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LMI-based robust sliding control for mismatched uncertain nonlinear systems using fuzzy models

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SUMMARY

We propose a robust sliding control design method for uncertain Takagi–Sugeno fuzzy models. The uncertain fuzzy systems under consideration have mismatched parameter uncertainties in the state matrix and external disturbances. We make the first attempt to relax the restrictive assumption that each nominal local system model shares the same input channel, which is required in the traditional VSS-based fuzzy control design methods. We derive the existence conditions of linear sliding surfaces guaranteeing the asymptotic stability in terms of constrained LMIs. We present an LMI characterization of such sliding surfaces. Also, an LMI-based algorithm is given to design the switching feedback control term so that a stable sliding motion is induced in finite time. Finally, we give two simulation results to show the effectiveness of the proposed method. Copyright © 2011 John Wiley & Sons, Ltd.

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KEY WORDS: uncertain nonlinear system; linear matrix inequality (LMI); sliding surface; variable structure system (VSS); switching feedback control

1. INTRODUCTION

Over the past two decades, fuzzy techniques have been widely and successfully exploited in nonlinear system modeling and control. The Takagi–Sugeno (T–S) model [1] is a popular and convenient tool for handling complex nonlinear systems. Correspondingly, the fuzzy feedback control design problem for a nonlinear system has been studied extensively by using the T–S model where simple local linear models are combined to describe the global behavior of the nonlinear system [2–7]. In practice, the inevitable uncertainties may enter a nonlinear system model in a very complicated way. The uncertainty may include modeling errors, parameter variations, external disturbances, and fuzzy approximation errors. In such a situation, the fuzzy feedback control design methods in [2–7] may not work well anymore. To deal with the problem, some authors [8, 9] have exploited the variable structure system (VSS) theory, which has provided an effective means to design robust controllers for uncertain nonlinear systems where the uncertainties are bounded by known scalar valued functions.

In the VSS, the control design of the plant is intentionally changed by using a viable high-speed switching feedback control to obtain a desired system response, from which the VSS arises in finite time. The VSS drives the trajectory of the system onto a specified and user-designed surface, which is called the sliding surface or the switching surface, and maintains the trajectory on this sliding surface for all subsequent times. The closed-loop response obtained from using a VSS control law comprises two distinct modes. The first is the reaching mode, also called nonsliding mode, in which

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the trajectory starting from anywhere on the state space is being driven towards the switching surface. The second is the sliding mode in which the trajectory asymptotically tends to the origin. The central feature of the VSS is the sliding mode on the sliding surface on which the system remains insensitive to internal parameter variations and external disturbance. In sliding mode, the order of the system dynamics is reduced. This enables simplification and the decoupling design procedure [10–13]. However, all the VSS-based fuzzy control system design methods are based on the assumption that each nominal local system model shares the same input channel. This assumption is very restrictive and inadequate to modeling uncertainty/nonlinearity in various mechanical systems.

Considering these facts above, we propose a robust sliding control design method for the mismatched uncertain T–S fuzzy model with parameter uncertainties and norm-bounded external disturbances. Each nominal local system model of the uncertain system under consideration may not share the same input channel. As the local controller, we use a sliding mode controller with a nonlinear switching feedback control term. We derive LMI conditions for the existence of linear sliding surfaces guaranteeing asymptotic stability of the reduced order equivalent sliding mode dynamics, and we give an explicit formula of the switching surface parameter matrix in terms of the solution of the LMI existence conditions. We also design the nonlinear switching feedback control term to drive the system trajectories so that a stable sliding motion is induced in finite time on the switching surface and the state converges to zero. To show the effectiveness of the proposed method, we give a numerical design example together with an LMI-based design algorithm. The rest of the paper is organized as follows. Section 2 describes the T–S fuzzy model and reviews some preliminary results. Section 3 presents an LMI existence condition of linear sliding surfaces and an explicit characterization of the sliding surface parameter matrices and a sliding control law. Section 4 gives two numerical design examples to demonstrate the validity and effectiveness. Finally, Section 5 offers some concluding remarks.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following uncertain T–S fuzzy model [14], including parameter uncertainties and external disturbances:

