

**Nonequilibrium noise correlations in a point contact of helical edge states**Yu-Wen Lee,<sup>1,\*</sup> Yu-Li Lee,<sup>2,†</sup> and Chung-Hou Chung<sup>3,4,‡</sup><sup>1</sup>*Department of Physics, Tunghai University, Taichung, Taiwan, Republic of China*<sup>2</sup>*Department of Physics, National Changhua University of Education, Changhua, Taiwan, Republic of China*<sup>3</sup>*Department of Electrophysics, National Chiao Tung University, Hsin-Chu, Taiwan, Republic of China*<sup>4</sup>*Physics Division, National Center for Theoretical Sciences, HsinChu, Taiwan, Republic of China 300*

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We investigate theoretically the nonequilibrium finite-frequency current noise in a four-terminal quantum point contact of interacting helical edge states at a finite bias voltage. Special focus is put on the effects of the single-particle and two-particle scattering between the two helical edge states on the fractional charge quasiparticle excitations shown in the nonequilibrium current noise spectra. Via the Keldysh perturbative approach, we find that the effects of the single-particle and the two-particle scattering processes on the current noise depend sensitively on the Luttinger liquid parameter. Moreover, the Fano factors for the auto- and cross correlations of the currents in the terminals are distinct from the ones for tunneling between the chiral edge states in the quantum Hall liquid. The current noise spectra in the single-particle-scattering-dominated and the two-particle-scattering-dominated regime are shown. Experimental implications of our results on the transport through the helical edges in two-dimensional topological insulators are discussed.

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**I. INTRODUCTION**

Ever since the discovery of the quantum Hall effect, there has been a growing interest in topological properties in certain quantum condensed-matter systems, especially when a model of the topological states in the absence of applied magnetic fields was constructed.<sup>1</sup> Recently, a new topological state of matter in two dimensions, the quantum spin Hall insulator (QSHI), was theoretically proposed in various systems with time-reversal symmetry and spin-orbit interactions.<sup>2,3</sup> The hallmark of the topological nature in QSHIs is the presence of a bulk gap together with gapless edge states.<sup>4</sup> These edge states propagate in opposite directions for opposite spins, and thus are usually dubbed as the helical liquids.<sup>5</sup> The stability of the helical liquid against the elastic backscattering is protected by the time-reversal invariance;<sup>2</sup> hence, the helical liquid forms a distinctive feature of this new topological state of matter. This state occurs in HgTe/CdTe quantum well structures,<sup>6</sup> and there has already been experimental evidence in transport properties of helical liquids, which may be considered as the confirmation of these unique one-dimensional (1D) systems.<sup>7,8</sup>

In the presence of electron-electron interactions, these helical edge states form a special type of Luttinger liquid (LL), the helical LL, in which the spins are associated with the directions of the momenta.<sup>5</sup> Therefore, it is interesting, both theoretically and experimentally, to look for the unique signatures of helical LLs and, in particular, to distinguish them from the usual LLs. Recently, it was proposed in Refs. 9 and 10 that a four-terminal quantum point contact (QPC) in the QSHI can be used as a probe of the helical LL. In particular, in Ref. 9, it was noted that the problem of the QPC in a QSHI can be mapped onto the model of a spinful LL with a weak tunneling link. The corresponding LL parameter of the charge mode  $K_c = K$  is the inverse of that of the spin mode  $K_s = 1/K_c = 1/K$ . Therefore, the edge states of the QSHI with a tunnel junction can realize phases which cannot exist for the spin-SU(2) invariant LL with a single impurity where  $K_s = 1$  there.<sup>11</sup> It was further shown

in Ref. 9 that there exists a quantum critical point which can be tuned by adjusting the value of the gate voltage.<sup>10</sup> As a result, the low temperature zero-bias conductance can be described by a universal scaling function of the temperature and the gate voltage. Later, a duality relation between the charge and spin sectors in such a four-terminal setup was found in the nonequilibrium situation.<sup>12</sup>

It is important to notice that in determining the phase diagram of the QPC in a QSHI, the two-particle scattering processes, which are naively regarded as less relevant than the single-particle one, play an important role. It is therefore interesting and important to realize a direct experimental probe of these two-particle scattering processes. One way to achieve this goal is to analyze the current noise of the tunnel junction. A pioneering work along this direction has been done recently in Ref. 13. There, via the cumulant generating function, the nonequilibrium spin-resolved tunnel current and its correlations at zero frequency are obtained by the perturbation theory in the tunneling strength. Particularly, the competition between the single-particle and the two-particle scattering processes has already been seen in the zero-frequency tunnel current noise. Further, fermionic Hanbury-Brown and Twiss (HBT) correlations between spin-up and spin-down tunnel currents are also examined in the same work, and it is shown that only the two-particle tunnelings contribute to the HBT correlations.

On the other hand, it was argued in the study of the noise measurement in the edge states of the fractional quantum Hall (FQH) liquid that more information can be extracted from the noise correlations of the currents in the terminals than from the noise in the tunnel current flowing through the junction.<sup>14,15</sup> In particular, fractional charge quasiparticle excitations have been suggested<sup>14-18</sup> and measured<sup>19,20</sup> in transport through FQH liquids (or chiral LLs) as well as in the nonchiral LLs.<sup>21</sup> Recently, it has been proposed that fractional charge quasiparticle excitations induced by electron interactions exist and may be probed in the helical edge states of the QSHI.<sup>22</sup>

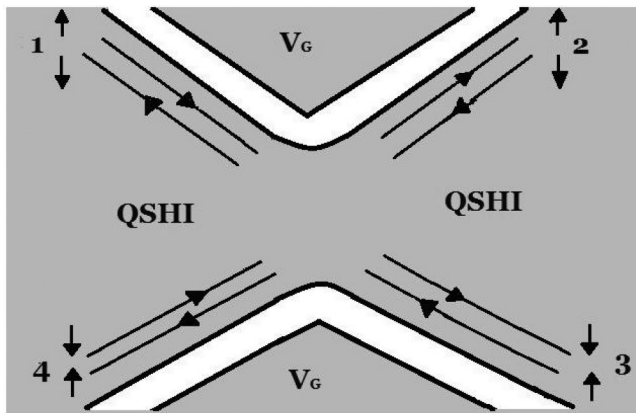


FIG. 1. A QPC in a QSHI. The value of the gate voltage is greater than its critical value so that the point contact is open.

It is therefore of great interest and fundamental importance to investigate further this issue which is relatively less studied. Furthermore, in addition to the zero-frequency shot noise, it has been suggested that even more information is stored in the finite-frequency current noise, such as the quantum statistics of the quasiparticle excitations,<sup>16–18</sup> the dynamics of correlations,<sup>14</sup> and the role of electron interactions.

Motivated by these observations, in the present work, we investigate the current noise of two weakly coupled helical LLs in a generic four-terminal setup in the presence of a finite bias  $V$  between the top and bottom edges of the point contact when it is open, as shown in Fig. 1. In particular, we extend the earlier studies on the zero-frequency tunnel current noise in Ref. 13 in two directions. First, instead of studying the correlations of the tunnel current directly, we investigate currents in the four terminals and the associated noise spectra. Second, instead of just calculating the noise spectrum at zero frequency, we also obtain the noise spectra at finite frequency. As we mentioned above, there is certain important physical information about this system, which is not probed directly by the tunnel current noise at zero frequency, that can be revealed through this approach. In particular, we show that the Fano factor obtained through the correlations of the currents in the terminals will depend on the LL parameter in a way exhibiting the fractional charge carried by the elementary excitations in the helical LL. Therefore, in addition to reproducing some results of Ref. 13, our work may provide a complementary point of view on the physics of the helical LL in such a four-terminal setup.

