

# The Heterostructure Photonic Crystal Waveguide Splitter

Ting-Hang Pei and Yang-Tung Huang

**Abstract**—We propose a two-dimensional heterostructure photonic crystal (PhC) waveguide splitter composed of different air-hole radii in the core and claddings. Two PhCs in the core and claddings are treated as two effective homogeneous media in the first photonic band. By using a triangular coupler in the input port, light can be efficiently coupled from the conventional waveguide into the PhC waveguide splitter. Total internal reflections take place at outer interfaces and confine light in the PhC waveguide very well. Finally, incident light is divided into three parts, and each of them couples to output waveguides with total transmission more than 90%.

**Index Terms**—Heterostructure, photonic crystal (PhC), splitter.

## I. INTRODUCTION

**I**N integrated optical circuits, waveguide branches based on planar photonic crystals (PhCs) are an attractive developing field recently. A device which can split the input energy into two or more channels is an important application. The design of waveguide branches in the PhC with perfect transmission and zero reflection has been widely discussed [1]–[3]. They are T-shaped or Y-shaped waveguides designed to equally transport half the input energy to two output ports. The coupled mode theory is shown to explain transmission of light through these waveguides very well [3].

Recently, the PhC heterostructure waveguide with 2-D triangular PhCs composed of air holes in a GaAs slab has been demonstrated. The waveguide is designed with the PhC core surrounded by two PhCs with smaller holes [4]. Its advantage is the large core width which can efficiently couple energy into conventional waveguides or from them. On the contrary, the PhC defect waveguide designing to support single mode is made up of such PhC removed a few lines of air holes or rods, where the core is so small that the coupling efficiency is often lower [4]–[7].

In our previous work [8], the high-transmission PhC heterostructure Y-branch waveguide is proposed. Unlike other defect-mode PhC waveguide operating at the photonic band gap (PBG), the designed waveguide operates at photonic band region. It is composed of two different PhCs, each of them

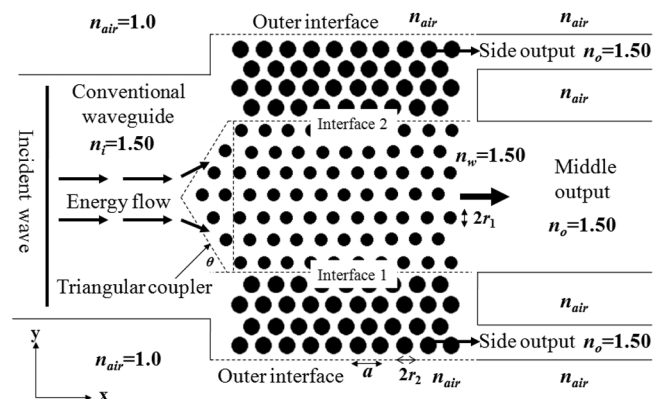


Fig. 1. Schematic diagram of the PhC heterostructure waveguide with a triangular coupler connecting with a conventional waveguide. The wave is incident from conventional waveguide into the PhC heterostructure waveguide. The arrow in the waveguide means the energy flow.

has different air-hole radius resulting in different effective refractive index within the first photonic band. Then these two effective media, just like the core and cladding of the conventional waveguide, confine light in the core region very efficiently. It can be explained by the mechanism of total internal reflection between two PhCs and the transverse resonance condition is also satisfied. This result tells us that even in complicated structures such as PhCs, the concept of effective medium still holds in certain frequency region without losing correctness of optical performances. We also propose the periodic-defect PhC and discuss its equivalent structure, the effective PhC [9], where the effective medium theory is further verified in some frequency region.

In this Letter we use the heterostructure PhC waveguide composed of a PhC core placing in between two PhCs to form the waveguide splitter. This structure removes the design of the T-shaped or Y-shaped waveguides and guide light into three output waveguides by the mechanism of total internal reflection at outer interfaces.