$$\dot{x}(t) = \sum_{i=1}^{r} \beta_i(\theta)([A_i + \Delta A_i(t)]x(t) + B_i[u(t) + h(t,x)]) \tag{1}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, A_i, B_i are constant matrices of appropriate dimensions, $\Delta A_i(t)$ represents the parameter uncertainties in $A_i, h(t,x) \in R^m$ denotes external disturbances. $\theta = [\theta_1, \ldots, \theta_s], \theta_j (j = 1, \ldots, s)$ are the premise variables, s is the number of the premise variables, $\beta_i(\theta) = \omega_i(\theta)/\sum_{j=1}^{r}\omega_j(\theta), \omega_i : R^s \to [0,1], i = 1, \ldots, r$ is the membership function of the system with respect to plant rule i, r is the number of the IF–THEN rules, β_i can be regarded as the normalized weight of each IF–THEN rule and it satisfies that $\beta_i(\theta) \geqslant 0. \sum_{i=1}^{r} \beta_i(\theta) = 1$. We will assume that the following are satisfied:

- A1: The $n \times m$ matrix B defined by $B = 1/r \sum_{i=1}^{r} B_i$ satisfies the rank constraint rank $(B) = m$, that is, the matrix B has full column rank m.
- A2: The function $h(t,x)$ is unknown but bounded as $\left\| h(t,x) - \hat{h}(t,x) \right\| \leqslant \sum_{k=0}^{l} \rho_k \left\| x \right\|^k$ where $\rho_0, \ldots, \rho_l$ are known constants, $\hat{h}(t,x)$ is an estimate of $h(t,x)$, and l is a known positive integer.
- A3: $\Delta A_i(t)$ is of the form $T_i \Pi_i(t)$ where $\Pi_i(t)$ is unknown but bounded as $\| \Pi_i(t) \| \leqslant 1$.

The system (1) does not have to satisfy the restrictive assumption that all the input matrices of the local system models are in the same range space. It should be noted that the assumption A1 implies that $rank(B_i) \leqslant m$ and each nominal local system model may not share the same input channel. The assumption A2 with $l = 1$ and $\hat{h}(t,x) = 0$ has been used in the literature [15]. We can set $\hat{h}(t,x)$ as the nominal value of $h(t,x)$. Using the above assumptions, the uncertain T–S fuzzy model (1) can be written as follows:

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$$\dot{x}(t) = \sum_{i=1}^{r} \beta_i(\theta)(A_i + T_i \Pi_i(t))x(t) + [B + HF(\beta)G][u + h(t,x)], \tag{2}$$

where $\beta = [\beta_1(\theta), \ldots, \beta_r(\theta)]$, and the matrices $H, G, F(\beta)$ are defined by

$$H = \frac{1}{2}[(B - B_1), \ldots, (B - B_r)], G = [I, \ldots, I]^T, F(\beta) = diag\,[(1 - 2\beta_1(\theta))I, \ldots, (1 - 2\beta_r(\theta))I]. \tag{3}$$

It should be noted that the system (1) does not have to satisfy $B_1 = B_2 = \cdots = B_r$, which is used in almost all published results on VSS design methods including the VSS-based fuzzy control design methods of [11, 12]. Hence, the methods [8] and [9] cannot be applied to the above model (1). Because $\beta_i(\theta) \geqslant 0$ and $\sum_{i=1}^{r} \beta(\theta) = 1$, we can see that the following inequality always holds:

$$F^T(\beta)F(\beta) = F(\beta)F^T(\beta) \leqslant I \tag{4}$$

Many examples in the literature and various mechanical systems such as motors and robots do not satisfy the restrictive assumptions that each nominal local system model shares the same input channel and they fall into the special cases of the above model [14].