Throughout this work, we assume that our system is far away from the quantum critical point so that the perturbative calculation becomes reliable. Further, when the point contact is pinched off, the corresponding noise spectra can be obtained by an appropriate duality transformation.<sup>10</sup> Our main results obtained via the Keldysh perturbation theory<sup>23</sup> are shown in Figs. 2–8. Naively, the two-particle scattering processes seem to be more irrelevant than the single-particle one. However, we find that the current and noise spectrum may be dominated by them, depending on the value of the LL parameter,<sup>24</sup> as shown in Figs. 2–6. To reveal the competition between the single-particle and the two-particle scattering processes clearly, we calculate the Fano factors for the auto- and cross correlations

of currents in the terminals. We find that they depend on the LL parameter and the relative strength of the tunneling processes, which can also be viewed as an indirect probe on the possible fractional charges in a helical liquid. What we find here is quite different from the corresponding one for tunneling between chiral edge states in the quantum Hall liquid.<sup>14,25</sup> Moreover, the Fano factors for the auto- and cross correlations approach different values in the zero-bias limit, depending on the LL parameter. As we discuss below, this result follows from the entanglement of the right and left movers in the final states for different scattering processes.

The rest of the paper is organized as follows. In Sec. II, we set up the model to fix our notation. The calculations on the currents and noise spectra are summarized in Sec. III. These results and comparison with the previous work are discussed in Sec. IV. The last section is devoted to conclusions.

## II. MODEL

At low energies, the system in Fig. 1 can be described by the Hamiltonian  $H = H_0 + \delta H$ , where

$$H_0 = \sum_{i=1}^4 \int_0^{+\infty} dx \mathcal{H}_0^{(i)}, \quad (1)$$

with

$$\begin{aligned} \mathcal{H}_0^{(i)} = & i v_F (\psi_{i,\text{in}}^\dagger \partial_x \psi_{i,\text{in}} - \psi_{i,\text{out}}^\dagger \partial_x \psi_{i,\text{out}}) + u_2 J_{i,\text{in}} J_{i,\text{out}} \\ & + \frac{u_4}{2} (J_{i,\text{in}} J_{i,\text{in}} + J_{i,\text{out}} J_{i,\text{out}}), \end{aligned} \quad (2)$$

and  $\delta H$  being defined below. Here  $\psi_{i,\text{in}}$ ,  $\psi_{i,\text{out}}$  are a time-reversed pair of fermion fields with opposite spin, which propagate toward and away from the junction,  $v_F$  is the bare Fermi velocity, and the  $u_2$ ,  $u_4$  terms are forward scattering. As pointed out in Ref. 9,  $H_0$  can be mapped onto the Hamiltonian of spin- $\frac{1}{2}$  fermions. To proceed, we define the spin- $\frac{1}{2}$  fermion fields as

$$\begin{aligned} \psi_{R\uparrow}(x) &= \begin{cases} \psi_{2,\text{out}}(x) & x > 0, \\ \psi_{1,\text{in}}(-x) & x < 0, \end{cases} \\ \psi_{R\downarrow}(x) &= \begin{cases} \psi_{3,\text{out}}(x) & x > 0, \\ \psi_{4,\text{in}}(-x) & x < 0, \end{cases} \\ \psi_{L\uparrow}(x) &= \begin{cases} \psi_{3,\text{in}}(x) & x > 0, \\ \psi_{4,\text{out}}(-x) & x < 0, \end{cases} \\ \psi_{L\downarrow}(x) &= \begin{cases} \psi_{2,\text{in}}(x) & x > 0, \\ \psi_{1,\text{out}}(-x) & x < 0. \end{cases} \end{aligned} \quad (3)$$

In terms of  $\psi_{L\sigma}$  and  $\psi_{R\sigma}$ , where  $\sigma = \uparrow, \downarrow = +, -$ ,  $H_0$  can be written as

$$H_0 = \int_{-\infty}^{+\infty} dx \mathcal{H}_0, \quad (4)$$

where

$$\begin{aligned} \mathcal{H}_0 = & \sum_{\sigma} [i v_0 (\psi_{L\sigma}^\dagger \partial_x \psi_{L\sigma} - \psi_{R\sigma}^\dagger \partial_x \psi_{R\sigma}) + u_2 J_{L\sigma} J_{R-\sigma}] \\ & + \frac{u_4}{2} \sum_{\sigma} (J_{L\sigma} J_{L\sigma} + J_{R\sigma} J_{R\sigma}). \end{aligned} \quad (5)$$

It is now clear that Eq. (4) is nothing but the Hamiltonian of the spin- $\frac{1}{2}$  fermions.

Using the bosonization formulas,<sup>26</sup>

$$\begin{aligned}\psi_{L\sigma} &= \frac{1}{\sqrt{2\pi a_0}} \eta_\sigma e^{-i\sqrt{4\pi}\phi_{L\sigma}}, \\ \psi_{R\sigma} &= \frac{1}{\sqrt{2\pi a_0}} \eta_\sigma e^{i\sqrt{4\pi}\phi_{R\sigma}},\end{aligned}$$

and defining the bosonic fields

$$\Phi_\sigma = \phi_{L\sigma} + \phi_{R\sigma}, \quad \Theta_\sigma = \phi_{L\sigma} - \phi_{R\sigma},$$

where  $a_0$  is the short-distance cutoff,  $H_0$  becomes

$$H_0 = \sum_{\alpha=c,s} \frac{v_\alpha}{2} \int_{-\infty}^{+\infty} dx: \left[ K_\alpha (\partial_x \Theta_\alpha)^2 + \frac{1}{K_\alpha} (\partial_x \Phi_\alpha)^2 \right], \quad (6)$$

where  $K_c = K$ ,  $K_s = 1/K$ ,  $v_c = v = v_s$ ,

$$\Phi_c = \frac{1}{\sqrt{2}} (\Phi_+ + \Phi_-), \quad \Phi_s = \frac{1}{\sqrt{2}} (\Phi_+ - \Phi_-),$$

and similar expressions for  $\Theta_{c,s}$ . The Klein factors  $\eta_\sigma$  are usually chosen to satisfy  $\eta_+ \eta_- = i$ . When spin is conserved at the junction, there are four fixed points.<sup>11</sup> These include the perfectly transmitting (CC) limit, in which both charge and spin conduct, the perfectly reflecting (II) limit, in which both charge and spin are insulating, and the mixed fixed points, denoted by CI (IC), in which charge (spin) is perfectly transmitting and spin (charge) is perfectly reflecting. According to the analysis in Refs. 9 and 10, the CC and II phases are separated by a quantum phase transition line by varying the gate voltage. This occurs when  $1/2 < K < 2$ . This is the region that we study.

With the help of  $\Phi_c$  and  $\Phi_s$ ,  $\delta H$  in the CC limit is given by

$$\begin{aligned}\delta H &= [v_e e^{i\sqrt{2\pi}\Phi_c(0)} + \text{H.c.}] \cos[\sqrt{2\pi}\Phi_s(0)] \\ &+ [v_\rho e^{i\sqrt{8\pi}\Phi_c(0)} + \text{H.c.}] + v_\sigma \cos[\sqrt{8\pi}\Phi_s(0)].\end{aligned} \quad (7)$$

In terms of the fermion fields, the various terms in  $\delta H$  can be written as

$$\begin{aligned}v_e &: \psi_{L\sigma}^\dagger \psi_{R\sigma} + \text{H.c.}, \\ v_\rho &: \psi_{L\uparrow}^\dagger \psi_{R\uparrow} \psi_{L\downarrow}^\dagger \psi_{R\downarrow} + \text{H.c.}, \\ v_\sigma &: \psi_{L\uparrow}^\dagger \psi_{R\uparrow} \psi_{R\downarrow}^\dagger \psi_{L\downarrow} + \text{H.c.}\end{aligned}$$

Thus,  $v_e$  represents the backscattering of a single electron across the point contact,  $v_\rho$  denotes the process involving the tunneling of spin (not charge) between the top and bottom edges, and  $v_\sigma$  represents the process involving the tunneling of charge  $2e$  between the top and bottom edges. For the weak potential strength, the three terms are irrelevant when  $1/2 < K < 2$ . In general, higher-order terms could also be included. However, those terms are less relevant. It suffices to keep the terms in Eq. (7) to determine the phase diagram. In the following, we compute the noise spectrum in the CC limit to study the effects of the two-particle scattering processes.

