Toward Optimal Multiuser Antenna Beamforming for Hierarchical Cognitive Radio Systems

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***Abstract*—In this paper, we present a joint antenna beamforming and power allocation technique to maximize the multiuser sum rate in an underlying microcellular system which reuses the same spectrum of a macrocellular system. One challenge in this kind of hierarchical cognitive radio (HCR) systems is to manage the interference between the macrocell and the microcell. The key contribution of this paper is to develop an optimization technique for antenna beamforming that can maximize the achievable sum rate of the underlying cognitive radio (CR) microcellular system and control the interference between the macrocell and the microcell with a satisfaction level. The proposed technique optimizes the sum rate performance by maximizing its lower bound and transfers the original non-convex problem into a convex optimization problem by introducing auxiliary variables to confine the intra-user interference power among the secondary system. Next, an iterative sum rate maximization (ISM) algorithm is developed to find the beamforming weights and the allocated power for each secondary user to simultaneously maximize system sum rate, coverage, and concurrent multiuser transmission probability in the HCR system. The developed joint design methodology provides valuable insights into the design of an optimal HCR system for various numbers of users as well as cell coverage, and can quantitatively optimize the performance tradeoffs in the hierarchical multiuser CR systems for current and future wireless communication applications.**

***Index Terms*—Hierarchical cognitive radio, multiuser beamforming, power allocation, convex optimization.**

I. Introduction

RECENTLY, cognitive radio (CR) has emerged as an important technology for future wireless communications [1][2] since it can improve the utilization of precious frequency spectrum. According to the amount of the required side information, CR systems have three ways to avoid interfering noncognitive users: underlying, interweaving, and overlaying [3][4]. In the underlay scenario, the CR system operates when the interference from CR systems to the primary users is below a prescribed threshold [5]–[17]. The interweave CR system, requiring considerable information about the activity of noncognitive users, opportunistically accesses the vacant frequency spectrum which is unoccupied by the noncognitive users at particular time or a specific geographic location [18]–[20]. The overlay CR system requires noncognitive users' information and applies sophisticated signal processing to control the interference to the noncognitive users [27].

In this paper, we investigate a hierarchical cognitive radio (HCR) system where an underlying microcellular system simultaneously shares the same spectrum resource with a macrocellular system. Wireless data service usually exhibits an asymmetric behavior with less data traffic generated in the uplink than that in the downlink, and thus the uplink spectrum is often underutilized. To improve the spectrum efficiency, an underlying microcellular CR system is deployed to recycle the uplink spectrum of an already existing macrocellular system. One major challenge for this kind of HCR systems is to manage the interference between the microcell and the macrocell. Another essential challenge is to optimize the secondary users' throughput, while protecting the primary users from being interfered by the secondary users. Moreover, the primary users in the macrocellular system posses higher priority to use the spectrum than the secondary users in the microcellular system.

The objective of this paper is to overcome the aforementioned challenges for the HCR system by using multiple antenna beamforming. In the last decade, multiple-antenna techniques have received a lot of attention due to the advantage of capacity increase [22] and interference reduction [23]. The objective in [23] was to apply beamforming and power control techniques to increase data throughput of homogeneous wireless networks, while maintaining total transmission power at the minimum required level and achieving signal-to-interference plus noise power ratio (SINR) thresholds for all users' links. Our work differs from [23] in two aspects. First, we design the beamforming and power control from a sum-rate maximization perspective, while [23] was from a viewpoint of power minimization. Second, we consider both the intra-cell interference and the cross-tier interference between the macrocell and the microcell, while [23] was only applied to a single cell scenario. In this paper, we suggest applying the beamforming or space-division multiple access (SDMA) techniques to maximize the sum rate of the underlying microcellular system, while serving a number of secondary users of the underlying CR systems by effectively overcoming the cross-tier interference between the microcellular CR system and the macrocellular noncognitive radio system. For an underlying CR system we take account of the following three constraints: interference power, SINR, and transmission power. With the interference power constraint, the primary user can use a single-user decoder without degrading data rates even in the presence of interfering secondary users. We are also interested in the numbers of serviceable secondary users with the minimum guaranteed data rate and the extra

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data rate that can be provided by the underlying CR system for the secondary users. Hence, the SINR constraint is set as the the minimum link quality requirement for concurrent multiuser transmissions, depending on signalling schemes and bit error rates. The transmission power constraint is to limit the maximum total transmission power at the underlying CR base station (BS).

The HCR system can be regarded as a paradigm of the traditional two-user interference channel problem where two users transmit independent messages without cooperation. The exact capacity region of the two-user interference channel is not yet fully understood and remains an open issue. In this channel model, each transmit-receive pair wishes to achieve reliable communications, but the two users interfere with each other. The capacity region is only known for a few special cases such as the Gaussian interference channel or discrete memoryless interference channel with strong interference [24][25]. The growth of CR has recently motivated many works to study the two-user interference channel with one cognitive transmitter [26]–[28]. The channel models considered in [26][29][30] assume that the cognitive transmitter could non-causally or causally access the message of the primary transmitter to enable interference reduction and improve transmission rates through some precoding strategies, e.g., Gel'fand-Pinsker coding in [31] and Costa's dirty paper coding in [32]. Due to the one-side transmitter cooperation, finding the capacity region of the two-user cognitive interference channel is much easier than that of the traditional two-user interference channel. The optimization framework to determine the sum capacity of the cognitive interference channel is in general a non-convex problem. Based on theoretical results in [29], the work of [33] thus attempts to optimize the sum capacity of a multi-input multi-output (MIMO) cognitive interference channel by applying duality techniques. However, the results invented by this work can only provide us an upper bound for the sum rate of the two-user interference channel (or equivalently, the HCR system) as it is assumed that the message of the primary transmitter is fully acquirable at the cognitive transmitter. Therefore, it is imperative to understand the achievable sum rate of the traditional two-user interference channel and to benchmark the sum rate performance of the HCR system from practical design viewpoints.

In this paper, we jointly design the beamforming weights and the allocated power for each CR user to maximize the sum rate of the underlying CR system, while satisfying the constraints of the interference power to the primary BS, the required SINR for the secondary users, and the total transmitted power from the secondary BS. The design framework for the HCR system discussed in this paper can help us understand: (1) how beamforming techniques can be used to manage the interference in the HCR system; (2) what sum rate can be achieved in the underlying CR system. Actually, the sum rate maximization problem is strongly NP-hard even for a standard multiuser power control problem [34]. Compared with those works without adopting beamforming or merely concerning power control [5]–[17], it is more challenging to find the optimal solution for the joint multiuser beamforming and power allocation with all the aforementioned considered constraints, which is indeed a non-convex optimization problem. Detailed review of these related works will be conducted in Section II.

The main contribution of this paper is to solve the aforementioned non-convex optimization problem by proposing an iterative sum rate maximization (ISM) algorithm. First, we optimize the sum rate of the microcellular system by alternatively maximizing its lower bound performance and discuss the feasibility of the three imposed constraints. A necessary condition for the choice of I_{max} and $\gamma_{min,k}$ to make the considered optimization problem feasible is provided, where I_{max} and $\gamma_{min,k}$ are the given values with respect to the interference power and SINR constraints, respectively. Besides, we also analyze the feasibility condition for the case where secondary users are ideally scheduled at different spatial angular directions. Second, we transform the problem of interest into a form of convex optimization by introducing auxiliary variables in the objective function to constrain the intra-user interference power among the secondary users. The transformation approach used for tackling the non-convex sum rate maximization problem is inherently different from the conventional epigraph approach in [15] although they look similar at the first glance. The idea of our approach originates from an observation that by fixing the values of the auxiliary variables, the non-convex optimization problem becomes a manageable convex optimization problem. In addition, an iterative procedure is provided for updating two lower bound-related coefficients so as to tighten the lower bound of the sum rate performance. It is proved that when the iterative procedure converges, the optimal solution of the transformed convex optimization problem with the two converged coefficients is a local maximizer of the original sum rate maximization problem. Third, we use the Lagrangian function and Karush-Kuhn-Tucker (KKT) conditions to analyze the optimal sum rate performance with respect to the perturbations of the auxiliary variables. Interestingly, we find that the maximum achievable sum rate of the transformed equivalent system can proportionally increase with respect to the introduced auxiliary variables when the intra-user interference power constraints among the secondary users are inactive for the derived optimal beamforming weights, i.e., the equality for the intra-user interference power constraints does not hold. When the intra-user interference power constraints are active, i.e., the equality for the intra-user interference power constraints becomes effective, one can increase the sum rate by trading off the objective of maximizing the matched output power and that of minimizing the intra-user interference power among the secondary users. This property helps the proposed ISM algorithm maximize the underlying microcellular sum rate, while complying with all the three aforementioned constraints. The developed methodology can enhance the underlying CR systems to achieve the maximum sum rate, and facilitate the coexistence with noncognitive systems.

The rest of this paper is organized as follows. Some related works are introduced in Section II. In Section III, we describe the HCR system model, where multiple antennas are used at the underlying CR system for downlink broadcasting. Section IV introduces the sum rate maximization problem by jointly designing multiuser beamforming and power allocation with the constraints of SINR, interference power, and transmission power and discuss the feasibility of the three imposed constraints. In Section V, we present the ISM algorithm, which can help reach the optimal solution for the non-convex optimization problem. Section VI shows some numerical results. Finally, Section VII concludes the paper.

The following notations are used throughout this paper. The uppercase and lowercase boldface letters are used to denote matrices and vectors, respectively. The notations $(\cdot)^{\dagger}$, $(\cdot)^{T}$, and $E[\cdot]$ denote the conjugate transpose, the transpose, and the expectation, respectively. The quantity $\|\mathbf{x}\|^2$ denotes the Euclidean norm of a vector $\mathbf{x}$. Let $\Re e(\cdot)$ and $\Im m(\cdot)$ denote the real and imaginary part, respectively. $\mathbf{I}_N$ represents the $N \times N$ identity matrix.

II. Related Works

Some recent research works [5]–[17] discussed the sum rate maximization and power minimization issues for the underlying CR systems. Two essential issues associated with allocating power and designing beamforming weights for the secondary users were investigated in these works. The works of [5]–[8] focused on minimizing the total transmission power of the CR system. In [5], a joint power control and beamforming algorithm was proposed to minimize the total transmission power of the CR system and to satisfy the SINR requirements for both the primary and secondary users. However, acquiring the channel information of the primary system for the underlying CR system is difficult because it requires a huge amount of feedback information between two heterogeneous systems. Hence, the works [6] and [7] considered a more practical design, in which an interference power constraint was used instead of the SINR constraint for the primary user. Two power control strategies were proposed in [6] to assure the aggregated interference power at the primary nodes with a satisfaction probability. However, antenna beamforming is not applied in [6]. Joint power control and beamforming was proposed in [7] to minimize the total transmission power of the CR system, subject to the following two constraints: (1) the interfering power to the primary user is below an acceptable value; (2) the SINR of the secondary users is above a required threshold. Paper [7] developed two suboptimal schemes based on the weighted least square principle to tackle the above optimization problem. Considering the same optimization problem as in [7], the authors in [8] utilized the Gram-Schmidt basis scheme to find the beamforming weights that place spatial nulls at the direction of primary users' receivers.

