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Decision-making for the best selection of suppliers by using minor ANP

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Abstract There are many studies on the application examples and the definitions of the super-matrix in the Analytic Network Process (ANP). The ANP applies network structures with the alternatives and the criteria to clarify a complex decision-making. It is important to obtain the consent from both the criteria and the alternatives because the evaluation is relative and reciprocal. In addition, there are very few studies on the interaction between the criteria and the alternatives. There is no realistic interpretation of the maximum eigenvalue in the ANP and the negative elements of the criteria matrix. Based on our study, the evaluation with only the alternatives consists of the missing values or the non-square matrix, it is not easy to make. The ANP that contains these irregular alternatives is defined as "Minor ANP". Then, we describe these methods of making the priority of only the alternative's matrix values by using the ANP. This study has proven that the missing values in the alternative matrix can be replaced by zeros such that the criteria matrix can be derived without using the inverse alternative matrix. Then, the priority of alternative selection can be obtained through the calculation of eigenvector. An empirical case of the best selection of supplier illustrates our proposed methods for applying to the manufacturing industry. The result reveals the methods can be well adopted in the real world.

Keywords Decision analysis · Analytic network process (ANP) · Multiple criteria decision making (MCDM) · Non-square matrix

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Introduction

There are many studies on the application examples and the definitions of the super-matrix in the Analytic Network Process (ANP) (Saaty 1996). The ANP completely relies on human side to assist in solving social decision problems. The ANP applies network structures with the alternatives and the criteria to clarify a complex decision-making. It is important to obtain the consent from both the criteria and the alternatives because the evaluation is relative and reciprocal. In addition, there are very few studies on the interaction between the criteria and the alternatives. Either, there is no realistic interpretation of the maximum eigenvalue in the ANP and the negative elements of the criteria matrix. Ozaki et al. (2009) proved the missing value in the alternative matrix which can be replaced as "zero" to be calculated without considering of criteria direct effect, instead of deriving the criteria matrix from inversed alternative matrix. Then, the priority of alternative selection can be obtained through the calculation of eigenvector.

The way of considering both criteria matrix and alternative matrix can be named as "Major ANP" or "standard ANP". As for the proposed "Minor ANP" which can be named as the "irregular ANP" with only gives a consideration on the alternative matrix. This study focuses on the "Minor ANP", and describes the types of the decision-making through the meaning of the eigenvalue, and the negative elements of the criteria matrix. And the experiments numeric calculations via case study of suppliers' selection are applied.

Since selecting the supplier that provides the best design and manufacturing expertise can make the product more competitive and cost effective. A supplier should offer more than just "parts that meet spec." Choosing the right supplier involves much more than scanning price lists in manufacture industry. The choice will depend on a wide range of factors such as value for money, quality, reliability and service. To weigh up the importance of these different factors will depend on the business's priorities and strategy as well as the preference of decision makers. Besides, selecting a supplier is both a quantitative and qualitative process. The way of calculation on strategically choosing suppliers can help understand when some missing values occurred. Additionally, it can also help the potential customers to weigh up their purchasing decisions in products.

The reminder of this paper is organized as follows. Next Section, features of "Minor ANP" are reviewed and discussed. Then, the evaluation value of the non-square matrix is developed. After Section, an empirical case of experimental numbers for the best selection of suppliers is illustrated to show and discuss our proposed methods for applying to the manufacturing industry. Finally, conclusion is presented in last Section.

Minor ANP—features of the method

The existing methods such as Harker (1987a) and Nishizawa (2005, 2007) are those using the characteristics of mathematical or AHP calculation Saaty (1980). However, there is still a problem which exist an unclear condition when both methods are used simultaneously. For example, the missing values in alternative matrix can be nearly presumed by Harker method, by means of a combination with Nishizawa method to presume the missing values. Obviously, the result is hardly to find the way out on calculation for its eigenvector. From Nishizawa method, the comparison values of two pairs or more are necessary to presume the missing values, relatively; the result also has raised a problem to reveal the method is appropriate and affected others at the same time.

