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DECENTRALIZED CONTROL OF A ROTOR SYSTEM SUPPORTED BY MAGNETIC BEARINGS

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Abstract—The controller design for a permanent/electromagnetic magnetic bearing system that suspends a high-speed rigid horizontal rotor is investigated. The key points to ensure the stability of a rotor using this type of magnetic bearing are discussed. To facilitate implementation, a decentralized state feedback algorithm is proposed to stabilize the inherently unstable magnetic suspension system. Experimental results for the controlled system show that the system operates well at rotor speeds of up to 10,000 rpm.

NOMENCLATURE

x, y	displacements of mass center of rotor along X- and Y-axes
θ_x, θ_y	angular displacements about the X- and Y-axes
x_a, y_a	rotor displacements at left magnetic bearing location
x_b, y_b	rotor displacements at right magnetic bearing location
a, b	distances to magnetic bearings from rotor center of mass
L	distance between two magnetic bearings
Ω	rotational speed about the spinning Z-axis
m	rotor mass
I_r	the transverse mass moments of inertia of rotor
I_a	the polar mass moments of inertia of the rotor
ϵ, ζ	mass eccentricity components of rotor corresponding to the X- and Y-axes
F_n	the magnetic force
i_n	the coil current
i_{j0}	static part of coil current
i_j^*	the controlled current
k_d	the force–displacement coefficient
k_i	the force–current coefficient
V_{ej}	voltage of error signal
V_{cj}	output voltage generated by the controller
g_s, g_d	constant gain of sensor and driver respectively
K_p, K_d	p-control and d-control feedback game matrices

1. INTRODUCTION

THE demand for high-speed/high-power machine tool spindles has led researchers to investigate alternatives to traditional rolling contact bearings. One such alternative is magnetic bearings, a recent high-technology development in the turbomachinery field. Magnetic bearings suspend a rotating shaft by controlling magnetic attractive force without mechanical contact and lubricaton. The major advantages of these bearings are that they eliminate traditional bearing problems such as wear and lubrication and, even more important, offer great potential for vibration reduction. Magnetic bearings have been the subject of much development work in laboratories and are gradually being put into use in industry in machine tool spindles, turbomolecular pumps, and other applications.

The attractive magnetic suspension system used in magnetic bearings is inherently unstable; therefore, artificial stabilization by means of feedback control is required. Much recent literature has investigated the stability of and vibration control in magnetic bearing rotor systems. Fumio et al. [1] considered a horizontal rotating shaft controlled by a magnetic bearing. They derived the equations of motion for a levitated rotating body and clarified the relations between the voltage, current, and attractive force of

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electromagnets. Takeshi and Toshiro [2] studied a totally active d.c.-type magnetic bearing rotor system. They derived the equations of motion for a vertically rigid rotor suspended by 10 magnetic bearings and rotating at a constant angular velocity, and they also designed a controller using linear optimal control theory and presented an analytic solution. Kenzou et al. [3] used a finite element method to obtain a mathematical model of a flexible rotor taking into acount the interaction between the mechanical system of the flexible rotor and a controller. They did not, however, consider the gyroscopic effect. The feedback control algorithms developed in these studies are very complicated, and each control input depends on all the degrees of freedom of the rotor. Furthermore, the stability analysis in the above studies was based on only a single, fixed speed of rotation of the rotor.

In this paper, we propose a new controller design method for a rigid symmetrical rotor supported by permanent/electromagnetic magnetic bearings (PEMBs). In our previous laboratory studies [4, 5] we found that a permanent-magnetic biased magnetic bearing has the merits of small size, light weight, and low power consumption. In addition, and perhaps even more important, a PEMB can be operated at almost a linear force vs control current, which makes controller design easy. To simplify the controller design and facilitate implementation, we propose a decentralized state feedback control algorithm. Furthermore, depending upon the necessary and sufficient conditions for the stabilization of a conservative system [6], we can design a controller so that the closed-loop stability will be maintained when the speed of rotation changes.

2. MODELING OF THE HORIZONTAL ROTOR BEARING SYSTEM

Figure 1 shows the schematic of the test rotor. The rotor is suspended by two sets of PEMBs, and a motor regulates the speed of the rotor. The rotor is assumed to be rigid and symmetric and to have uniform mass unbalance. The shaft is suspended horizontally by contact-free magnetic bearings at both sides and the rotor positions of the magnetic bearings are measured by four eddy current-type sensors. The rotor has four degrees of freedom including two translational motions in the radial directions and two rotational motions about its center of mass, and is controlled by eight electromagnets. Furthermore, all displacements from the desired position are assumed to be small. Figure 2 shows the basic structure of the attractive-type magnetic bearing rotor system. The attractive forces supplied by the electromagnets acting on the rotor are expressed as F_n ($n = 1, ..., 8$). Figure 2 also shows the direction and point of action of each force. Here, we define a coordinate frame O-XYZ fixed in space. The origin point O corresponds to the center of mass of the rotor and the Z-axis corresponds to its spinning axis. The parameters x and y denote the linear displacements of the center of mass of the rotor along the X- and Y-axis, respectively, and θ_x and θ_y represent the angular displacements of the spinning axis about the X- and Y-axis individually. x_a, y_a, x_b, and y_b are the displacements of the rotor at the magnetic bearing locations, and a and b are the distances to the magnetic bearings from the center of mass of the rotor.
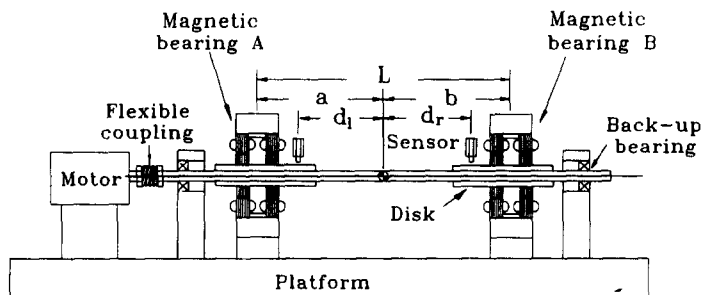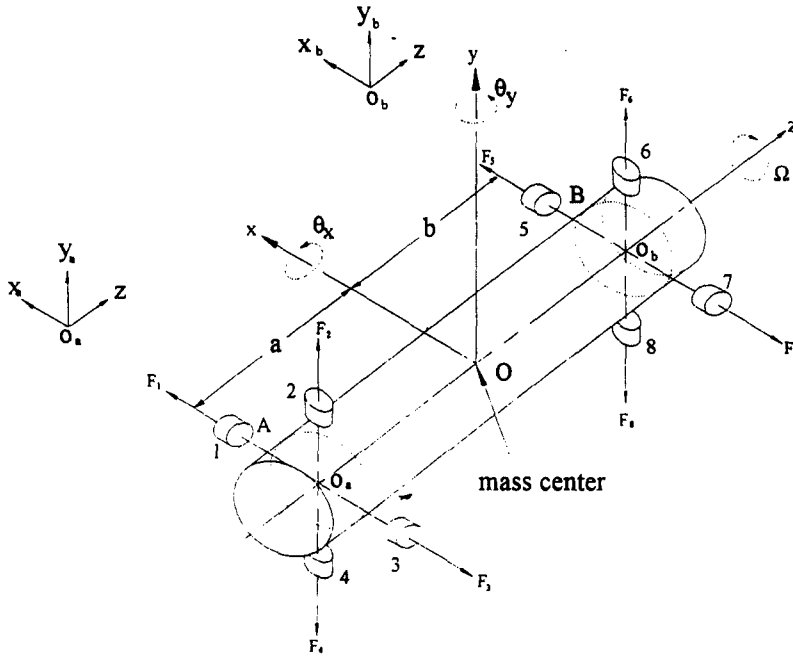

