SCATTERING OF PLANE WAVES BY METALLIC GRATINGS

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Abstract

A new and unifying approach is presented for the analysis of plane-wave scattering by metal-strip gratings with a complex permittivity to account for their finite conductivity. A new set of metal modes is discovered, and the method of mode matching is employed for the formulation of the boundary-value problem. The effect of grating conductivity and thickness on the scattering characteristics are systematically examined, including the current distribution and power absorption within the metal strips.

A. Introduction

Grating structures have been used for many applications ranging from microwave to optical frequencies[1-8]. In particular, metal-strip gratings offer many advantages, such as easy fabrication and flexible design to achieve desired electromagnetic characteristics and good mechanical strength. In the past, metal-strip gratings have been often assumed to have an infinite conductivity and also to have a zero thickness, in order to simplify mathematical analyses[6-8]. These simplifying assumptions may be justifiable at the microwave frequencies, but not at millimeter-wave frequencies and beyond.

We present here a new and unifying approach to the class of metal-strip gratings which are realistically characterized by a finite conductivity and therefore must have a non-zero thickness. Specifically, the finite conductivity of metal is incorporated as the imaginary part of a complex dielectric constant, so that a metal-strip grating may be treated as a dielectric structure for which a rigorous formulation by the method of mode matching has been well developed[2,3]. Thus, the present approach is expected to yield accurate electromagnetic fields everywhere within a grating structure. The main purposes of this work are threefold: (1) to establish a theoretical foundation for the analysis of metal-strip gratings from a rigorous point of view, (2) to develop a clear physical picture of wave processes associated with metal-strip gratings, in order to gain a better physical understanding for design considerations, and (3) to evaluate accurately the effects of finite conductivity and finite thickness of the metal strips on the scattering characteristics of metal-strip grating, so that benchmark results can be established for a wide frequency range, as needed.

In order to employ the method of mode matching for the analysis of metal-strip gratings, we have examined the modes in the grating layer, and have discovered a new set of modes, in addition to an old (or well-known) set, in the case of metal-strip gratings with a high conductivity. A mode in the new set has its energy confined mostly inside the metal strips, while a mode in the well-known set has its energy confined mostly in the air spaces separating the metal strips. Therefore, the former is called a metal mode and the latter an air mode. Mathematically, it takes both sets, new and old, together to form a complete set of modes, so that the electromagnetic fields in the structure can be judiciously represented. Physically, since the metal modes are mostly confined inside the metal strips, they could contribute significantly to the absorption effect of the gratings, which remains to be accurately assessed. In the presence of a finite conductivity, the energy of an incident plane wave will be partially absorbed by the metal strips. With the present approach, we can calculate accurately the reflected and transmitted energy in the uniform regions on the two sides the grating layer; thus, the energy absorbed by the grating can be simply and accurately determined from the law of energy conservation. The mathematical techniques and physical understanding gained from the treatment of the plane-wave scattering problem will be very valuable for the analysis and design of metal-strip gratings for use as filters or antennas.

In this paper, we begin, in Section B, with a statement of the problem and the method of analysis for the grating problem under consideration. The Floquet solutions of the grating layer are carefully examined in Section C. The physical implications of the air and metal modes are explained in terms of the current distributions on the metal strips in Section D. Numerical results are given, in Section E, for the scattering of a plane wave, first by a single interface between a semi-infinite periodic array of metal layers and a semi-infinite uniform medium and then by a grating of finite thickness sandwiched between two semi-infinite uniform dielectric media. Finally, conclusion and discussions are given in Section F.

B. Statement of problem and Method of analysis

The structure of interest here is sketched in Fig. 1, with all the relevant parameters indicated. A grating is sandwiched between two semi-infinite media of the (relative) dielectric constant ε_f and ε_c,. The grating consists of metal strips of rectangular cross-section separated by air spaces which may be filled with another dielectric and our formulation still holds. The air regions have a dielectric constant ε_a and a width w_a; the metal strips have a width w_m and a conductivity σ. Including the effect of conductivity, the complex dielectric constant of the metal can be written as:

$$\varepsilon_m = 1 - j\,60\sigma\lambda. \qquad (1)$$

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where λ is the free-space wavelength. Such a characterization of metal may also apply to ground plane, if present; thus, the ohmic loss of the ground plane can be included in our analysis without any extra effort. Finally, the grating has a thickness t_g, and its period is given by: $d = w_a + w_m$.