### III. NONEQUILIBRIUM CURRENT AND NOISE

To analyze the transport properties of this system, we apply a voltage bias  $V$  between the upper and lower edges of the

point contact. In such a case,  $H_0$  becomes

$$\begin{aligned}H_0 &= \sum_{i=1}^4 \int_0^{+\infty} dx \mathcal{H}_0^{(i)} - \sum_{i=1,2} \int_0^{+\infty} dx \mu_+(J_{i,\text{in}} + J_{i,\text{out}}) \\ &- \sum_{i=3,4} \int_0^{+\infty} dx \mu_-(J_{i,\text{in}} + J_{i,\text{out}}) \\ &= \int_{-\infty}^{+\infty} dx [\mathcal{H}_0 - \mu_+(J_{R\uparrow} + J_{L\downarrow}) - \mu_-(J_{R\downarrow} + J_{L\uparrow})],\end{aligned}$$

where  $\mu_+ - \mu_- = -eV$ . (Here we assume that the charge carried by an electron is  $-e$ .) To proceed, it is convenient to move the dependence on the chemical potentials to  $\delta H$ . This is achieved by the time-dependent gauge transformation (throughout the calculations, we set  $\hbar = 1$ ),

$$\begin{aligned}\psi_{R\uparrow}(\psi_{L\downarrow}) &\rightarrow e^{i\mu_+ t} \psi_{R\uparrow}(\psi_{L\downarrow}), \\ \psi_{R\downarrow}(\psi_{L\uparrow}) &\rightarrow e^{i\mu_- t} \psi_{R\downarrow}(\psi_{L\uparrow}),\end{aligned}$$

leading to  $\delta H = \sum_{i=1}^3 \delta H_i$ , where

$$\begin{aligned}\delta H_1 &= [v_e e^{i\sqrt{2\pi}K_c \tilde{\Phi}_c(t,0)} + \text{H.c.}] \cos[\sqrt{2\pi}K_s \tilde{\Phi}_s(t,0) - \omega_0 t], \\ \delta H_2 &= v_\rho e^{i\sqrt{8\pi}K_c \tilde{\Phi}_c(t,0)} + \text{H.c.}, \\ \delta H_3 &= v_\sigma \cos[\sqrt{8\pi}K_s \tilde{\Phi}_s(t,0) - 2\omega_0 t].\end{aligned} \quad (8)$$

Here  $\tilde{\Phi}_\alpha = \Phi_\alpha / \sqrt{K_\alpha}$  and  $\omega_0 = eV$ . The  $\omega_0$  dependence of the various terms reflects the numbers of transferred charges involved in the corresponding process.

Let  $\hat{J}_i$  denote the particle current operator flowing into terminal  $i$ . Then we have

$$\begin{aligned}\hat{J}_1(t, x_1) &= J_{1,\text{in}}(t, -x_1) - J_{1,\text{out}}(t, -x_1) \\ &= J_{R\uparrow}(t, x_1) - J_{L\downarrow}(t, x_1), \\ \hat{J}_2(t, x_2) &= J_{2,\text{in}}(t, x_2) - J_{2,\text{out}}(t, x_2) \\ &= J_{L\downarrow}(t, x_2) - J_{R\uparrow}(t, x_2), \\ \hat{J}_3(t, x_3) &= J_{3,\text{in}}(t, x_3) - J_{3,\text{out}}(t, x_3) \\ &= J_{L\uparrow}(t, x_3) - J_{R\downarrow}(t, x_3), \\ \hat{J}_4(t, x_4) &= J_{4,\text{in}}(t, -x_4) - J_{4,\text{out}}(t, -x_4) \\ &= J_{R\downarrow}(t, x_4) - J_{L\uparrow}(t, x_4),\end{aligned}$$

where  $x_1, x_4 < 0$  and  $x_2, x_3 > 0$ . In terms of the bosonic fields,  $\hat{J}_i$  can be written as

$$\begin{aligned}\hat{J}_1 &= \sqrt{\frac{K_s}{2\pi}} \partial_x \tilde{\Phi}_s - \frac{1}{\sqrt{2\pi}K_c} \partial_x \tilde{\Theta}_c = -\hat{J}_2, \\ \hat{J}_3 &= \sqrt{\frac{K_s}{2\pi}} \partial_x \tilde{\Phi}_s + \frac{1}{\sqrt{2\pi}K_c} \partial_x \tilde{\Theta}_c = -\hat{J}_4,\end{aligned} \quad (9)$$

where  $\tilde{\Theta}_\alpha = \sqrt{K_\alpha} \Theta_\alpha$ . The current flowing into terminal  $i$  is given by  $I_i = -e v_F \langle \hat{J}_i \rangle$ . According to our convention,  $I_i$  is positive when the current flows out of the terminal.

The noise spectrum is defined by

$$S_{ij}(\omega; x, x') \equiv e^2 v_F^2 \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \{ \Delta \hat{J}_i(t, x), \Delta \hat{J}_j(0, x') \} \rangle, \quad (10)$$

where  $\Delta \hat{J}_i = \hat{J}_i - \langle \hat{J}_i \rangle$ . We would like to calculate  $I_i$  and  $S_{ij}$  in terms of the perturbative expansion in the tunneling amplitude

$v_l$  ( $l = e, \rho, \sigma$ ) within the Keldysh formalism.<sup>23</sup> We shall see later that  $\langle \hat{J}_i \rangle = O(|v_l|^2)$ . Thus, to order of  $|v_l|^2$ ,  $S_{ij}$  can be written as

$$S_{ij}(\omega; x, x') = e^2 v_F^2 \int_{-\infty}^{+\infty} dt e^{i\omega t} \{ \langle \hat{J}_i(t, x), \hat{J}_j(0, x') \rangle \}.$$

The perturbative calculations of the current and noise spectrum can be straightly performed using the Keldysh functional integral formulation, as was done in Ref. 14 for tunneling between the chiral LLs. To the order of  $|v_l|^2$ , the currents at zero temperature are given by

$$I_1(t) = -\frac{e}{2} \text{sgn}(\omega_0) [ |v_e|^2 \text{Re}(\mathcal{A}) |\omega_0|^{K+1/K-1} + |v_\sigma|^2 \text{Re}(\mathcal{B}_s) |2\omega_0|^{4/K-1} ] = I_2(t) = -I_3(t) = -I_4(t), \quad (11)$$

and the noise spectra at zero temperature are given by

$$\begin{aligned} S_{ii}(\omega) = e^2 \left\{ \frac{K}{\pi} |\omega| + \left[ \frac{1-K^2}{2} |v_e|^2 \text{Im}(\mathcal{A}) |\omega_0|^{K+1/K-1} + |v_\sigma|^2 \text{Im}(\mathcal{B}_s) |2\omega_0|^{4/K-1} \right] \sin\left(\frac{2|\omega x|}{v}\right) \right. \\ + \frac{|v_e|^2}{8} (1-K^2) (\mathcal{A} e^{i\frac{2|\omega x|}{v}} + \text{c.c.}) [ |\omega + \omega_0|^{K+1/K-1} + |\omega - \omega_0|^{K+1/K-1} ] \\ + \frac{|v_\sigma|^2}{4} (\mathcal{B}_s e^{i\frac{2|\omega x|}{v}} + \text{c.c.}) [ |\omega + 2\omega_0|^{4/K-1} + |\omega - 2\omega_0|^{4/K-1} ] \\ + |v_e|^2 \text{Re}(\mathcal{A}) (|\omega_0| - |\omega|)^{K+1/K-1} \left[ \sin^2\left(\frac{\omega x}{v}\right) + K^2 \cos^2\left(\frac{\omega x}{v}\right) \right] \theta(|\omega|) \theta(|\omega_0| - |\omega|) \\ \left. + 2|v_\sigma|^2 \text{Re}(\mathcal{B}_s) (|2\omega_0| - |\omega|)^{4/K-1} \sin^2\left(\frac{\omega x}{v}\right) \theta(|\omega|) \theta(|2\omega_0| - |\omega|) - 2K^2 |v_\rho|^2 (\mathcal{B}_c e^{i\frac{2|\omega x|}{v}} + \text{c.c.}) |\omega|^{4K-1} \right\}, \quad (12) \end{aligned}$$