In the literature, another direction for the underlying CR systems was only focused on maximizing the achievable data rates, e.g. [9]–[17]. A power loading scheme for multicarrier CR systems was investigated by the convex optimization approach in [9] without applying beamforming techniques and only considering the interference introduced to the primary user. Considering a cognitive multiple access channel, a weighted sum rate maximization problem for power allocation was investigated in [10] under the constraints of interference power and transmission power, in which an efficient iterative algorithm was proposed to decouple the complicated two-constraint optimization problem into two single-constraint problems [10]. The works in [11]–[17] utilized the beamforming for overcoming the interference problem, and the idea of user scheduling was exploited in [11]–[12] to improve the sum rate performance opportunistically. A zero-forcing beamforming (ZFB) scheme incorporated with a user selection algorithm was proposed to maximize the sum rate of the secondary system, satisfy the SINR requirements of the secondary users, and limit the interference power to the primary users [11]. The proposed scheme in [11] was suboptimal and may violate the two aforementioned constraints in some cases. In addition, a joint ZFB and user scheduling scheme was proposed in [12] to manage the cross-tier interference among the primary and secondary users. It was shown that the combination of beamforming and user scheduling can mitigate the interference and increase the sum rate of the underlying CR system. Similarly, the work in [13] intended to maximize the sum rate of an underly paradigm CR network by iteratively performing transmit beamforming and power control. A duality relation between broadcast and multiple access channels was utilized to devise an minimum mean square error (MMSE) beamformer by further attending the interference power to primary users. The non-convex sum rate formulation was then approximated by a convex function, and the power control was optimized by a sub-gradient method with a fixed beamforming weight at each iteration. Based on the noncooperative game theory, a novel decentralized approach was investigated in [14], where each CR transmit-receive pair competes against the others to maximize its own information rate.

Some works, however, focused on the worst SINR criterion which is intimately associated with the sum rate maximization, and they casted the beamforming design problem via standard conic optimization frameworks or genetic algorithms [15][16]. From the perspective of random matrix theory, an antenna beamforming scheme was designed to maximize a cognitive SIR by deriving the lower bound on the average interference to the primary users and the upper bound on the average cognitive SIR of the cognitive user [17]. Although the worst SINR maximization problem is mathematically easier than the sum rate maximization problem, optimizing the worst SINR does not necessarily maximize the sum rate performance.

From the above discussion, we observe that the interference problem between the macrocellular systems and the microcellular systems has not been well addressed by simultaneously considering all the constraints of interference power, SINR, and transmission power. Moreover, the optimal solution for jointly designing antenna beamforming and power allocation for underlying CR systems has not been implemented from the perspective of maximizing achievable sum rates.

III. Hierarchical Cognitive Radio Systems

Fig. 1 illustrates the considered HCR system model, where a frequency division duplex (FDD) macrocellular system consists of a BS and a primary user equipped with one single antenna. In the same area, an underlying time division duplex (TDD) microcellular BS with multiple antennas is deployed. The underlying CR microcellular BS is equipped with M multiple antennas to serve a group of K secondary users concurrently by utilizing the uplink spectrum of the primary macrocellular system, where $M > K$. The HCR system aims

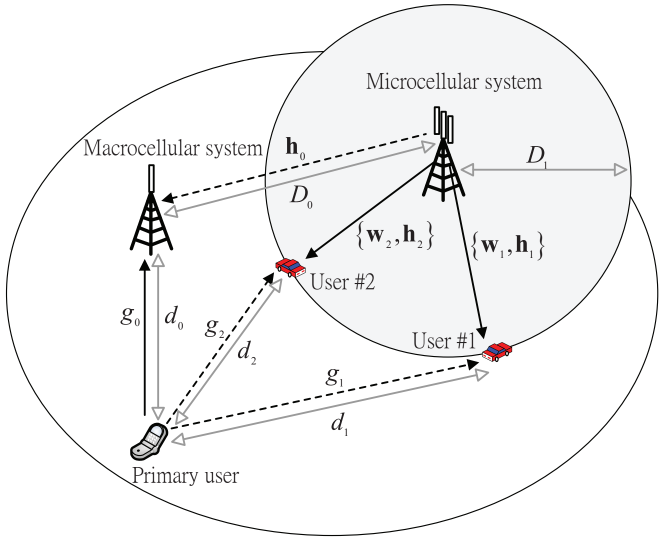

Fig. 1. An HCR system model.

at providing reliable communications for the secondary users and avoids yielding severe interference to the primary user.

Denote s_j and x as the transmitted signals from the secondary BS to the j^{th} secondary user and that from the primary user to the primary BS, respectively. Assume that the transmitted signals s_j and x are uncorrelated with each other, i.e., $E\left[xs_j^*\right]=0$ for all j, and $E\left[s_is_j^*\right]=0$ for all $j\neq i$. Furthermore, the energy of signal sources is assumed to be normalized to one, i.e., $E\left[|s_j|^2\right]=E\left[|x|^2\right]=1$ for all j. An $M\times 1$ column vector $\mathbf{w}_j$ represents the beamforming weight for the j^{th} secondary user, for $j=1,\ldots,K$. Then, the received signal at the k^{th} secondary user is given by

$$r_k=\left(\sum_{j=1}^{K}\mathbf{w}_js_j\right)^{\dagger}\mathbf{h}_k+\sqrt{Q}g_kx+z_k,\ k=1,\ldots,K \ , \quad (1)$$

where $\mathbf{h}_k$ denotes the channel response corresponding to M antennas between the secondary BS and the k^{th} secondary user; Q denotes the transmitted power for the primary user; g_k represents channel response between the primary user and the k^{th} secondary user; and z_k is Gaussian noise at the k^{th} receiver side with zero mean and variance σ_z^2. On the other hand, the received signal at the primary BS can be written as

$$r_0=\sqrt{Q}g_0x+\left(\sum\nolimits_{j=1}^{K}\mathbf{w}_js_j\right)^{\dagger}\mathbf{h}_0+z_0\ , \quad (2)$$

where z_0 denotes the noise at the primary BS, and $\mathbf{h}_0$ and g_0 represents the channel from the secondary BS to the primary BS and that from the primary user to the primary BS, respectively. Let D_0 and d_0 be the distance from the primary BS to the secondary BS and to the primary user, respectively, and represent d_k as the distance between the primary user and the k^{th} secondary user ($k=1,\ldots,K$). For simplicity, we assume that the secondary users are uniformly distributed within a circle of a fixed radius D_1. Assume that long-term power control is adopted to compensate path loss in both the macrocellular and microcellular systems. Hence, in (1) and (2) the channel responses g_k with the effects of shadowing and multipath fading are represented as

$$g_k=\begin{cases}\sqrt{\beta_k}a_k,\ k=0\\ \sqrt{\xi_k\beta_k}a_k,\ k=1,\ldots,K\ ,\end{cases} \quad (3)$$

where a_k is a Rayleigh-faded channel gain, β_k stands for the log-normal shadowing with a variance of S, ξ_k characterizes the effective interference power boost for the k^{th} secondary user resulting from the perfect power control mechanism at the primary user. Due to the long-term power control for compensating path loss, the power emitted by the primary user is scaled by a factor of $(d_0/d_{ref})^{\alpha_1}$, where d_{ref} is the reference distance, and α_1 is the path loss exponent of the primary system. Thus, the effective interference power gain to the k^{th} secondary user is given by $\xi_k\!=\!(d_0/d_{ref})^{\alpha_1}(d_{ref}/d_k)^{\alpha_1}\!=\!(d_0/d_k)^{\alpha_1}$. Similarly, we can express the channel responses $\mathbf{h}_k$ as

$$\mathbf{h}_k=\begin{cases}\sqrt{\xi\tilde{\beta}_k}\tilde{a}_k\mathbf{v}_k,\ k=0\\ \sqrt{\tilde{\beta}_k}\tilde{a}_k\mathbf{v}_k,\ k=1,\ldots,K\ ,\end{cases} \quad (4)$$

where $\mathbf{v}_k$ is the steering vector characterizing the relative phase response of each antenna; $\tilde{a}_k$ and $\tilde{\beta}_k$ have the same definitions as a_k and β_k in (3), respectively; $\xi\!=\!(D_1/D_0)^{\alpha_2}$ models the effective interference power gain for the relative distance of D_1 and D_0 with the path loss exponent α_2 in the secondary system. Note that for a linear array with antenna spacing of half a wavelength and a far-field narrowband source assumption, the steering vector can be expressed as $\mathbf{v}_k=\left[1,e^{-j\pi\sin\theta_k},\ldots,e^{-j(M-1)\pi\sin\theta_k}\right]^T$, where θ_k is the spatial angle for the k^{th} secondary user and $-\frac{\pi}{2}\le\theta_k<\frac{\pi}{2}$[35].

IV. Joint Multiuser Beamforming and Power Allocation

A. Sum Rate Maximization Problem

From (1), the average SINR for the k^{th} secondary user is given by

$$\Upsilon_k=\frac{\left|\mathbf{w}_k^{\dagger}\mathbf{h}_k\right|^2}{\sum_{j=1,j\neq k}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_k\right|^2+B_k},\ k=1,\ldots,K\ , \quad (5)$$

where $B_k=Q\left|g_k\right|^2+\sigma_z^2$ is the noise plus the interference power resulted from the primary user. The achievable sum rate for all the secondary users in the underlying CR system can be lower bounded by

$$\begin{aligned}R_{sum}&=\sum\nolimits_{k=1}^{K}\log_2(1+\Upsilon_k)\\&\ge\sum\nolimits_{k=1}^{K}p_k\log_2(\Upsilon_k)+\rho_k\triangleq R_{sum,lb}\ ,\end{aligned} \quad (6)$$

where the two coefficients p_k and ρ_k can be chosen as $p_k=\varsigma_k/(1+\varsigma_k)$ and $\rho_k=\log_2(1+\varsigma_k)-p_k\log_2(\varsigma_k)$, for any given $\varsigma_k>0$ [13]. In fact, the equality in (6) holds when $p_k=\Upsilon_k/(1+\Upsilon_k)$ and $\rho_k=\log_2(1+\Upsilon_k)-p_k\log_2(\Upsilon_k)$, and the equality holds for $(p_k,\rho_k)=(1,0)$ if Υ_k approaches plus infinity.