According to the issue from Nishizawa method, the problem solving is demonstrated by using the proposed method. As a token cases are shown in Table 1 where the missing values are arbitrarily generated. The decision makers

Table 1 Case of alternative selection

		Criteria		
		C_1	C_2	
	Alternative\weight	w_1 (0.8)	w_2 (0.2)	Value
D_1	A_1	0.7	0.6	0.68 (a_1)
	A_2	0.3	0.4	0.32 (a_2)
	Alternative\weight	w_1 (0.5)	w_2 (0.5)	Value
D_2	A_1	0.1	0.7	0.40 (b_1)
	A_3	0.9	0.3	0.60 (b_2)
	Alternative\weight	w_1 (0.3)	w_2 (0.7)	Value
D_3	A_2	0.2	0.7	0.55 (c_1)
	A_3	0.8	0.3	0.45 (c_2)

Table 2 Alternative selection by decision makers

Alternative\decision maker	D_1	D_2	D_3
A_1	a_1	b_1	□
A_2	a_2	□	c_1
A_3	□	b_2	c_2

$D_{i,}(i = 1, 2, 3)$ give selection to alternatives A_1, A_2, and A_3 upon given criteria $(C_1, C_{2)}$ by subjective weights (w_1, w_2) for priority measuring via calculation of performance value.

According to Table 1, the matrix of alternative selection by decision makers, as Table 2, is formed and the missing values have appeared with the sign of "□".

Priority level of alternatives matrix with missing value

The priority calculation by using the ANP via the alternative's matrix, it is a beginning of the "Minor ANP" as part of the ANP characteristics. The missing values also were mentioned by Harker (1987b) which based upon the graph-theoretic structure of the pairwise comparison matrix and the gradient of the right Perron vector. The alternatives matrix $\boldsymbol{U}_{\square}$ replaced the missing values with "zero" to be matrix $\boldsymbol{U}_{AHP}$ are given as follows.

$$\boldsymbol{U}_{\square} = \begin{bmatrix} a_1 & b_1 & \square \\ a_2 & \square & c_1 \\ \square & b_2 & c_2 \end{bmatrix} \Rightarrow \boldsymbol{U}_{AHP} = \begin{bmatrix} a_1 & b_1 & 0 \\ a_2 & 0 & c_1 \\ 0 & b_2 & c_2 \end{bmatrix} \quad (1)$$

In this case, Harker method can not obtain the priority through eigenvector calculation. Due to Harker method (Harker & Vargas 1987), the eigenvector $\boldsymbol{x} = (x_1, x_2, x_3, x_4)$ of the alternative matrix is the imperfect evaluation matrix which adds the numbers of missing values as "one" to the diagonal. Nevertheless, the eigenvalue λ is not able to be calculated. Sugiura & Kinoshita (2005, 2007) proposed a solution to calculate eigenvector by transforming the matrix $\boldsymbol{U}_{AHP}$ as Eq. (2).

$$U_{AHP} = \begin{bmatrix} a_1/a_1 & a_1/a_2 & b_1/b_2 \\ a_2/a_1 & a_2/a_2 & c_1/c_2 \\ b_2/b_1 & c_2/c_1 & c_2/c_2 \end{bmatrix} \tag{2}$$

where the calculation Eq. (3) can be written as:

$$U_{AHP} = \begin{bmatrix} w_1/w_1 & w_1/w_2 & w_1/w_3 \\ w_2/w_1 & w_2/w_2 & w_2/w_3 \\ w_3/w_1 & w_3/w_2 & w_3/w_3 \end{bmatrix} = \begin{bmatrix} a_1/a_1 & a_1/a_2 & b_1/b_2 \\ a_2/a_1 & a_2/a_2 & c_1/c_2 \\ b_2/b_1 & c_2/c_1 & c_2/c_2 \end{bmatrix} = \begin{bmatrix} 1 & A & B \\ 1/A & 1 & C \\ 1/B & 1/C & 1 \end{bmatrix} \tag{3}$$

This study refers to the methods from Harker & Vargas (1987); Sugiura & Kinoshita (2005) and Ozaki et al. (2009) which use the matrix inverse of the matrix $\boldsymbol{U}_{AHP}$ to define the criteria matrix $\boldsymbol{W}$ by replacing the missing values' position as "zero". And the eigenvectors to the eigenvalue k of the ANP are assumed to be $\boldsymbol{x}, \boldsymbol{z}$.

$$\begin{bmatrix} 0 & \boldsymbol{W} \\ \boldsymbol{U} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} = k \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} \tag{4}$$

The eigenvector $\mathbf{z} = (z_1, z_2, z_3)$ of the above matrix as the Eq. (4) which calculates the equivalent of the eigenvector of $\boldsymbol{U}_{AHP}$. From this study, the eigenvector can be obtained only by way of deriving from the alternatives matrix $\boldsymbol{U}$.

