Multiresolution Coding for Digital Transmission

Shiang-Whea Lin   Wen-Thong Chang[1]

Department of Communication Engineering
National Chiao Tung University, Hsin-Chu, Taiwan 30050

ABSTRACT

Joint consideration for signal bit stream priority in source coding and the Euclidean distance between signal points in modulation achieves stepwise degradation in the picture quality. In this multiresolution strategy, system performance is affected by the way how the original images are decomposed, and how many signal levels are used in modulation. With 1:2 ratio of bit-rates between two components in multiresolution 64 QAM transmission, we investigate subband coder, pyramid coder, and decomposed DCT coder in multiresolution source coding. Optimum quantization is employed with nonuniform quantizer matched to Laplacian distribution. Simulation reveals that decomposed DCT coder is a better candidate for multiresolution transmission, which has the advantages that source coder is simple and not sensitive to bit-ratio variation.

1 Introduction

In multi-level QAM digital transmission, each symbol consists of a finite number of bits and is transmitted by quadrature amplitude modulated signals. These transmitted signals form a constellation map that relates the transmitted bit patterns and the transmitted quadrature signal levels. The reliability of this transmission scheme depends highly on the distance between two signal points. The arrangement of the signal points in the constellation map will determine the distances among the bit patterns and then create a relative importance among the bits in a bit stream.

The different bits in two bit patterns corresponding to adjacent signal points will be the bits that are most likely to be in error and can be seen as the least significant bits. On the contrary, these bits corresponding to the farthest signal points can be seen as the most significant bits. Therefore, bits in a multiresolution QAM possess unequal error probability in nature. Based on this characteristics, the constellation map can be arranged in such a way that these bit patterns corresponding to adjacent signal points differ only in one bit. A 64 QAM constellation arranged in this way is shown in Figure 1. The rightmost bits in the bit sequence formed in this way will have highest probability in error rate. Therefore, each bit pattern can be partitioned into parts of different significance. This phenomenon has a very signficant impact on video signal transmission. Since, in video transmission, the effect of each bit on the image quality in the bit streams to be transmitted differs very much. Therefore, partition the bit stream into groups with relative importance is applicable such that bits from important group in video stream are assigned to significant part of the bit pattern. This partitions of bit pattern and bit stream form the underlying structure of the concept of joint multiresolution video coding and modulation.

An important characteristics in this multiresolution modulation is that signal points in the constellation map can be seen to be partitioned into groups. Within each group, the most significant bits for these signal points are the same. As shown in Figure 1, the signal points are partitioned into four groups. The most significant bits corresponding to each of the four groups are 00, 01, 11, 10, respectively. For each group, there are four least significant bits. Based on this concept, to further emphasize the usefulness of this multiresolution modulation, signal point distances can be divided into two types. The intra-distance is used to indicate the signal point distances within a group. The inter-distance is used to indicate the distance between the groups. For these two distance measures, extra parameters can be used to adjust the ratio between the intra-distance and

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the inter-distance. Thus, depending on the application domain, the ratio can be changed to meet the transmission requirement.

This concept has been proposed by Cover called as embedded modulation. Cover [1] showed in theory that the efficiency of embedded broadcast as compared with independent sharing of the broadcast channel resources in time or frequency among the receivers is superior. The concept is applied on real end-to-end system design by Vetterli's nonuniform modulation[2]. In this modulation scheme, the coarse bit stream is embedded into the added detail bit stream. In signal constellation of modulator, coarse bit has longer Euclidean distance than detail ones, thus higher protection for coarse bits is provided.

In this paper, under the 1:2 ratio of bit-rates partition in this multiresolution 64 QAM transmitter, is to investigate the behavior of multiresolution source coding schemes, which include subband coder, pyramid coder, and decomposed DCT coder. In different quantization accuracy, nonuniform quantizer matched to Laplacian distribution is used for optimum quantization. Bit allocation algorithms are applied according to the variance of the coefficients. The behavior of multiresolution 64 QAM modulator in AWGN environment is analyzed.

2 Multiresolution Source Coding

Multiresolution (MR) source coding schemes can be seen as successive refinement methods. They are suboptimal in term of compression and signal-to-noise ratio as compared with the single resolution (SR) scheme that achieves the same full resolution quality for point to point transmission. But they can be superior in digital broadcast situation, which is a multiuser communication problem.