with  $i = 1, 2, 3, 4$ , and

$$\begin{aligned} S_{12}(\omega) = -e^2 \left\{ \frac{K|\omega|}{\pi} \cos\left(\frac{2\omega x}{v}\right) - \left[ \frac{1+K^2}{2} |v_e|^2 \text{Im}(\mathcal{A}) |\omega_0|^{K+1/K-1} + |v_\sigma|^2 \text{Im}(\mathcal{B}_s) |2\omega_0|^{4/K-1} \right] \sin\left(\frac{2|\omega x|}{v}\right) \right. \\ - \frac{|v_e|^2}{8} (1+K^2) (\mathcal{A} e^{i\frac{2|\omega x|}{v}} + \text{c.c.}) [ |\omega + \omega_0|^{K+1/K-1} + |\omega - \omega_0|^{K+1/K-1} ] \\ - \frac{|v_\sigma|^2}{4} (\mathcal{B}_s e^{i\frac{2|\omega x|}{v}} + \text{c.c.}) [ |\omega + 2\omega_0|^{4/K-1} + |\omega - 2\omega_0|^{4/K-1} ] \\ + |v_e|^2 \text{Re}(\mathcal{A}) (|\omega_0| - |\omega|)^{K+1/K-1} \left[ \frac{i}{2} \sin\left(\frac{2|\omega x|}{v}\right) + K^2 \cos^2\left(\frac{\omega x}{v}\right) \right] \theta(|\omega|) \theta(|\omega_0| - |\omega|) \\ \left. + i|v_\sigma|^2 \text{Re}(\mathcal{B}_s) (|2\omega_0| - |\omega|)^{4/K-1} \sin\left(\frac{2|\omega x|}{v}\right) \theta(|\omega|) \theta(|2\omega_0| - |\omega|) - 2K^2 |v_\rho|^2 (\mathcal{B}_c e^{i\frac{2|\omega x|}{v}} + \text{c.c.}) |\omega|^{4K-1} \right\}, \quad (13) \end{aligned}$$

where  $S_{ii}(\omega) = S_{ii}(\omega; x, x)$ ,  $S_{ij}(\omega) = S_{ij}(\omega; x, -x)$  for  $i \neq j$ ,

$$\begin{aligned} \text{Re}(\mathcal{A}) = \frac{\pi a_0^{K+1/K}}{v^{K+1/K} \Gamma(K+1/K)}, \quad \text{Im}(\mathcal{A}) = -\frac{\pi a_0^{K+1/K} \tan[\pi(K+1/K)/2]}{v^{K+1/K} \Gamma(K+1/K)}, \\ \text{Re}(\mathcal{B}_s) = \frac{\pi a_0^{4/K}}{v^{4/K} \Gamma(4/K)}, \quad \text{Im}(\mathcal{B}_s) = -\frac{\pi a_0^{4/K} \tan(2\pi/K)}{v^{4/K} \Gamma(4/K)}, \end{aligned}$$

and

$$\begin{aligned} (\mathcal{A} e^{i\frac{2|\omega x|}{v}} + \text{c.c.}) = \frac{2\pi a_0^{K+1/K}}{v^{K+1/K} \Gamma(K+1/K)} \left\{ \cos\left(\frac{2|\omega x|}{v}\right) + \tan\left[\frac{\pi}{2}(K+1/K)\right] \sin\left(\frac{2|\omega x|}{v}\right) \right\}, \\ (\mathcal{B}_s e^{i\frac{2|\omega x|}{v}} + \text{c.c.}) = \frac{2\pi a_0^{4/K}}{v^{4/K} \Gamma(4/K)} \left[ \cos\left(\frac{2|\omega x|}{v}\right) + \tan(2\pi/K) \sin\left(\frac{2|\omega x|}{v}\right) \right]. \end{aligned}$$

On account of current conservation, the tunneling current  $I_t$  is given by  $I_t = -(I_1 + I_2) = I_3 + I_4$ . This has been verified by directly calculating  $I_t = -e \langle \hat{J}_t \rangle$  through the tunnel current operator

$$\hat{J}_t = -[v_e e^{i\sqrt{2\pi}\Phi_c(0)} + \text{H.c.}] \sin[\sqrt{2\pi}\Phi_s(0)] - 2v_\sigma \sin[\sqrt{8\pi}\Phi_s(0)]. \quad (14)$$

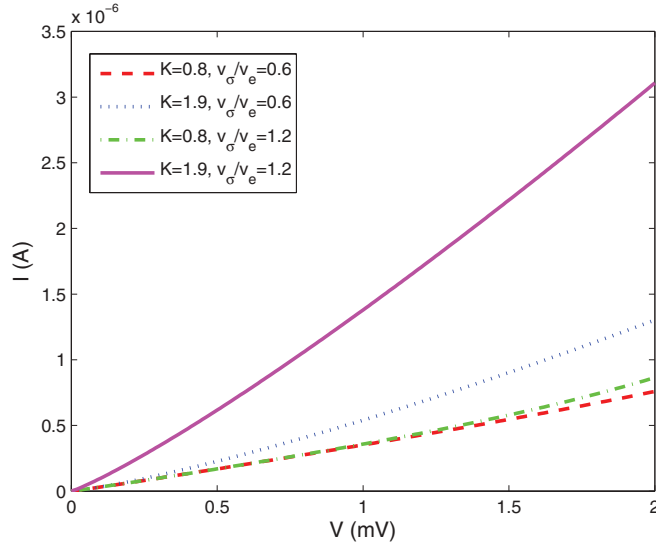


FIG. 2. (Color online) Dependence of the current  $I = |I_i|$  on the bias  $V$ . We use the parameters  $a_0 = 10^{-7}$  m,  $v = 5.5 \times 10^5$  m/s, and  $|v_e| = \hbar v/a_0$ .

#### IV. RESULTS AND DISCUSSION

We now discuss our results. The result for the current is shown in Fig. 2. Although what we are considering is the nonequilibrium transport, it is interesting to see that the dependence of each term in Eq. (11) on the bias follows from the scaling dimension of each scattering process. [The scaling dimensions of the  $v_e$ ,  $v_\rho$ , and  $v_\sigma$  terms about the LL fixed point are  $\Delta_e = (K + 1/K)/2$ ,  $\Delta_\rho = 2K$ , and  $\Delta_\sigma = 2/K$ , respectively.] We notice that the  $v_\rho$  term completely disappears in Eq. (11) because it does not involve net charge transport. Although this term plays an important role in determining the phase diagram, our perturbative calculations show that its effects on the electrical transport can only be probed through the current correlations at finite frequency, as shown in Eqs. (12) and (13).