FIG. 1. Schematic of test rotor.

FIG. 2. Basic structure of the rotor-bearing system (1–8: electromagnets).

On the basis of the preceding description, the dynamic equations of the rotor about the mass center are as follows:

$$m\ddot{x} = F_1 - F_3 + F_5 - F_7 + m\epsilon\Omega^2\cos\Omega t - m\zeta\Omega^2\sin\Omega t$$

$$m\ddot{y} = -mg + F_2 - F_4 + F_6 - F_8 + m\epsilon\Omega^2\sin\Omega t + m\zeta\Omega^2\cos\Omega t$$

$$I_r\ddot{\theta}_y - \Omega I_a\dot{\theta}_x = b(F_5 - F_7) - a(F_1 - F_3)$$

$$I_r\ddot{\theta}_x + \Omega I_a\dot{\theta}_y = a(F_2 - F_4) - b(F_6 - F_8), \tag{1}$$

where Ω is the rotational speed about the spinning axis Z, m is the rotor mass, I_r is the transverse mass moment of inertia of the rotor, I_a is the polar mass moment of inertia of the rotor, and ϵ and ζ are the mass eccentricity components of the rotor corresponding to the X- and Y-axes.

Our previous research showed that the magnetic forces F_1 to F_8 provided by the two sets of PEMBs are a function of the length of the air gap and the magnitude of the magnetic flux, which is the sum of the electromagnet (EM) flux and permanent magnet (PM) bias flux. The length of the air gap is dependent upon the nominal length of the gap and the displacement from the center position of the magnetic bearing. Because the EM flux due to the coil current i_n is far less than the PM bias flux and the rotor displacement d_n is confined to less than 1/10 of the gap length under normal operation, the magnetic force may be approximated by a Taylor expansion about the normal operating point $i_n = 0$ and $d_n = 0$, taking only those terms of less than second order into consideration, i.e. the magnetic force can be expressed as

$$F_n = f_0 + k_d d_n + k_i i_n, \tag{2}$$

where f_0 is the static magnetic force when $i_n = 0$ and $d_n = 0$, k_d is the force–displacement factor, and k_i is the force–current factor.

For simplicity, we let the eight electromagnets have the same coefficients k_d and k_i. Then the eight input forces can be written as follows:

$$F_1 = f_0 + k_i i_1 + k_d x_a \qquad F_5 = f_0 + k_i i_5 + k_d x_b$$

$$F_2 = f_0 + k_i i_2 + k_d y_a \qquad F_6 = f_0 + k_i i_6 + k_d y_b$$

$$F_3 = f_0 + k_i i_3 - k_d x_a \qquad F_7 = f_0 + k_i i_7 - k_d x_b$$

$$F_4 = f_0 + k_i i_4 - k_d y_a \qquad F_8 = f_0 + k_i i_8 - k_d y_b. \tag{3}$$

Substituting these control forces (3) into equation (1), we obtain the dynamic equations of the rotor mass center:

$$m\ddot{x} - 2k_d(x_a+x_b) = k_i(i_1 - i_3 + i_5 - i_7) + m\epsilon\Omega^2\cos\Omega t - m\zeta\Omega^2\sin\Omega t$$

$$m\ddot{y} - 2k_d(y_a+y_b) = -mg + k_i(i_2 - i_4 + i_6 - i_8) + m\epsilon\Omega^2\sin\Omega t + m\zeta\Omega^2\cos\Omega t$$

$$I_r\ddot{\theta}_y - \Omega I_a\dot{\theta}_x - 2k_d(bx_b - ax_a) = k_i b(i_5 - i_7) - k_i a(i_1 - i_3)$$

$$I_r\ddot{\theta}_x + \Omega I_a\dot{\theta}_y - 2k_d(ay_a - by_b) = k_i a(i_2 - i_4) - k_i b(i_6 - i_8). \tag{4}$$

To simplify the dynamic equations (4), we will express the system equations in terms of displacements in the magnetic bearing locations. Since the rotor is assumed to be rigid and the displacement from the desired position is assumed to be small, the relationships between x_1, x_2, y_1, and y_2 and x, y, θ_x, and θ_y can be expressed as follows:

$$x = \frac{bx_a + ax_b}{L} \qquad\qquad y = \frac{by_a + ay_b}{L}$$

$$\theta_x \cong \tan\theta_x = \frac{y_a - y_b}{L} \qquad \theta_y \cong \tan\theta_y = \frac{x_b - x_a}{L}. \tag{5}$$