3. SLIDING CONTROL DESIGN VIA LMI APPROACH

In this section, we demonstrate the problem of designing a robust sliding controller via LMI approach.

3.1. LMI characterization of linear sliding surfaces

The sliding mode control (SMC) design is decoupled into two independent tasks of lower dimensions: the first involves the design of $m(n-1)-$ dimensional switching surfaces for the sliding mode such that the reduced order sliding mode dynamics satisfies the design specifications such as stabilization, tracking, regulation, etc. The second is concerned with the selection of a switching feedback control for the reaching mode so that it can drive the system's dynamics into the switching surface [11]. We first characterize linear sliding surfaces using LMIs.

Let us define the linear sliding surface as $\sigma = Sx = 0$ where S is an $m \times n$ matrix. Referring to the previous results [11] and [16], we can see that for the system (2) it is reasonable to find a sliding surface such that

P1 $[SB + SHF(\beta)G]$ is nonsingular for any β satisfying $\beta_i(\theta) \geqslant 0, i = 1, \ldots, r$, and $\sum_{i=1}^{r} \beta_i(\theta) = 1$.
P2 The reduced $(n-m)$ order sliding mode dynamics restricted to the sliding surface $Sx = 0$ is asymptotically stable for all admissible uncertainties.

It should be noted that P1 is necessary for the existence of the unique equivalent control [11] and the assumption A1 is necessary for the nonsingularity of SB.

Theorem 1
Consider the following LMIs:

$$\begin{bmatrix} \Lambda^T[(A_i + T_i \Pi_i(t))Y + *]\Lambda & * & * \\ \eta H^T \Lambda & -I & * \\ (A_i + T_i \Pi_i(t))Y\Lambda & \eta H & -I \end{bmatrix} < 0, \forall i \tag{5}$$

$$\begin{bmatrix} Y & I & 0 \\ I & c_1 I & 0 \\ 0 & 0 & c_2 I - Y \end{bmatrix} > 0, \tag{6}$$

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$$\begin{bmatrix} 2\eta\kappa & * & * \\ rc_1 & r\eta & 0 \\ rc_2 & 0 & r\eta \end{bmatrix} > 0 \tag{7}$$

where $Y \in R^{n\times n}, c_1 \in R, c_2 \in R, \eta \in R$ are decision variables, $\kappa = \lambda_{\min}(B^T B), \Lambda \in R^{n\times(n-m)}$ is any full rank matrix such that $B^T\Lambda = 0, \Lambda^T\Lambda = I$, and * represents blocks that are readily inferred by symmetry. Suppose that the LMIs (5)–(7) have a solution vector (Y, c_1, c_2, η), then there exists a linear sliding surface parameter matrix S satisfying P1–P2 and by using a solution matrix Y to (5)–(7), S can be parameterized as follows:

$$\sigma(x) = Sx = (B^T Y^{-1} B)^{-1} B^T Y^{-1} x. \tag{8}$$

Proof
By using Schur complement formula [17], we can easily show that in fact the following LMIs are incorporated in the LMIs (5)–(7)

$$c_1 > 0,\ c_2 > 0,\ \eta > 0,\ \eta^2 HH^T < I,\ 2\eta^2\kappa > r(c_1^2 + c_2^2). \tag{9}$$

It is clear that if the following inequality (10) holds, then $SB + SHF(\beta)G = I + SHF(\beta)G$ is nonsingular and hence P1 holds

$$SHF(\beta)GG^T F^T(\beta)H^T S < I. \tag{10}$$

Using (3), (4), (9) and $GG^T \leqslant \|G\|^2 I = rI$, we can obtain

$$SHF(\beta)GG^T F^T(\beta)H^T S^T \leqslant \frac{r}{\eta^2} SS^T. \tag{11}$$

By using the Schur complement formula [17], we can see that (6) and (9) imply

$$0 < c_1^{-1} I < Y < c_2 I,\ 0 < c_2^{-1} I < Y^{-1} < c_1 I \tag{12}$$

And this leads to

$$SHF(\beta)GG^T F^T(\beta)H^T S^T \leqslant \frac{r}{\eta^2} SS^T \leqslant \frac{rc_1c_2}{\eta^2}(B^T B)^{-1} \leqslant \frac{rc_1c_2}{\kappa\eta^2} I. \tag{13}$$

