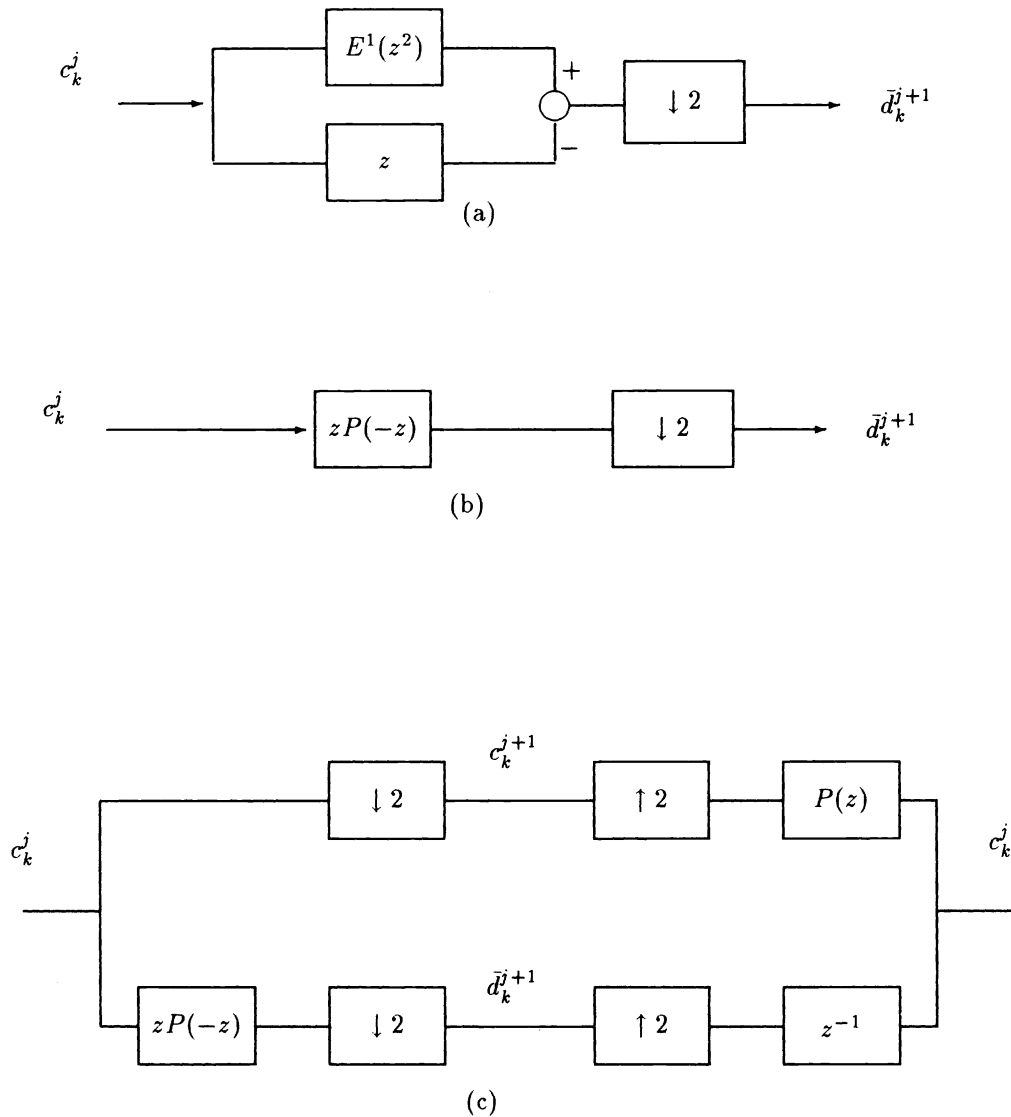


Figure 3: Filter bank implementation of multiresolution interpolative DPCM. (a) The structure to generate the prediction error  $\bar{d}_k^{j+1}$ . (b) Equivalent structure of (a). (c) Filter bank implementation structure.