Substituting equations (5) into equations (4) yields the dynamics of the system as follows:

$$m\frac{b}{L}\ddot{x}_a + m\frac{b}{L}\ddot{x}_b - 2k_d(x_a+x_b) = k_i(i_1 - i_3 + i_5 - i_7)$$

$$+ m\epsilon\Omega^2\cos\Omega t - m\zeta\Omega^2\sin\Omega t$$

$$m\frac{b}{L}\ddot{y}_a + m\frac{b}{L}\ddot{y}_b - 2k_d(y_a+y_b) = -mg + k_i(i_2 - i_4 + i_6 - i_8)$$

$$+ m\epsilon\Omega^2\sin\Omega t + m\zeta\Omega^2\cos\Omega t$$

$$\frac{I_r}{L}(\ddot{x}_b - \ddot{x}_a) - \frac{\Omega I_a}{L}(\dot{y}_a - \dot{y}_b) - 2k_d(bx_b - ax_a) = k_i b(i_5 - i_7) - k_i a(i_1 - i_3)$$

$$\frac{I_r}{L}(\ddot{y}_a - \ddot{y}_b) + \frac{\Omega I_a}{L}(\dot{x}_b - \dot{x}_a) - 2k_d(ay_a - by_b) = k_i a(i_2 - i_4) - k_i b(i_6 - i_8). \tag{6}$$

3. STABILITY ANALYSIS AND CONTROLLER DESIGN

In our design, the currents flowing through the electromagnets, as shown in Fig. 3, are assigned as $i_1 = -i_3$, $i_2 = -i_4$, $i_5 = -i_7$ and $i_6 = -i_8$. Furthermore, the coil current i_j, $j = 1$–8, consists of a static part i_{j0} and a dynamic part i_j^* (i.e. the controlled current). We can represent the incremental current as

$$i_j = i_j^* + i_{j0} \qquad j = 1, ..., 8. \tag{7}$$

To provide an initial force to suspend the rotor without inducing any initial torque, we set
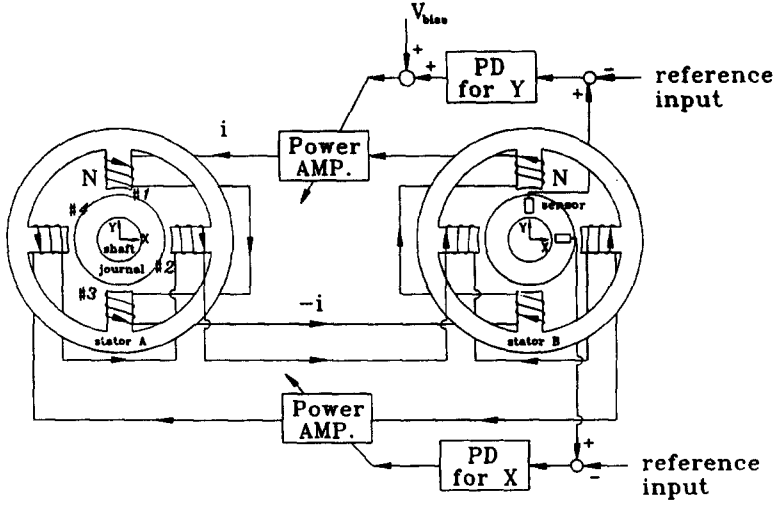
FIG. 3. Control circuit for a permanent-magnet biased magnetic bearing.

$$i_{20} = -i_{40} = \frac{bmg}{2(a+b)\,k_i},$$

$$i_{60} = -i_{80} = \frac{amg}{2(a+b)\,k_i}$$

and $i_{10} = i_{30} = i_{50} = i_{70} = 0$; then the equations of motion (6) can be simplified and rewritten as follows:

$$m\frac{b}{L}\ddot{x}_a + m\frac{a}{L}\ddot{x}_b - 2k_d\,(x_a+x_b) = 2k_i\,(i_1^*+i_5^*) + m\epsilon\Omega^2\cos\Omega t - m\zeta\Omega^2\sin\Omega t$$

$$m\frac{b}{L}\ddot{y}_a + m\frac{a}{L}\ddot{y}_b - 2k_d\,(y_a+y_b) = 2k_i\,(i_2^*+i_6^*) + m\epsilon\Omega^2\sin\Omega t + m\zeta\Omega^2\cos\Omega t$$

$$\frac{I_r}{L}(\ddot{x}_b - \ddot{x}_a) - \frac{\Omega I_a}{L}(\dot{y}_a-\dot{y}_b) - 2k_d\,(bx_b-ax_a) = 2k_j\,bi_5^* - 2k_i\,ai_1^*$$

$$\frac{I_r}{L}(\ddot{y}_a - \ddot{y}_b) + \frac{\Omega I_a}{L}(\dot{x}_b-\dot{x}_a) - 2k_d\,(ay_a-by_b) = 2k_i\,ai_2^* - 2k_i\,bi_6^*. \tag{8}$$

The above equations can be rearranged as follows:

$$\ddot{x}_a + \frac{\Omega I_a a}{I_r L}(\dot{y}_a-\dot{y}_b) - \frac{2k_d}{m}(x_a+x_b) - \frac{2k_d}{I_r}(a^2 x_a - abx_b)$$

$$= \frac{2k_i}{m}(i_1^*+i_5^*) + \frac{2k_i}{I_r}(a^2 i_1^* - abi_5^*) + \epsilon\Omega^2\cos\Omega t - \zeta\Omega^2\sin\Omega t$$

$$\ddot{x}_b - \frac{\Omega I_a b}{I_r L}(\dot{y}_a-\dot{y}_b) - \frac{2k_d}{m}(x_a+x_b) - \frac{2k_d}{I_r}(b^2 x_b - abx_a)$$

$$= \frac{2k_i}{m}(i_1^*+i_5^*) + \frac{2k_i}{I_r}(b^2 i_5^* - abi_1^*) + \epsilon\Omega^2\cos\Omega t - \zeta\Omega^2\sin\Omega t$$

$$\ddot{y}_a - \frac{\Omega I_a a}{I_r L} (\dot{x}_a - \dot{x}_b) - \frac{2k_d}{m} (y_a + y_b) - \frac{2k_d}{I_r} (a^2 y_a - ab y_b)$$

$$= \frac{2k_i}{m} (i_2^* + i_6^*) + \frac{2k_i}{I_r} (a^2 i_2^* - ab i_6^*) + \epsilon\Omega^2 \sin\Omega t + \zeta\Omega^2 \cos\Omega t$$

$$\ddot{y}_b + \frac{\Omega I_a b}{I_r L} (\dot{x}_a - \dot{x}_b) - \frac{2k_d}{m} (y_a + y_b) - \frac{2k_d}{I_r} (b^2 y_b - ab y_a)$$

$$= \frac{2k_i}{m} (i_2^* + i_6^*) + \frac{2k_i}{I_r} (b^2 i_6^* - ab i_2^*) + \epsilon\Omega^2 \sin\Omega t + \zeta\Omega^2 \cos\Omega t. \tag{9}$$