Calculating the eigenvectors in AHP and ANP

Since the matrix $\boldsymbol{U}_{AHP}$ is represented by $A = a_1/a_2$, $B = b_1/b_2$ and $C = c_1/c_2$. The maximum eigenvalue and the eigenvector are assumed to be α and $\mathbf{y}$ $(y_1, y_2, y_3 = 1)$.

$$y_1 = \frac{B\{AC(1-\alpha) - B\}}{B(1-\alpha) - AC}$$

and

$$y_2 = \frac{C\{B(1-\alpha) - AC\}}{AC(1-\alpha) - B}$$

are the eigenvector of $\boldsymbol{U}_{AHP}$. On the other hand, the maximum eigenvalue and the eigenvector in $\boldsymbol{UW}$ based on Eq. (4) are assumed to be k and $\mathbf{z}$ $(z_1, z_2, z_3 = 1)$

$$z_1 = \frac{B\left\{(B+AC)\left(1-k^2\right) - AC\right\}}{(B+AC)\left(1-k^2\right) - B} z_3$$

$$z_2 = \frac{C\left\{(B+AC)\left(1-k^2\right) - B\right\}}{(B+AC)\left(1-k^2\right) - AC} z_3$$

are obtained. When equation z_1 and equation z_2 are rewritten by $1 - \frac{1}{1-k^2} = \alpha - 1$, then $z_1 = y_1$ and $z_2 = y_2$ are obtained. Therefore, the eigenvector $\boldsymbol{UW}$ becomes equivalent with the eigenvector of $\boldsymbol{U}_{AHP}$.

Presuming the missing values in the imperfect alternative matrix

A presumption method of the missing values is introduced by four dimensions' matrix in an imperfect AHP (as Eq. 5). The imperfect evaluation matrix $\boldsymbol{U}_{AHP}$ is defined as follows.

$$\boldsymbol{U}_{AHP} = \begin{bmatrix} 1 & 0 & a & b \\ 0 & 1 & c & d \\ a^{-1} & c^{-1} & 1 & 0 \\ b^{-1} & d^{-1} & 0 & 1 \end{bmatrix} \tag{5}$$

In this Section, we propose a new presumption method of the missing values, which can be distinguishable from the methods of Harker & Vargas (1987).

As Eq. (5), the missing values presumption to the criteria matrix $\boldsymbol{W}$ s obtained from the missing value matrix. And the eigenvector of $\boldsymbol{UW}_{ANP}$ in the ANP matrix is calculated, as Eq. (6).

$$\boldsymbol{W} = \frac{1}{a^2d^2+abcd+b^2c^2} \begin{bmatrix} abcd & 0 & ab^2c^2 & a^2bd^2 \\ 0 & abcd & a^2cd^2 & b^2c^2d \\ ad^2 & b^2c & abcd & 0 \\ bc^2 & a^2d & 0 & abcd \end{bmatrix}$$
$$\boldsymbol{UW} = k \begin{bmatrix} 1/(bc+ad) & ab & abc & abd \\ cd & 1/(bc+ad) & acd & bcd \\ d & b & 1/(bc+ad) & bd \\ c & a & ac & 1/(bc+ad) \end{bmatrix} \tag{6}$$

where, k is constant when the eigenvector $\mathbf{z}$ (z_1, z_2, z_3, z_4) in the matrix $\boldsymbol{UW}$ is assumed to be $z_4 = 1$. Then, the missing value z_1/z_2 can be calculated as $\sqrt{abc^{-1}d^{-1}}$, and missing value z_3/z_4 to be solved as $\sqrt{a^{-1}bc^{-1}d}$.
On the other hand, when Eq. (5) is described as maximum eigenvalue λ_H and eigenvector $\mathbf{y}$ (y_1, y_2, y_3, y_4) by the matrix of Harker, $(\lambda_H - 2)y_1 = ay_3 + by_4$ and $(\lambda_H - 2)y_2 = cy_3 + dy_4$ are obtained from row one and row two of the matrix. Simultaneously, $y_1/y_2 = \sqrt{abc^{-1}d^{-1}}$ and $y_3/y_4 = \sqrt{a^{-1}bc^{-1}d}$ are obtained as the missing value by the geometric mean calculation..

As a result, the estimations by this proposed method reveals the same values as the Harker method (Harker & Vargas 1987). Therefore, the validity of this method can be proved and its effective calculations.