Our objective is to maximize the sum rate of the secondary system by tightening the lower bound in (6) subject to the following three constrains: (1) to ensure the interference power to the primary BS; (2) to guarantee the SINR requirement for each secondary user; (3) to limit the transmission power from the secondary BS. From (6), the problem can be formulated as follows:

$$\begin{aligned} &\hat{\mathbf{W}} = \arg\max_{\mathbf{W}} \sum_{k=1}^{K} p_k \cdot \log_2\left(\frac{\left|\mathbf{w}_k^{\dagger}\mathbf{h}_k\right|^2}{\sum_{j=1,j\neq k}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_k\right|^2+B_k}\right) + \rho_k \\ &\text{subject to} \\ &(C.1)\ \sum_{j=1}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_0\right|^2 \leq I_{\max} \\ &(C.2)\ \frac{\left|\mathbf{w}_k^{\dagger}\mathbf{h}_k\right|^2}{\sum_{j=1,j\neq k}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_k\right|^2+B_k} \geq \gamma_{\min,k},\ k=1,\ldots,K \\ &(C.3)\ \sum_{j=1}^{K}\left\|\mathbf{w}_j\right\|^2 \leq P_{\max}\ , \end{aligned} \tag{7}$$

where $\mathbf{W} = [\mathbf{w}_1,\ldots,\mathbf{w}_K]$; $0 < p_k < 1$, for all k; $I_{\max}$, $\gamma_{\min,k}$, and $P_{\max}$ are given with respect to the interference power, SINR, and transmission power constraints, respectively, The selection of the coefficients p_k and ρ_k will affect the lower bound performance, which will be discussed later in Section V. Note that the primary user's link quality often relies on its SINR value. However, using an instantaneous SINR as a performance constraint is impractical for HCR systems since it is difficult to track instantaneous channels of the primary link at the microcellular BS. For practical applications, an attractive performance metric is the outage probability of the primary user's SINR. In Appendix A, we establish the equivalence relationship between the outage probability constraint and the interference power constraint of (*C.1*). Moreover, the power allocated to the secondary users is implicitly defined in the power norm of $\mathbf{w}_j$ $(j = 1,\ldots,K)$. Since the amplitude and phase of the beamforming weight vectors for all the secondary users need to be jointly optimized, we refer (7) as a joint optimization problem for multiuser beamforming and power allocation in the rest of this paper. The joint multiuser beamforming and power control optimization problem can maximize the overall microcellular sum rate by making tradeoffs among the three imposed constraints in (7). When the constraint set is feasible, all the secondary users are allowed to transmit concurrently. In general, the concurrent transmission opportunity strongly depends on the secondary users' spatial angles or correlations. Solving this sum rate optimization problem can provide us some important insights into the performance of the concurrent transmission probability and the achievable sum rate for various spatial scheduling schemes and different numbers of transmit antennas.

B. Feasibility

This subsection discusses the feasibility of the optimization problem (7). By applying the Cauchy-Schwarz inequality and the constraint (*C.3*), it follows that $\sum_{j=1}^{K}\left|\mathbf{w}_j^{H}\mathbf{h}_0\right|^2 \leq \sum_{j=1}^{K}\left\|\mathbf{w}_j\right\|^2\left\|\mathbf{h}_0\right\|^2 \leq \left\|\mathbf{h}_0\right\|^2 P_{\max}$. Then, from (*C.1*) and (*C.3*), we obtain that any beamforming weights $\mathbf{w}_k$ subject to (*C.3*) must also satisfy the interference constraint (*C.1*) if

$$I_{\max} \geq P_{\max}\left\|\mathbf{h}_0\right\|^2. \tag{8}$$

Besides, by substituting (*C.3*) into (*C.2*), a necessary condition is provided in (B.4) in Appendix B. We prove in (B.7) that if secondary users are ideally scheduled at distinct angular directions in the spatial domain, there exists beamforming weights $\mathbf{w}_k$ such that

$$\begin{aligned} &\Upsilon_k \geq \gamma_{\min},\ \text{for all}\ \ k, \\ &\qquad\text{only if}\ \ \frac{P_{\max}}{K}\cdot\max_k\left(\frac{\|\mathbf{h}_k\|^2}{B_k}\right) \geq \gamma_{\min}. \end{aligned} \tag{9}$$

where we assume $\gamma_{\min,k} = \gamma_{\min}$ for all k. Finally, combining (8) and (9), we can conclude that the problem (7) is feasible for a given P_{max} only if I_{max} and $\gamma_{\min}$ are appropriately chosen to meet the two conditions (8) and (9).

V. Iterative Sum Rate Maximization Algorithm

A. Transformation into Convex Optimization Problem

As seen in (7), it is nontrivial to solve the joint optimization problem because the objective function is apparently non-concave. In this section, we propose an iterative approach to find the optimal solution of (7). First we introduce auxiliary variables in the objective function of (7) to limit the intra-user interference power among the secondary users. Then, we can rewrite the considered joint optimization problem as

$$\begin{aligned} &\hat{\mathbf{W}} = \arg\max_{\mathbf{W}} \sum_{k=1}^{K} p_k \cdot \log_2\left(\frac{1}{\Omega_k+B_k}\left|\mathbf{w}_k^{\dagger}\mathbf{h}_k\right|^2\right) + \rho_k \\ &\text{subject to} \\ &(C.1)\ \sum_{j=1}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_0\right|^2 \leq I_{\max} \\ &(C.2)\ \frac{\left|\mathbf{w}_k^{\dagger}\mathbf{h}_k\right|^2}{\sum_{j=1,j\neq k}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_k\right|^2+B_k} \geq \gamma_{\min,k},\ k=1,\ldots,K \\ &(C.3)\ \sum_{j=1}^{K}\left\|\mathbf{w}_j\right\|^2 \leq P_{\max} \\ &(C.4)\ \sum_{j=1,j\neq k}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_k\right|^2 \leq \Omega_k,\ k=1,\ldots,K\ , \end{aligned} \tag{10}$$

where Ω_k is an auxiliary variable with a non-negative value which bounds the intra-user interference power to the k^{th} secondary user, for $k = 1,\ldots,K$, and $\mathbf{\Omega} = [\Omega_1,\ldots,\Omega_K]^T$. Since the beamforming vectors in (10) appear in terms of the norm square expression, it is possible to forget the phase without affecting the optimal solution. Without loss of generality, we restrict that the matched output between the beamforming weight $\mathbf{w}_k$ and the channel response $\mathbf{h}_k$ for each secondary user is merely a non-negative real value, i.e., producing the amplitude gain. Thus, we can reformulate (10) as follows:

$$\begin{aligned} &\hat{\mathbf{W}} = \arg\max_{\mathbf{W}} \sum_{k=1}^{K} p_k\left(2\log_2\left(\mathbf{w}_k^{\dagger}\mathbf{h}_k\right) - \log_2\left(\Omega_k+B_k\right)\right) \\ &\qquad\qquad + \sum_{k=1}^{K}\rho_k \\ &\text{subject to} \\ &(C.1)\ \sum_{j=1}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_0\right|^2 \leq I_{\max} \\ &(C.2)\ \sum_{j=1,j\neq k}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_k\right|^2 + B_k \\ &\qquad\qquad \leq \frac{1}{\gamma_{\min,k}}\left(\mathbf{w}_k^{\dagger}\mathbf{h}_k\right)^2,\ k=1,\ldots,K \\ &(C.3)\ \sum_{j=1}^{K}\left\|\mathbf{w}_j\right\|^2 \leq P_{\max} \\ &(C.4)\ \sum_{j=1,j\neq k}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_k\right|^2 \leq \Omega_k,\ k=1,\ldots,K \\ &(C.5)\ \Re e\left(\mathbf{w}_k^{\dagger}\mathbf{h}_k\right) \geq 0,\ k=1,\ldots,K \\ &(C.6)\ \Im m\left(\mathbf{w}_k^{\dagger}\mathbf{h}_k\right) = 0,\ k=1,\ldots,K\ . \end{aligned} \tag{11}$$

Until now, it is still very difficult to simultaneously optimize the transformed optimization problem in (11) for both variables $\mathbf{W}$ and $\mathbf{\Omega}$ because the objective function in (10) or (11) is the logarithm of a quadratic-over-linear function, which appears as a non-concave one. The non-concavity can be intuitively checked by rewriting the objective function in (10) as the minus of two convex functions $2\log_2\left(\mathbf{w}_k^\dagger \mathbf{h}_k\right)$ and $\log_2(\Omega_k + B_k)$. However, we show that (11) is indeed a convex optimization problem for a given $\mathbf{\Omega}$. Clearly, the constraints (*C.1*), (*C.3*), and (*C.4*) in (11) are convex since they appear in the formulation of the non-negative weighted sums of quadratic functions. The constraint functions of (*C.5*) and (*C.6*) are linear. We can further express (*C.2*) in (11) as a second-order cone convex constraint like [38]. Since the above optimization problem of (11) for any fixed value of $\mathbf{\Omega}$ is convex, one can obtain the optimal beamforming weight vectors to achieve the maximum sum rate $R_1(p_k, \rho_k, \mathbf{\Omega})$.

B. Tightness of Sum Rate Lower Bound Performance

The lower bound performance of (6) is strictly related to p_k and ρ_k, and we tighten the performance by iteratively updating the two coefficients. Define $\hat{\mathbf{w}}_k^{(i)}$ as the optimal solution of (11) for a given $\left(p_k^{(i)}, \rho_k^{(i)}, \Omega_k^{(i)}\right)$ at the i^{th} iteration. In the next iteration, we update $\Upsilon_k^{(i+1)} = \Upsilon_k\left(\mathbf{w}_k \leftarrow \hat{\mathbf{w}}_k^{(i)}\right)$, $\Omega_k^{(i+1)} = \sum_{j=1,j\neq k}^K \left|\hat{\mathbf{w}}_j^{(i)} \mathbf{h}_k\right|^2$, and the two coefficients as

$$\begin{aligned} p_k^{(i+1)} &= p_k\left(\varsigma_k \leftarrow \Upsilon_k^{(i+1)}\right); \\ \rho_k^{(i+1)} &= \rho_k\left(\varsigma_k \leftarrow \Upsilon_k^{(i+1)}\right), \end{aligned} \tag{12}$$

where $\left(p_k^{(0)}, \rho_k^{(0)}, \Omega_k^{(0)}\right)$ are initialized as $(1, 0, \infty)$, and the notation $f(x \leftarrow y)$ means that the variable x in $f(\cdot)$ is replaced by y. We repeat the above procedure until the values of p_k and ρ_k converge, i.e., $p_k^{(i+1)} = p_k^{(i)} = p_k^*$, $\rho_k^{(i+1)} = \rho_k^{(i)} = \rho_k^*$, and $\Omega_k^{(i+1)} = \Omega_k^{(i)} = \Omega_k^*$ and it possesses the following two important properties:

1) *Convergence*: The update of $p_k^{(i)}$ and $\rho_k^{(i)}$ monotonically increases the lower bound performance. That is, we have $\sum_{k=1}^K p_k^{(i+1)} \log_2\left(\Upsilon_k^{(i+1)}\right) + \rho_k^{(i+1)} \geq \sum_{k=1}^K p_k^{(i)} \log_2\left(\Upsilon_k^{(i+1)}\right) + \rho_k^{(i)}$. This can be easily verified from (6), and thus, the convergence can be obtained.
2) *Local maximizer*: When the iterative procedure converges, the optimal solution of (11) with $(p_k, \rho_k, \Omega_k) = (p_k^*, \rho_k^*, \Omega_k^*)$ can reach a local maximizer of the original sum rate maximization problem, i.e., the problem that the objective function $R_{sum} = \sum_{k=1}^K \log_2(1 + \Upsilon_k)$ is adopted in (11). See Appendix C for the proof.