Finally, the equations of motion can be written in matrix form as follows:

$$M\ddot{x} + D\dot{x} + Kx = Bu + Ew, \tag{10}$$

where

$$M = I_{4\times4}$$

$$D = \begin{bmatrix} 0 & 0 & \alpha_1 & -\alpha_1 \\ 0 & 0 & -\alpha_2 & \alpha_2 \\ -\alpha_1 & \alpha_1 & 0 & 0 \\ \alpha_2 & -\alpha_2 & 0 & 0 \end{bmatrix}, K = -\begin{bmatrix} \beta_1 + a^2\beta_2 & \beta_1 - ab\beta_2 & 0 & 0 \\ \beta_1 - ab\beta_2 & \beta_1 + b^2\beta_2 & 0 & 0 \\ 0 & 0 & \beta_1 + a^2\beta_2 & \beta_1 - ab\beta_2 \\ 0 & 0 & \beta_1 - ab\beta_2 & \beta_1 + b^2\beta_2 \end{bmatrix}$$

$$B = \begin{bmatrix} B & 0 & 0 \\ & 0 & 0 \\ 0 & 0 & B_1 \\ 0 & 0 & \end{bmatrix}, B_1 = \begin{bmatrix} \gamma_1 + a^2\gamma_2 & \gamma_1 - ab\gamma_2 \\ \gamma_1 - ab\gamma_2 & \gamma_1 + b^2\gamma_2 \end{bmatrix}, E = \begin{bmatrix} \epsilon & -\zeta \\ \epsilon & -\zeta \\ \zeta & \epsilon \\ \zeta & \epsilon \end{bmatrix}$$

$$x = [x_a \, x_b \, y_a \, y_b]^T, u = [i_1^* \, i_5^* \, i_2^* \, i_6^*]^T, w = [\Omega^2 \cos\Omega t \, \Omega^2 \sin\Omega t]^T$$

$$\alpha_1 = \frac{\Omega I_a a}{I_r L} \qquad\qquad \alpha_2 = \frac{\Omega I_a b}{I_r L}$$

$$\beta_1 = \frac{2k_d}{m} \qquad\qquad \beta_2 = \frac{2k_d}{I_r}$$

$$\gamma_1 = \frac{2k_i}{m} \qquad\qquad \gamma_2 = \frac{2k_i}{I_r}$$

Furthermore, the damping matrix D can be separated into two parts, a symmetric part D_c and a skew-symmetric part G_c, expressed as follows:

$$D_c = \begin{bmatrix} 0 & 0 & 0 & \frac{\alpha_2 - \alpha_1}{2} \\ 0 & 0 & \frac{\alpha_1 - \alpha_2}{2} & 0 \\ 0 & \frac{\alpha_1 - \alpha_2}{2} & 0 & 0 \\ \frac{\alpha_2 - \alpha_1}{2} & 0 & 0 & 0 \end{bmatrix}$$

$$G_c = \begin{bmatrix} 0 & 0 & \alpha_1 & -\dfrac{\alpha_1 + \alpha_2}{2} \\[2ex] 0 & 0 & -\dfrac{\alpha_1 + \alpha_2}{2} & \alpha_2 \\[2ex] -\alpha_1 & \dfrac{\alpha_1 + \alpha_2}{2} & 0 & 0 \\[2ex] \dfrac{\alpha_1 + \alpha_2}{2} & -\alpha_2 & 0 & 0 \end{bmatrix}.$$

The equations of motion can then be rewritten as:

$$M\ddot{x} + D_c\dot{x} + G_c\dot{x} + Kx = Bu + Ew. \tag{11}$$

Table 1 shows the eigenvalues of the rotor bearing system for the case of $a = 0.083$, $b = 0.083$, and $\Omega = 10{,}000$ rpm. We found that the magnetic bearing rotor system is inherently unstable, and thus a controller is necessary to make the system stable.

Earlier studies [1–3] have shown that the feedback control matrix is very complicated and coupled with all states. Furthermore, the stability of the closed-loop system is dependent on the speed of rotation Ω, and consequently the control gain must change according to changes in Ω. Here, we hope to control the directions x and y and hopefully the left and right magnetic bearings separately to simplify the controller, i.e. our aim is a decentralized state feedback control, where the control gain is independent of Ω.

According to the discussion by Lee and Chen [6], in order to stabilize a conservative system with rigid-body modes using a collocated direct position feedback design with a symmetric and positive definite feedback gain matrix, the necessary and sufficient conditions for the rigid-body modes to be stabilized are that the number of sensors/ actuators be at least equal to the number of multiple eigenvalues of the uncontrolled system, and if the full-order closed loop equation can be expressed as

$$M\ddot{q}(t) + D_c\dot{q}(t) + G_c\dot{q}(t) + Kq(t) = 0, \tag{12}$$

where

M = the mass matrix
D_c = the symmetric part of the damping matrix
G_c = the skew-symmetric part of the damping matrix
K = the stiffness matrix,

then the necessary and sufficient conditions for the system (12) to be asymptotically stable are that the stiffness matrix must be symmetric and positive definite and the symmetric part of the damping matrix must be semi-positive definite.