2.1 Source Decomposition

In order to match the multi-level modulation, we decompose the original image into coarse component and detail component in each coding scheme. The coarse component will be put in the more robust channel and will carry more information sensitive to human vision. The detail component will be put in a less reliable channel. So, low priority information will be put in this component for improving reconstructed quality. In the following, we will investigate the characteristics of signal decomposition in subband(SBC), pyramid(PMC), and DCT coding schemes.

1:Subband/DCT hybrid coder :
The subband/DCT hybrid coder uses separable 2-D QMF to decompose the original images. After filtering, $x_c(i,j)$ is taken from the LL-subband as coarse component, the other subbands are taken as detail component $x_d(i,j)$. For both components, DCT transform is used to compress the signals.

2:Pyramid/DCT hybrid coder :
Pyramid/DCT hybrid coder represents a low-pass filtering and decimating system. The relationship between original image $g_0(i,j)$ and coarse component $L_c(i,j)$ is expressed as :

$$\begin{aligned} L_c(i,j) &= \sum_{m=-2}^{2}\sum_{n=-2}^{2} w(m,n) * g_0(2i+m, 2j+n) \\ &\quad \text{where } 1 \le i,j \le M \end{aligned} \qquad (1)$$

where $M \times M$ is the image size. And the weighting pattern $w(m,n) = \hat{w}(m)\hat{w}(n)$, called the generating kernel, is defined as :

$$\begin{aligned} \hat{w}(0) &= a \\ \hat{w}(-1) &= \hat{w}(1) = 1/4 \\ \hat{w}(-2) &= \hat{w}(2) = 1/4 - a/2. \end{aligned} \qquad (2)$$

The filter is chosen subject to certain constraints. [3] and [4] have more detail discussion. In math, the detail component $L_d(i,j)$ is represented as :

$$L_d(i,j) = g_0(i,j) - G(L_c(i,j)) \tag{3}$$

where

$$G(L_c(i,j)) = 4 \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m,n) * L_c(\frac{i-m}{2}, \frac{j-n}{2}) \tag{4}$$

Only these terms for which $(i-m)/2$ and $(j-n)/2$ are integers are included in the sum. We choose $a = 0.6$ for our experiment , which has perceptual as well as computational advantages.

3:DCT/BD coder :

In DCT/BD coder, the transformed cofficients is partitioned into the coarse component and detail component in accordance with bit-rate ratio γ. It is known that the frequencies from low to high within a block are arranged in zigzag order. The boundary between the coarse component and detail component is determined by both the bit allocation table and the parameter γ of transmitter. Let $N_B(u,v)$ be the bit allocation table, and $\gamma = \alpha_c/\alpha_d$, then the boundary is the minimum integer B that satisfies the following inequality:

$$\sum_{k=0}^{B} N_B[zigzagx(k), zigzagy(k)] \geq \frac{\alpha_c}{\alpha_c + \alpha_d} R \tag{5}$$

where R is the number of bits assigned to a block. In our transmitter, $\alpha_c = 1, \alpha_d = 2$. k is the zigzag order and the value of k falls in the range of 0 to 63. Given the zigzag order k, the functions of *zigzagx* and *zigzagy* return the x index and y index in a block, respectively. Once the boundary is determined, the coarse component $X_c(u,v)$ and detail component $X_d(u,v)$ are obtained by the following expression :

$$X_c(u,v) = X(u,v) \qquad \text{for } \{(u,v)|zigzag(\tfrac{u}{N}, \tfrac{v}{N}) \leq B\} \tag{6}$$

$$X_d(u,v) = X(u,v) \qquad \text{for } \{(u,v)|zigzag(\tfrac{u}{N}, \tfrac{v}{N}) > B\} \tag{7}$$

where the function *zigzag* returns the value of zigzag order when given the x index and y index. Table 1(a) lists the $SNR0$ of half-resolution in different bit-rate. The definition of $SNR0$ is

$$SNR0 = -10\log_{10}[\frac{1}{M*M} \sum_{i=1}^{M} \sum_{j=1}^{M} \frac{[x(i,j) - \hat{x}(i,j)]^2}{(255)^2}] \tag{8}$$

where $x(i,j)$ and $\hat{x}(i,j)$ are the original and reconstructed image, respectively. And $M \times M$ is the image size. The half-resolution $SNR0$ is nearly identical as that of SBC and PMC. On the average, 3 dB gain in full-resolution are obtained with fewer overheads.