After examining the voltage dependence of the current, we now turn to the voltage and frequency dependence of the noise spectrum. Since the behavior of  $\text{Re}\{S_{12}(\omega)\}$  is similar to that of  $S_{11}(\omega)$ , we just plot the frequency dependence of  $\Delta S_{11} = S_{11} - S_{11}^{(0)}$  in Figs. 3 and 4, where  $S_{11}^{(0)}$  is the noise spectrum in the absence of tunneling, that is,  $v_l = 0$ . We would like to emphasize a few features. First of all, the noise spectrum at finite frequency is sensitive to the position of the probe, with the overall oscillatory behavior determined by the sine or cosine functions. Next, in addition to the singularity at  $\omega = 0$ ,  $S_{ij}(\omega)$  also exhibits singularities at  $\omega = \omega_0$  and  $\omega = 2\omega_0$ , corresponding to the single-particle and two-particle tunneling, respectively. We expect that these singularities remain intact even by taking into account the nonperturbative effects.<sup>14</sup> For  $1/2 < K < \sqrt{3}$  (the regime dominated by the single-particle scattering  $v_e$  term), the singularity at  $\omega = \omega_0$  reveals itself in the guise of a cusp in  $dS_{11}/d\omega$ , as shown in the inset of Fig. 3. However, the subleading singularity at  $\omega = 2\omega_0$  in this region can only be seen in the higher-order derivatives of  $S_{11}$  due to its higher powers. For example, near  $2\omega_0$ ,  $S_{11} \sim |\omega - 2\omega_0|^{3.71}$

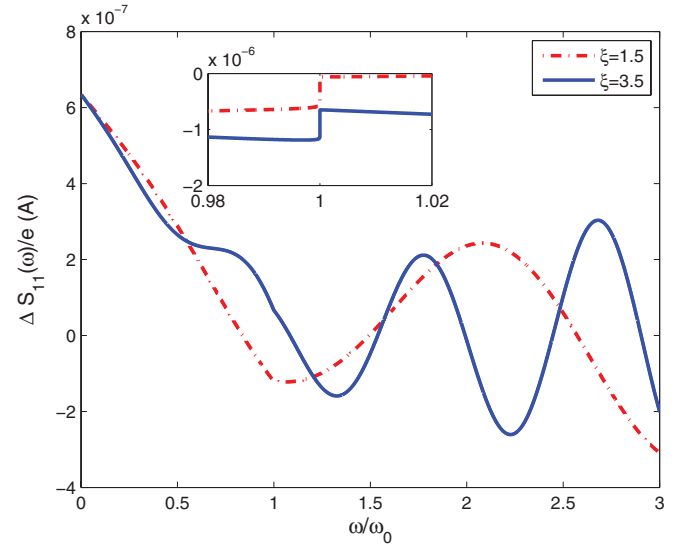


FIG. 3. (Color online) Dependence of  $\Delta S_{11} = S_{11} - S_{11}^{(0)}$  on the frequency  $\omega$  at  $K = 0.85$ , where  $S_{11}^{(0)}$  is the noise spectrum in the absence of tunneling, that is,  $v_l = 0$ . As explained in the text, the singularity at  $\omega = \omega_0$  is much stronger than that at  $\omega = 2\omega_0$  in the region where the single-particle scattering is dominated ( $1/2 < K < \sqrt{3}$ ), and thus a clear structure can be seen in the figure near  $\omega = \omega_0$ . To reveal the singularity at  $\omega = \omega_0$ , a plot of  $d\Delta S_{11}/d\omega$  around  $\omega = \omega_0$  is shown in the inset. We use the parameters  $a_0 = 10^{-7}$  m,  $v = 5.5 \times 10^5$  m/s,  $|v_e| = \hbar v/a_0 = |v_\sigma| = |v_\rho|$ , and  $\xi = x/(\hbar v/\omega_0)$ .

at  $K = 0.85$ ; hence, one can see the singular behavior at  $\omega = 2\omega_0$  at least in the fourth-order derivative  $d^4 S_{11}/d\omega^4$ . On the other hand, for  $\sqrt{3} < K < 2$  (the regime dominated by the two-particle scattering  $v_\sigma$  term), the singular behaviors

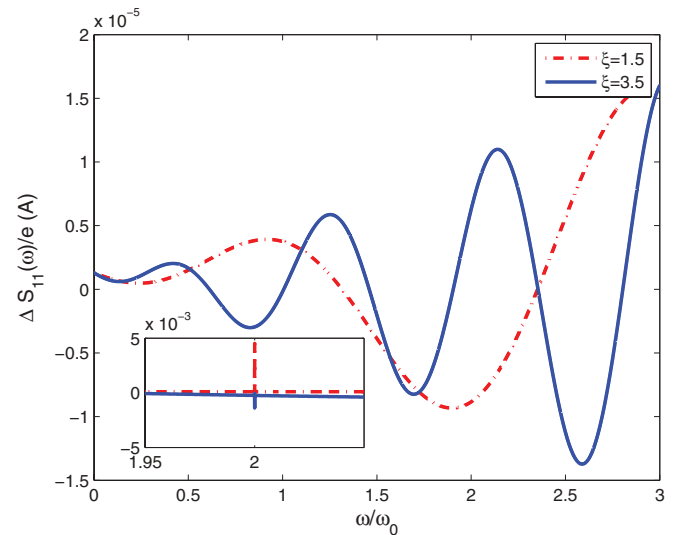


FIG. 4. (Color online) Dependence of  $\Delta S_{11} = S_{11} - S_{11}^{(0)}$  on the frequency  $\omega$  at  $K = 1.85$ , where  $S_{11}^{(0)}$  is the noise spectrum in the absence of tunneling, that is,  $v_l = 0$ . As explained in the text, the singularity at  $\omega = 2\omega_0$  is relatively weaker than that at  $\omega = \omega_0$  in the region where the two-particle scattering is dominated ( $\sqrt{3} < K < 2$ ). To reveal the former, a plot of  $d^2 \Delta S_{11}/d\omega^2$  around  $\omega = 2\omega_0$  is shown in the inset. We use the parameters  $a_0 = 10^{-7}$  m,  $v = 5.5 \times 10^5$  m/s,  $|v_e| = \hbar v/a_0 = |v_\sigma| = |v_\rho|$ , and  $\xi = x/(\hbar v/\omega_0)$ .



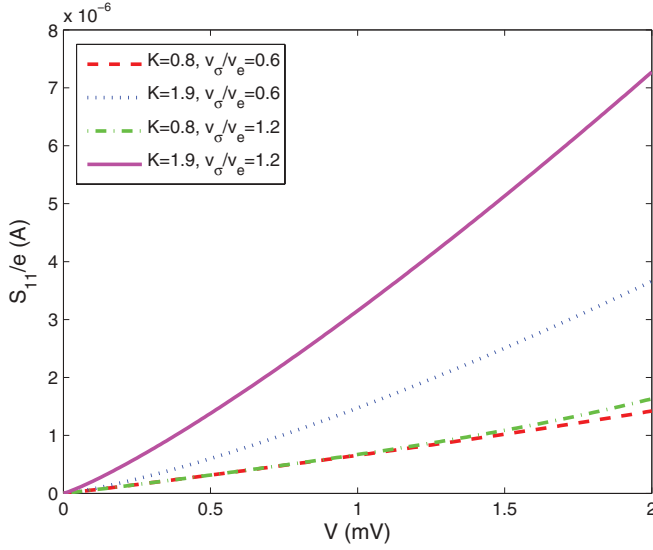


FIG. 5. (Color online) Dependence of the autocorrelation at zero frequency  $S_{11}(0)$  on the bias  $V$ . We use the parameters  $a_0 = 10^{-7}$  m,  $v = 5.5 \times 10^5$  m/s, and  $|v_e| = \hbar v/a_0$ .

of  $S_{11}$  around  $\omega = \omega_0$  and  $\omega = 2\omega_0$  are not clear as shown in Fig. 4 due to its high powers in  $|\omega - \omega_0|$  and  $|\omega - 2\omega_0|$  in this region. Nevertheless, the dominated singular behavior at  $\omega = 2\omega_0$  can still be revealed through the second-order derivative  $d^2 S_{11}/d\omega^2$ , as shown in the inset of Fig. 4. These qualitative features shown in  $S_{ij}(\omega)$  can be used in future experiments to probe the dynamical current correlations of the helical LLs in the presence of both single-particle and two-particle scattering.