Using the inequality $2ab \leqslant a^2 + b^2$ where a and b are scalars, we can show that (13) implies

$$SHF(\beta)GG^T F^T(\beta)H^T S^T \leqslant \frac{r}{2\kappa\eta^2}(c_1^2 + c_2^2) I. \tag{14}$$

Finally, by using the above inequalities (9) and (13), we can obtain

$$SHF(\beta)GG^T F^T(\beta)H^T S^T \leqslant \frac{r}{\eta^2} SS^T < I, \tag{15}$$

which implies that $[SB + SHF(\beta)G]$ is nonsingular, that is, P1 holds. □

Now, we will show that S of (8) guarantees P2. Define a transformation matrix and the associated vector v as $M = [\Lambda(\Lambda^T Y\Lambda)^{-1}, Y^{-1}B(B^T Y^{-1} B)^{-1}]^T = [V^T, S^T]^T, v = [v_1^T, v_2^T]^T = Mx$ where $v_1 \in R^{n-m}, v_2 \in R^m$.

Then we can see that $M^{-1} = [Y\Lambda, B]$ and $v_2 = \sigma$. By the above transformation we can obtain

$$\begin{aligned} \begin{bmatrix} \dot{v}_1 \\ \dot{\sigma} \end{bmatrix} &= \sum_{i=1}^{r} \beta_i(\theta) \begin{bmatrix} V(A_i + T_i\Pi_i(t))Y\Lambda & V(A_i + T_i\Pi_i(t))B \\ S(A_i + T_i\Pi_i(t))Y\Lambda & S(A_i + T_i\Pi_i(t))B \end{bmatrix} \begin{bmatrix} v_1 \\ \sigma \end{bmatrix} \\ &\quad + \begin{bmatrix} VHF(\beta)G \\ I + SHF(\beta)G \end{bmatrix} [u + h(t,x)]. \end{aligned} \tag{16}$$

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From the equivalent control method [11], we can see that the equivalent control is given by $u_{eq}(t) = -\sum_{i=1}^{r} \beta_i(\theta)[I + SHF(\beta)G]^{-1} S(A_i + T_i \Pi_i(t))x - h(t,x)$. By setting $\dot{\sigma} = \sigma = 0$ and substituting $u(t)$ with $u_{eq}(t)$, we can show that the reduced $(n-m)$ order sliding mode dynamics restricted to the switching surface $\sigma = Sx = 0$ is given by

$$\dot{v}_1 = \sum_{i=1}^{r} \beta_i(\theta)(\Lambda^T Y \Lambda)^{-1} \Lambda^T D(\beta)(A_i + T_i \Pi_i(t)) Y \Lambda v_1 \tag{17}$$

where $D(\beta) = I - HF(\beta)G[I + SHF(\beta)G]^{-1}S$.

Using the matrix inversion lemma

$$(I + AB)^{-1} = I - A(I + BA)B,$$

where A and B are compatible constant matrices such that $(I + AB)$ is nonsingular, we can show that the sliding mode dynamics is equivalent to

$$\dot{v}_1 = \sum_{i=1}^{r} \beta_i(\theta)(\Lambda^T Y \Lambda)^{-1} \Lambda^T C(\beta)(A_i + T_i \Pi_i(t)) Y \Lambda v_1, \tag{18}$$

where $v_1 = (\Lambda^T Y \Lambda)^{-1} \Lambda^T x$ and $C(\beta) = I - H[I + F(\beta)GSH]^{-1} F(\beta)GS$.

The previous results [18, 19] imply that sliding mode dynamics (18) is asymptotically stable.