TABLE 1. EIGENVALUES OF THE
OPEN-LOOP SYSTEM

$-5.833 + j0.0633$
$-5.833 - j0.0633$
$5.833 + j0.0633$
$5.833 - j0.0633$
-5.833
5.833
5.833
-5.833

Comparing the dynamic equations (11) and (12), we see that position and velocity feedback control is necessary to satisfy the conditions for asymptotic stability.

As shown in Fig. 3, the control system controls the position of the rotor by providing current to the electromagnet according to the signals from the position sensors. The distances between the magnet poles and the shaft of the rotor are measured by eddy current-type position sensors. Signals from the position sensors are then compared with the reference signals, which define the rotor's central position. Usually, the sensors cannot be placed directly in the bearing planes, under the assumption of a rigid rotor, so the position of the radial bearing is compensated for by a geometric relationship as follows:

$$x_a = \frac{(a+d_r)x_1 - (a-d_1)x_r}{d_1 + d_r}$$

$$x_b = \frac{(b-d_r)x_1 + (b+d_1)x_r}{d_1 + d_r}$$

$$y_a = \frac{(a+d_r)y_1 - (a-d_1)y_r}{d_1 + d_r}$$

$$y_b = \frac{-(b-d_r)y_1 + (b+d_1)y_r}{d_1 + d_r}, \tag{13}$$

where x_1, y_1, x_r, and y_r are the displacements of the rotor at the left and right side sensor positions and d_1 and d_r are the distances of the sensors from the mass center of the rotor.

The error signals measured from magnetic bearings A and B are proportional to the difference between the central position and the actual position of the rotor at any given time. If both error signals are zero, the rotor position is in the center of the stator. In response to the magnitude of the error signals, the controller generates a suitable low power voltage signal to drive the power amplifier and then provides the control currents to the winding coils so that suitable bearing forces are generated and the desired rotor position is maintained.

Because an inexpensive and reliable sensor for measuring the rotor displacement velocity has not yet been developed, a differential circuit or a velocity observer is commonly used to produce a pseudo velocity from the displacement measurement. In this study, a differential analog circuit is used.

Furthermore, since the time constants of the sensor and driver are usually very small, we can neglect the delay phenomenon. The dynamic equations of the sensor and driver can be expressed as

$$V_{ej} = g_s \mathbf{x}(j) \qquad j = 1, 2, 3, 4 \tag{14}$$

and

$$\mathbf{u}(j) = g_d V_{cj}, \qquad j = 1, 2, 3, 4, \tag{15}$$

where V_{ej} is the voltage of error signal which is the difference between the measured voltage of the sensor and the reference input, V_{cj} is the output voltage generated by the controller, g_s and g_d are the constant gain of the sensor and driver, respectively, and $\mathbf{x}(j)$ expresses the displacement of the rotor at magnetic bearings A and B.

The control law can then be expressed as

$$V_{cj} = -K_p V_{ej} - K_d \dot{V}_{ej}, \tag{16}$$

where K_p and K_d are the position and velocity feedback control gain matrices, respectively.

Thus, the overall dynamics of the system is:

$$m\ddot{x} + G_c\dot{x} + (D_c + g_d g_s BK_d)\dot{x} + (K + g_d g_s BK_p)x = Ew. \tag{17}$$

The position feedback control gain matrix K_p is designed to make the matrix $K + g_d g_s BK_p$ symmetric and positive definite, and since the symmetric part of the damping matrix D_c is non-negative, the velocity feedback control gain matrix K_d is set to be a symmetric and positive semi-definite matrix. Hence the system is asymptotically stable.

For the sake of simplicity and ease of implementation, we shall propose a decentralized controller. Define $K_p = B_1^{-1} K_p^*$ and $K_d = B_1^{-1} K_d^*$, and choose both K_p^* and K_d^* to be diagonal matrices:

$$K_p^* = \text{diag}\,(k_{p1}^*\, k_{p2}^*\, k_{p3}^*\, k_{p4}^*) \tag{18}$$

$$K_d^* = \text{diag}\,(k_{d1}^*\, k_{d2}^*\, k_{d3}^*\, k_{d4}^*). \tag{19}$$

To satisfy the necessary and sufficient conditions for asymptotic stability, we set

$$k_{dj}^* \geq 0, \qquad j = 1\text{–}4$$

$$g_d g_s k_{p1}^* > \beta_1 + a^2\beta_2,\, g_d g_s k_{p2}^* > \beta_1 + b^2\beta_2$$

$$g_d g_s k_{p3}^* > \beta_1 + a^2\beta_2,\, g_d g_s k_{p4}^* > \beta_1 + b^2\beta_2$$

$$(g_d g_s)^2 k_{p1}^* k_{p2}^* - \beta_1 g_d g_s (k_{p1}^* + k_{p2}^*) - \beta_2 g_d g_s (b^2 k_{p1}^* + a^2 k_{p2}^*) + \beta_1\beta_2 (a^2+b^2) > 0$$

$$(g_d g_s)^2 k_{p3}^* k_{p4}^* - \beta_1 g_d g_s (k_{p3}^*+k_{p4}^*) - \beta_2 g_d g_s (b^2 k_{p3}^* + a^2 k_{p4}^*) + \beta_1\beta_2 (a^2+b^2) > 0. \tag{20}$$

Then the symmetric part of the damping matrix will be non-negative and the stiffness matrix will be positive definite and symmetric.

As stated above, the control law can be expressed as

$$\begin{bmatrix} i_1^* \\ i_5^* \end{bmatrix} = -g_d g_s \begin{bmatrix} \gamma_1+a^2\gamma_2 & \gamma_1-ab\gamma_2 \\ \gamma_1-ab\gamma_2 & \gamma_1+b^2\gamma_2 \end{bmatrix}^{-1} \left(\begin{bmatrix} k_{p1}^* & 0 \\ 0 & k_{p2}^* \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} + \begin{bmatrix} k_{d1}^* & 0 \\ 0 & k_{d2}^* \end{bmatrix} \begin{bmatrix} \dot{x}_a \\ \dot{x}_b \end{bmatrix} \right)$$

$$\begin{bmatrix} i_2^* \\ i_6^* \end{bmatrix} = -g_d g_s \begin{bmatrix} \gamma_1+a^2\gamma_2 & \gamma_1-ab\gamma_2 \\ \gamma_1-ab\gamma_2 & \gamma_1+b^2\gamma_2 \end{bmatrix}^{-1} \left(\begin{bmatrix} k_{p3}^* & 0 \\ 0 & k_{p4}^* \end{bmatrix} \begin{bmatrix} y_a \\ y_b \end{bmatrix} + \begin{bmatrix} k_{d3}^* & 0 \\ 0 & k_{d4}^* \end{bmatrix} \begin{bmatrix} \dot{y}_a \\ \dot{y}_b \end{bmatrix} \right). \tag{21}$$