2.2 Block Statistical Model

After the coarse component and detail component are obtained, we set up a reliable statistical model for subsequent bit allocation and quantization. In theory we know that each DCT coefficient is formed from the cosine weighted sum of all the pixels in the original image. [5] propose that the Laplacian distribution is most suitable as the probability desity function(pdf).. The transformed coefficients in the coarse component and detail component are analyzed. Except the DC coefficients, the other coefficients distribute in

Laplacian characteristics. The DC coefficients are always assigned 8 bits to avoid blocking effect.

For more efficient bit allocation, coefficient statistics is used in the model [6]. The ac energy within each block in transform domain as statistical measure is used to analyze the input image activity. The DC level in transform domain is excluded in this measure since it only represents the brightness level.
Once the ac energies for all blocks are obtained, these blocks are classified into K classes in accordance to their ac energies. After the classification, the mean and variance matrices for each class is also estimated from the transform coefficients. In our implementation, the cases of $K = 1$ and $K = 4$ are tested. In latter we will see that with more complicated model, better $SNR0$ performance is achieved.

2.3 Bit Allocation and Quantization

Bit allocation determines the bits assigned to each coefficient for quantization. Better bit allocation will minimize the quantization distortion to achieve better $SNR0$ performance at a fixed bitrate. From the rate distortion theory [7], the optimum bit allocation depends on the variance of coefficients. Since the statistics of the coarse component and detail component are very different, one is low-pass characteristics and the other is high-pass, two bit allocation tables(BAT) are designed.

After the bits assigned to the coefficients are determined, the corresponding quantizer will be chosen to map a given coefficient $X(u,v)$ into $\hat{X}(u,v)$ taken from a finite set of values[8]. As analyzed previously, the coefficients except DC are Laplacian distributed. A set of optimum nonuniform quantizers matched to Laplacian distribution is employed here[9]. Their decision and reconstruction intervals are pre-determined and saved, thus, the operation of quantization is reduced to a simple table look-up process. Since these quantizers are optimum when the distribution of input coefficients are zero mean and unit variance, we normalize the input coefficients to meet this requirement by a normalization factor before quantization. The nomalization factor of each coefficient is estimated by

$$\sigma'_k(u,v) = c2^{N_{B,k}(u,v)-1} \tag{9}$$

where c is equal to the maximum standard deviation of those elements in the variance matrix which were assigned one bit. Since $\sigma'_k(u,v)$ is simply the estimation of $\sigma(u,v)$, the maximun value is selected rather than the average to avoid excess clipping.

We investigate both one class and four classes methods for each coding scheme. The four classes method used in SBC scheme has two alternatives;one is classified by ac energies and the other by its splitted subbands. Their $SNR0$ performance is listed in Table 1. There are some overheads in the more complicated method. The comparison table of overheads with these methods are listed in Table 2. In these two tables, we observe that more complicated bit allocation increases both half resolution and full resolution in DCT/BD but only increases full resolution in SBC/DCT and PMC/DCT. The ac energy and splitted-band alternatives achieve almost the same $SNR0$ performance but the latter does not need a classification map overhead. The detail component in PMC/DCT needs one more class than that in SBC/DCT to achieve nearly the same $SNR0$ performance. The one-class DCT/BD achieves nearly the same $SNR0$ performance as the four-class SBC/DCT with saving three BATs. Morever, the four-class DCT/BD obtains 1 dB gain in half resolution and 4.5 dB gain in full resolution, on the average, over than four-class SBC/DCT with only a classification map overhead.

2.4 Coding Schemes Comparison

From the above analysis, under the constraint in 1:2 bitrate ratio between coarse component and detail component, for SBC, the detail component are divided into three separate band-pass signals. This is a frequency division approach. In PMC, the blocks in the detail component are classified into four categories according to their ac energy of the transform coefficients. This corresponds to a space domain division appproach. In both SBC and PMC, bit allocation is done after their source image decomposition. Therefore, bit allocation is constrained by the bit-rate ratio. As compared with SBC and PMC, DCT coder is simple and near the standard. And bits are assigned before source decomposition, the efficiency of bit allocation is less sensitive to the bit-rate ratio parameter and is superior to the two schemes. It achieves almost the same $SNR0$ with fewer overhead or several dB gain with a little more overheads.