3.2. *Sliding control law design*

After the switching surface parameter matrix S is designed so that the reduced $(n-m)$ order sliding mode dynamics has a desired response, the next step of the SMC design procedure is to design a switching feedback control law for the the reaching mode such that the reachability condition is met. If the switching feedback control law satisfies the reachability condition, it drives the state trajectory to the switching surface $\sigma = Sx = 0$ and maintain it there for all subsequent times. In this section, with σ of (8), we design a sliding fuzzy control law guaranteeing that σ converges to zero. We will use the following nonlinear sliding switching feedback control law as the local controller.

Control rule i: IF θ_1 is μ_{i1} and ... and θ_s is μ_{is}, THEN

$$u(t) = -\hat{h}(t,x) - \chi_i \sigma - S(A_i + T_i \Pi_i(t))x - \frac{1}{1-\omega} \delta_i(t,x) \frac{\sigma}{\|\sigma\|}$$

where

$$\delta_i(t,x) = \alpha_i + \omega \|S(A_i + T_i \Pi_i(t))x\| + (1+\omega) \sum_{k=0}^{l} \rho_k \|x\|^k \tag{19}$$

and $\sigma = Sx, \omega = \sqrt{r}\,\|SH\|, \alpha_i > 0, \chi_i > 0$. It should be noted that (15) implies $\omega = \sqrt{r}\,\|SH\| \leqslant \sqrt{r}\,\|S\| \cdot \|H\| \leqslant \eta \|H\|$. This and (9) guarantee $0 \leqslant \omega < 1$. The final controller inferred as the weighted average of the each local controller is given by

$$u(t) = -\hat{h}(t,x) - \sum_{i=1}^{r} \beta(\theta) \left(\chi_i \sigma + S(A_i + T_i \Pi_i(t))x + \frac{1}{1-\omega} \delta_i(t,x) \frac{\sigma}{\|\sigma\|} \right) \tag{20}$$

and we can establish the following theorem.

Theorem 2

Suppose that the LMIs (5)–(7) have a solution vector (Y, c_1, c_2, η) and the linear sliding surface is given by (8). Consider the closed-loop control system of the uncertain system (2) with control (20). Then the state converges to zero.

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Proof
Because Theorem 1 implies that the linear sliding surface (8) guarantees P1–P2, we only have to show that σ converges to zero. Define a Lyapunov function as $E(t) = 0.5\sigma^T\sigma$. The time derivative of $E(t)$ is $\dot{E} = \sigma^T\dot{\sigma}$. From (2), (8), (20), $\|SHF(\beta)G\| \leqslant \sqrt{r}\,\|SH\| = \omega, 0 \leqslant \omega < 1$, and A2, we obtain

$$\sigma^T\dot{\sigma} = \sigma^T\sum_{i=1}^{r}\beta_i(\theta)S(A_i + T_i\Pi_i(t))x(t) + \sigma^T[I + SHF(\beta)G][u + h(t,x)]$$

$$\leqslant \sum_{i=1}^{r}\beta_i(\theta)\sigma^T S(A_i + T_i\Pi_i(t))x(t) + \sigma^T u + \{\omega\,\|u\| + (1+\omega)\,\|h(t,x)\|\}\,\|\sigma\|\,.$$

□

This implies that $\dot{E} \leqslant -(1-\omega)\sum_{i=1}^{r}\beta_i(\theta)\chi_i\,\|\sigma\|^2 - \sum_{i=1}^{r}\beta_i(\theta)\alpha_i\,\|\sigma\| \leqslant 0$, which indicates that $E \in L_2 \cap L_\infty, \dot{E} \in L_\infty$. Finally, by using Barbalat's lemma, we can conclude that σ converges to zero.

4. NUMERICAL EXAMPLE

In this section, two examples are used to illustrate the effectiveness of the proposed method and to compare with the existing method.