C. Iterative Sum Rate Maximization (ISM) Algorithm

Now, we determine the optimal value for $\mathbf{\Omega}$ under the converged values of (p_k^*, ρ_k^*):

$$\hat{\mathbf{\Omega}} = \arg\max_{\mathbf{\Omega}} R_1(p_k^*, \rho_k^*, \mathbf{\Omega}) \ . \tag{13}$$

Although $\hat{\mathbf{\Omega}}$ in (13) can be obtained by the exhaustive search, it is still preferable to develop an efficient way to reduce the computational complexity. First, according to (10), we define

$$\begin{aligned} R_2(\mathbf{\Omega}, \mathbf{W}) &= \sum_{k=1}^K p_k^* \cdot \log_2\left(\frac{1}{\Omega_k + B_k}\left(\mathbf{w}_k^\dagger \mathbf{h}_k\right)^2\right) + \rho_k^* \\ &\triangleq f(\mathbf{\Omega}) + g(\mathbf{W}) \ , \end{aligned} \tag{14}$$

where $f(\mathbf{\Omega}) = \sum_{k=1}^K -p_k^* \cdot \log_2(\Omega_k + B_k)$ and $g(\mathbf{W}) = \sum_{k=1}^K 2p_k^* \cdot \log_2\left(\mathbf{w}_k^\dagger \mathbf{h}_k\right) + \rho_k^*$. For convenience of notation, we omit p_k^* and ρ_k^* in $R_1(p_k^*, \rho_k^*, \mathbf{\Omega})$. From (11) and (14), it follows that

$$\begin{aligned} R_1(\mathbf{\Omega}) &= \max_{\mathbf{W}\in\mathbf{\Theta}(\mathbf{\Omega})} R_2(\mathbf{\Omega}, \mathbf{W}) \\ &= \max_{\mathbf{W}\in\mathbf{\Theta}(\mathbf{\Omega})} \{f(\mathbf{\Omega}) + g(\mathbf{W})\} \ , \end{aligned} \tag{15}$$

where $\mathbf{\Theta}(\mathbf{\Omega})$ is the feasible set associated with the optimization problem of (11) and its size depends on the value of $\mathbf{\Omega}$. Consider two vectors $\mathbf{\Omega}^{(1)} = \left[\Omega_1^{(1)}, \ldots, \Omega_K^{(1)}\right]^T$ and $\mathbf{\Omega}^{(2)} = \left[\Omega_1^{(2)}, \ldots, \Omega_K^{(2)}\right]^T$, leading to two non-empty feasible sets $\mathbf{\Theta}^{(1)}$ and $\mathbf{\Theta}^{(2)}$, respectively. By letting $\mathbf{\Omega}^{(2)} \leq \mathbf{\Omega}^{(1)}$ (i.e., $\Omega_k^{(2)} \leq \Omega_k^{(1)}\ \forall k$), we can observe that

$$f\left(\mathbf{\Omega}^{(2)}\right) \geq f\left(\mathbf{\Omega}^{(1)}\right) \ . \tag{16}$$

Define $R_g(\mathbf{\Omega}) = \max_{\mathbf{W}\in\mathbf{\Theta}(\mathbf{\Omega})} g(\mathbf{W})$. Since $\mathbf{\Theta}^{(2)}$ is a subset of $\mathbf{\Theta}^{(1)}$ if $\mathbf{\Omega}^{(2)} \leq \mathbf{\Omega}^{(1)}$, one can see that

$$R_g\left(\mathbf{\Omega}^{(2)}\right) \leq R_g\left(\mathbf{\Omega}^{(1)}\right) \ . \tag{17}$$

To further analyze the optimal value of $R_g(\mathbf{\Omega})$ with respect to the perturbations of the auxiliary vector $\mathbf{\Omega}$, we can write a Lagrangian function by incorporating the constraint of (*C.4*) in (11) into the objective function $g(\mathbf{W})$. Hence, we transform the optimization problem of $R_g(\mathbf{\Omega}) = \max_{\mathbf{W}\in\mathbf{\Theta}(\mathbf{\Omega})} g(\mathbf{W})$ into its equivalent form:

$$\begin{aligned} R_g(\mathbf{\Omega}) = -\max_{\boldsymbol{\lambda}\geq\mathbf{0}} \min_{\mathbf{W}\in\tilde{\mathbf{\Theta}}} & -g(\mathbf{W}) \\ & + \sum_{k=1}^K \lambda_k\left(\sum_{j=1,j\neq k}^K \left|\mathbf{w}_j^\dagger \mathbf{h}_k\right|^2 - \Omega_k\right), \end{aligned} \tag{18}$$

where $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_K]^T$ is a Lagrange multiplier vector with respect to the constraint of (*C.4*), and $\tilde{\mathbf{\Theta}}$ is a convex set formed by all constraints in (11), exclusive of (*C.4*). From (18), we then show that

$$\begin{aligned} & R_g\left(\mathbf{\Omega}^{(2)}\right) \\ &= \max_{\boldsymbol{\lambda}\geq\mathbf{0},\mathbf{W}\in\tilde{\mathbf{\Theta}}} g(\mathbf{W}) - \sum_{k=1}^K \lambda_k\left(\sum_{j=1,j\neq k}^K \left|\mathbf{w}_j^\dagger \mathbf{h}_k\right|^2 - \Omega_k^{(2)}\right) \\ &= g\left(\tilde{\mathbf{W}}\right) - \sum_{k=1}^K \tilde{\lambda}_k\left(\sum_{j=1,j\neq k}^K \left|\tilde{\mathbf{w}}_j^\dagger \mathbf{h}_k\right|^2 - \Omega_k^{(2)}\right) \\ &\geq g\left(\widehat{\mathbf{W}}\right) - \sum_{k=1}^K \widehat{\lambda}_k\left(\sum_{j=1,j\neq k}^K \left|\widehat{\mathbf{w}}_j^\dagger \mathbf{h}_k\right|^2 - \Omega_k^{(2)}\right) \\ &= g\left(\widehat{\mathbf{W}}\right) - \sum_{k=1}^K \widehat{\lambda}_k\left(\sum_{j=1,j\neq k}^K \left|\widehat{\mathbf{w}}_j^\dagger \mathbf{h}_k\right|^2 - \Omega_k^{(1)}\right) \\ &\qquad + \sum_{k=1}^K \widehat{\lambda}_k\left(\Omega_k^{(2)} - \Omega_k^{(1)}\right) \\ &= R_g\left(\mathbf{\Omega}^{(1)}\right) + \sum_{k=1}^K \widehat{\lambda}_k \Delta_k \ , \end{aligned} \tag{19}$$

where $\Delta_k = \Omega_k^{(2)} - \Omega_k^{(1)}$, $\tilde{\mathbf{W}}$ and $\tilde{\lambda}_k$ are the optimal variables in (18) for $\mathbf{\Omega} = \mathbf{\Omega}^{(2)}$, and $\widehat{\mathbf{W}}$ and $\widehat{\lambda}_k$ are the optimal variables in (18) for $\mathbf{\Omega} = \mathbf{\Omega}^{(1)}$. From (17) and (19), we provide an upper bound and a lower bound on the perturbed optimal value of $R_g\left(\mathbf{\Omega}^{(2)}\right)$. The optimal Lagrange multipliers $\widehat{\lambda}_k$ are exactly the local sensitivity of the lower bound for the optimal value $R_g\left(\mathbf{\Omega}^{(2)}\right)$ with respect to the perturbed vector $\mathbf{\Omega}^{(2)}$. In fact, $\widehat{\lambda}_k$ can be regarded as a subgradient of $R_g\left(\mathbf{\Omega}\right)$ at $\mathbf{\Omega} = \mathbf{\Omega}^{(1)}$ [38]. If $\widehat{\lambda}_k$ is small, the lower bound for $R_g\left(\mathbf{\Omega}^{(2)}\right)$ will not decrease too much for a small perturbation on Δ_k. On the contrary, the influence on the lower bound becomes significant if $\widehat{\lambda}_k$ is large. By applying the KKT condition [38], we can conclude that (as shown in Appendix D)

$$R_1\left(\mathbf{\Omega}^{(2)}\right) \geq R_1\left(\mathbf{\Omega}^{(1)}\right), \quad \text{for} \quad \sum_{j=1, j\neq q}^{K} \left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_q\right|^2 < \Omega_q^{(1)} \; ; \tag{20a}$$

$$R_1\left(\mathbf{\Omega}^{(2)}\right) \geq R_1\left(\mathbf{\Omega}^{(1)}\right) + \widehat{\lambda}_q \Delta_q - p_q^* \cdot \log_2\left(\frac{\Omega_q^{(1)} + \Delta_q + B_q}{\Omega_q^{(1)} + B_q}\right), \quad \text{for} \quad \sum_{j=1, j\neq q}^{K} \left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_q\right|^2 = \Omega_q^{(1)} \; . \tag{20b}$$

The above analyses indicate two important properties. First, when the q^{th} intra-user interference power $\sum_{j=1, j\neq q}^{K} \left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_q\right|^2 < \Omega_q^{(1)}$, we can increase $R_1\left(\mathbf{\Omega}\right)$ by decreasing the value of $\Omega_q^{(1)}$ until the intra-user interference power $\sum_{j=1, j\neq q}^{K} \left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_q\right|^2 = \Omega_q^{(1)}$. Second, when the q^{th} intra-user interference power $\sum_{j=1, j\neq q}^{K} \left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_q\right|^2 = \Omega_q^{(1)}$, the lower bound of the optimal value is determined by the values of $\widehat{\lambda}_q$ and Δ_q. This lower bound can not be improved by decreasing $\Omega_q^{(1)}$ because the value of $\widehat{\lambda}_q \Delta_q - p_q^* \cdot \log_2\left(\frac{\Omega_q^{(1)} + \Delta_q + B_q}{\Omega_q^{(1)} + B_q}\right)$ could be smaller than zero. Intuitively, the lower bound for (20) shows that the sum rate is affected by the secondary users' matched output power and the intra-user interference power among the secondary users. Based on the observation found in (20), we propose an iterative algorithm to increase the sum rate by decreasing Ω_k ($k = 1, \ldots, K$).