3 Multiresolution Modulation

The fundmental concept of multiresolution modulation is that the coarse bits have longer Euclidean distance than the detail bits on the constellation. So, ever in highly noisy environment, coarse bits are well received. However, the full-resolution is obtained by refinement of the added detail bits only in low noise environment. And, the transmission channel is more robust to errors caused by noise. This concept can be applied in conventional digital modulation techniques such as PSK, PAM and QAM.

The constellation of a 64 QAM is shown in Figure 1. In this figure, every symbol is composed of 2 coarse bits and 4 detail bits. The 2 coarse bits determine one of the four regions, while the remaining 4 detail bits determine one of the 16 signal points falling in that regoin. In such modulation scheme, the ratio of bitrates between coarse component and detail component is 1:2. This is the parameter $\gamma = \alpha_c/\alpha_d = 1/2$ metioned in chapter 2;γ is an important parameter in this transmission system. In 1, the ratio between $D1$ and $D2$ is a design parameter λ, which can be utilized to regulate the half resolution coverage and the full resolution coverage depending on the need of the practical situations. As λ goes from 0 to 1, the coarse Euclidean distance decreases and the detail Euclidean distance increases for a fixed power budget. In other words, λ indicates the quantitive tradeoffs involved in coarse and detail channel robustness. The $SNRi$ is the channel quality measure and is defined as

$$SNRi = 10\log_{10}(\frac{E_{av}}{\sigma_n^2}) \tag{10}$$

where E_{av} is the average energy of the equivalent constellation. σ_n^2 is the variance of added white Gaussian noise. The threshold for λ ranging from 0.1 to 1.0 is listed in Figure 2. The coarse threshold and detail threshold are set at the value such that the corresponding bit error rate(BER) is less than 10^{-4}. From the table, we can see that as λ increases, the coarse threshold shifts left and the detail threshold shifts right, thus the coverage between half and full resolutions can be adapted by properly choosing the value of λ. Simulation for channels verifies this trade-off between the coarse channel and detail channel as shown in Figure 3. This figure shows that such modulation scheme will split up the original physical broadcast AWGN channel into two logical channels with different robustness, which is adapted by the parameter λ. The dotted line is the performance of conventional multiplexing 64 QAM.

4 Simulation

4.1 Combined Systems

The effectiveness of above three source coding schemes combined with the matched transmitter is simulated on the broadcast AWGN channel. Figure 4 shows the block diagram of system A, which is the conventional SR DCT-based coder combined with the traditional 64 QAM. Figure 5 shows the block diagram of system B; the MR 64 QAM combined system. We choose $\lambda = 0.3$ as the parameter in the modulation. The SBC/DCT, PMC/DCT, and DCT/BD alternatives are chosen in the block of "MR Source Coder". In Figure 5, the block of "Pre-MR Processor" takes two bits from the coarse bit stream and four bits from the detail bit stream to form a signal point. The block of "Post-MR Processor" splits the received six bits into two bits for the coarse bit stream and four birs for the detail bit stream.

For the above several combined systems, four different bit-rates are experimented, which include 0.5bpp, 1.0bpp, 1.5bpp, and 2.0bpp.

4.2 Results

Figure 6 shows the $SNR0$ performances for system A and system B under the channel input power SNR_i ranging from $3dB$ to $20dB$. The definition of the $SNR0$ and SNR_i are given in Eq (8) and Eq (10) respectively. The $SNR0$ is the source quality measure and the SNR_i is the channel quality measure. From Table 3 we know that the threshold of SNR_i is approximate $13.5dB$ in traditional 64 QAM transmission. So, faraway receivers suffer from a sharp threshold effect when the SNR_i is less than $13.5dB$. But in system B, the coarse component and detail component from source coder are so sophiscated arranged in the constellation that stepwise $SNR0$ performance curves are achieved in broadcast AWGN channel. From this figure, it is observed that both SBC/DCT and PMC/DCT schemes need more complicated bit allocation algorithm to obtain nearly the same full-resolution as system A. However, the DCT/BD scheme always has the same full-resolution as system A. Morever, since the source decomposition is done after bit allocation such that the more complicated bit allocation algorithm increases both the half-resolution and full-resolution, better bit allocation efficiency is achieved.