In contrast to the tunneling between chiral LLs in a similar four-terminal configuration where only the cross correlations for the chiral currents depend on the position of the probe,<sup>14</sup> both the auto- and the cross correlations in the present case are sensitive to the position of the probe  $x$ . It turns out that their zero-frequency limits are the most robust measurements of fluctuations in the present situation because the resulting expressions in this limit are independent of the position of the probe. The dependence of  $S_{11}(0)$  and  $S_{12}(0)$  on the bias  $V$  is shown in Figs. 5 and 6. We notice that the bias dependence of each term in  $S_{11}(0)$  and  $S_{12}(0)$  also follows from the scaling dimension of each scattering process. Therefore, among the three scattering terms, only one scattering process will dominate the behavior of  $S_{ij}$  at low bias, depending on the LL parameter  $K$ , though the introduction of two-particle scattering will, in general, enhance the strength of the current correlation. It follows from Eqs. (12) and (13) that  $S_{ij}(0)$  at low bias are dominated by the single-particle scattering term (the  $v_e$  term) at  $1/2 < K < \sqrt{3}$ , while at  $\sqrt{3} < K < 2$  it is the  $v_\sigma$  term that is dominant. A direct consequence of this is that, as shown in Figs. 5 and 6, the increase of the magnitude of  $S_{ij}(0)$  with  $K = 1.9$  at low bias is much larger than the one of  $S_{ij}(0)$  with  $K = 0.8$  for the same amount of the increment of the ratio  $|v_\sigma/v_e|$ . A similar situation also occurs for the current, as shown in Fig. 2.

It should be noted here that, at  $\sqrt{3} < K < 2$ , the exponents  $K + 1/K - 1$  and  $4/K - 1$  are numerically quite close to one another. This indicates that both the  $v_e$  and the  $v_\sigma$  terms will

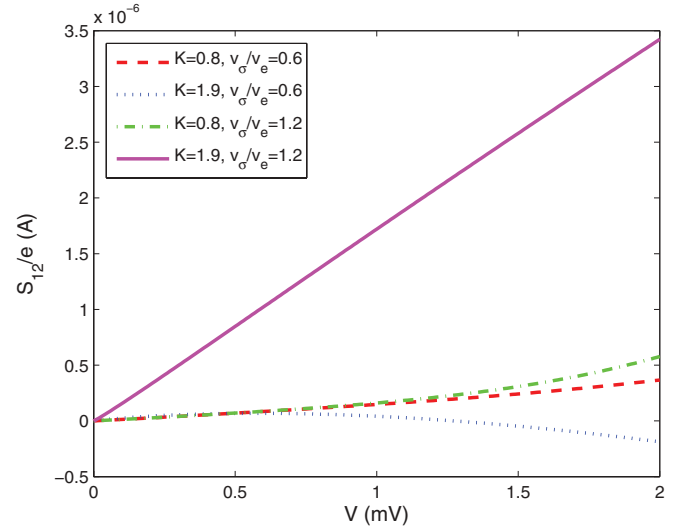


FIG. 6. (Color online) Dependence of the cross correlation at zero frequency  $S_{12}(0)$  on the bias  $V$ . We use the parameters  $a_0 = 10^{-7}$  m,  $v = 5.5 \times 10^5$  m/s, and  $|v_e| = \hbar v/a_0$ .

contribute significantly to  $S_{ij}(0)$ , except for the case when the bias is extremely low. For example, at  $K = 1.9$ , we need to take  $V$  to be about 0.01 meV in order to see that the contribution of the  $v_\sigma$  term is 10 times larger than that of the  $v_e$  term, assuming that  $|v_\sigma/v_e| = O(1)$ . Therefore, a better way to reveal the competition between the single-particle and the two-particle scattering processes is to investigate the Fano factor, which is defined by

$$F_{ij}(V) = \frac{S_{ij}(0)}{2e|I|}. \quad (15)$$

Since the Fano factor is directly related to the charge fluctuations in the terminals, as we shall demonstrate later, it contains the information about the fractional charge excitations in the helical LL. From the noises and currents calculated above, we have

$$F_{ii}(V) = \frac{1}{2} \left[ \frac{1 + K^2 + 2\eta|v_\sigma/v_e|^2|\omega_0|^{3/K-K}}{1 + \eta|v_\sigma/v_e|^2|\omega_0|^{3/K-K}} \right], \quad (16)$$

$$F_{12}(V) = \frac{1}{2} \left[ \frac{1 - K^2 + 2\eta|v_\sigma/v_e|^2|\omega_0|^{3/K-K}}{1 + \eta|v_\sigma/v_e|^2|\omega_0|^{3/K-K}} \right],$$

where

$$\eta \equiv 2^{4/K-1} \frac{\text{Re}(\mathcal{B}_s)}{\text{Re}(\mathcal{A})}$$

$$= 2^{4/K-1} \left( \frac{a_0}{v} \right)^{3/K-K} \frac{\Gamma(K + 1/K)}{\Gamma(4/K)}$$

is a nonuniversal constant.

Since the  $v_\rho$  dependence completely disappears in the zero-frequency limit,  $F_{ij}$  is very sensitive to the single ratio  $|v_\sigma/v_e|$ . In general,  $F_{ij}(V)$  consists of terms exhibiting a power law in  $V$  with exponents related to the scaling dimension of each scattering process. We plot  $F_{11}$  and  $F_{12}$  as functions of the bias  $V$  in Figs. 7 and 8. The effects of the  $v_e$  and the  $v_\sigma$  terms are disentangled at the zero-bias limit. By taking  $V \rightarrow 0$ , we

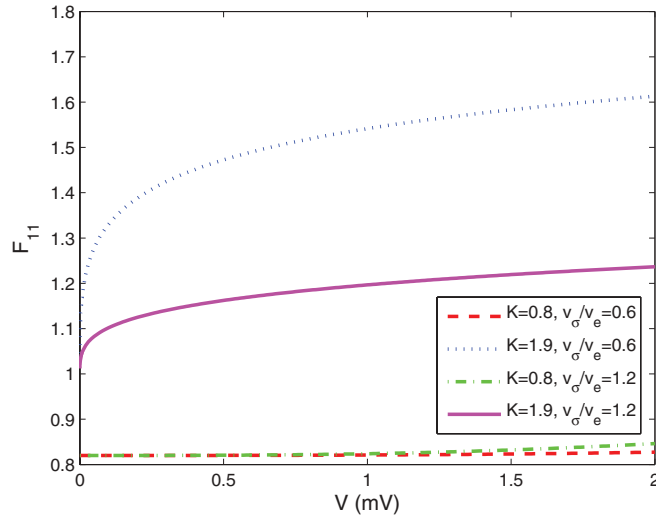


FIG. 7. (Color online) Dependence of  $F_{11}$  on the bias  $V$ . We use the parameters  $a_0 = 10^{-7}$  m and  $v = 5.5 \times 10^5$  m/s.

find that

$$F_{ii}(0) = \begin{cases} \frac{1+K^2}{2} & 1/2 < K < \sqrt{3}, \\ 1 & \sqrt{3} < K < 2, \end{cases} \quad (17)$$

and

$$F_{12}(0) = \begin{cases} \frac{1-K^2}{2} & 1/2 < K < \sqrt{3}, \\ 1 & \sqrt{3} < K < 2. \end{cases} \quad (18)$$

Note that in the region where the  $v_\sigma$  term dominates,  $F_{ij}(0)$  takes the classical Schottky result, while it depends on the LL parameter  $K$  in the region where the single-particle tunneling is dominant. As a by-product, the behaviors of  $F_{ij}(V)$  may provide us with a way to measure the value of  $K$  for the helical liquid.