In an attempt to understand the general wave phenomena taking place in a grating structure, particularly the induced currents and the absorption effect due to the finite conductivity of the metal strips, we shall restrict ourselves to the class of gratings which consist of metal strips of rectangular cross-section. The approach is to characterize the metal realistically with a finite conductivity which is incorporated as the imaginary part of a complex dielectric constant, as given by (1). Therefore, metal gratings may be treated as dielectric structures which have been extensively studied in the past[1-5]. In particular, a rigorous formulation of dielectric gratings by the method of mode matching has been presented recently for the general case of oblique incidence[5]. Specifically, in the rigorous formulation, a grating structure is decomposed into constituent parts or regions. A general solution for every constituent region can be expressed as a superposition of a complete set of modes which are easily determined, and the overall structure is then formulated as a boundary-value problem from which the scattering characteristics of grating structures are determined. Here, the structure under consideration consists of a periodic layer sandwiched between two semi-infinite uniform media. The electromagnetic fields in the uniform media on the two sides of the grating layer are well known; in fact, each medium can be represented by an input impedance or admittance for a given plane wave. Therefore, there remain only two key steps to the construction of field solutions in a grating structure: (1) the determination of a complete set of characteristic solutions or mode functions of the periodic layer, and (2) the formulation of grating Waveguides as a rigorous boundary-value problem[5]. While these steps have been well understood, the characteristic solutions for metal gratings of high conductivity remain to be carefully examined, as explained next.

C. Modes in the grating region

The Floquet modes of a periodic arrays of dielectric layers have been well known in the literature. Since the dielectric constant of a good conductor can be represented by a complex number with a large imaginary part, as given in (1), a periodic array of metal layers can be viewed as an infinite medium with a periodically varying dielectric constant, and the Floquet modes of the periodic medium can readily be obtained. From such a viewpoint, we show here that there exist two different sets of characteristic or Floquet modes for the grating layer. To determine the characteristic solutions of a grating with two alternating dielectric strips of rectangular cross-section, it is sufficient to consider a periodic array of alternating dielectric layers for which a closed-form dispersion relation has been obtained[1-3,5] in the form:

$$\sin \kappa_a d_a \sin \kappa_m d_m = Q(\cos \kappa_a d_a \cos \kappa_m d_m - \cos \kappa d) \quad (2)$$

$$Q = \frac{2 Z_a Z_m}{Z_a^2 + Z_m^2} \quad (3)$$

where Z_a is the wave impedance (with respect to the x-direction) of the air region and Z_m is that of the metal region.

In the case where one of the two media is a good conductor and the other is a good dielectric, we have: $Z_m << Z_a$ and $Q << 1$. Therefore, the term on the right-hand side of the equality in (2) is numerically small. In particular, in the limiting case of perfect conductor, $Z_m = 0$ and Q as defined by (3) vanishes. From (2), we then obtain the two independent equations:

$$\sin \kappa_a d_a = 0 \quad (4)$$

$$\sin \kappa_m d_m = 0 \quad (5)$$

Evidently, each of the two equations may be regarded as the dispersion relation of an ideal parallel-plate waveguide, and will yield a set of modes. For simplicity, the modes of the air region will be referred to as the air modes and those of the metal region as the metal modes. The air modes form a set of ideal waveguide modes, and so do the metal modes. The set of air modes should have been expected, but the existence of the metal modes which exist inside the perfect conductors is quite surprising at the first sight. In the extreme case of infinite conductivity, the set of metal modes still exists, but its existence is immaterial and may be ignored. However, such a new set of metal modes can not be ignored in the case of finite conductivity. The inclusion of the new set of metal modes makes the present work unique in the analysis of metal-strip gratings. The air modes and the metal modes together form a complete set of mode functions for judicious representations of fields in the grating layer, in order to formulate metal-strip gratings as a boundary-value problem.