Evaluation in the non-square matrix

The square matrix of the alternatives is described in the preceding Section, and the non-square matrix as Eq. (7) of the alternatives describe in this Section.

$$\boldsymbol{U} = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix} \tag{7}$$

Kinoshita & Sugiura (2008) applied the idea of "Dominant AHP" (Kinoshita & Nakanishi 1999; Kinoshita et al. 2002; Kinoshita & Sugiura 2008) with non-square matrix to the evaluation.

$$\begin{bmatrix} 1 & 1 \\ a_2/a_1 & b_2/b_1 \\ a_3/a_1 & b_3/b_1 \end{bmatrix} \cdot \begin{bmatrix} a_1/(a_1+b_1) \\ b_1/(a_1+b_1) \end{bmatrix} = \begin{bmatrix} (a_1+b_1)/(a_1+b_1) \\ (a_2+b_2)/(a_1+b_1) \\ (a_3+b_3)/(a_1+b_1) \end{bmatrix}$$

as of

$$\begin{bmatrix} (a_1+b_1)/\sum(a_i+b_i) \\ (a_2+b_2)/\sum(a_i+b_i) \\ (a_3+b_3)/\sum(a_i+b_i) \end{bmatrix} \tag{8}$$

This study draws an institution process of the problem-solving where the alternative matrix and criteria matrix are regularized. And Kinoshita method adopts a computational approach and considers the weights from each criterion and alternative.

While, Nishizawa (2007) explained that the evaluation value of the regularized super-matrix $\boldsymbol{S}$ was corresponding to Kinoshita's solution. The evaluation value of the non-square matrix was calculated with the same way as normalized weights as the Saaty type ANP. Besides, Nishizawa's proposal provides a possible solution which can be obtained from this problem solving if a suitable criteria matrix is given.

The proposed method

We take the matrix inverse of the alternative matrix $\boldsymbol{U}$, and then define the left inverse $\boldsymbol{U'}$ as $\boldsymbol{U'U=I}$. Hence, the eigenvectors in the ANP calculation to the eigenvalue k is assumed to be $\boldsymbol{x}$ and $\boldsymbol{z}$.

$$\begin{bmatrix} \boldsymbol{0} & \boldsymbol{U'} \\ \boldsymbol{U} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} = \boldsymbol{k} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} \tag{9}$$

Then, we propose the method by assuming an eigenvector $\boldsymbol{z}$ of $\boldsymbol{UU'}$ to be the evaluation value in Eq. (10).

$$\boldsymbol{U'} = \begin{bmatrix} \dfrac{a_2b_3 - a_3b_2 + b_2}{a_1b_2 - a_2b_1} & \dfrac{-a_1b_3 + a_3b_1 - b_1}{a_1b_2 - a_2b_1} & 1 \\ \dfrac{-a_2 + a_2b_3 - a_3b_2}{a_1b_2 - a_2b_1} & \dfrac{a_1 - a_1b_3 + a_3b_1}{a_1b_2 - a_2b_1} & 1 \end{bmatrix} \tag{10}$$

Additionally, we assume the eigenvector $\boldsymbol{z}$ and the homogeneous simultaneous linear equations $(\boldsymbol{UW} - k^2\boldsymbol{I})\boldsymbol{z} = 0$.

$$\boldsymbol{UWz} = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{bmatrix} \begin{bmatrix} \dfrac{a_2b_3 - a_3b_2 + b_2}{a_1b_2 - a_2b_1} & \dfrac{-a_1b_3 + a_3b_1 - b_1}{a_1b_2 - a_2b_1} & 1 \\ \dfrac{-a_2 + a_2b_3 - a_3b_2}{a_1b_2 - a_2b_1} & \dfrac{a_1 - a_1b_3 + a_3b_1}{a_1b_2 - a_2b_1} & 1 \end{bmatrix} = k^2 z$$

as of

$$\frac{(a_2b_3 - a_3b_2 + b_2)\,a_i + (a_2b_3 - a_3b_2 - a_2)\,b_i}{a_1b_2 - a_2b_1} z_1 + \frac{(a_3b_1 - a_1b_3 - b_1)\,a_i + (a_1 - a_1b_3 + a_3b_1)\,b_i}{a_1b_2 - a_2b_1} z_2 + (a_i + b_i)z_3 = k^2 z_i = 0 \quad (i = 1, \cdots, 3)$$

where $k = 0$. The homogeneous simultaneous linear equations $z_1 = \frac{a_1+b_1}{a_3+b_3-1} z_3 \cong \frac{a_1+b_1}{a_3+b_3} z_3$ and $z_2 = \frac{a_2+b_2}{a_3+b_3-1} z_3 \cong \frac{a_2+b_2}{a_3+b_3} z_3$ are obtained. $z_3 = (a_3 + b_3)/\sum(a_i + b_i)$ is substituted, $z_1 = (a_1 + b_1)/\sum(a_i + b_i)$ and $z_2 = (a_2 + b_2)/\sum(a_i + b_i)$, $z_2 = (a_2 + b_2)/\sum(a_i + b_i)$ are converted.