In Ref. 27 (see also Refs. 21, 22, and 28), it was shown that when a charge is injected into a LL, it will break up into two counterpropagating—left-moving and right-moving—quasiparticles carrying fractional charges. We now

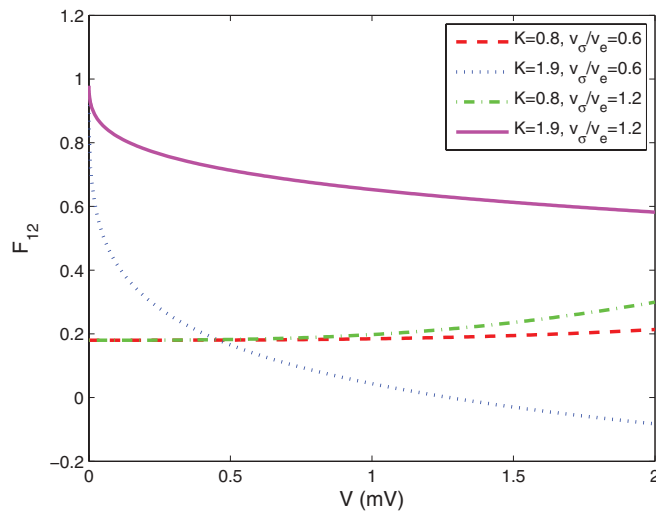


FIG. 8. (Color online) Dependence of  $F_{12}$  on the bias  $V$ . We use the parameters  $a_0 = 10^{-7}$  m and  $v = 5.5 \times 10^5$  m/s.

apply this idea to interpret our results [Eqs. (17) and (18)]. Since the effects of the  $v_e$  and the  $v_\sigma$  terms are disentangled in the limit  $V \rightarrow 0$ , we consider this limit first. Without loss of generality, we assume that  $V > 0$ . Then the  $v_e$  term implies a single-electron tunneling from the bottom edge to the top one, whereas the  $v_\sigma$  term implies the simultaneous tunneling of a spin-up electron and a spin-down electron from the bottom edge to the top one. The former ( $v_e$ ) process generates the following state:

$$\begin{aligned} \sum_{\sigma} \Psi_{\sigma}^{\dagger}(x=0)|O_{LL}\rangle \\ = \sum_{\sigma} \psi_{R\sigma}^{\dagger}(x=0)|O_{LL}\rangle + \sum_{\sigma} \psi_{L\sigma}^{\dagger}(x=0)|O_{LL}\rangle, \end{aligned}$$

while the state produced by the latter ( $v_\sigma$ ) is

$$\psi_{R\uparrow}^{\dagger}(x=0)\psi_{L\downarrow}^{\dagger}(x=0)|O_{LL}\rangle,$$

where  $|O_{LL}\rangle$  denotes the ground state of the LL. In the above, the terms with higher scaling dimensions are neglected. To proceed, we define the new chiral bosonic fields

$$\phi_{\alpha l} = \frac{1}{2}(\tilde{\Phi}_{\alpha} + \tilde{\Theta}_{\alpha}), \quad \phi_{\alpha r} = \frac{1}{2}(\tilde{\Phi}_{\alpha} - \tilde{\Theta}_{\alpha}),$$

where  $\alpha = c, s$ .  $\phi_{\alpha l}$  and  $\phi_{\alpha r}$  describe the elementary excitations of the spin- $\frac{1}{2}$  LL propagating with speed  $v$  along the left and the right directions, respectively. In terms of  $\phi_{\alpha l}$  and  $\phi_{\alpha r}$ , we may define the chiral fields carrying a unit of U(1) charge<sup>27,29</sup>:

$$\tilde{\psi}_{cl} = \exp\left[-i\sqrt{\frac{2\pi}{K}}\phi_{cl}\right], \quad \tilde{\psi}_{cr} = \exp\left[i\sqrt{\frac{2\pi}{K}}\phi_{cr}\right]. \quad (19)$$

Then we have for the single-particle ( $v_e$ ) process

$$\begin{aligned} \sum_{\sigma} \Psi_{\sigma}^{\dagger}(x=0)|O_{LL}\rangle \\ = [\tilde{\psi}_{cl}^{\dagger}(x=0)]^{Q_-} [\tilde{\psi}_{cr}^{\dagger}(x=0)]^{Q_+} O_{s1}(x=0)|O_{LL}\rangle \\ + [\tilde{\psi}_{cl}^{\dagger}(x=0)]^{Q_+} [\tilde{\psi}_{cr}^{\dagger}(x=0)]^{Q_-} O_{s2}(x=0)|O_{LL}\rangle, \end{aligned} \quad (20)$$

and for the two-particle ( $v_\sigma$ ) process we obtain

$$\begin{aligned} \psi_{R\uparrow}^{\dagger}(x=0)\psi_{L\downarrow}^{\dagger}(x=0)|O_{LL}\rangle \\ = \tilde{\psi}_{cl}^{\dagger}(x=0)\tilde{\psi}_{cr}^{\dagger}(x=0)O_{s3}(x=0)|O_{LL}\rangle, \end{aligned} \quad (21)$$

where

$$Q_{\pm} = \frac{1 \pm K}{2},$$

and

$$\begin{aligned} O_{s1} &= e^{i\frac{\pi}{16K}(1-K^2)} \sum_{\sigma} \frac{\eta_{\sigma}}{\sqrt{2\pi a_0}} e^{-i\sqrt{\frac{\pi}{2}}\sigma(\Phi_s - \Theta_s)}, \\ O_{s2} &= e^{i\frac{\pi}{16K}(1-K^2)} \sum_{\sigma} \frac{\eta_{\sigma}}{\sqrt{2\pi a_0}} e^{i\sqrt{\frac{\pi}{2}}\sigma(\Phi_s + \Theta_s)}, \\ O_{s3} &= \frac{\eta_{\uparrow}\eta_{\downarrow}}{2\pi a_0} e^{i\frac{(1+K)\pi}{4K}} e^{-i\sqrt{\frac{\pi}{2}}(\Phi_s - \Theta_s)} e^{-i\sqrt{\frac{\pi}{2}}(\Phi_s + \Theta_s)}. \end{aligned}$$

Since the operators  $O_{s1}$ ,  $O_{s2}$ , and  $O_{s3}$  are charge neutral, by focusing only on the charge states we may reexpress Eqs. (20)

and (21) as

$$\sum_{\sigma} \Psi_{\sigma}^{\dagger}(x=0)|O_{LL}\rangle \sim |Q_{+}, Q_{-}\rangle + |Q_{-}, Q_{+}\rangle,$$

$$\psi_{R\uparrow}^{\dagger}(x=0)\psi_{L\downarrow}^{\dagger}(x=0)|O_{LL}\rangle \sim |1, 1\rangle,$$

where  $|Q_l, Q_r\rangle$  denotes the charge state in which the left and the right movers carry charge  $Q_l$  and  $Q_r$ , respectively. Both the above expressions can be understood as the consequence of fractionalization of charge upon its injection into a LL as discussed in Ref. 27.

In one spatial dimension, the current fluctuations amount to the measurement of charge fluctuations. Accordingly, we get

$$S_{ii}(0) \propto Q_{+}^2 + Q_{-}^2 = \frac{1 + K^2}{2},$$

$$S_{12}(0) \propto 2Q_{+}Q_{-} = \frac{1 - K^2}{2},$$

when the  $v_e$  term dominates, while for the  $v_{\sigma}$  term being dominant,  $S_{ij}(0)$  is proportional to a  $K$ -independent constant. From the above analysis, we see that the dependence of  $F_{ij}(0)$  on the LL parameter  $K$  follows from the fact that the final state of the single-particle scattering is an entangled state of the left and the right mover  $-Q_{\pm}e$ . On the other hand, the classical Schottky result arises from the final state of the two-particle scattering, which is a direct product state of the left- and the right-mover both carrying charge  $-e$ . (This state is not a direct product state of the single-electron states because here the left- and the right-mover carry fractional spins,  $\pm 1/K$  in units of  $\hbar/2$ .) At finite bias, both the  $v_e$  and the  $v_{\sigma}$  terms will contribute to the current and the current noise so that the Fano factor depends on the ratio  $|v_{\sigma}/v_e|$ .