## II. SIMULATED SYSTEM

### A. Structure of the PhC Heterostructure Waveguide

Consider a two-dimensional PhC heterostructure waveguide splitter lying in the  $x$ - $y$  plane as shown in Fig. 1. It is composed of air holes embedded into the dielectric material with refractive index  $n_w = 1.50$  in a triangular array with lattice constant  $a$ . All the lattice constants in the PhC core and claddings are the same. The elementary lattice vectors are  $\vec{a}_1 = (a, 0)$  and  $\vec{a}_2 = (a/2, \sqrt{3}a/2)$ , and the elementary reciprocal lattice vectors in the  $k$ -space are  $\vec{G}_1 = 2\pi/a(1, -1/\sqrt{3})$  and  $\vec{G}_2 = 2\pi/a(0, 2/\sqrt{3})$ , respectively. The radius  $r_1$  of air holes

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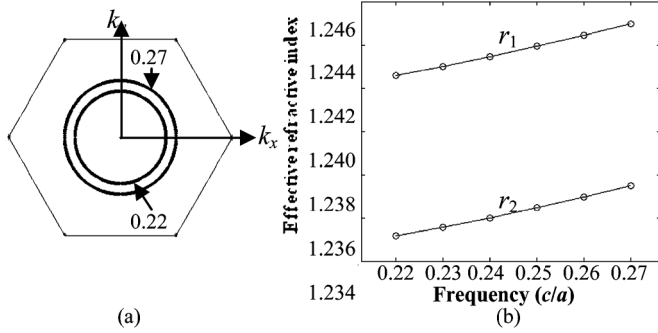


Fig. 2. (a) EFSs for  $r_1$ . The frequencies of the inner and the outer circles are  $0.22(c/a)$  and  $0.27(c/a)$ , respectively. (b) The frequency- $N_{eff}$  relations in (a) for  $r_1$  and  $r_2$ , respectively.

in the PhC core is  $0.395a$  and the radius  $r_2$  in the claddings  $0.40a$ . A triangular coupler made up of the PhC, whose composition is the same as the PhC core, is designed at the input port. Its purpose is to couple light efficiently from the conventional waveguide into the PhC heterostructure waveguide splitter and divide light into two parts. One is toward Interface 1 and the other is toward Interface 2. In Fig. 1, the inclined angle  $\theta$  of the triangular coupler is  $30^\circ$ . The dielectric constant  $n_i$  of the conventional waveguide is 1.50 and the surrounding of the waveguide is air.

### B. Effective Refractive Index

The refracted angle of a light beam from one material into the PhC is determined by the equifrequency surfaces (EFSs) of light in the material and PhC [10]. Each EFS corresponds to certain frequency. The group velocity is normal to the EFS at a certain wave vector and defined as  $\vec{v}_g = \nabla_{\vec{k}}\omega$  in which  $\vec{k}$  and  $\omega$  are wave vector and frequency, respectively. Because the propagation direction is parallel to the group velocity, we can determine it in the PhC once  $\vec{k}$  is given. According to the conservation rule, the incident and refracted wave vectors are continuous for the tangential components parallel to the interface. Given the incident wave vector with frequency and the incident angle, the refracted wave vector as well as the refracted angle can be determined. By analyzing the EFS, we can find out the operating frequency at which optical response of the PhC is like that of a homogeneous medium.

From the EFS calculations of the first photonic band as shown in Fig. 2(a), the shapes of EFSs are round the same as those in homogeneous materials. We can define an effective refractive index  $N_{eff}$  from each EFS at certain frequency as shown in Fig. 2(b). The upper and lower curves are the frequency- $N_{eff}$  relations corresponding to  $r_1$  and  $r_2$ , respectively. The radius of each EFS increases as the frequency increase, so  $N_{eff}$  is positive in the first photonic band [10]. The operating frequency  $0.24(c/a)$  is chosen for studying. At this frequency, the  $N_{eff}$  for the PhC core is 1.2437 and that for PhC claddings is 1.2360.