D. Current Distributions on Metal Strips

With the electric field of a mode determined in the previous sections, the current on the metal strip can be related simply to the local electric field by:

$$\underline{J}(\underline{r}) = \sigma \underline{E}(\underline{r}) \quad (6)$$

where $\underline{J}$ is the current-density vector, $\underline{E}$ is the electric-field vector, $\underline{r}$ is the position vector, and σ is the conductivity of the metal. For a good conductor, σ is very large; as an example, for copper, $\sigma = 6 \times 10^4$ S/mm in the millimeter wavelength range. For a TE mode, the electric field is polarized in the y-direction, so that the induced current can flow freely along the metal strip and is expected to have a considerable effect on the ohmic loss of the microstrip lines.

With this approach, the electromagnetic fields inside the grating layer can be conveniently represented as a superposition of the complete set of modes, including the two subsets of air modes and metal modes. Once the fields in the grating layer are determined, the induced currents on the metal strips can be obtained simply by multiplying the electric field by the conductivity of the metal. Since the mode functions of the periodic array of metal layers can be determined exactly, the superposition of the air and metal modes will make it very easy to visualize the current distributions on the metal strips, thereby establishing a clear physical picture of the induced currents on the metal strips.

Consider now the contributions of the air modes to the current distribution on the metal strips. For simplicity, let us restrict ourselves here to the case of TE polarization. In this case, we have only a single component of the electric field and the current flows only along the strips. The electric fields due to the air modes near the metal edges are proportional to the skin depth, which is very small, as expected. However, the current due to such a weak field can be rather large, because of the high conductivity. As the conductivity is increased, so is the current density. In the limit of infinite conductivity, the current density becomes singular, as is well known. Since the total singular current must include the contributions from all the air modes, the singular behavior of the current density on a metal strip can be easily attributed to the air modes, and this provides a simple explanation of the existence of the large edge currents on the metal strips for the case of TE modes. On the other hand, the electric field of a metal mode varies sinusoidally over the broad surfaces of the strip, and each metal mode will induce a current distribution in the same fashion. Once the amplitudes of the modes are determined for a given source of excitation, the total surface current will be a superposition of the individual currents as contributed from all the metal modes. This explain that the induced current varies sinusoidally on the broad surfaces of the metal strips.

E. Numerical Results

We have carried out extensive numerical data for the scattering of plane waves by metal-strip gratings, under various operating conditions. The convergence of the numerical analysis has been carefully investigated and the criterion for convergence has been established. The scattered power and absorbed power are systematically evaluated for both polarizations, and they are shown to agree exceedingly well with available data for idealized structure[6]. With the present approach, we are able to study the effect of conductivity and thickness of the grating. As an illustration, Fig. 2 shows the frequency dependence of the reflected power for the scattering of a plane wave by a periodic array of semi-infinite metal layers in air, for both polarizations. The plane wave is incident from the uniform air region at the normal incidence. At low frequencies, all modes are below cutoff, except for the TEM mode. Thus, the TE incident wave is almost totally reflected, while the TM incident wave may be transmitted through the TEM mode, depending on the width of the metal strips. On the other hand, at high frequencies, many higher-order modes may be propagating in the air regions of the periodic array, and the reflected power is almost proportional to the ratio of the width of the metal strip to the period of the grating, as physically expected. Furthermore, it should be pointed out that the rapid variations of the curves can be identified with the cutoff conditions of both modes of the parallel-plate waveguide and the space harmonics in the uniform air region.

Fig. 3 shows the results for the scattering of a plane wave by a metal grating of finite thickness, for different values of conductivity. The grating thickness chosen here is $t_g = 10^{-6}$ mm, which is extremely small. In the case of very high conductivity, $\sigma = 3.96\times10^5$ S/mm, the reflected and transmitted power are plotted with the solid lines and they are found to be in agreement with those of perfect-conductor case. When the conductivity is reduced to the value corresponding to that of aluminum, $\sigma = 1.45\times10^4$ S/mm, the reflected and transmitted power are plotted with the dashed lines. In this case, the reflected power is decreased, while the transmitted power is increased, as expected. Finally, when the conductivity is further reduced to the value, $\sigma = 2.0\times10^3$ S/mm for a relatively poor conductor, the reflected and transmitted power are plotted with the dotted lines. In this case, the effect of the grating is greatly diminished, as expected.

Fig. 4 shows the surface distributions of the current component flowing across the metal strips, for the case of TM incident wave. The current distributions is symmetrical for the case of normal incidence and become asymmetrical for an oblique incidence. This can be easily explained in terms of the symmetries of the metal modes. These results demonstrate the powerfulness of the present approach. More results together with their physical meanings will be discussed in the presentation.