Example 1
To illustrate the performance of the proposed SMC fuzzy control design method, consider the following two-rule fuzzy model from a vertical take-off and landing helicopter model [20]

Plant Rule 1: IF x_1 is about 0, THEN

$$\dot{x} = (A_1 + T_1\Pi_1(t))x + B_1[u + h(t,x)]$$

Plant Rule 2: IF x_1 is about ± 2, THEN

$$\dot{x} = (A_2 + T_2\Pi_2(t))x + B_2[u + h(t,x)]$$

where

$$A_1 = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.3181 & -0.7070 & 1.4100 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad T_1 = \begin{bmatrix} 0 \\ 0.1 \\ 0 \\ 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.4181 & -0.7070 & 1.4300 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0 \\ 0.1 \\ 0 \\ 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.6446 & -7.5922 \\ -5.5200 & 4.4900 \\ 0 & 0 \end{bmatrix},$$

$$\Pi_1(t) = \Pi_2(t) = \begin{bmatrix} 0 & 0 & \sin t & 0 \end{bmatrix}$$

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$$h(t,x) = d(t) + \begin{bmatrix} 0.9\sin 3t & 0.9\sin 3t \end{bmatrix}^T, \beta_1 = \frac{1 - 1/(1+e^{-14(x_1-1)})}{1+e^{-14(x_1+1)}}, \beta_2 = 1 - \beta_1. \quad (21)$$

It should be noted that T_1 and T_2 are not matched and thus the previous VSS-based fuzzy control design methods cannot be applied to the above system (21). Via LMI optimization with (21), we can obtain the sliding surface $\sigma = Sx$.

By setting $\hat{h}(t,x) = \begin{bmatrix} 0.9\sin 3t & 0.9\sin 3t \end{bmatrix}^T, \chi_i = 5, \alpha_i = 0.1, r = 2, l = 1, \rho_k = 1$, we can obtain the following nonlinear controller:

Control Rule 1: IF x_1 is about 0, THEN

$$u(t) = \begin{bmatrix} -0.9\sin 3t & -0.9\sin 3t \end{bmatrix}^T - 5\sigma - S(A_1 + T_1\Pi_1(t))x - \frac{1}{1-\omega}\delta_1 sgn(\sigma).$$

Control Rule 2: IF x_1 is about ± 2, THEN

$$u(t) = \begin{bmatrix} -0.9\sin 3t & -0.9\sin 3t \end{bmatrix}^T - 5\sigma - S(A_2 + T_2\Pi_2(t))x - \frac{1}{1-\omega}\delta_2 sgn(\sigma).$$

The final controller inferred as the weighted average of each local controller is given by

$$u(t) = \begin{bmatrix} -0.9\sin 3t & -0.9\sin 3t \end{bmatrix}^T - \sum_{i=1}^{r} \beta(\theta)\left[5\sigma + S(A_i + T_i\Pi_i(t))x + \frac{1}{1-\omega}\delta_i sgn(\sigma)\right]. \quad (22)$$

To assure the effectiveness of our fuzzy controller, we apply the controller to the two-rule fuzzy model (21) with nonzero $d(t)$. We assume that $d(t) = \begin{bmatrix} x_1 \sin 2t - 0.5sgn(x_4) & x_1 \sin 2t - 0.5sgn(x_4) \end{bmatrix}^T$. The time histories of the state, the sliding variable σ, and the input (22) are shown in Figure 1 when $x_1(0) = x_2(0) = x_4(0) = 0,\ x_3(0) = 10$.

From Figure 1, the proposed controller is applicable to low order fuzzy control synthesis for uncertain fuzzy systems with mismatched parameter uncertainties in the state matrix and external disturbances. The control performances are satisfactory. It should be noted that all existing VSS-based fuzzy control system design methods cannot be applied to the two-rule fuzzy model (21) because B_1 is not in the range space of B_2.