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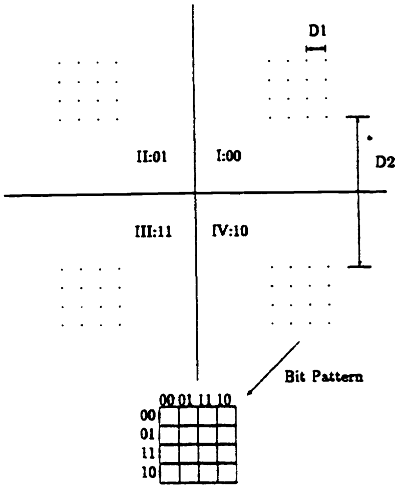


$\lambda = D1/D2$

Figure 1: MR 64 QAM Constellation where D1 is Intra-Distance and D2 is Inter-Distance.

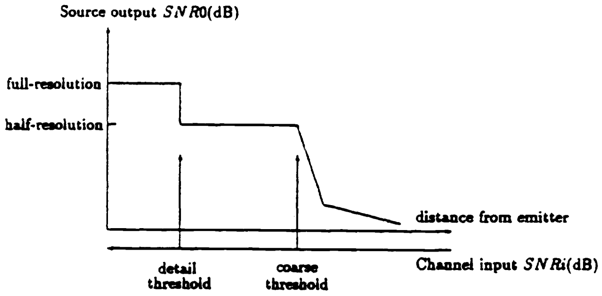
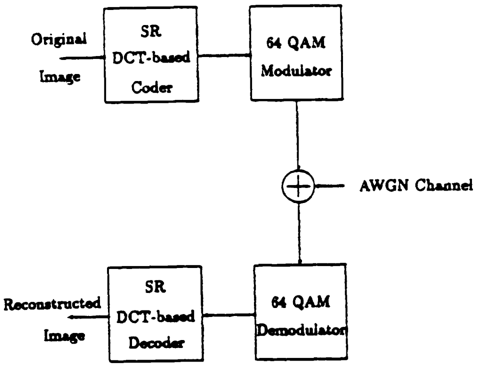
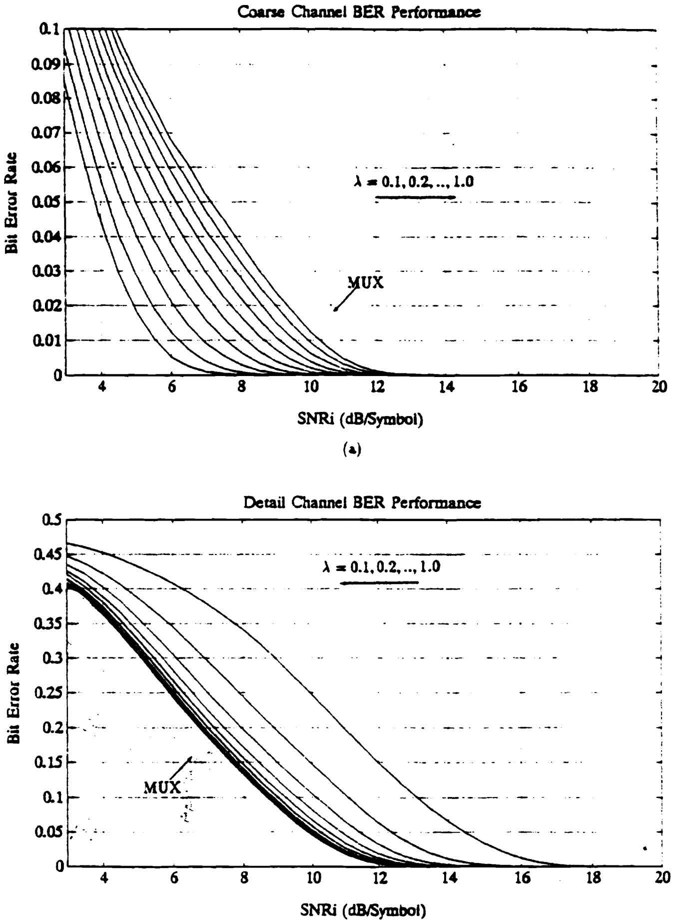


Figure 2: Stepwise Broadcast Channel in Multiresolution Modulation





(a)

(b)

Figure 3: The bit error rate Performance of MR 64 QAM (a)coarse channel.(b)detail channel.