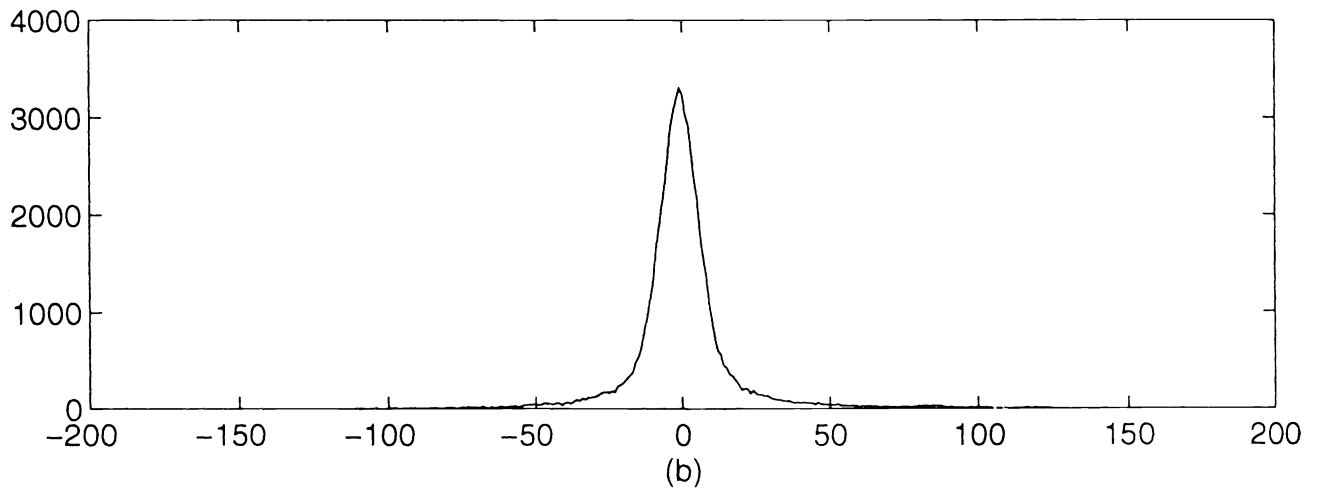
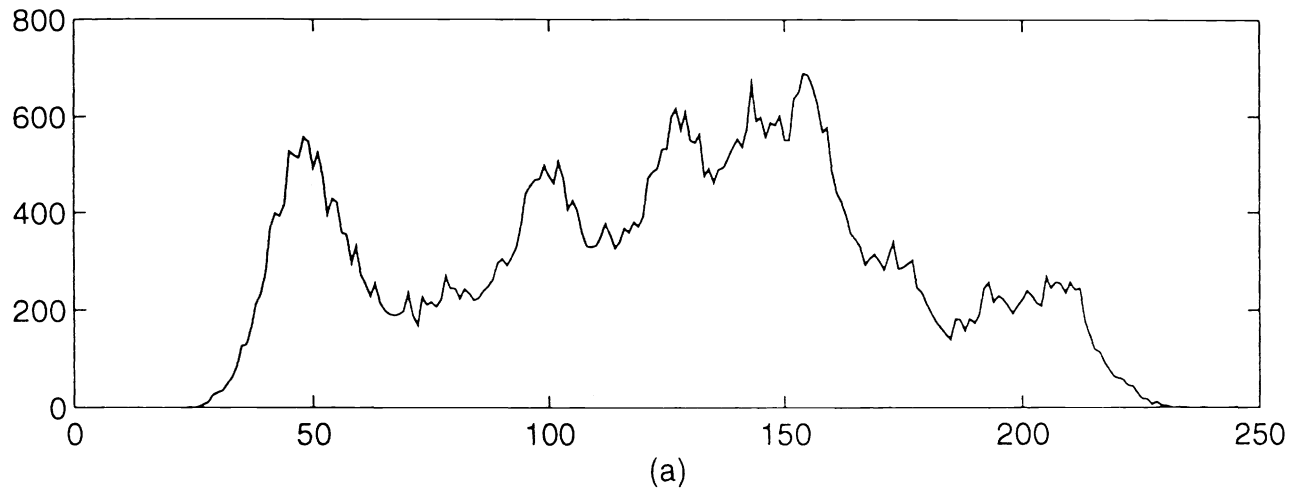


Figure 4: (a) The histogram of the original image "Lena". (b) The histogram of the prediction errors with 7-tap maximally flat filter.

k	$c_k^0$	L=3	L=7	L=11
$2^4 \times 0$	0.000	0.000	0.000	0.000
$2^4 \times 1$	1.250	1.250	1.250	1.250
$2^3 \times 1$	1.406	0.781	0.703	0.674
$2^3 \times 3$	0.469	-0.156	-0.234	-0.264
$2^2 \times 1$	0.957	0.254	0.244	0.250
$2^2 \times 3$	1.465	0.137	0.000	0.046
$2^2 \times 5$	0.879	0.020	0.000	0.009
$2^2 \times 7$	0.137	-0.098	-0.049	-0.032
$2^1 \times 1$	0.549	0.071	0.099	0.109
$2^1 \times 3$	1.238	0.056	0.000	-0.018
$2^1 \times 5$	1.477	0.042	0.000	0.000
$2^1 \times 7$	1.384	0.027	0.000	0.000
$2^1 \times 9$	1.077	0.012	0.000	0.000
$2^1 \times 11$	0.671	-0.002	0.000	0.000
$2^1 \times 13$	0.286	-0.017	0.000	0.002
$2^1 \times 15$	0.037	-0.032	-0.011	-0.008
$2^0 \times 1$	0.293	0.019	0.044	0.050
$2^0 \times 3$	0.770	0.017	0.000	-0.008
$2^0 \times 5$	1.112	0.015	0.000	0.000
$2^0 \times 7$	1.335	0.013	0.000	0.000
$2^0 \times 9$	1.453	0.011	0.000	0.000
$2^0 \times 11$	1.480	0.009	0.000	0.000
$2^0 \times 13$	1.432	0.008	0.000	0.000
$2^0 \times 15$	1.323	0.006	0.000	0.000
$2^0 \times 17$	1.167	0.004	0.000	0.000
$2^0 \times 19$	0.980	0.002	0.000	0.000
$2^0 \times 21$	0.775	0.000	0.000	0.000
$2^0 \times 23$	0.569	-0.002	0.000	0.000
$2^0 \times 25$	0.374	-0.003	0.000	0.000
$2^0 \times 27$	0.206	-0.005	0.000	0.000
$2^0 \times 29$	0.080	-0.007	0.000	0.001
$2^0 \times 31$	0.009	-0.009	-0.003	-0.002

Table 1: Multiresolution interpolative DPCM of  $c_k^0$  with different length of interpolation filter:L=3, L=7 and L=11.

length	3	7	11	15
mean	1.025	0.794	0.751	0.733
standard deviation	16.672	16.763	16.972	17.136

Table 2: Mean and standard deviation of multiresolution interpolative DPCM with maximally flat filters of different length.

$\rho$	0.95	0.90	0.85	0.80	0.75	0.70
mean	0.355	1.321	2.901	5.092	7.896	11.313
standard deviation	15.684	15.009	14.445	14.020	13.772	13.748

Table 3: Mean and standard deviation of traditional two dimensional DPCM.