Thus, the semi-decentralized controllers of the four pairs of electromagnets can be implemented independently without horizontal and vertical interaction. A block diagram of the overall system for the semi-decentralized case is shown in Fig. 4.
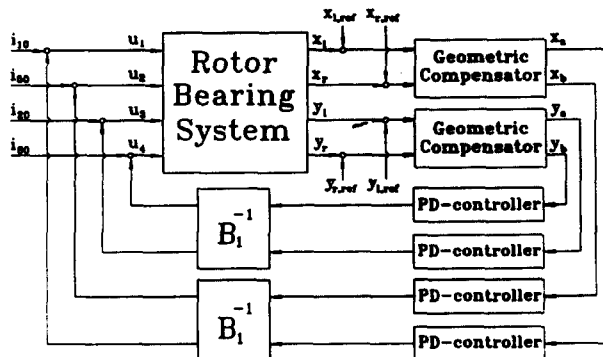

FIG. 4. Block diagram of the closed-loop system for semi-decentralized case.

If $a = b = L/2$, then $\alpha_1 = \alpha_2$ and $D_c = 0$. Thus the necessary and sufficient conditions for an asymptotically stable system can be simplified as follows:

$$k_{dj}^* \geq 0, \qquad j = 1-4,$$

$$g_d g_s k_{pj}^* > \beta_1 + a^2\beta_2, j = 1-4$$

$$(g_d g_s)^2 k_{p1}^* k_{p2}^* - g_d g_s (\beta_1 + a^2\beta_2)(k_{p1}^* + k_{p2}^*) + 2a^2 \beta_1\beta_2 > 0$$

$$(g_d g_s)^2 k_{p3}^* k_{p4}^* - g_d g_s (\beta_1 + a^2\beta_2)(k_{p3}^* + k_{p4}^*) + 2a^2 \beta_1\beta_2 > 0. \tag{22}$$

And the control law becomes:

$$\begin{bmatrix} i_1^* \\ i_5^* \end{bmatrix} = -\frac{g_d g_s}{4a^2 \gamma_1 \gamma_2} \begin{bmatrix} \gamma_1+a^2\gamma_2 & -\gamma_1+a^2\gamma_2 \\ -\gamma_1+a^2\gamma_2 & \gamma_1+a^2\gamma_2 \end{bmatrix} \left(\begin{bmatrix} k_{p1}^* & 0 \\ 0 & k_{p2}^* \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} + \begin{bmatrix} k_{d1}^* & 0 \\ 0 & k_{d2}^* \end{bmatrix} \begin{bmatrix} \dot{x}_a \\ \dot{x}_b \end{bmatrix} \right)$$

$$\begin{bmatrix} i_2^* \\ i_6^* \end{bmatrix} = -\frac{g_d g_s}{4a^2 \gamma_1 \gamma_2} \begin{bmatrix} \gamma_1+a^2\gamma_2 & -\gamma_1+a^2\gamma_2 \\ -\gamma_1+a^2\gamma_2 & \gamma_1+a^2\gamma_2 \end{bmatrix} \left(\begin{bmatrix} k_{p3}^* & 0 \\ 0 & k_{p4}^* \end{bmatrix} \begin{bmatrix} y_a \\ y_b \end{bmatrix} + \begin{bmatrix} k_{d3}^* & 0 \\ 0 & k_{d4}^* \end{bmatrix} \begin{bmatrix} \dot{y}_a \\ \dot{y}_b \end{bmatrix} \right). \tag{23}$$

Furthermore, if we choose $a = b = L/2$ and $4I_r/L^2 = m$, then we have $D_c = 0$, $B = I_{4 \times 4}$, and $K = -\frac{4k_d}{m} I_{4 \times 4}$, and the equations of motion (18) can be simplified as follows:

$$I\ddot{x} + G_c\dot{x} - \frac{4k_d}{m} Ix = \frac{4k_i}{m} Iu + Ew, \tag{24}$$

where I is a 4×4 identity matrix.

We design both K_p and K_d to be diagonal matrices, expressed as follows:

$$K_p = \text{diag}(k_{p1} \, k_{p2} \, k_{p3} \, k_{p4}) \tag{25}$$

$$K_d = \text{diag}(k_{d1} \, k_{d2} \, k_{d3} \, k_{d4}). \tag{26}$$

For the stiffness matrix, let

$$4k_i g_d g_s k_{pj} - 4k_d > 0, \qquad j = 1, 2, 3, 4, \tag{27}$$

then the positive definite and symmetric conditions will be satisfied.

For the damping matrix, set

$$k_{dj} \geq 0, \qquad j = 1, 2, 3, 4, \tag{28}$$

then the symmetric part of the damping matrix will be non-negative.

Therefore, the controllers of the four pairs of electromagnets can be implemented independently and the control law can be expressed as follows:

$$i_1^* = -g_d g_s k_{p1} x_a - g_d g_s k_{d1} \frac{dx_a}{dt}$$

$$i_5^* = -g_d g_s k_{p2} x_b - g_d g_s k_{d2} \frac{dx_b}{dt}$$

$$i_2^* = -g_d g_s k_{p3} y_a - g_d g_s k_{d3} \frac{dy_a}{dt}$$

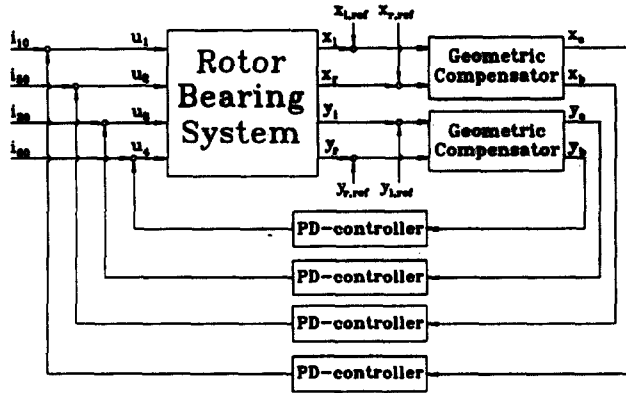$$i_6^* = -g_d g_s k_{p4} y_b - g_d g_s k_{d4} \frac{dy_b}{dt}. \tag{29}$$

FIG. 5. Block diagram of the closed-loop system for full decentralized case.

