$$s^*_{ni} = \left| \frac{1}{M/m_n}(X - |X|_{M/m_n}) \right|_{m_{ni}} = s^* < m_n$$

To prove sufficiency, suppose that, for $\tau \geq t + 1$ syndrome digits:

$$s^*_{ni} = s^* \quad 0 \leq s^* < m \quad ni \in \{n, ..., n + 2t\} \quad i = 1, ..., \tau$$

Recalling that, from the syndrome definition,

$$s^*_{ni} = \left| \frac{1}{M/m_n}(\overline{x}_{ni} - |\overline{X}|_{M/m_n}) \right|_{m_{ni}} = s^*$$

and substituting

$$x^*_{n+j} = \left| \frac{M}{m_n} s^* + |\overline{X}|_{M/m_n} \right|_{m_{n+j}}$$

for the $2t + 1 - \tau \leq t$ residue digits $\bar{x}_{n+j}$, $n+j \neq ni$, a legitimate integer X^* is obtained. As the code is t-correcting, the error is uniquely localised in $\{m_n, m_{n+1}, ..., m_{n+2t}\}$.

(iii) *Theorem 2:* The same syndrome cannot originate from two different errors.

(iv) *Proof of theorem 2:* By contradiction, consider any two legitimate integers X' and X'', $0 < X', X''; < M$, and two different errors E' and E'' such that

$$\left\lfloor \frac{X' + E'}{M/m_n} \right\rfloor = \left\lfloor \frac{X'' + E''}{M/m_n} \right\rfloor$$

This implies that:

$$|X' + E' - (X'' + E'')| \leq \frac{M}{m_n} - 1$$

and

$$\left| (X' - X'') + a' \frac{MM_R}{\prod_{i=1}^{t} m_{ik}} - a'' \frac{MM_R}{\prod_{i=1}^{t} m_{ih}} \right|$$

$$= \left| (X' - X'') + \alpha \frac{MM_R}{\prod_{i=1}^{\tau} m_{iu}} \right| \leq \frac{M}{m_n} - 1$$

where $\tau \leq 2t$, $iu \in \{1, .., n+2t\}$. Then, for arbitrary errors and for arbitrary integers:

$$\frac{MM_R}{\prod_{i=1}^{\tau} m_{iu}} \leq \frac{M}{m_n} - 1 + M - 1$$

i.e.

$$M_R \leq \prod_{i=1}^{\tau} m_{iu} + \left(\frac{1}{m_n} - \frac{2}{M} \right) \prod_{i=1}^{\tau} m_{iu} \qquad (4)$$

(v) *Case 1:* $\Pi^{\tau}_{i=1} m_{iu} \neq \Pi^{2t}_{i=1} m_{n+i}$. We obtain

$$M_R \geq (m_n + 1) \prod_{i=2}^{2t} m_{n+i} \geq \prod_{i=1}^{\tau} m_{iu} + \prod_{i=2}^{2t} m_{n+i}$$

$$> \prod_{i=1}^{\tau} m_{iu} + \left(\frac{1}{m_n} - \frac{2}{M} \right) \prod_{i=1}^{\tau} m_{iu}$$

which contradicts the inequality (eqn. 4).

(vi) *Case 2:* $\Pi^{\tau}_{i=1} m_{iu} = \Pi^{2t}_{i=1} m_{n+i}$, i.e. errors affect the complete set of redundant moduli. From theorem 1, two sets of $(t+1)$ identical syndrome digits should be consistent with $2t + 1$ moduli in the set $\{m_n, m_{n+1}, ..., m_{n+2t}\}$ and a contradiction arises.

(vii) *Theorem 3:* For any error E, there is a set of $m_n + i$ $(i = 0,1)$ syndromes in the range

$$\left[\left\lfloor \frac{E}{M/M_n} \right\rfloor, \left\lfloor \frac{E}{M/M_n} \right\rfloor + m_n - 1 + i \right]$$

Conclusions: The preceding results lead to considerable enhancements in the error decoding procedures. In fact, all previous methods produced a set of possible solutions and a lengthy verification procedure was necessary to obtain the final result. In contrast, the above theorems show that to any given syndrome S there corresponds a unique error

$$E(S) \equiv \{e_1, e_2, ..., e_{n+2t}\}$$

and a number is corrected by performing, in parallel, $n+2t$ modular subtractions:

$$|X|_{m_i} = |\overline{X} - E|_{m_i} = |\overline{x_i} - e_i|_{m_i}$$

It is worth noting that the procedure can be carried out by means of a table look-up approach. All possible error occurrences can be stored in a memory and accessed by using the residue digits of the syndrome as an address. The only limitation is dictated by the dimensions of memory modules which are available.