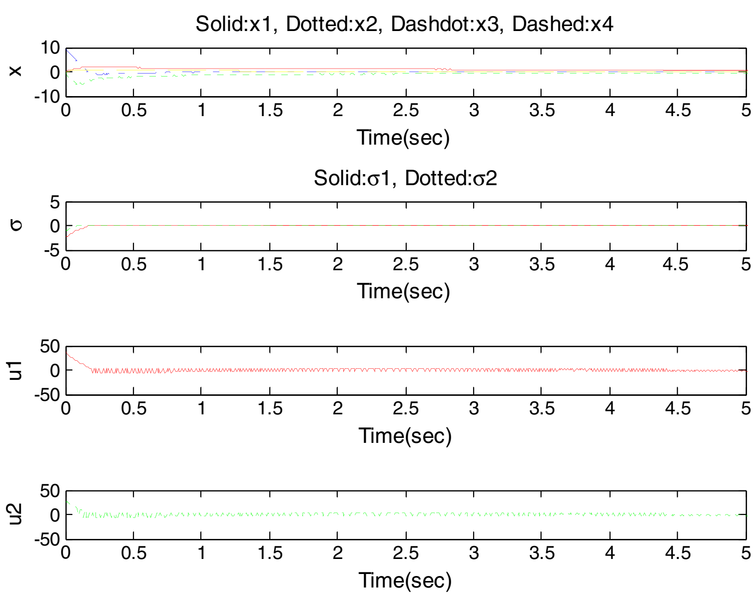

Figure 1. Simulation results for initial conditions $x_1(0) = x_2(0) = x_4(0) = 0,\ x_3(0) = 10$.

Example 2
For the special case of $\Pi_i(t) \equiv 0$, the robust sliding controller design is proposed in [19]. Consider the following inverted pendulum on a cart [19]

$$\dot{x}_1 = x_2, \quad \dot{x}_3 = x_4$$

$$\dot{x}_2 = \frac{1}{l\psi}(3g\sin x_1 - 3a\cos x_1[u+d(t)+\phi]), \dot{x}_4 = -\frac{1}{\psi}(1.5mag\sin 2x_1 - 4a[u+d(t)+\phi]) \quad (23)$$

where x_1 is the angle (rad) of the pendulum from the vertical, $x_2 = \dot{x}_1, x_3$ is the displacement (m) of the cart, $x_4 = \dot{x}_3,\ \psi = 4 - 3ma\cos^2 x_1, \phi = mlx_2^2 \sin x_1, u$ is the input, and $d(t)$ is related to external disturbances, which may be caused by the frictional force. $a = 1/(m+M), m$ is the mass of the pendulum, M is the mass of the cart, $2l$ is the length of the pendulum, and $g = 9.8m/s^2$ is the gravity constant. We set $M = 9\ kg, m = 1\ kg, l = 1$ m. We assume that $d(t)$ is bounded as $|d(t)| \leqslant \rho_0 + \rho_1 \|x\|$ where ρ_0 and ρ_1 are known constants. Here, we approximate the system (23) by the following two-rule fuzzy model.

Plant Rule 1: IF x_1 is about 0, THEN

$$\dot{x} = A_1 x + B_1[u + h(t,x)]$$

Plant Rule2: IF x_2 is about $\pm 60°(\pm\pi/3^r ad)$, THEN

$$\dot{x} = A_2 x + B_2[u + h(t,x)]$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 7.9459 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.7946 & 0 & 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -0.0811 \\ 0 \\ 0.1081 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 6.1945 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.3097 & 0 & 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -0.0382 \\ 0 \\ 0.1019 \end{bmatrix},$$

$$h(t,x) = d(t) + x_2^2 \sin x_1, \beta_1 = \frac{1 - 1/(1 + e^{-14(x_1 - \pi/8)})}{1 + e^{-14(x_1 + \pi/8)}}, \beta_2 = 1 - \beta_1. \quad (24)$$

Because B_1 is not in the range space of B_2, all existing VSS-based fuzzy control system design methods cannot be applied to the above system (24). Via LMI optimization with (24), we can obtain the sliding surface $\sigma = Sx$.