A block diagram of the full decentralized control system is shown in Fig. 5.

Figure 6 shows the root loci for three types of decentralized controllers with different speeds of rotation Ω. The figure indicates that the stability of all three types of controllers will not be affected by the rotating speed Ω; that is, the controllers can be used for any rotating speed Ω without requiring that the control gain be adjusted to maintain stability.
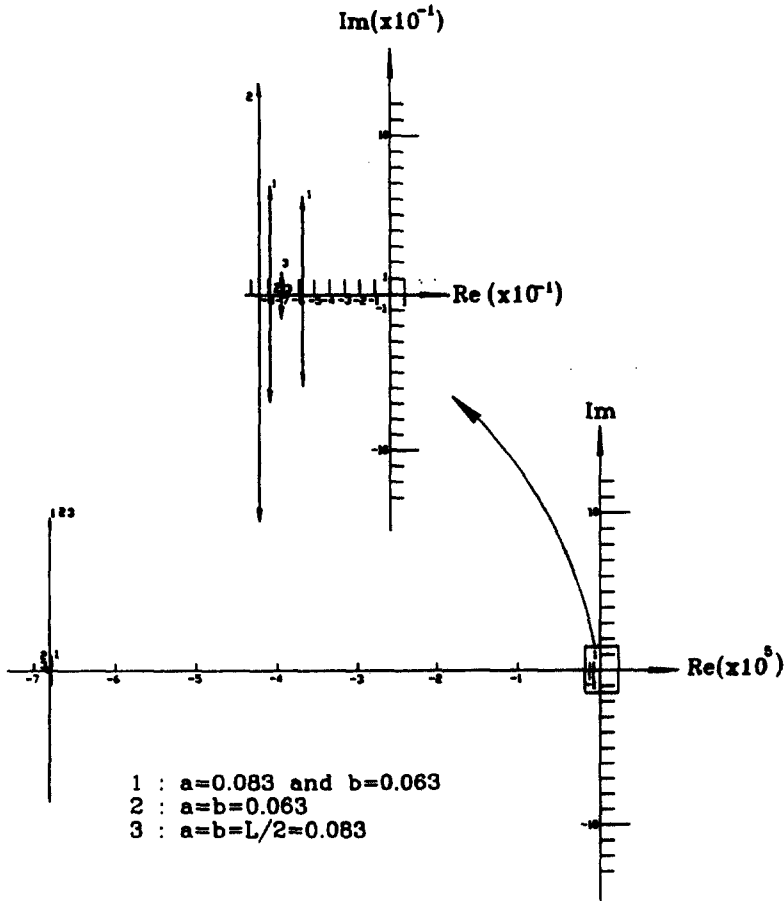

1 : a=0.083 and b=0.063
2 : a=b=0.063
3 : a=b=L/2=0.083

FIG. 6. Root loci for three decentralized controlled systems with rotating speed Ω from 0 to 10,000 rpm

4. EXPERIMENTAL RESULTS

4.1. *Test apparatus*

The laboratory test rotor is shown in Fig. 7. The rigid shaft, which was an 11-mm-diameter drill rod 320 mm long, had two disks, as indicated in Fig. 7. The total mass of the rotor was about 0.852 kg. The disks were used for two magnetic bearings and four sensors.

The shaft was connected to the driving motor by a flexible coupling. The gap between the electromagnet and stator was about 1.1 mm. The backup bearings provided protection in an emergency if the amplitude exceeded 0.5 mm. The displacements were measured by gap sensors of the eddy current type. These signals and the signals of the sensor offset input voltages were fed to the geometry compensator, and then the error signals were sent to the decentralized state feedback controller. The analog controllers consisted of proportional and differential circuits. The control signals from the analog controllers were supplied to the electromagnets through power amplifiers.

In this experiment, the prototype magnetic bearing used was that designed by Lee *et al.* [4, 5]. The displacement stiffness k_d was designed as 65,000 N/m and the current stiffness k_i was designed as 13 N/A; the sensitivity of the sensor was 2000 V/m, i.e. g_s was equal to 2000, and g_d was driven to 2.0. The gains of the four PD-controllers were chosen to be $k_{pj} = 3.0$, and $k_{dj} = 2.5$, $j = 1, 2, 3, 4$.