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Fast timing recovery scheme for class IV partial response channels

J.-Y. Lin and C.-H. Wei

Indexing terms: Magnetic recording, Clock recovery

A timing recovery scheme with fast acquisition is proposed for a class IV partial response (PR-IV) magnetic recording system. The timing information is obtained directly from the coefficients of the channel estimator. For initial adjustment of the timing phase, a known preamble is sent ahead of the user data and a modified threshold decision is employed to avoid the hangup effect.

Introduction: Recovering a symbol-rate clock with an appropriate timing phase is crucial to the detection of synchronous data signals. A baud-rate timing recovery technique was first proposed by Mueller and Müller [1] for the PAM system; to operate the timing clock on the optimum sampling phase, they defined a timing function as the linear combination of the channel response. Based on the Mueller and Müller method, a timing recovery scheme with a channel estimator was proposed to extract the timing information from the tap coefficients of the channel estimator [2]. However, many applications require fast convergence for tap coefficients of the channel estimator. A special preamble sequence is often sent prior to the user data as a training sequence for the channel estimator. This also corresponds to the initial phase adjustment for the timing recovery circuit. During the reception of the preamble sequence, it could occasionally hang up for an extended period of time at an unstable equilibrium point half way between the desired sampling timings. To avoid the hangup effect, Dolivo [3] used a variable threshold decision method for fast acquisition from a preamble signal.

In this Letter, a timing recovery circuit with channel estimator for a class IV partial response (PR-IV) channel is presented. The timing function is expressed as a function of tap coefficients of the channel estimator. A modified threshold decision is used to eliminate the hangup effects.

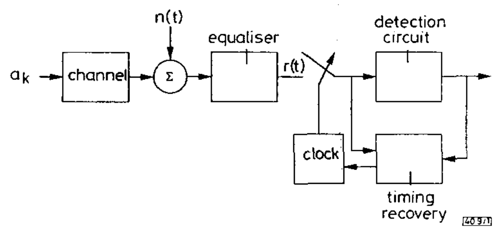

Fig. 1 *Block diagram of class IV partial response recording system*

Partial response systems: A partial response system with partial response signalling can be modelled as shown in Fig. 1. The data sequence $a_k \in \{+1, -1\}$ is sent at rate $1/T$ through the transmission filter. For PR-IV signalling [4], the pulse response of the overall channel is given by

$$h(t) = p(t) - p(t-2T) \tag{1}$$

where

$$p(t) = \frac{\sin \pi t/T}{\pi t/T} \tag{2}$$

The best sampling for a given system depends on the overall pulse response and thus on the characteristic of the recording channel [4]. In the Mueller and Müller method [1], depending on the pulse response shape, the timing function is defined as the linear combination of the channel pulse response:

$$f(\tau) = \sum u_i h_i(\tau) \tag{3}$$

where u_i are scalars and $h_i(\tau)$ are the sampled pulse response of the recording channel. They show that, depending on the pulse shape $h(t)$, either $f_A(\tau) = (h_1(\tau) - h_{-1}(\tau))/2$ or $f_B(\tau) = h_1(\tau)$ can give a useful estimate of timing error. At the ideal sampling instant nT, the PR-IV channel response will be $\{\ldots h_{-1}, h_0, h_1, h_2, h_3, \ldots\} = \{\ldots 0, 1, 0, -1, 0, \ldots\}$. Hence, $f(\tau)$ is chosen as the timing function:

$$f_B(\tau) = h_1(\tau) \tag{4}$$

Channel estimator based timing recovery: The timing function (eqn. 4) will determine the transfer characteristic of the sampling clock control loop. Because the sampled timing information is unknown, we need to extract the timing information from the received signal [1]. With the channel estimator mentioned [2], the coefficients of the channel estimator will be the same as the channel response if a well estimated channel response is assumed, i.e.

$$c_i = h_i \qquad \text{for all } i \tag{5}$$

where c_i and h_i are the coefficients of the channel estimator and the sampled PR-IV channel, respectively. Therefore, in eqn. 4, the timing function can be directly defined as [2]

$$f(\tau) = c_1 \tag{6}$$

During data reception, the data signal is reconstructed by a simple binary symbol-by-symbol decision with fixed threshold at zero:

$$\hat{a}_k = \begin{cases} +1 & \text{for } r_k(\tau) + \hat{a}_{k-2} \geq 0 \\ -1 & \text{for } r_k(\tau) + \hat{a}_{k-2} < 0 \end{cases} \tag{7}$$