An empirical case of experimental numbers for the best selection of suppliers

Supplier or vendor selections are complex due to the fact that various criteria must be considered in decisions making process (Spekman 1988). The analysis of criteria for selection and measuring the performance of suppliers has been the focus of many studies. Therefore, we take the most effective empirical study on high-tech companies of suppliers' selection, and focus its famous sector on wafer foundry manufacturing in Taiwan. The considerations on choosing right suppliers as of first tier supplier and second tier supplier could be important to ensure the production line stability. Therefore, the aspects of demand side, supplier selection may concern several criteria, such as supplier status, delivery capability, processes, technical status, supplier culture, financial/commercial, support. From the aspects of supply side, there are a number of key characteristics that should look for when identifying and short listing possible suppliers. Good supplier should be able to demonstrate that they can offer the benefits, such as reliability, quality, value for money, strong service and clear communication, financial security, a partnership approach, etc.

Decision making on selecting suppliers

Several methods have been proposed in the literature for single sourcing supplier selection. Although, selecting a supplier is both a quantitative and qualitative process. A strategic approach (Spekman 1988) to choosing suppliers can also help to understand how the own potential customers weigh up their purchasing decisions. The most effective suppliers are those who offer products or services that match—or exceed—the needs of the business. So when we are looking for suppliers, it is best to be sure of our business needs and what you want to achieve by buying, rather than simply paying for what suppliers want to sell you. For example, if we want to cut down the time it takes us to serve our customers, suppliers that offer us faster delivery will rate higher than those that compete on price alone.

The supplier selection process occurs after the buyer or sourcing team has determined the decision criteria on which to base the decision and gathered data on each potential supplier. Supplier selection is the point at which the buyer or sourcing team decides how much volume to place with each specific supplier(s). The criteria to the supplier selection process are driven primarily by the value of the purchase to the organization and the risk of acquiring the purchase in the marketplace. The higher the value and risk of the purchase, the more likely that cross-functional sourcing teams will make the supplier selection decision. If the logic behind the selection process is flawed or something missing to be included into the decision process, these errors can be costly for both the buying and selling organizations in terms of unacceptable supplier performance and a negative impact on operational and/or strategic capabilities.

The criteria of selecting supplier

The criteria of selecting supplier (Nydick & Hill 1992) who will meet production needs are to make manufacturing easier, obtain higher quality, increase reliability, lower cost, improve performance, increase life, lower maintenance, lower parts count, reduce size and weight, a more saleable product, increase energy efficiency, add intelligence (Kahraman et al. 2003; Ghodsypour & O'Brien 2001).

As for how many suppliers should a company maintain, it is well worth examining how many suppliers really need. In the best case of choosing the right supplier as well as buying from a carefully targeted group, it could have a number of

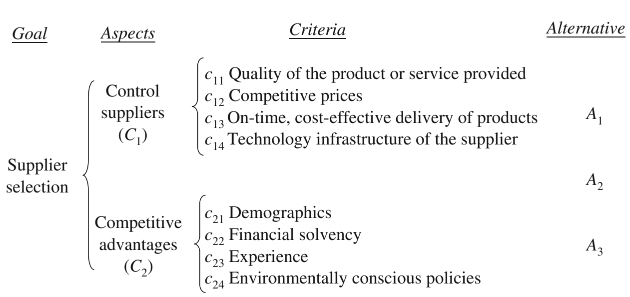

Fig. 1 Hierarchy Structure

Table 3 Supplier selection data

Alternative\criteria	C_1	C_2	Total	Ratio
A_1	36	16	52	0.3662
A_2	12	32	44	0.3099
A_3	42	4	46	0.3239
Total	90	52	142	1

benefits (Ghodsypour & O'Brien 1998) as the concerns of criteria.

It will be easier to *control suppliers* (C_1) and business will become more important to them, may be able to make deals that give an extra *competitive advantage* (C_2).