Fig. 2 illustrates the procedures of implementing the ISM algorithm. In the initialization step, we first release the intra-user interference power constraints among the secondary users. Subject to the constraints of the primary user's interference power, the secondary users' SINR, and the total transmission power, we find the lower bound coefficients p_k^* and ρ_k^*, and maximize the sum-log of the matched output power for secondary users. Let $\hat{\mathbf{W}}^{(0)}$ denote the obtained beamforming weight. Then, the intra-user interference power constraints in (20) and the corresponding $R_1\left(\mathbf{\Omega}^{(0)}\right)$ are updated by setting $\Omega_k^{(0)} = \sum_{j=1, j\neq k}^{K} \left|\hat{\mathbf{w}}_j^{(0)\dagger}\mathbf{h}_k\right|^2$ for all k. When there is no feasible solution $\hat{\mathbf{W}}^{(0)}$ at the initialization, it is implied that the required values $I_{\max}$, $\gamma_{\min,k}$, and $P_{\max}$ for the three constraints in (7) are too strict to allow concurrent transmissions for all the secondary users. Set $i = 1$ and $q = 1$, and choose a fixed step

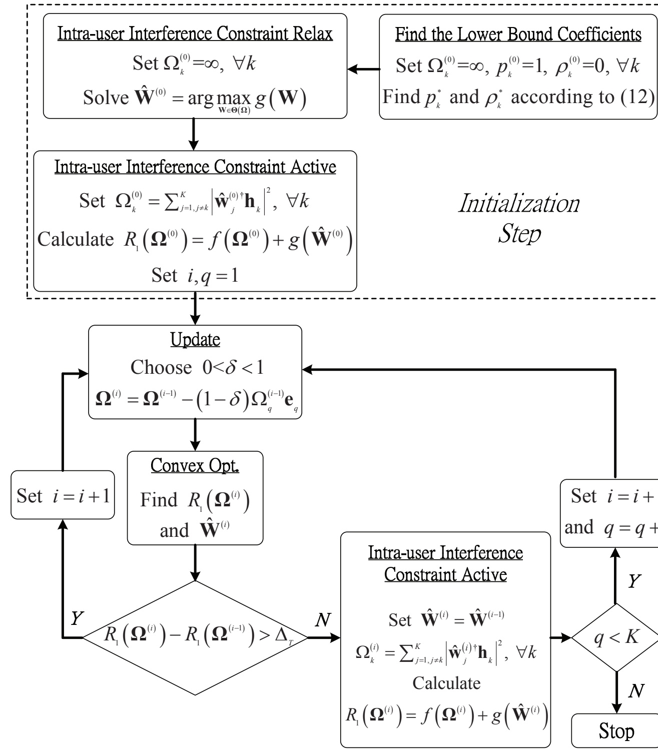

Fig. 2. Procedures for the ISM algorithm.

size δ, where $0 < \delta < 1$. To increase the achievable sum rate, we make a tradeoff between the objective of maximizing the matched output power and that of minimizing the intra-user interference power for the secondary users. We first compute a new vector $\mathbf{\Omega}^{(i)}$ from $\mathbf{\Omega}^{(i-1)}$ based on an updating rule $\mathbf{\Omega}^{(i)} = \mathbf{\Omega}^{(i-1)} - (1-\delta)\,\Omega_q^{(i-1)}\mathbf{e}_q$, where $\mathbf{e}_q$ is the q^{th} column of a $K \times K$ identity matrix $\mathbf{I}_K$. By solving (11) via convex optimization, if $\mathbf{\Omega}^{(i)}$ is feasible, we can obtain $R_1\left(\mathbf{\Omega}^{(i)}\right)$ and continue to the next iteration by increasing the value of i when the improvement in $R_1\left(\mathbf{\Omega}^{(i)}\right) - R_1\left(\mathbf{\Omega}^{(i-1)}\right)$ is larger than a specified threshold $\Delta_T \geq 0$. Otherwise, we have $\hat{\mathbf{W}}^{(i)} = \hat{\mathbf{W}}^{(i-1)}$, and then activate the intra-user interference power constraints by setting $\Omega_k^{(i)} = \sum_{j=1, j\neq k}^{K} \left|\hat{\mathbf{w}}_j^{(i)\dagger}\mathbf{h}_k\right|^2$ for all k. Next, we calculate the corresponding achievable sum rate $R_1\left(\mathbf{\Omega}^{(i)}\right)$, and then proceed to the next secondary user by adding the values of q and i. The algorithm terminates until q reaches the maximum value of K and no further improvement in $R_1\left(\mathbf{\Omega}\right)$ can be achieved. Many path-following interior-point algorithms have been investigated for efficiently achieving the optimality of the convex optimization problem of (11). The discussion of the convergence and complexity of these interior-point algorithms can be found in [36].

It is assumed that the microcellular BS has to know the channel state information $\mathbf{h}_k$ ($k = 0, \ldots, K$), and the noise plus interference power at each secondary user B_k ($k = 1, \ldots, K$). This can be accomplished by periodically broadcasting beacon signals from the microcellular BS in some dedicated beacon time slots. Thus, the secondary users and the macrocellular BS can individually measure the corresponding channel $\mathbf{h}_k$. The beacon signal can be transmitted by using spread spectrum techniques such as pseudo random sequences to avoid disturbing the primary user's transmission at the preliminary stage. Also, each secondary user is assumed to be equipped with a cellular signal strength monitoring device to monitor the noise plus interference power B_k coming from its surrounding primary user. Afterwards, all these information is reported to the microcellular BS in turn through their uplink channels [37].

It is worthwhile to discuss the relationship between the proposed scheme and the ZFB scheme in [11] and [12]. From (10), we can observe that when the auxiliary variables Ω_k are all equal to zero, the proposed scheme is degenerated into a ZFB-based sum rate maximization problem with optimal user power allocation, i.e., replace $\mathbf{w}_j$ in (10) by $\sqrt{\beta_j}\hat{\mathbf{w}}_{ZF,j}$, as follows:

$$\begin{aligned} &\hat{\boldsymbol{\beta}} = \arg\max_{\boldsymbol{\beta}} \sum_{k=1}^{K} p_k \cdot \log_2\left(\frac{\beta_k}{B_k}\right) + \rho_k \\ &\text{subject to} \\ &(C.1)\ \sum_{j=1}^{K} \beta_j \left|\hat{\mathbf{w}}_{ZF,j}^{\dagger}\mathbf{h}_0\right|^2 \le I_{\max} \\ &(C.2)\ \frac{\beta_k}{B_k} \ge \gamma_{\min,k},\ k = 1,\ldots,K \\ &(C.3)\ \sum_{j=1}^{K} \beta_j \left\|\hat{\mathbf{w}}_{ZF,j}\right\|^2 \le P_{\max}\,, \end{aligned} \tag{21}$$

where $\hat{\mathbf{W}}_{ZF}^{\dagger} = [\hat{\mathbf{w}}_{ZF,1},\ldots,\hat{\mathbf{w}}_{ZF,K}]^{\dagger} = \left(\mathbf{H}^{\dagger}\mathbf{H}\right)^{-1}\mathbf{H}^{\dagger}$ is the ZFB solution, $\mathbf{H} = [\mathbf{h}_1,\ldots,\mathbf{h}_K]$, $\boldsymbol{\beta} = [\beta_1,\ldots,\beta_K]$, and β_j is the power allocated to the j^{th} secondary user. The degenerated optimization problem of (21) is a convex one with affine function constraints, and the zero-forcing condition will rigorously limit the degree of design freedom for achieving the optimal sum rate, where the allocated power β_j should satisfy the three constraints in (21). Our proposed scheme, however, bears resemblance to the MMSE solution by replacing $\hat{\mathbf{W}}_{ZF}^{\dagger}$ in (21) with $\hat{\mathbf{W}}_{MMSE}^{\dagger} = \left(\mathbf{H}^{\dagger}\mathbf{H} + \boldsymbol{\Phi}\right)^{-1}\mathbf{H}^{\dagger}$, where $\boldsymbol{\Phi}$ complies with the intra-user interference power constraints of (*C.4*) in (10). Due to the relaxation of $\Omega_k = 0$, the proposed scheme offers more flexibility to obtain better performance than the ZFB scheme.

VI. Performance and Discussion

A. Simulation Setup

Here we describe the simulation environments for evaluating the performance of the proposed ISM algorithm. Table 1 lists the simulation parameters, and the 3GPP-based system parameters and channel models are adopted in our simulations [39][40]. Consider a Cartesian coordinate system with X and Y axes as shown in Fig. 3. Assume that the BSs of the macrocellular and microcellular systems are located at points $(0\,m, 300\,m)$ and $(400\,m, 0\,m)$, respectively. A primary user randomly appears within the circle centered at $(0\,m, 300\,m)$ with the radius of $d_0 \le 100\,m$, and K secondary users uniformly locate at a semicircle with angle of arrival (AOA) $\theta_k = -90^\circ + [90^\circ + 180^\circ\,(k-1)]/K$, for $k = 1,\ldots,K$, and the radius of $D_1 = 100\,m$, $200\,m$ or $300\,m$ to the secondary BS. Particularly, we also simulate the case where the secondary users are randomly distributed within the circle with the radius of $D_1 \le 300\,m$, i.e., random spatial scheduling. The secondary BS is equipped with an array of $M = K+1$ antenna elements which are separated by half a wavelength. The thermal noise density and system bandwidth are set as $-174\,dBm/Hz$ and $10\,MHz$, respectively. Assume that perfect power control is used for both macrocellular and microcellular systems, and let the maximum uplink transmission power of the primary user be $23\,dBm$. The maximum downlink total transmission power for the secondary BS is set to be $(49 + 10\log_{10} K)\ dBm$, which is proportional to the number of secondary users. For each secondary user, the minimum link quality requirement to achieve a BER=10^{-6} under BPSK modulation with coding rate 1/2 is given by $6.4\,dB$ in terms of SINR [41]. The interference power constraint parameter, I_{max}, is calculated according to (A.3). For example, I_{max} is given by $-91.85\,dBm$ for $P_{out} = 0.01$, $\gamma = 6.4\,dB$, and $d_0 = 100\,m$. The variance for the log-normal shadow fading is $8\,dB$, and the spatial correlation of the shadowing in the microcell is modeled by $(0.3)^{\Delta/10}$, where Δ is the distance between any two secondary users with respect to the reference distance of $10\,m$. The non-light-of-sight (NLOS) path loss model is used for each channel link [39]. We also simulate the case where the BS-to-BS link is light-of-sight (LOS) in Fig. 8 [39]. The Rayleigh fading is adopted, which is very often observed in cellular applications. We evaluate the proposed algorithm in terms of the achievable sum rate R_{sum} and the concurrent transmission probability P_{ct} in the secondary system. The concurrent transmission probability is defined as the probability that the underlying CR BS can simultaneously serve all K users in the presence of the primary user [42]. The step size and adjustment parameters δ and Δ_T in the ISM algorithm are set to be 0.1 and 0.02, respectively. In addition, the ZFB method and the beamforming method, referred to as "MaxMin SINR" in [15], are simulated to compare with our proposed ISM algorithm.

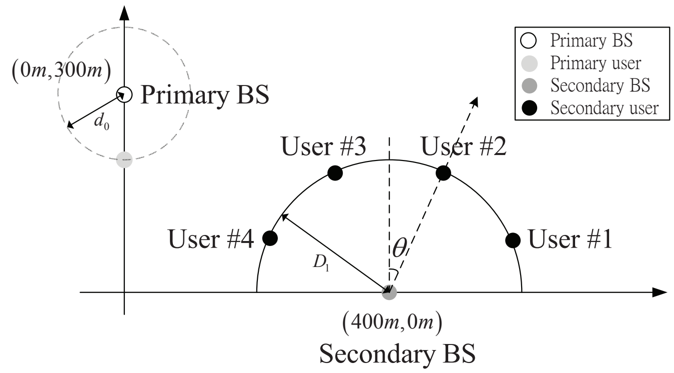

Fig. 3. Geographical locations of the HCR systems.