For fast initial acquisition of the timing phase, the preamble sequence $\{a_k\} = \{\ldots, +1, +1, -1, -1, +1, +1, -1, -1, \ldots\}$ is sent ahead of the user data. The received data signal $x(t)$ will be a sinewave of frequency $1/4T$ [3]. If the sampling occurs halfway between the desired instants, namely with a timing phase $\tau = T/2$, the data signal $x_k = a_k - a_{k-2}$ is zero and the decision data signal is $\hat{a}_k = \text{sgn}[\hat{a}_{k-2} + n_k]$. Therefore, the decision signal $\hat{a}_k = \hat{a}_{k-2}$ will be obtained when the noise signal is small. This will prevent the tap coefficients of the channel estimator from converging to the optimum points. It also corresponds to an unstable equilibrium point where phase adjustment can hang up for the initial acquisition.

However, if the preamble sequence is selected to be

$$a_k = -a_{k-2} \tag{8}$$

the receiving signal $\hat{a}_k$ can be determined when $\hat{a}_{k-2}$ is known. That is, if the prior decision datum $\hat{a}_{k-2}$ is +1 (or −1), the likelihood that the decision datum $\hat{a}_k$ is −1 (or +1) is significantly increased. Therefore, the proposed acquisition algorithm avoids the hangup effects by reconstructing the preamble sequence with

$$\hat{a}_k = \begin{cases} +1 & \text{for } r_k(\tau) - \hat{a}_{k-2} \geq 0 \\ -1 & \text{for } r_k(\tau) - \hat{a}_{k-2} < 0 \end{cases} \tag{9}$$

Note that this algorithm is similar to the variable threshold decision method in [3].

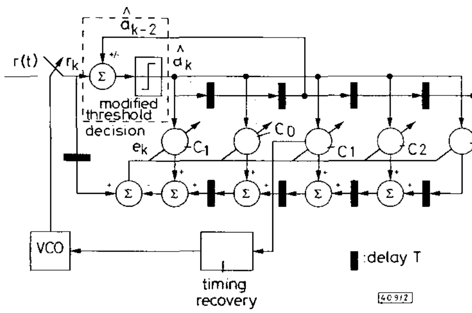

Fig. 2 *Channel estimator based timing recovery using modified threshold decision with 5 tap transposed-form FIR filter*

Simulation results: The behaviour of the proposed timing recovery circuit is investigated by computer simulation. A channel estimator using 5 tap transposed-form filter [5] is accompanied with the modified threshold decision, as shown in Fig. 2. The well known LMS algorithm with the adaption step size $\mu = 2^{-4}$ is used to adapt tap coefficients of the channel estimator. For initial acquisition of the timing phase, a 100 symbol preamble sequence is sent ahead of the user data. In the timing recovery circuit, the phase is updated by the algorithm

$$\tau_{n+1} = \tau_n + \alpha c_1 \tag{10}$$

where $\alpha = 0.07$ in the simulation. Fig. 3 shows the normalised timing phase τ_n/T starting at $\tau_0 = 0.5$ for 100 runs of different sequences. The results obtained without modified threshold decision, as shown in Fig. 3*a*, has hangup effets during the initial acquisition. In Fig. 3*b*, it shows that the sampling phases with modified threshold decision will converge after 50 iterations. Note that no hangup effects have been observed when the modified threshold decision method is employed.

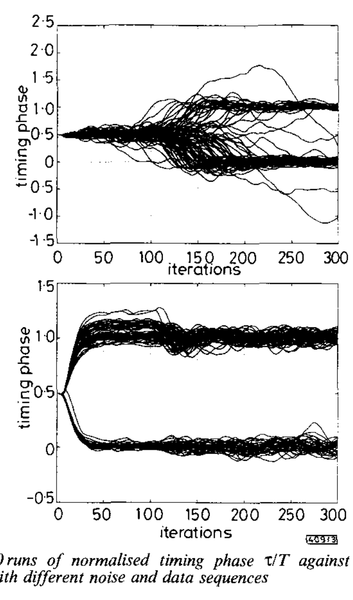

Fig. 3 *100 runs of normalised timing phase τ/T against iteration n obtained with different noise and data sequences*

SNR = 15dB
a Without modified threshold decision
b With modified threshold decision

Conclusion: A baud-rate timing recovery circuit with channel estimator in PR-IV digital recording channel is presented. By using a modified threshold decision method, the hangup effects can be eliminated. This scheme provides fast acquisition of the timing phase and fast training of tap coefficients of the channel estimator with simple circuits.