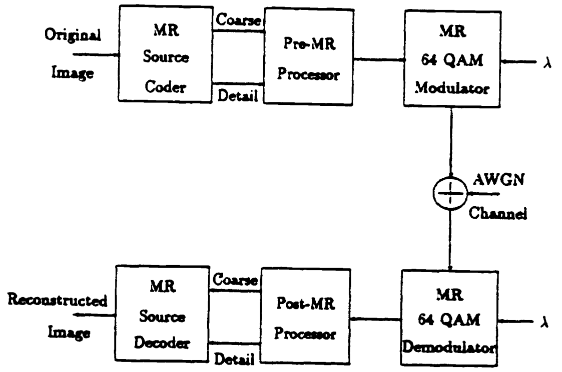


Figure 5: System B : The MR 64 QAM Communication System

Figure 4: System A : A Conventional SR 64 QAM Communication System

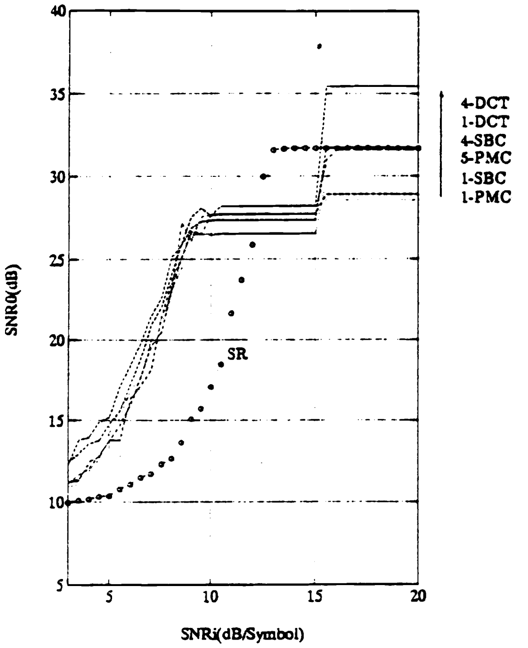


Figure 6: MR 64 QAM Combined Systems Performance with Bit-rate 2.0bpp.

(a)One-class Bit Allocation method

	SBC/DCT		PMC/DCT		DCT/BD	
BitRate	half	full	half	full	half	full
0.5bpp	23.14	22.17	22.61	22.60	22.78	25.10
1.0bpp	24.76	24.14	24.32	24.81	25.44	27.39
1.5bpp	26.36	25.42	25.60	26.66	26.53	29.48
2.0bpp	27.72	28.23	26.55	28.57	27.36	31.73

(b)Four-class Bit Allocation method

Bitrate	SBC/DCT				PMC/DCT		DCT/BD	
	ac energies		splitted-band					
	half	full	half	full	half	full	half	full
0.5bpp	23.14	23.40	23.14	23.20	22.61	23.31	24.95	27.69
1.0bpp	24.76	26.41	24.76	25.60	24.32	26.24	26.73	31.23
1.5bpp	26.36	29.27	26.36	28.79	25.60	29.25	27.95	33.85
2.0bpp	27.72	31.58	27.72	31.05	26.55	31.37	28.20	35.45

Table 1: The half-resolution and full-resolution $SNR0$(dB) for each scheme (a) one class (b) four classes

λ	Coarse Threshold(dB)	Detail Threshold(dB)
0.1	8	19
0.2	9.5	17
0.3	10	15.5
0.4	10.5	15
0.5	11	14.5
0.6	11.5	14.5
0.7	12	14.5
0.8	12.5	14
0.9	13	14
1.0	13.5	13.5

Table 3: Relation of Coarse Threshold and Detail Threshold v.s. λ

(a)List of One-class overheads

	Classification Maps	BAT	Normalization Factors
SBC/DCT	0	2	2
PMC/DCT	0	2	2
DCT/BD	0	1	1

(b)List of Four-class overheads

		Classification Maps	BAT	Normalization Factors
SBC/DCT	ac energies	1	4	4
	splitted-band	0	4	4
PMC/DCT		1	5	5
DCT/BD		1	4	4

Table 2: Summary of overheads for each coding scheme (a) one class (b) four classes