The propagation wave in the infinite PhC is the Bloch wave which satisfies the periodic condition. It is sum of all the Fourier series which can be expressed as [11]

$$E_{z,kn}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \sum_{\vec{G}} E_{z,\vec{k}n}(\vec{G}) e^{i\vec{G}\cdot\vec{r}}, \quad (1)$$

where  $\vec{k}$  is the wave vector in the first Brillouin zone,  $n$  is a band index, and  $\vec{G} = \vec{G}_{pq} = p\vec{G}_1 + q\vec{G}_2$  with integers  $p$  and  $q$ . The  $k$  vectors are  $0.2985$  and  $0.2966$  ( $2\pi/a$ ) for  $r_1$  and  $r_2$ , respectively. From our calculations at frequency  $0.24(c/a)$ , the main term for both  $r_1$  and  $r_2$  is the  $\vec{G}_{00}$  one, and both Fourier coefficients in (1) are all about 99.6%. All higher ones are very small and less than 0.01 which can be ignored. It is an extremely good approximation that describes the wave in the PhC only by  $\vec{G}_{00}$  term in (1), which is  $e^{i\vec{k}\cdot\vec{r}}$  the same as the wave in the homogeneous medium. So the effective refractive index can reflect the optical performance of the PhC very well.

## III. SIMULATION RESULTS

### A. FDTD Simulation

In order to investigate the transport phenomenon, the FDTD method [12] is used to simulate light coupling from the conventional waveguide to the PhC heterostructure waveguide and propagating in it. A TM-mode Gaussian wave with the electric field perpendicular to the  $x$ - $y$  plane is excited in the conventional waveguide. By using Snell's law and the effective refractive indices of the PhC core and claddings, the propagation directions can be obtained. The incident wave from the conventional waveguide is along the  $x$  direction and then incident on the triangular coupler, so the refracted angles in the triangular coupler and PhC core are both  $7.09^\circ$  formed with the  $x$ -axis. After passing through the triangular coupler, the wave is divided into two parts and each of them propagates toward Interface 1 and Interface 2, respectively. Incident angles formed with the  $y$ -axis at Interface 1 and Interface 2 are both  $82.91^\circ$  which is less than the critical angle  $83.62^\circ$ . Then one part of each wave reflects from the interface, and the other penetrates through the interface into the PhC cladding, where the propagation direction is  $3.10^\circ$  formed with the  $x$ -axis. By further calculations, total internal reflections can take place at two outer interfaces which are between the PhC claddings and the outer media. In this case, most waves are confined in the PhC core and claddings very well. Finally, the incident wave in the conventional waveguide is divided into three parts, and then each of them enters into three output waveguides all with refractive indices  $n_o = 1.50$ . Once the propagation directions in the PhC core and claddings are obtained, the width and length of the waveguide in each region can be determined. In our simulation, the number of air-hole layers in the PhC core along the  $y$  direction is 33, and that in each PhC cladding is 36. The width of the conventional waveguide is a little larger than the PhC core. The number of air-hole layers in the PhC core along the  $x$  direction is 460. The width of the middle output waveguide is the same as the PhC core, and that of each side output waveguide is 21 air-hole layers in the  $y$  direction. The distribution of the electric field in the whole structure is shown in Fig. 3. It is obvious that the incident wave is split into three parts which enter three output waveguides eventually.

### B. Direction of Averaged Energy Velocity

Next, in order to understand the energy flow near the outer interface, the propagation direction of light is calculated. This purpose also helps to further verify the concept of effective medium in PhCs. It has been showed that the group velocity  $\vec{v}_g$  is equal