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Improvement of the multipoint-to-point link of the GSM system

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Indexing terms: Mobile radio systems, Mobile communication systems, Cellular radio

The authors outline an overview of the Group Spécial Mobile (GSM) access protocol and highlight the principle of an algorithm which exploits the difference in the distance separation between the mobile stations and the base station as an added control dimension to reduce the number of possible contentions on the dedicated control channel. A comparison of the performance of the original and modified protocols at various traffic loads is presented.

Introduction: A considerable number of public land mobile networks (PLMNs) are currently in operation all over Europe operating in various frequency bands. These first generation cellular systems employ analogue radio technology to support the services (mainly voice) they provide to their subscribers [1].

These first generation systems have reached their upper limit and they cannot accommodate any further increase in traffic demand. This limitation, coupled with the incompatibility of various systems used throughout Europe, has motivated the European Conference of Postal and Telecommunications Administrations (CEPT) to search for a common single cellular network for the whole of Europe. This research has resulted in a new standard known as Group Special Mobile (GSM) [Note 1], scheduled for the early 1990s [2].

The GSM system operates in the 900MHz frequency bands and relies on digital technology to provide an end-to-end digital connectivity. The GSM uses burst mode transmission based on a slot and frame structure and offers a variety of services which a user can access by a standard set of interfaces at the mobile station (MS). In addition, the GSM system has adopted time division multiple access/frequency division multiple access (TDMA/FDMA) as an access protocol to the wireless medium [3].

Like any other radio system, the GSM subsystem will provide two categories of logical channel according to the information streams they carry. The two categories are: traffic channels and control channels. The traffic channels are exclusively used to carry two types of user information: speech and data. The control channels, on the other hand, will support the signalling information exchange between the mobile station and base station (BS) on the radio interface [5].

The control channels that exist in the GSM system can broadly be subdivided into three main channel types: broadcast channels (BCCH), dedicated control channels (DCCH), and common control channels (CCCH).

In GSM, the procedure adopted on the CCCH will not allow a point-to-point connection to be established on them. Instead, the up-link part of the CCCH (MS to BS direction) is used for a multipoint-to-point connection establishment while the down-link (BS to MS direction) is devoted for paging and access grant messages. The access grant messages are sent to allocate a dedicated channel (traffic or stand-alone dedicated control channel) to an MS after its channel request message has been successfully received by the BS. The MS will use the traffic channel (TCH) as a DCCH if a TCH is allocated to it in the access grant message [4, 5].

Simulation assumptions: To explore the capabilities and perform the comparison between the two access protocols (GSM access protocol and the modified version), each protocol was simulated on a computer using a special oriented language known as simulation language for alternative modelling (SLAM). In this simulation, there are some parameters which have been assumed as follows: one simulation time is normalised to 1µs real time; the system has been simulated for 5s of real time; MS chooses its reference number from a uniform distribution between 1 and 129; three radius cells were considered, 1, 3 and 5km; the spatial distribution of the mobiles is assumed to be uniformly distributed all over the radio cells; each MS will be fitted with a new timer known as contention avoidance timer T_{ca}; a perfect capture is assumed; the perfect capture allows the BS to read one access burst out of several contending access bursts.

GSM access protocol: Prior to establishing a radio resource connection, a mobile station has to transmit an access burst containing a channel request message on the RACH. Radio resource connection enables the MS to establish a point-to-point communication. Furthermore, the sending of the access burst on the RACH is scheduled by using a timer known as T3120. The timer is started and at its expiry the channel request message is dispatched. The value of the timer is drawn randomly according to a specific statistical law.

The channel request message will carry two important pieces of information: one indicates the reason why a radio resource connection is needed and the other is a random reference number. The former will assist the network to give priority to services such as emergency calls, while the latter is used by the MS as a temporary identification for the accessing mobile stations.

The MS starts the timer T3120 with a new value as soon as the sending of the initial channel request message is completed. While the T3120 is running, the MS will keep listening to the full down-link CCCH (i.e. be ready to receive an access grant message which might arrive immediately). However, if the T3120 expires and no answer from the network is received, the MS repeats the sending of the channel request message if the 'maximum re-transmission' allowed in the radio cell is not exceeded. The information concerning the 'maximum retransmission' allowed in the radio cell is obtained from the system information broadcast on the BCCH. It is worth mentioning here that the MS draws a new random reference value prior to every re-attempt of a re-transmission of the channel request message. In the case when both T3120 expires and the 'maximum re-transmission' allowed has been reached, the MS waits for some time to allow the network to answer. A cell re-selection will then be performed prior to any future request attempt.

The network side will respond, if it succeeded in reading a channel request message, by sending an immediate assignment message on the down-link part of the CCCH. The ability of the network to read a channel request message depends on the probability that

Note 1: The initials now stand for "Global System for Mobile Communications"