4.2. *Results*

Figure 8 shows the peak-to-peak responses of the rotor in both magnetic bearings A and B. This figure shows that this rotor-bearing system operates well. Figure 9 shows the orbits of the shaft in the positions of the left and right disks at rotating speeds $\Omega = 2750$, 4400, 9860 and 10,020 rpm with a sampling time of 1 msec. These experimental results indicate that the displacements remain in the range of 15% of the air gap; hence the motion of the rotor is maintained in an acceptably small range and the rotor does not collide with the backup bearings. The motion of the left end of the
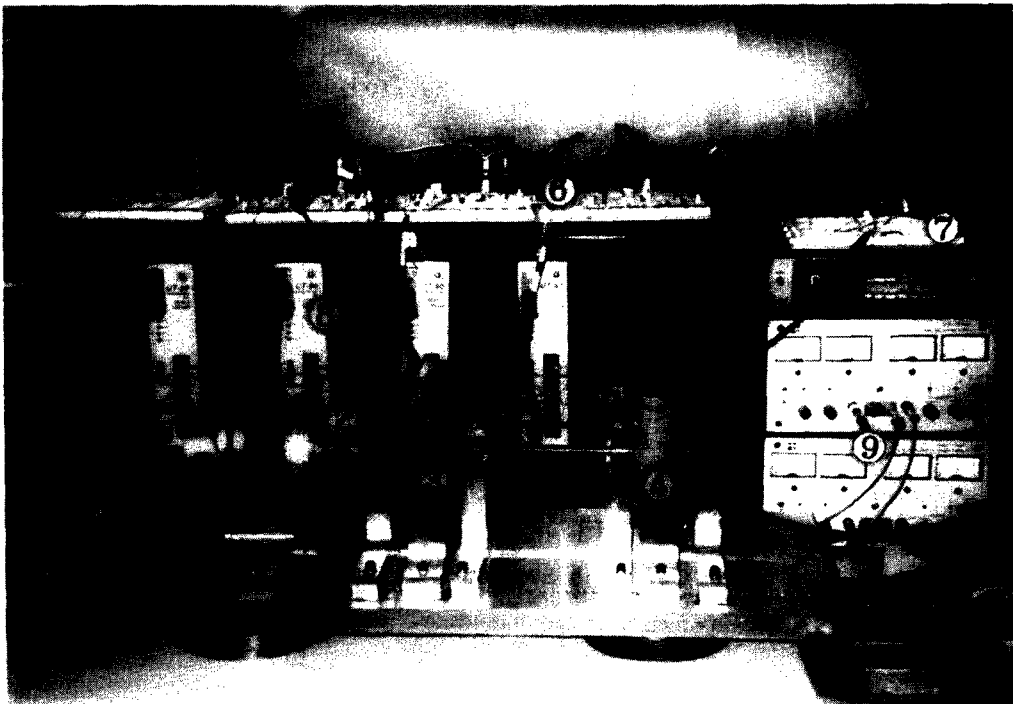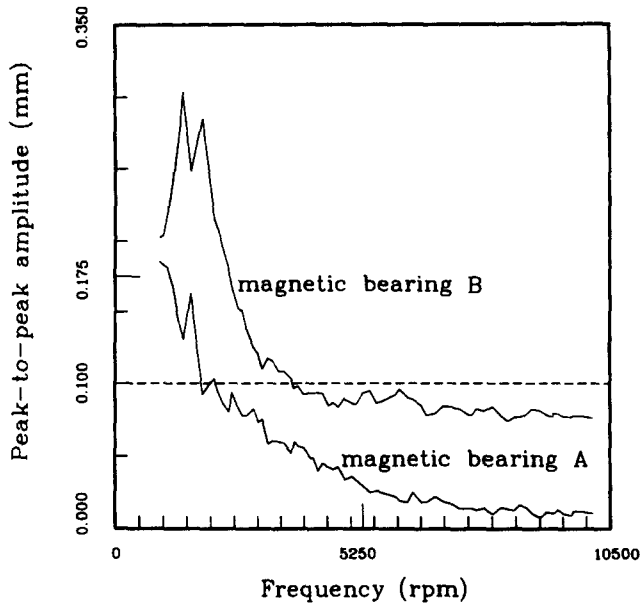

FIG. 7. Laboratory test apparatus.

FIG. 8. Spectrum of the peak-to-peak vibration amplitude.

(a) 2750 rpm

(b) 4400 rpm

(c) 9860 rpm

(d)10140 rpm

---------- orbit of left disk
——————— orbit of right disk
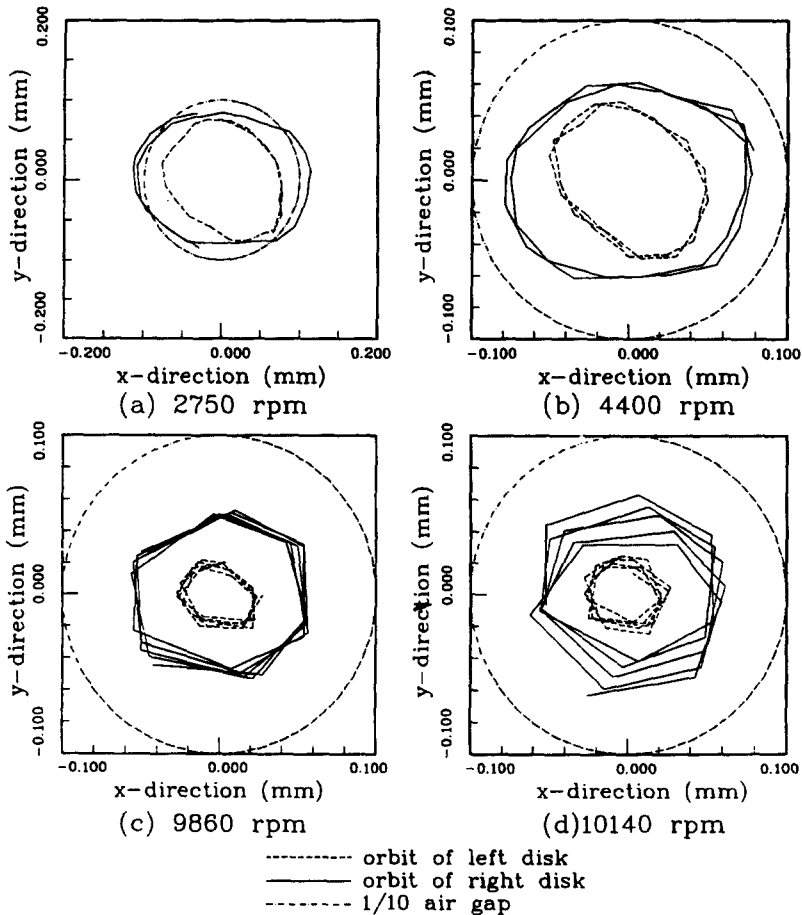--------- 1/10 air gap

FIG. 9. The orbit of the rotor at different speeds of rotation.

shaft is constrained by the flexible coupling, so there is no quick position changing, but at the free end of the shaft the orbits show relatively large variation.

5. CONCLUSIONS

This paper has presented a controller design for an attractive force-type magnetic bearing rotor system. According to the analysis of stability, three types of state feedback controllers have been proposed. Decentralized control can be achieved by suitably chosen location of the magnetic bearings. Experimental results show that the stability of the closed-loop system is maintained, even when the speed of rotation of the rotor is varied.

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