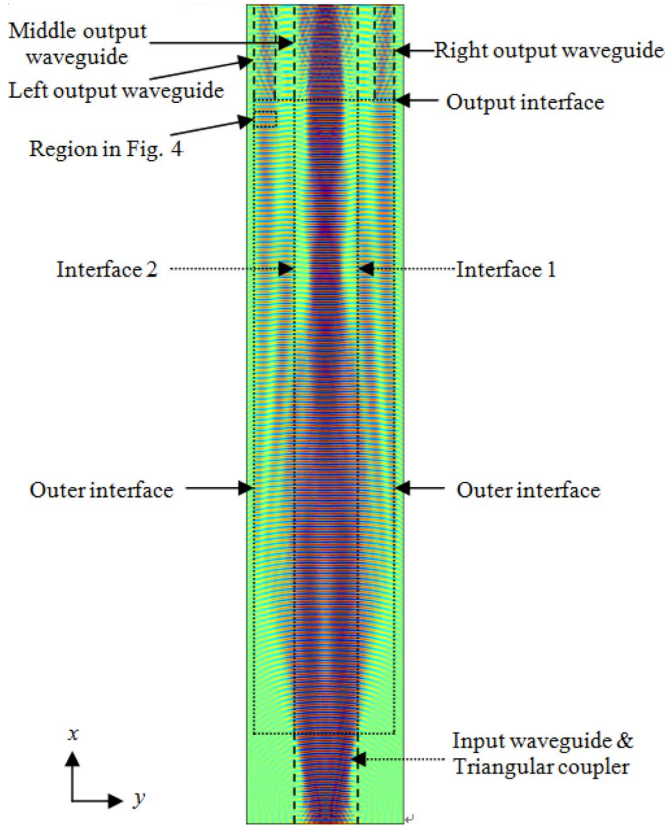


Fig. 3. FDTD simulation of the wave propagating through the PhC heterostructure waveguide splitter.

to the averaged energy velocity  $\vec{v}_g$  in a unit cell, that is,  $\langle \vec{v}_e \rangle = \vec{v}_g = \nabla_{\vec{k}\omega}$  [13].  $\vec{v}_g$  is defined as

$$\langle \vec{v}_e \rangle = \frac{\frac{1}{S} \int \vec{P}_k d\vec{r}_{//}}{\frac{1}{S} \int U_k d\vec{r}_{//}} \equiv \frac{\langle \vec{P}_k \rangle}{\langle U_k \rangle}, \quad (2)$$

where  $S$  is the area of a unit cell and the integral range,  $\vec{r}_{//}$  is the 2-D position vector,  $\vec{P}_k$  is the time-averaged Poynting vector, and  $U_k$  is time-averaged energy density. The distribution of energy directions denoted in the left-upper rectangle region of Fig. 3 is shown in Fig. 4, in which the arrow orientation means the direction of averaged energy velocity. From calculations, all directions are almost parallel to the outer interface. The averaged propagation angle relative to the  $x$  direction in this region is about  $3^\circ$ , which is very close to the calculation by using the effective refractive indices and Snell's law. Furthermore, the ratio of energy in three output waveguides is 1:8:1. Energy in the middle output waveguide occupies about 80% of total output energy. The transmitted efficiency from the triangular coupler to three output waveguides is as high as 90%.

#### IV. CONCLUSION

In summary, the heterostructure PhC waveguide splitter can transport light very efficiently and guide light into three output waveguides. The triangular coupler in the input port plays a role

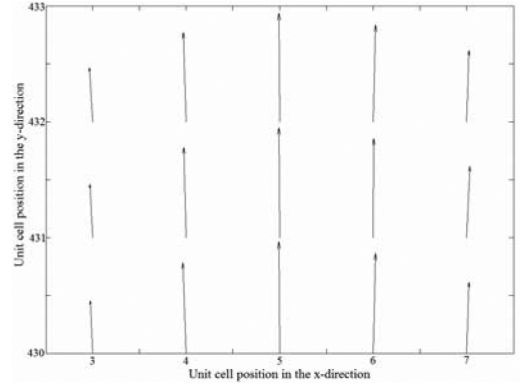


Fig. 4. Energy flow directions near the outer interface denoted by arrows.

which divides the incident light into two parts. It has been verified that the effective refractive index is a very good approximation to treat the PhC core and claddings at the operating frequency. This structure realizing energy branches is very different from T-shaped or Y-shaped waveguides. Unlike these defect PhC waveguides, the wave propagating in it obeys Snell's law where refraction, reflection, and total internal reflection take place at interfaces. The averaged energy velocity near the outer interface indeed matches the prediction by Snell's law. It indicates that varying the radius of air holes in the PhC can change the effective refractive index, which broadens the application of the PhC.

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