Key factors (Choi & Hartley 1996) when selecting a supplier/service provider, there are considered as:

1. Quality of the product or service provided (c_{11})
2. Competitive prices (c_{12})
3. On-time, cost-effective delivery of products (c_{13})
4. Technology infrastructure of the supplier (c_{14})
5. Demographics (c_{21})
6. Financial solvency (c_{22})
7. Experience (c_{23})
8. Environmentally conscious policies (c_{24})

Figure 1 has summarized above statement as of the hierarchy structure for the discussion on missing value while the evaluation work to be proceeding.

Kinoshita's method

According to (Kinoshita & Nakanishi 1999; Kinoshita et al. 2002; Kinoshita & Sugiura 2008) which evaluation in non-square matrix method have applied ideas on "Dominant AHP" with non-square matrix evaluation. The weights of the criteria are obtained from the ratios of the evaluation from the dominant alternatives. When two or more dominant alternatives and two or more criteria are weighted, the evaluation value also can be obtained by the concurrent convergence method (CCM) (Sugiura & Kinoshita 2005).

Table 3 provides a simple case that Kinoshita method sum up the values to calculate the ratios from two criteria (C_1, C_2) and three alternatives (A_1, A_2, A_3), i.e. $A_1 = 52/142 = 0.3662$, $A_2 = 44/142 = 0.3099$ and $A_3 = 46/142 = 0.3239$. This study draws an institution process of the problem solving where the alternative matrix and criteria matrix are regularized. Kinoshita method adopts a computational approach and considers the weights from each criterion and alternative.

Nishizawa's method

Nishizawa explained that the evaluation value of the regularized super-matrix $\boldsymbol{S}$ was corresponding to Kinoshita's solution and perform the criteria matrix $\boldsymbol{W}$ by using the transposed matrix from alternative matrix $\boldsymbol{U}$. However, the evaluation value of the non-square matrix has been calculated as same the normalized weights as the Saaty type ANP.

$$S = \begin{bmatrix} 0 & 0 & 0.6923 & 0.2727 & 0.9130 \\ 0 & 0 & 0.3077 & 0.7273 & 0.0870 \\ 0.4 & 0.3077 & 0 & 0 & 0 \\ 0.1333 & 0.6154 & 0 & 0 & 0 \\ 0.4667 & 0.0769 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{matrix} 0.3662 \\ 0.3099 \\ 0.3239 \end{matrix} \tag{11}$$

Therefore, as Eq. (11) the Nishizawa's proposal provides a possible solution which can be obtained from this problem solving if a suitable criteria matrix is given.

The proposed method

We take the matrix inverse of the alternative matrix $\boldsymbol{U}$, and then define the left inverse $\boldsymbol{U}'$ as $\boldsymbol{U}'\boldsymbol{U}=\boldsymbol{I}$. Hence, the eigenvectors in the ANP calculation to the eigenvalue k is assumed to be $\boldsymbol{x}$ and $\boldsymbol{z}$.

Then, we applied the numbers from Table 3 to calculate $\boldsymbol{U}'\boldsymbol{U}$ as follow.

$$\boldsymbol{UU}' = \begin{bmatrix} 36 & 16 \\ 12 & 32 \\ 42 & 4 \end{bmatrix} \begin{bmatrix} -1.3167 & 0.5333 & 1 \\ -1.3625 & 0.5875 & 1 \end{bmatrix} = \begin{bmatrix} -69.2 & 28.6 & 52 \\ -59.4 & 25.2 & 44 \\ -60.75 & 24.75 & 46 \end{bmatrix} \tag{12}$$

The Eq. (12) yields (0.3688, 0.3121, 0.3191) as an eigenvector when the minimum eigenvalue is $k = 0$. The result reveals that this eigenvector calculation is extremely close to Kinoshita's solution, i.e. (0.3662, 0.3099, 0.3239).

$$\boldsymbol{UU}' = \begin{bmatrix} 1.1391 & 1.1556 & 1.1304 \\ 0.9778 & 1.0182 & 0.9565 \\ 1 & 1 & 1 \end{bmatrix} \tag{13}$$

In Eq. (13), the elements of $\boldsymbol{UU}^{l}$ of the third row are assumed to be "one". Therefore, the column in the center of this vector shows the highest value, and the right column reveals the same result as Kinoshita's normalized solution.

Validity of this method in the non-square matrix

The study clarifies the meaning of minimum eigenvalue we have proposed. Then, numbers of the token cases (as Table 4) of the evaluation matrix $\boldsymbol{U}$ are generated. Directly, the inverse matrix $\boldsymbol{U'}$ is defined to be criteria matrix $\boldsymbol{W}$, and then the eigenvector of $\boldsymbol{UW}$ to be calculated.