F. Conclusion and Discussions

We have carried out extensive numerical data for the scattering of plane waves by metal-strip gratings, under various operating conditions. The convergence of the numerical analysis has been carefully investigated and the criterion for convergence has been established. The scattered power and absorbed power are systematically evaluated for both polarizations, and they are shown to agree exceedingly well with available data for idealized structure. With the present approach, we are able to study the effect of conductivity and thickness of metal gratings; for the first time in the literature, we are able to determine the actual current distributions in the metal strips.

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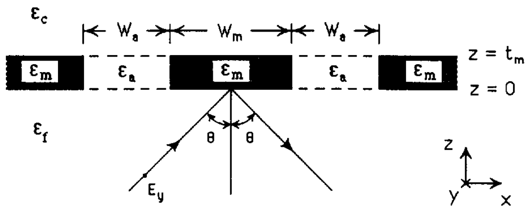

Fig. 1. Configuration of metallic grating

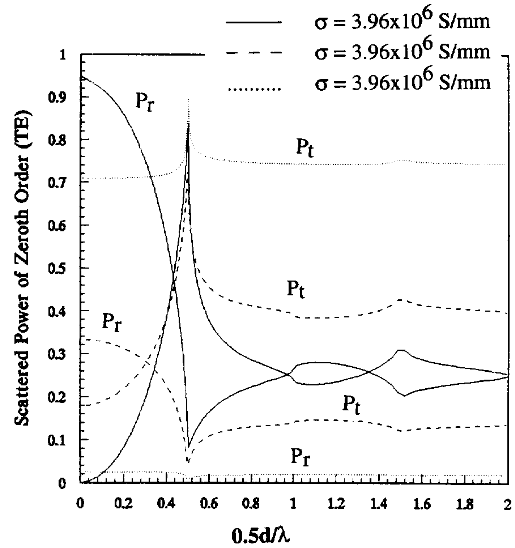

Fig. 3 Scattered power of zeroth order vs. wavelength for a plane wave at normal incidence. P_r is the

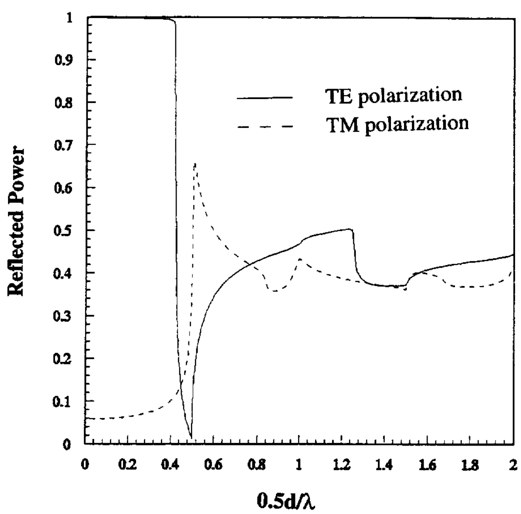

Fig. 2 Reflected power vs. wavelength for the scattering of a plane wave at normal incidence by a semi-infinite grating. The parameters of the grating are: $w_a = 0.6$mm and $w_m = 0.4$ mm.

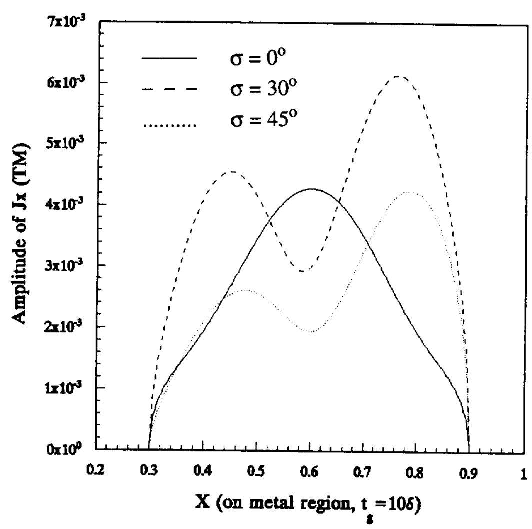

Fig. 4 Current distributions on a metal strip. The parameters of the grating are:
$w_a = w_m = \lambda = 0 = 0.6$ mm.