B. Numerical Results

Fig. 4 shows the sum rate improvement by using the ISM algorithm for a cell radius $D_1 = 200\,m$, compared to the case without considering the sum rate maximization. The initial sum rate is small and limited by the intra-user interference power among the secondary users because the ISM algorithm initially maximizes the total matched output power for the secondary users, subject to the three HCR system constraints (*C.1*)-(*C.3*) in (7). Moreover, the sum rate performance under the converged coefficient $(p_k^*, \rho_k^*, \Omega_k^*)$ is better than the initial sum rate, and the gap becomes significant

TABLE I
SIMULATION PARAMETERS

Position & Coverage	
Position of primary BS	$(0\,m, 300\,m)$
Position of secondary BS	$(400\,m, 0\,m)$
AOA of secondary users	$-90° + [90° + 180°\,(k-1)]/K$ or random distribution
Antenna Configuration for Secondary BS	
Number of antennas	$K+1$
Antenna spacing	$\lambda/2$ (λ: wavelength)
Power	
Transmit power of primary user	$23\,dBm$
Transmit power of secondary BS	$(49 + 10\log_{10} K)\ dBm$
Interference power at primary BS	$-91.85\,dBm$ ($P_{out} = 0.01$ and $d_0 = 100\,m$)
SINR of secondary users	$6.4\,dB$
Channel Model	
Path loss exponent	NLOS: $131.1 + 42.8 \cdot log_{10}(d)\,dB$ LOS: $103.4 + 24.2 \cdot log_{10}(d)\,dB$ (d: distance in km)
Shadow effect	Log-normal, $\sim N(0, 8\,dB)$
Shadow correlation in microcell	$0.3^{\Delta/10}$ (Δ: distance in m)
Rayleigh fading	$\sim CN(0,1)$

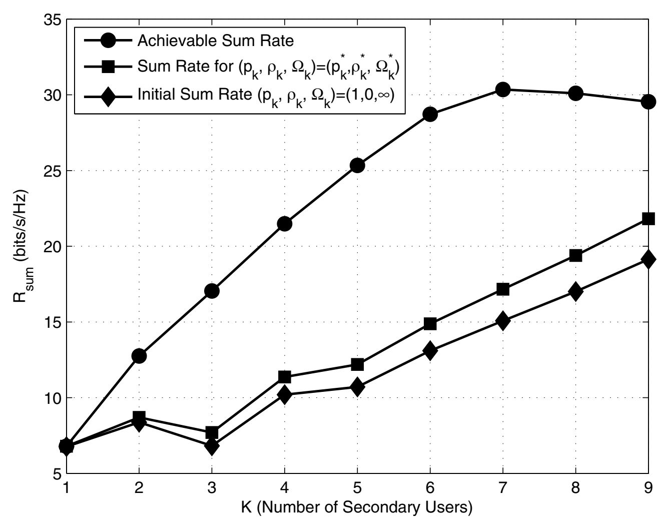

Fig. 4. Comparison of the initial sum rate and achievable sum rate for a cell radius $D_1 = 200\,m$.

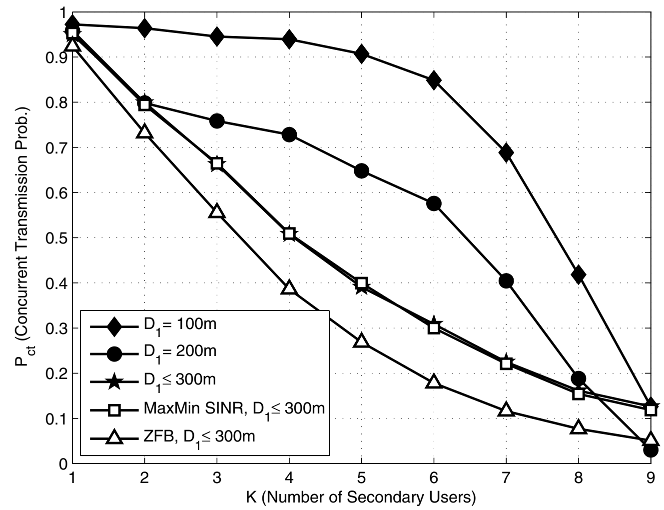

Fig. 5. Concurrent transmission probability for various numbers of secondary users K with cell radiuses $D_1 = 100$, 200, and $D_1 \leq 300$ meters.

as K increases because a more tight lower bound is obtained in severe interference scenarios. We can observe that the ISM algorithm can dramatically increase the achievable sum rate two to three folds for $K \geq 3$, compared with the initial phase. Particularly, we can see that the intra-user interference significantly dominates the performance of the initial sum rate for $K = 3$ since the secondary users have highly correlated spatial signatures in this case, while the achievable sum rate can be greatly improved after several iterations of the ISM algorithm.

Fig. 5 shows the concurrent transmission probability for various numbers of secondary users K with microcell radiuses $D_1 = 100\,m$, $200\,m$ and $D_1 \leq 300\,m$. It is found that for $D_1 = 100\,m$ and $200\,m$, the concurrent transmission probability monotonically decreases when the number of secondary users or the cell radius increase. For a target probability of $P_{ct} = 0.8$, the underlying CR system can simultaneously support six and two users with cell radiuses $100\,m$ and $200\,m$, respectively. When the radius changes from $200\,m$ to $300\,m$ and the users are uniformly distributed within the circle, the concurrent transmission probability degrades significantly as K increases. This is because some secondary users on the left-side of the microcell are closer to the macrocell, thereby experiencing more interference contributed from the primary user. Moreover, the limitation of interference power to the primary BS will rigorously restrain these secondary users from being allocated enough power, resulting in a high possibility of violating the SINR requirements. From the figure, we observe that the concurrent transmission probability of the proposed algorithm is identical to that of the MaxMin SINR method and is superior to that of the ZFB method because the intra-user interference power is strictly limited to zero in the ZFB method.

Fig. 6 shows the achievable sum rate versus the number of secondary users for various cell radius D_1. We can observe that the achievable sum rate degrades a little bit as the cell radius is expanded from $100\,m$ to $200\,m$. Another interesting observation is the improvement of the achievable sum rate resulting from beamforming techniques. As one can expect, the spatial domain can provide another resource to increase the sum rate of the secondary system. Thus, the sum rate increases linearly with the number of users up to six even when the secondary users simultaneously appear on the cell edge of $100\,m$ and $200\,m$. However, there exists an optimal achievable sum rate against the number of secondary users in the cell edge cases. Note that for $K = 7$, the secondary system can achieve the sum rate 32.2 and 30.4 $bits/s/Hz$ for the cell radius $100\,m$ and $200\,m$, respectively. For the two considered cell edge cases, each user has an average rate of 4.6 and 4.3 $bits/s/Hz$. When the number of users

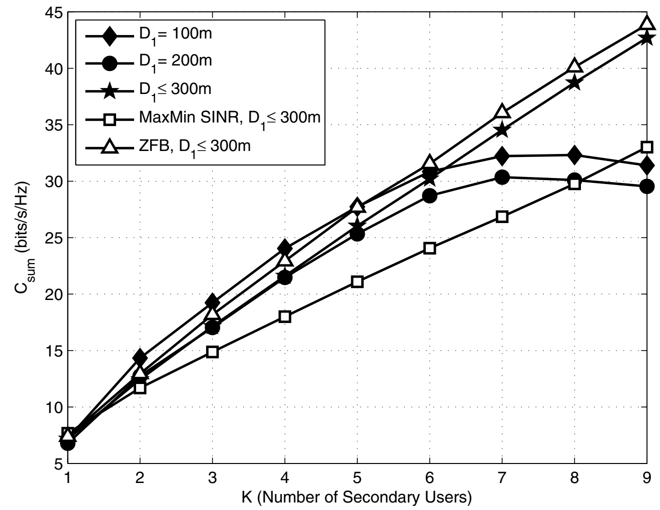

Fig. 6. Achievable sum rates for various numbers of secondary users K with cell radiuses $D_1 = 100$, 200, and $D_1 \leq 300$ meters.

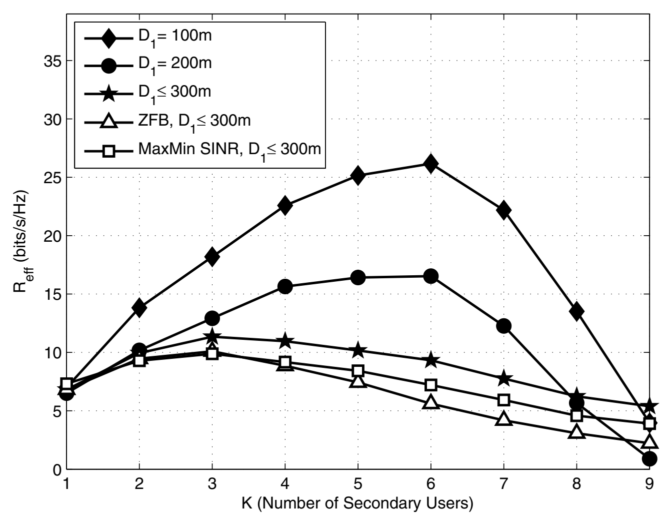

Fig. 7. Effective sum rates for various numbers of secondary users K with cell radiuses $D_1 = 100$, 200, and $D_1 \leq 300$ meters.

is larger than seven, the achievable sum rate is saturated even if more antennas are deployed at the underlying BS. While users are uniformly distributed within the cell radius of $300\,m$, the achievable sum rate is increased proportional to the number of secondary users K, and the proposed ISM algorithm can improve sum rate performance by about 8 and 10 $bits/s/Hz$ for $K = 7$ and $K = 9$, respectively, compared to the method that maximizes the worst SINR. This significant gap is due to the fact that the sum rate formula is a logarithm function of SINR, and maximizing the worst SINR only does not necessarily maximize the sum rate performance. With a great sacrifice of the concurrent transmission probability, the ZFB method performs slightly better than the proposed ISM algorithm.

Fig. 7 shows the effective sum rate versus the number of secondary users. The effective sum rate R_{eff} is defined as the product of the achievable sum rate R_{sum} and the concurrent transmission probability P_{ct}. For $D_1 = 100\,m$ and $200\,m$, the effective sum rate decreases when the number of users is larger than six. From the viewpoint of maximizing the effective sum rate at the cell edge, we suggest deploying an antenna array with at most seven antennas at the underlying CR BS, covering an area with the cell radius of $200\,m$ when considering the use of beamforming for SDMA in an HCR system. Besides, we also simulate the effective sum rate performance of the ZFB with optimum power allocation in (21) for comparison. It is found that the proposed scheme performs better than the ZFB scheme, and the performance gap becomes obvious as K increases. This is because when K increases, the zero-forcing criterion will lower the probability for multiuser concurrent transmission. We can also observe that for $D_1 \leq 300\,m$ and $K \leq 9$, the maximum effective sum rate offered by the underlying CR system is not more than 12 $bits/s/Hz$. The MaxMin SINR method is inferior to our proposed algorithm, and the performance gap of the effective sum rate between the two methods is about 3 $bits/s/Hz$.