It is a minimum eigenvalue when the eigenvector stabilizes to be calculated and proved through several cases (as Table 4). The eigenvector of $\boldsymbol{UW}$ are obtained according to the follows Eq. (14).

$$z_1 = \frac{a_1 + b_1}{a_3 + b_3 - 1} z_3 \cong \frac{a_1 + b_1}{a_3 + b_3} z_3$$

and

$$z_2 = \frac{a_2 + b_2}{a_3 + b_3 - 1} z_3 \cong \frac{a_2 + b_2}{a_3 + b_3} z_3 \tag{14}$$

when a minimum eigenvalue is assumed to be zero. The eigenvector in this method yields the same solution as Kinoshita's when assuming $z_3 = (a_3 + b_3)/\sum (a_i + b_i)$. Moreover, the correlation coefficient of our cases by this method and Kinoshita's solutions is 0.999318, obviously, they are almost the same. Therefore, it seems to be no obstacle, even if the left inverse is assumed to be criteria matrix $\boldsymbol{W}$. It is clarified that the eigenvector to the minimum eigenvalue of $\boldsymbol{UW}$ yields the evaluation of the non-square matrix.

For the purpose of showing the difference from those mentioned methods. We simply utilized our proposed method to have the comparison with Harker method and Nishizawa method which shows as Table 4. And the advantages of the proposed method have its calculation approach in difference from the Harker method which is an analytical technique for requesting it by the simultaneous equations. On the other hand, our method demonstrates the matrix with the missing value which is caught with the decision making problem as an uncertain evaluation procession. And it is shown that we need mutual evaluation with entirely opposite to the evaluation in order to make the proper decision in these samples.

Discussion and Implications

Missing values presumption by ANP in an imperfect evaluation matrix

Even if the eigenvector in the imperfect matrix is obtained, the missing values cannot be always presumed. This problem has been ignored up to now. From our proposed model, the solution in the ANP means "an agreement" with the alternative matrix $\boldsymbol{U}$ and the criteria matrix $\boldsymbol{W}$. The situation of "Negotiation breakdown" might be occurred, when the maximum eigenvalue and the eigenvector do not exist. Therefore,

Table 4 Examples of calculations

Case	***U***		***UW***			***UW*** Column			Eigenvalue	Eigenvector		
						1st	2nd	3rd		Max. λ_1	Mid. λ_2	Min. λ_3
1	31	38	−73.29	−72.08	69	0.7563	0.7667	0.7582	1.00001	0.1919	0.7704	0.7667
	14	2	−17.23	−15.71	16	0.1778	0.1671	0.1758	0.99849	0.7594	0.1633	0.1778
	48	43	−96.89	−94.02	91	1	1	1	1.50E-03	1	1	1
2	20	13	−57.65	3.92	33	0.6360	0.6471	0.6346	None	None	0.6351	0.6470
	13	43	−99.52	7.65	56	1.0980	1.2633	1.0769	0.99650		1.0841	1.0980
	34	18	−90.63	6.05	52	1	1	1	0.00354		1	1
3	49	16	32.93	−113.37	65	1.0641	1.0317	1.0156	1.01535	0.9884	−2.1702	1.0324
	31	24	27.01	−94.93	55	0.8730	0.8639	0.8594	0.99999	0.8516	−0.0378	0.8733
	30	34	30.94	−109.88	64	1	1	1	−1.54E-02	1	1	1
4	7	13	145.86	−60.54	20	0.3197	0.3175	0.3125	1.03056	0.3228	−0.1162	0.3174
	26	43	499.78	−207.86	69	1.0952	1.0900	1.0781	1.00000	1.1026	0.0522	1.0953
	28	36	456.32	−190.70	64	1	1	1	−3.07E-02	1	1	1
5	21	42	−187.06	200.81	63	1.2533	1.2600	1.2353	1.10364	1.2547	0.0000	1.2605
	9	34	−128.36	138.06	43	0.8600	0.8663	0.8431	1.10364	0.8619	−0.3137	0.8598
	34	17	−149.26	159.38	51	1	1	1	-0.10414	1	1	1

Eigenvalue—Maximum (Upper) λ_1, Middle λ_2, Minimum (Lower) λ_3 then $\lambda_1 \geq \lambda_2 \geq \lambda_3$
Remark
(1) whole agreement with value positive and more than 1
(2) negotiation breakdown happens when any eigenvector cannot be obtained
(3) partial agreement to be with value negative or close to 0

this method provides the ways of judgement and presuming the existence of the missing value, also might check the validity of the Harker method.