We may now compare our results with the main results in Ref. 13. First of all, in terms of current conservation, that is,  $I_t = -I_1(0^-) - I_2(0^+)$ , we can obtain the tunnel current noise at finite frequency:

$$S_t(\omega) \equiv e^2 \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \{\Delta \hat{J}_t(t), \Delta \hat{J}_t(0)\} \rangle$$

$$= S_{11}(\omega; 0^-, 0^-) + S_{22}(\omega; 0^+, 0^+) + S_{12}(\omega; 0^-, 0^+) + S_{21}(\omega; 0^+, 0^-), \quad (22)$$

where  $\Delta \hat{J}_t = \hat{J}_t - \langle \hat{J}_t \rangle$  and  $\hat{J}_t$  is given by Eq. (14). Inserting Eqs. (12) and (13) into Eq. (22), we find that

$$S_t(\omega) = e^2 [ |v_e|^2 \text{Re}(\mathcal{A}) (|\omega + \omega_0|^{K+\frac{1}{k}-1} + |\omega - \omega_0|^{K+\frac{1}{k}-1}) + 2|v_{\sigma}|^2 \text{Re}(\mathcal{B}_s) (|\omega + 2\omega_0|^{4/K-1} + |\omega - 2\omega_0|^{4/K-1}) ] \quad (23)$$

to  $O(|v_l|^2)$ . Equation (23) can also be obtained by a direction calculation using  $\hat{J}_t$  defined in Eq. (14). The zero-frequency limit,  $S_t(0)$ , coincides with the result in Ref. 13 [see Eq. (21) of Ref. 13]. It should be emphasized that although we may obtain the total tunnel current noise in Ref. 13 from our  $S_{ij}$ , the reverse is not true. This is simply because the currents in the four terminals,  $I_1$ ,  $I_2$ ,  $I_3$ , and  $I_4$ , cannot be expressed by the tunnel currents  $I_{t\uparrow}$  and  $I_{t\downarrow}$ , where  $I_{t\uparrow}$  and  $I_{t\downarrow}$  are spin-up and spin-down tunnel currents, respectively. The other way to see the difference between the two approaches can be seen from the Fano factor for the tunnel current, which is defined

as  $F_t(V) = S_t(0)/(2e|I_t|)$ . To  $O(|v_l|^2)$ , we have

$$F_t(V) = \frac{1 + 2\eta|v_{\sigma}/v_e|^2|\omega_0|^{3/K-K}}{1 + \eta|v_{\sigma}/v_e|^2|\omega_0|^{3/K-K}}, \quad (24)$$

leading to

$$F_t(0) = \begin{cases} 1 & 1/2 < K < \sqrt{3}, \\ 2 & \sqrt{3} < K < 2, \end{cases}$$

in the zero-bias limit. We see that the Fano factor in the zero-bias limit  $F_t(0)$ , corresponding to the effective quasiparticle charge transporting in the tunneling process, exhibits the classical Schottky result: For the single-particle-process-dominated region ( $1/2 < K < \sqrt{3}$ ),  $F_t(0) = 1$  (corresponding to charge  $e$ ), whereas for the region dominated by the two-particle process ( $\sqrt{3} < K < 2$ ),  $F_t(0) = 2$  (corresponding to charge  $2e$ ). This must be the case since only electrons can tunnel between the two edges. By contrast, the currents in the terminals consist of quasiparticles which may carry fractional charge; the Fano factors for the currents in the terminals can therefore be used to detect the fractionally charged elementary excitations in the helical LL, as discussed. Hence, our work contains unique information about the nature of the fractional charge elementary excitations of the helical edge states, which is not seen in the tunnel current noises as studied in Ref. 13. On the other hand, as shown in Ref. 13, the cross correlation between the spin-up and spin-down tunnel currents can be used to study the fermionic HBT correlations. Since the currents  $I_i$  studied here are not spin-resolved, our present results cannot be used to address such an issue, and it is beyond the scope of our present work.

It is interesting to notice that in the case of tunneling between the chiral LLs, the Fano factor takes the classical Schottky result.<sup>14</sup> In the present case, however, the Fano factor is a function of the LL parameter  $K$  even in the absence of the two-particle tunneling. Similar results also occur for tunneling into a nanotube.<sup>28</sup> Hence, our work offers a way to distinguish the spin- $\frac{1}{2}$  LL from the chiral LL.

Finally, we would like to point out that, as noticed in Ref. 10, there exists a duality relation between the CC and the II limits. Therefore, the noise spectrum in the II limit in the presence of a bias between the left and right edges can be obtained from our results by interchanging the LL parameters of the charge and spin modes, that is,  $K \leftrightarrow 1/K$ .

## V. CONCLUSIONS

To summarize, we have studied the current and the noise spectrum of a four-terminal QPC in the QSHI at finite bias. Special emphasis is put on the fractional charge quasiparticle excitations shown in the noise correlations of the currents in the terminals (in contrast to the tunneling current noise spectrum) and examining how the single-particle and the two-particle scattering processes compete with each other. Via the Keldysh perturbative approach, we obtained noise spectra of the currents in the terminals, which are, in general, sensitive to the ratios of the tunneling strength and consist of terms exhibiting power law in bias voltage  $V$  with the exponents determined from the scaling dimension of each scattering process. We find that both auto- and cross correlations of



the noise spectra  $S_{ij}(\omega)$  are sensitive to the positions of the probe with an overall oscillatory behavior. Meanwhile,  $S_{ij}(\omega)$  exhibits singularities at  $\omega = \omega_0$  and  $\omega = 2\omega_0$ , corresponding to the single-particle and the two-particle scattering processes, respectively. It is a unique feature of the helical LL that the two-particle scattering process dominates the electrical transport at low bias in some parameter regime. The observation of the corresponding singularity in the finite-frequency noise spectra is a direct probe of this mechanism.

In addition to revealing the main characteristics of the current correlations at finite frequency, we also point out the difference between the noise spectra of the helical LL and the chiral LL. The correlations between the currents in the terminals studied in this paper furnish us with important information about the fractionally charged elementary excitations in the helical LL. In particular, we find from the Fano factors of the currents in the terminals that the fractional charge excitations show up in the single-particle scattering dominated regime ( $1/2 < K < \sqrt{3}$ ), whereas the classical Schottky result is obtained in the two-particle scattering dominated regime ( $\sqrt{3} < K < 2$ ). We provide further analytical understanding of these results via the idea of charge fractionalization in LLs with an electron injection as shown in Refs. 28, 21, and 22. Note that this information cannot be extracted from the previous study on the tunnel current noise.<sup>13</sup> In fact, we

have calculated the Fano factor  $F_t(V = 0)$ , corresponding to the effective charge transporting through the junction, and found that the result in the zero-bias limit is nothing but the classical Schottky result for both the single-particle- and the two-particle-tunneling-process-dominated regimes. This is in sharp contrast to our results for the Fano factors of the currents in the terminals. Therefore, our results offer a useful guide for the experimental identification of the helical LL, and thus the interaction effects in the QSHI.

Recently we became aware of the work by Souquet and Simon,<sup>30</sup> which has partial overlap with the present work. The finite-frequency tunneling current noise was calculated, and both singularities associated with the one-particle ( $\omega = \omega_0$ ) and the two-particle ( $\omega = 2\omega_0$ ) processes were also found in their results.

### ACKNOWLEDGMENTS

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