Fig. 8 depicts the effective sum rate performance for different SINR outage probabilities of the primary user when the BS-to-BS link is LOS or NLOS. The simulation is carried out with a required SINR of the primary user $\gamma = 6.4\,dB$ or $11.2\,dB$ for BPSK and QPSK modulation, respectively [41]. It can be generally observed that as γ decreases or P_{out} increases, the microcellular system can attain better effective sum rate performance. As compared with the NLOS channel, the microcellular system is subject to a performance loss from 3.5 $bits/s/Hz$ to 5 $bits/s/Hz$ in the LOS channel, and the slope of the improvement across P_{out} is relatively steeper.

Fig. 9 illustrates the power allocation ratio for each individual secondary user in the case of $K = 4$ and $D_1 = 100\,m$, $200\,m$, and $300\,m$. We can observe that for $D_1 = 100\,m$, the power distributed to each user is almost identical since the two-tier interference between the microcell and the macrocell is not a dominant factor in this case. Actually, users closer to the primary system can consume a little bit more power. When the cell radius D_1 increases to $200\,m$, two users on the central location are assigned more power (about $27\% \sim 29\%$) than the others on the two sides (about $19\% \sim 25\%$). For a larger cell radius of $300\,m$, the power allocated to the fourth user is immediately boosted to a ratio of 39%, thereby decreasing the accessible power to the other three users. We also find that the first user away from the primary system acquires the least power allocation ratio among the four users, and the ratio tends to decrease as the cell radius increases. When $D_1 = 300\,m$, the power ratio for user #1 is merely about 14%. This phenomenon can be explained as follows. In addition to the primary user, the fourth secondary user is interfered with other secondary users. Beamforming technique mainly exploits the information of the steering vectors, characterized by the AOA, to separate the secondary users. Since the steering vectors for the two users on the two sides of the microcell are highly correlated, the first user is assigned much less power to ensure the SINR requirement of the fourth user.

Fig. 10 illustrates the achievable sum rate for each secondary user in the case of $K = 4$ and $D_1 = 100\,m$, $200\,m$, and $300\,m$. Except for the fourth user, the secondary system can provide each secondary user with a user rate ranging between 4 and 8 $bits/s/Hz$, and a slight degradation can be observed

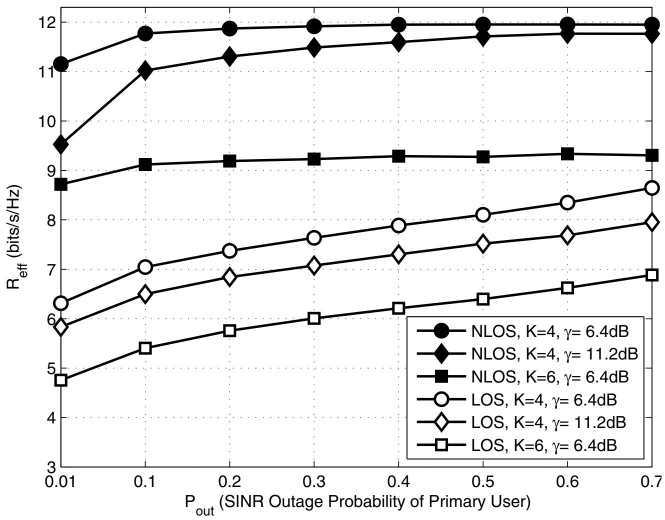

Fig. 8. Effective sum rates for various SINR outage probabilities of the primary user P_{out} with a cell radius $D_1 \leq 300$ meters when the BS-to-BS link is either LOS or NLOS.

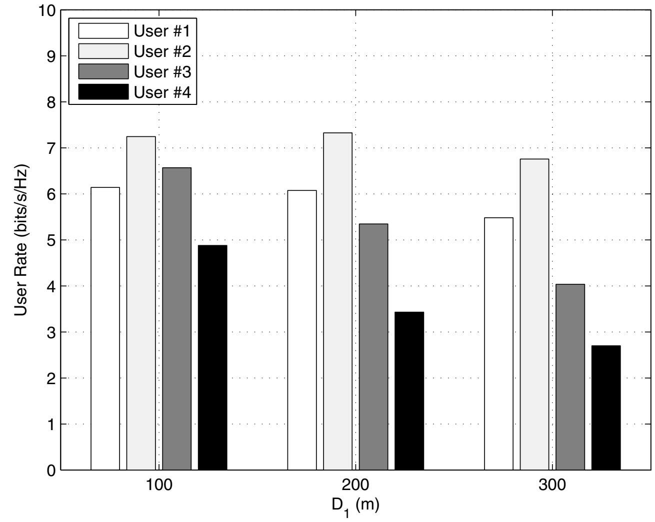

Fig. 10. Achievable rates for each secondary user in the case of $K = 4$ and $D_1 = 100\,m$, $200\,m$, and $300\,m$.

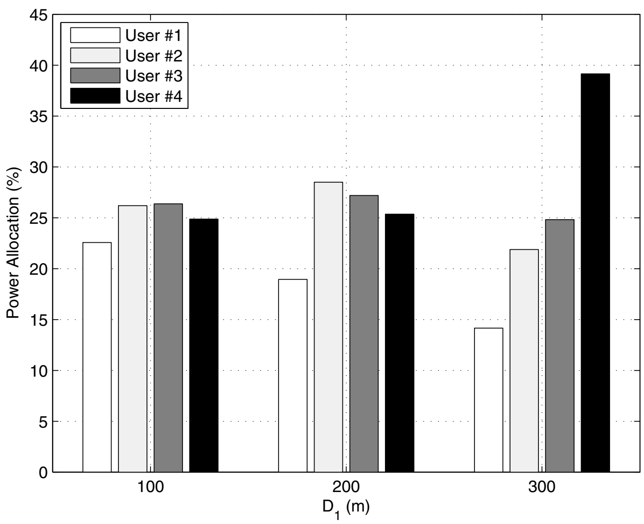

Fig. 9. Power allocation ratio for each secondary user in the case of $K = 4$ and $D_1 = 100\,m$, $200\,m$, and $300\,m$.

for an increasing radius. Indeed, the second user can attain the highest user rate. In spite of obtaining the highest power, the fourth user can only gain a user rate of about $2.56\ bits/s/Hz$, which is slightly higher than the minimum guaranteed rate $2.42\ bits/s/Hz$ with respect to the minimum required SINR $6.4\,dB$.

VII. Conclusion

In this paper, we have developed a design methodology for joint antenna beamforming and power allocation in the hierarchical multiuser CR system. The developed methodology can maximize the sum rate of the underlying CR system. Numerical results show the performance tradeoff of the HCR system for various cell coverage, achievable sum rate, and concurrent transmission probability, which can provide useful insights into the deployment of an HCR system for current and future wireless communication applications, e.g., long-term evolution (LTE), WiMAX, etc. With random spatial scheduling, it is possible to deploy an underlying microcellular CR system to improve the spectrum efficiency by using antenna beamforming. However, the improvement of the sum rate and spectrum utilization contributed by antenna beamforming is gradually saturated and even becomes worse when a large number of secondary cognitive users concurrently transmit data. In the worst case of all secondary users being at the cell edge, it is suggested to deploy at most seven antennas at the microcellular BS since there is no further improvement on the effective sum rate when the number of antennas is beyond seven. Our numerical results also implicate that for a cell coverage of $300\,m$, the maximum spectrum efficiency provided by the underlying CR system is lower than 12 $bits/s/Hz$, roughly between 5 and 12 $bits/s/Hz$. For a large cell radius, users' locations have influence on power allocation, system sum rate as well as concurrent transmission probability, and therefore, the spatial scheduling is a crucial factor for further improving the system performance when the underlying microcellular system is applied for next-generation wireless communications. An interesting research direction that can be extended from this work is to investigate the location-based scheduling to further improve the performance.

Appendix A.

From (2), the SINR of the primary user at the macrocellular BS is given by

$$\Upsilon_0 = \frac{Q\,|g_0|^2}{\sum_{j=1}^{K} \left|\mathbf{w}_j^{\dagger}\mathbf{h}_0\right|^2 + \sigma_z^2}\,, \tag{A.1}$$

where $|g_0|^2$ is a composite shadowing-Rayleigh fading random variable which can be approximated as the log-normal distribution with the mean $m_0 = \ln\left(1/\sqrt{2}\right)$ and variance $\sigma_0^2 = \left(S\ln(10)/10\right)^2 + \ln(2)$ [43]. The SINR outage probability for the primary user is therefore calculated as

$$\begin{aligned}\Pr\left(\Upsilon_0 \le \gamma\right) &= \Pr\left(|g_0|^2 \le \frac{\gamma\left(\sum_{j=1}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_0\right|^2 + \sigma_Z^2\right)}{Q}\right) \\ &= \Phi\left(\frac{\ln\left(\frac{\gamma}{Q}\left(\sum_{j=1}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_0\right|^2 + \sigma_Z^2\right)\right) - m_0}{\sigma_0}\right),\end{aligned} \quad (A.2)$$

where $\Phi(x)$ is referred to as the cumulative distribution function (CDF) of the standard normal distribution. We observe from (A.2) that to provide the primary user with a target SINR threshold γ and an outage probability upper bounded by P_{out} is equivalent to restricting the interference power to the following upper limit:

$$\begin{aligned}\sum_{j=1}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_0\right|^2 &\le I_{max} \\ &= \sigma_z^2\left(\frac{Q}{\gamma\sigma_z^2}\exp\left\{\sigma_0\Phi^{-1}\left(P_{out}\right) + m_0\right\} - 1\right),\end{aligned} \quad (A.3)$$

where the macrocellular BS only needs to know a preset average SNR of the primary link, Q/σ_z^2, after power control.

Appendix B.

To ease analysis, we assume that $\gamma_{\min,k} = \gamma_{\min}$, for all k. Define $\zeta_k = \frac{\left|\mathbf{w}_k^{\dagger}\mathbf{h}_k\right|^2}{\sum_{j=1}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_k\right|^2}$ and $\mu_k = \frac{\left|\mathbf{w}_k^{\dagger}\mathbf{h}_k\right|^2}{B_k}$. Using (5), we get

$$\begin{aligned}\min_k \Upsilon_k &= \min_k \frac{1}{\frac{1}{\zeta_k} + \frac{1}{\mu_k} - 1} = \frac{1}{\max_k\left(\frac{1}{\zeta_k} + \frac{1}{\mu_k}\right) - 1} \\ &\ge \frac{1}{\max_k\left(\frac{1}{\zeta_k}\right) + \max_k\left(\frac{1}{\mu_k}\right) - 1} \triangleq \tilde{\Upsilon},\end{aligned} \quad (B.1)$$

where we use $\max_k(a_k + b_k) \le \max_k a_k + \max_k b_k$. It can be verified from (B.1) that the SINR constraint (*C.2*) in (7) is feasible if there exists beamforming weights $\mathbf{w_k}$ such that $\tilde{\Upsilon} \ge \gamma_{\min}$. Note that $\max_k\left(\frac{1}{\mu_k}\right) = \frac{1}{\min_k \mu_k}$ and $\max_k\left(\frac{1}{\zeta_k}\right) = \frac{1}{\min_k \zeta_k}$. From (*C.3*) in (7) and applying the Cauchy-Schwarz inequality, we can write

$$\begin{aligned}\min_k \mu_k &\le \frac{1}{K}\sum_{k=1}^{K}\frac{1}{B_k}\left|\mathbf{w}_k^{\dagger}\mathbf{h}_k\right|^2 \\ &\le \frac{1}{K}\sum_{k=1}^{K}\frac{1}{B_k}\left\|\mathbf{w}_k\right\|^2\left\|\mathbf{h}_k\right\|^2 \le \frac{P_{\max}}{K}\max_k\left(\frac{1}{B_k}\left\|\mathbf{h}_k\right\|^2\right).\end{aligned} \quad (B.2)$$