By setting $\hat{h}(t,x) = x_2^2 \sin x_1, \chi_i = 5, \alpha_i = 1, r = 2, l = 1, \rho_k = 1$, we can obtain the following nonlinear controller:

Control Rule 1: IF x_1 is about 0, THEN

$$u(t) = -x_2^2 \sin x_1 - 5\sigma - SA_1 x - \frac{1}{1-\omega}\delta_1 sgn(\sigma).$$

Control Rule 2: IF x_1 is about $\pm 60°(\pm\pi/3$ rad), THEN

$$u(t) = -x_2^2 \sin x_1 - 5\sigma - SA_2 x - \frac{1}{1-\omega}\delta_2 sgn(\sigma).$$

The final controller inferred as the weighted average of each local controller is given by

$$u(t) = -x_2^2 \sin x_1 - \sum_{i=1}^{r} \beta(\theta) \left[5\sigma + SA_i x + \frac{1}{1-\omega}\delta_i sgn(\sigma) \right]. \quad (25)$$

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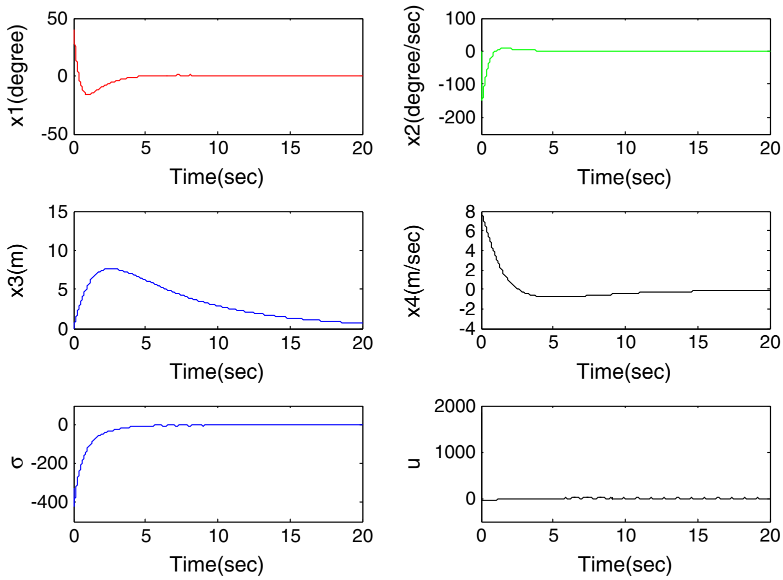

Figure 2. Simulation results for initial conditions $x_1(0) = 40^\circ(2\pi/9 \text{ rad }), x_2(0) = x_3(0) = x_4(0) = 0$.

To assure the effectiveness of our fuzzy controller, we apply the controller to the two-rule fuzzy model (24) with nonzero $d(t)$. We assume that $d(t) = x_1 \sin 2\pi t - 0.5\text{sgn}(x_4)$. The time histories of the state, the sliding variable σ, and the input (25) are shown in Figure 2 when $x_1(0) = 40^\circ(2\pi/9 \text{ rad }),\ x_2(0) = x_3(0) = x_4(0) = 0$.

5. CONCLUSIONS

A robust sliding fuzzy control design method was developed for the uncertain T–S fuzzy model, which includes mismatched parameter uncertainties and external disturbances. We relaxed the restrictive assumption that each nominal local system model shares the same input channel, which is always invoked in the traditional VSS-based fuzzy control design methods. As the local controller, an SMC law with a nonlinear switching feedback control term is used. We gave an LMI condition for the existence of linear sliding surfaces guaranteeing the asymptotic stability of the reduced order equivalent sliding mode dynamics. An explicit formula of the switching surface parameter matrix is derived in terms of the solution of the LMI existence condition and an LMI-based algorithm is developed to design the nonlinear switching feedback control term guaranteeing the reachability condition. Besides, two numerical design examples are given to show the effectiveness of our method. Finally, by using the proofs of Theorem 1 and Theorem 2, the previous VSC-based fuzzy control system design methods can be easily extended to include a T–S fuzzy model where each nominal local system model does not share the same input channel.

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