Meaning of eigenvalue and Validity of Kinoshita's method

Kinoshita method adopts a computational approach and considers the weights from each criterion and alternative. According to this method, the solution of the super-matrix of the ANP is an eigenvector to the maximum eigenvalue which becomes an index of an agreement. That is, if this value is large, it means that an agreement becomes near complete agreement. The minimum eigenvalue is used in Table 4 for the evaluation. It indicates a minimum unit for agreement. The agreement seems to collapse extremely easily when a minimum eigenvalue is close to zero. However, the correlation coefficient with the evaluation values of this method is 0.999318 with the same result as Kinoshita's solutions, and naturally think that both solutions are a nearly same.

Negative evaluation values in the criteria matrix

From our proposed method, the elements of the criteria matrix might become negative. Therefore, the elements of $\boldsymbol{U}^l$ adjust to a positive values is an option. That means the negative elements does not affect directly to the eigenvector calculation. For example, if all elements become a positive for the criteria matrix i.e. that adds 2 and 10 in the Eq. (14). The result shows the eigenvector z does not change anyway. Then, the eigenvector z_{+2} that adds 2 and the eigenvector z_{+10} that adds 10 are obtained. Therefore, the relationship between the eigenvector z_{+2} and the eigenvector z_{+10} become the same $z_{+2} = z_{+10} = (0.3662, 0.3099, 0.3239)$. While, the eigenvector $z = (0.3688, 0.3121, 0.3191)$ and $z \cong z_{+2} = z_{+10}$ exist little differences in the eigenvectors. An effective conversion regime could be found out, the criteria matrix might be changed by converting "the standard" of the evaluation.

Classification type of the decision-making

From this study, we figure out the fact that exists an agreement between the alternative matrix and the criteria matrix when the maximum eigenvalue exists in the "Major ANP". Whereas, the maximum eigenvalue and the eigenvector do not exist, then, there are no agreement between the alternative matrix and the criteria matrix. This situation named as "Negotiation breakdown" when any eigenvector cannot be obtained. The "whole agreement" happens when exists positive eigenvalue and more than 1. "partial agreement" are with value negative or close to 0. This study states that there is a minimum unit in an agreement between the criteria and alternatives. "Minor ANP" becomes a tool to give a classification on the cases of selection decisions that accepts a realistic type on decision-making process. And these three types of decision-making enable to reflect a realistic human behavior in decision-making.

A choice with missing values towards to the supplier selection

It is important to have a choice of sources via the supplier selection. Buying from only one supplier can be dangerous—where do you go if they let you down, or even go out of business? Equally, while exclusivity may spur some suppliers to offer a better service, others may simply become complacent and drop their standards. But the judgments of supplier selection still exist quite unknown with subjective or objective way of choosing. Missing parts of consideration from decision makers may cause both tangible and intangible loss. Also, it may count on the missing parts as an unknown message which still push decision makers ahead to move business forward by reducing to a minimum risk (as of giving the missing values a reasonable assumptions and predicting the possible way to go through).

Conclusion

This study describes "Minor ANP" which the method of making the priority by using irregular ANP which utilize the calculation with only the alternative matrix, i.e. with the missing value and non-square matrix. Also, this paper shows that there are three types of decision makings of "Whole agreement", "Negotiation breakdown", and "Partial agreement". Besides, this proposed method examined the validity of Harker method or Nishizawa method, and clarified the meaning without presumption.

With the case of supplier selection when there are some limitations in suppliers' capacity, quality, etc. In other words, no one supplier can satisfy the buyer's total requirements and the buyer needs to purchase some part of his/her demand from one supplier and the other part from another supplier to compensate for the shortage of capacity or low quality of the first supplier. And prior to the strategic decision may exist fuzzy or uncertainty and lack sufficient information to access the decision making process.

This method suggests that the elements of the criteria matrix need not necessarily be always positive. These conclusions are the results of approaching the restoration function that exists inside the ANP by the interaction between the alternative matrix and the criteria matrix. Finally, "Minor ANP" introduces user-friendliness and reflects the behavior of human decision-making. "Minor ANP" also allows for the intuitive recognition and a range of missing values to be possibly included into decision procedures. Furthermore, we expect to advance the research of "Minor ANP" and benefit its simplified calculation in the real world.

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