We now appeal to the results in [15], showing that $\min_k \xi_k \le \frac{rank(\mathbf{H})}{K}$, where $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K]$. From (B.1) and (B.2), it follows that

$$\tilde{\Upsilon} \le \frac{1}{\frac{K}{rank(\mathbf{H})} + \frac{K}{P_{\max}}\cdot\min_k\left(\frac{B_k}{\left\|\mathbf{h}_k\right\|^2}\right) - 1}. \quad (B.3)$$

Therefore, from (B.1) and (B.3), we conclude that there exist beamforming weights such that

$$\Upsilon_k \ge \gamma_{\min},\ \text{for all}\ \ k, \quad \text{only if} \quad \frac{1}{\frac{K}{rank(\mathbf{H})} + \frac{K}{P_{\max}}\cdot\min_k\left(\frac{B_k}{\left\|\mathbf{h}_k\right\|^2}\right) - 1} \ge \gamma_{\min}. \quad (B.4)$$

Notice that the rank of $\mathbf{H}$ depends on the steering vectors of secondary users. From (4), the determinant of $\mathbf{H}$ can be computed by applying the Vandermonde determinant [44]. For the case of $K \le M$ and $\sqrt{\tilde{\beta}_k}\tilde{a}_k \neq 0$ for all k, we can get

$$\det(\mathbf{H}) \neq 0 \ \text{ if and only if } \ e^{-j\pi\sin\theta_m} \neq e^{-j\pi\sin\theta_k}, \ \text{for all} \ \ k \neq m. \quad (B.5)$$

Since $\frac{-\pi}{2} \le \theta_m < \frac{\pi}{2}$ and $|\sin\theta_m - \sin\theta_k| < 2$ for any k and m, it is then straightforward to rewrite (B.5) as

$$rank(\mathbf{H}) = K \ \text{ if and only if } \ \theta_m \neq \theta_k, \ \text{for all} \ \ k \neq m. \quad (B.6)$$

Hence, if secondary users are ideally scheduled at distinct angular directions in the spatial domain, i.e., $rank(\mathbf{H}) = K \le M$, the necessary condition degenerates into

$$\Upsilon_k \ge \gamma_{\min}, \ \text{for all} \ \ k, \quad \text{only if} \quad \frac{P_{\max}}{K}\cdot\max_k\left(\frac{\|\mathbf{h}_k\|^2}{B_k}\right) \ge \gamma_{\min}. \quad (B.7)$$

Appendix C.

For simplicity, we express the constraint functions in (11), excluding (*C.4*), as $\psi_k(\mathbf{W})$‡. We now prove that the optimal solution of (11) with $(p_k^*, \rho_k^*, \Omega_k^*)$ satisfies KKT conditions of the original sum rate maximization problem:

$$\begin{aligned}&\{\hat{\mathbf{W}}, \hat{\mathbf{\Omega}}\} = \arg\max_{\mathbf{W},\mathbf{\Omega}}\sum_{k=1}^{K}\log_2\left(1 + \frac{1}{\Omega_k + B_k}\left|\mathbf{w}_k^{\dagger}\mathbf{h}_k\right|^2\right) \\ &\text{subject to} \\ &(C.1 - C.3, C.5, C.6)\ \psi_k(\mathbf{W}) \le 0, \quad k = 1, \ldots, 4K + 2, \\ &(C.4)\ \sum_{j=1, j\neq k}^{K}\left|\mathbf{w}_j^{\dagger}\mathbf{h}_k\right|^2 \le \Omega_k, \ k = 1, \ldots, K.\end{aligned} \quad (C.1)$$

Define $v_k^* \ge 0$ and $\lambda_k^* \ge 0$ as the Lagrangian multipliers for the constraints $\psi_i(\mathbf{W}) \le \mathbf{0}$ and (*C.4*), respectively, and $\mathbf{w}_k^*$ as the corresponding optimal solution of (11) when $(p_k, \rho_k, \Omega_k) = (p_k^*, \rho_k^*, \Omega_k^*)$. By applying KKT conditions, the partial derivative of the Lagrangian dual function of (11) with respect to $\mathbf{w}_j$ at $(\mathbf{w}_k^*, v_k^*, \lambda_k^*)$ yields

$$\begin{aligned}&\frac{-2\mathbf{h}_j\mathbf{h}_j^{\dagger}\mathbf{w}_j^*}{\sum_{i=1, i\neq j}^{K}\left|\mathbf{w}_i^{*\dagger}\mathbf{h}_j\right|^2 + B_j + \left(\mathbf{w}_j^{*\dagger}\mathbf{h}_j\right)^2} \\ &+ \sum_k v_k^*\,\nabla_{\mathbf{w}_j}\left[\psi_k(\mathbf{W})\right]\Big|_{\mathbf{w}_k = \mathbf{w}_k^*} + \sum_{k=1, k\neq j}^{K} 2\lambda_k^*\mathbf{h}_k\mathbf{h}_k^{\dagger}\mathbf{w}_j^* = 0,\end{aligned} \quad (C.2)$$

for $j = 1, \ldots, K$. On the other hand, the KKT conditions for the Lagrangian dual function of the problem (C.1) with respect to $\mathbf{w}_j$ and Ω_j are given by

$$\begin{aligned}&\frac{-2\mathbf{h}_j\mathbf{h}_j^{\dagger}\mathbf{w}_j}{\Omega_j + B_j + \left(\mathbf{w}_j^{\dagger}\mathbf{h}_j\right)^2} + \sum_k \tilde{v}_k\nabla_{\mathbf{w}_j}\left[\psi_k(\mathbf{W})\right] \\ &+ \sum_{k=1, k\neq j}^{K} 2\tilde{\lambda}_k\mathbf{h}_k\mathbf{h}_k^{\dagger}\mathbf{w}_j = 0, \ j = 1, \ldots, K,\end{aligned} \quad (C.3)$$

and

$$\tilde{\lambda}_j = \frac{\left(\mathbf{w}_j^{\dagger}\mathbf{h}_j\right)^2}{(\Omega_j + B_j)^2 + \left(\mathbf{w}_j^{\dagger}\mathbf{h}_j\right)^2(\Omega_j + B_j)}, \ j = 1, \ldots, K, \quad (C.4)$$

where $\tilde{v}_k \ge 0$ and $\tilde{\lambda}_k \ge 0$ are the Lagrangian multipliers. Without loss of generality, $\tilde{\lambda}_k > 0$ as $\gamma_{\min,k}$ is usually

‡The K equality constraints in (*C.6*) can be represented by $2K$ affine inequality constraints.

larger than zero. The complementary slackness condition $\tilde{\lambda}_j\left(\sum_{i=1,i\neq j}^{K}\left|\mathbf{w}_i^{\dagger}\mathbf{h}_j\right|^2-\Omega_j\right)=0$ yields

$$\Omega_j=\sum_{i=1,i\neq j}^{K}\left|\mathbf{w}_i^{\dagger}\mathbf{h}_j\right|^2,\ j=1,\ldots,K. \quad \text{(C.5)}$$

Substituting (C.5) into (C.3) and letting $(\mathbf{w}_k,\tilde{v}_k,\tilde{\lambda}_k)=(\mathbf{w}_k^*,v_k^*,\lambda_k^*)$, we can show that the conditions (C.2) and (C.3) are identical. Consequently, the optimal solution $\mathbf{w}_k^*$ is also a local maximizer of the problem (C.1).

APPENDIX D.

By applying the KKT conditions [38], three conditions associated with λ_k and Ω_k are obtained :

$$\widehat{\lambda}_k\left(\sum_{j=1,j\neq k}^{K}\left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_k\right|^2-\Omega_k^{(1)}\right)=0,\ k=1,\ldots,K \quad \text{(D.1)}$$

$$\sum_{j=1,j\neq k}^{K}\left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_k\right|^2\leq\Omega_k^{(1)},\ k=1,\ldots,K\ , \quad \text{(D.2)}$$

and

$$\widehat{\lambda}_k\geq 0,\ k=1,\ldots,K\ . \quad \text{(D.3)}$$

Hence, from (D.1)-(D.3), we can conclude that

$$\widehat{\lambda}_k=0,\quad \text{for}\quad \sum_{j=1,j\neq k}^{K}\left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_k\right|^2<\Omega_k^{(1)}\ ; \quad \text{(D.4a)}$$

$$\widehat{\lambda}_k>0,\quad \text{for}\quad \sum_{j=1,j\neq k}^{K}\left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_k\right|^2=\Omega_k^{(1)}\ . \quad \text{(D.4b)}$$

It is implied that the k^{th} optimal Lagrange multiplier is zero unless the k^{th} intra-user interference power $\sum_{j=1,j\neq k}^{K}\left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_k\right|^2=\Omega_k^{(1)}$. By picking a particular vector $\mathbf{\Omega}^{(2)}=\mathbf{\Omega}^{(1)}$, of which the q^{th} element is replaced by a non-negative value ω, where $w<\Omega_q^{(1)}$, and thus $\Delta_q<0$, we can obtain from (17), (19) and (D.4) that:

$$R_g\left(\mathbf{\Omega}^{(2)}\right)=R_g\left(\mathbf{\Omega}^{(1)}\right),\quad \text{for}\quad \sum_{j=1,j\neq q}^{K}\left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_q\right|^2<\Omega_q^{(1)}\ ; \quad \text{(D.5a)}$$

$$R_g\left(\mathbf{\Omega}^{(2)}\right)\geq R_g\left(\mathbf{\Omega}^{(1)}\right)+\widehat{\lambda}_q\Delta_q,\quad \text{for}\quad \sum_{j=1,j\neq q}^{K}\left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_q\right|^2=\Omega_q^{(1)}\ . \quad \text{(D.5b)}$$

From (15), (16) and (D.5), it follows that

$$R_1\left(\mathbf{\Omega}^{(2)}\right)\geq R_1\left(\mathbf{\Omega}^{(1)}\right),\quad \text{for}\quad \sum_{j=1,j\neq q}^{K}\left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_q\right|^2<\Omega_q^{(1)}\ ; \quad \text{(D.6a)}$$

$$R_1\left(\mathbf{\Omega}^{(2)}\right)\geq R_1\left(\mathbf{\Omega}^{(1)}\right)+\widehat{\lambda}_q\Delta_q-p_q^*\cdot\log_2\left(\frac{\Omega_q^{(1)}+\Delta_q+B_q}{\Omega_q^{(1)}+B_q}\right),\quad \text{for}\quad \sum_{j=1,j\neq q}^{K}\left|\widehat{\mathbf{w}}_j^{\dagger}\mathbf{h}_q\right|^2=\Omega_q^{(1)}\ . \quad \text{(D.6b)}$$

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