Fuzzy Modeling and Synchronization of Two Totally Different Chaotic Systems via Novel Fuzzy Model

Shih-Yu Li and Zheng-Ming Ge

***Abstract*—In this paper, a new fuzzy model is presented to simulate and synchronize two totally different and complicated chaotic systems, namely, 1) quantum cellular neural networks nanosystem (Quantum-CNN system) and 2) Qi system. Through the new fuzzy model, the following three main advantages can be obtained: 1) only two linear subsystems are needed; 2) the numbers of fuzzy rules can be reduced from 2^N to $2 \times N$ (comparing with the Takagi–Sugeno fuzzy model), where N is the number of nonlinear terms; 3) fuzzy synchronization of two different chaotic systems with different numbers of nonlinear terms can be achieved with only two sets of gain K. There are two examples in numerical simulation results to show the effectiveness and feasibility of our new model.**

***Index Terms*—New fuzzy synchronization scheme, novel fuzzy model, synchronization of different fuzzy systems.**

I. Introduction

In the last few years, synchronization in chaotic dynamical systems has received a great deal of interest among scientists from various fields. The phenomenon of synchronization of two chaotic systems is fundamental in science and has a wealth of applications in technology. Increasingly more applications of chaos synchronization were proposed. There are many control techniques to synchronize chaotic systems, such as fuzzy control [1]–[8], fuzzy logic control [9]–[15], linear and nonlinear error feedback control [16], [17], active control [18], [19], backstepping control method [20]–[22], impulsive control [23], [24], adaptive control [25], [26], and sliding mode control [27]–[29].

In recent years, fuzzy logic proposed by Zadeh [30] has received much attention from the control theorists as a powerful tool for the nonlinear control. Among various kinds of fuzzy methods, the Takagi–Sugeno fuzzy system [31] is widely accepted as a useful tool for design and analysis of fuzzy control system [10, 32]–[35]. Currently, some chaos control and synchronization based on T–S fuzzy systems have been proposed, such as the fuzzy sliding mode controlling technique [27]–[30], linear matrix inequality (LMI)-based synchronization [36]–[38], and fuzzy robust control [39]. These works are all focused on two identical nonlinear systems.

In the traditional T–S fuzzy model, we focus on the whole system. The number of the linear subsystem is decided by how many minimum nonlinear terms should be linearized in original system. As a result, there are 2^N linear subsystems (according to 2^N fuzzy rules) and $m \times 2^N$ equations in the T–S fuzzy system, where N is the number of minimum nonlinear terms, and m is the order of the system. If N is large, the number of linear subsystems in the T–S fuzzy system is huge. It becomes very inefficient and complicated. In addition, two different nonlinear systems may have different numbers of nonlinear terms. It causes different numbers of linear subsystems. Furthermore, when synchronization is studied, the traditional method—employing the idea of parallel distributed compensation (PDC) to design the fuzzy control law for stabilization of the error dynamics cannot be used here. This is due to the mismatch of the numbers of subsystems.

Consequently, the new fuzzy model proposed in this paper gives a new way of linearizing complicated nonlinear systems, in which only two subsystems are included. Further, through our new model, the idea of PDC and LMI-based method can both be applied to achieve synchronization. In simulation illustrations, the synchronization of the fuzzy Qi system based on our new model and the T–S fuzzy Lorenz system is also presented to show the feasible character of our new model.

The layout of the rest of this paper is as follows. In Section II, the theory of new fuzzy model is introduced. In Section III, a new fuzzy model of two chaotic systems is proposed. In Section IV, a fuzzy synchronization scheme is given. In Section V, two examples are proposed to show the effectiveness and feasibility of our new model. In Section VI, a detailed discussion of comparison with the new and T–S fuzzy model is proposed. In Section VII, conclusions are given.

II. New Fuzzy Model Theory

In this section, the core of the new fuzzy model is introduced. It can be divided into the following three steps, which we can abbreviate as TFS: 1) Transforming (T)—transforming each nonlinear equation to two linear subequations by fuzzy IF–THEN rules; 2) First (F)—taking all the first linear subequations to form the first linear subsystem; 3) Second (S)—taking all the second linear subequations to form the second one. The overall fuzzy model of the system is achieved by fuzzy blending of this two linear subsystem models.

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Consider a continuous-time nonlinear dynamic system as follows:

Equation i:

Rule 1:

$$\text{IF } z_i(t) \text{ is } M_{i1}$$
$$\text{THEN } \dot{x}_i(t) = A_{i1}x(t) + B_{i1}u(t)$$

Rule 2:

$$\text{IF } z_i(t) \text{ is } M_{i2}$$
$$\text{THEN } \dot{x}_i(t) = A_{i2}x(t) + B_{i2}u(t) \tag{2-1}$$

where

$$x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$$
$$u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T$$

where $i = 1, 2, \ldots, n$ (n is the number of nonlinear terms). M_{i1} and M_{i2} are fuzzy sets, A_i and B_i are column vectors, and $\dot{x}_i(t) = A_{ij}x(t) + B_{ij}u(t)$, $j = 1, 2$, is the output from the first and the second IF–THEN rules. Given a pair of $(x(t), u(t))$ and taking all the first linear subequations to form one linear subsystem and all the second linear subequations to form another linear subsystem, the final output of the fuzzy system is inferred as follows:

$$\dot{x}(t) = M_1 \begin{bmatrix} A_{11}x(t) + B_{11}u(t) \\ A_{21}x(t) + B_{21}u(t) \\ \vdots \\ A_{i1}x(t) + B_{i1}u(t) \end{bmatrix} + M_2 \begin{bmatrix} A_{12}x(t) + B_{12}u(t) \\ A_{22}x(t) + B_{22}u(t) \\ \vdots \\ A_{i2}x(t) + B_{i2}u(t) \end{bmatrix} \tag{2-2}$$

where M_1 and M_2 are diagonal matrices defined as follows:

$$\text{dia}\,(M_1) = [\, M_{11} \quad M_{21} \quad \ldots \quad M_{i1} \,]$$
$$\text{dia}\,(M_2) = [\, M_{12} \quad M_{22} \quad \ldots \quad M_{i2} \,].$$

Note that for each equation i, we have

$$\sum_{j=1}^{2} M_{ij}\,(z_i(t)) = 1$$
$$M_{ij}\,(z_i(t)) \geq 0, \quad i = 1, 2, \ldots, n; \quad j = 1, 2.$$

Via this fuzzy model, the final form of the fuzzy model becomes very simple. The new model provides a much more convenient approach for fuzzy model research and fuzzy application. The simulation results of complicated chaotic systems are discussed in Section III.

III. New Fuzzy Model of Complicated Chaotic Systems

In this section, the new fuzzy models of two different chaotic systems, namely, 1) two-cell quantum cellular neural networks nanosystem (Quantum-CNN system) and 2) Qi system, are shown in models 1 and 2.

Model 1—New Fuzzy Model of the Quantum-CNN System: For a two-cell Quantum-CNN [40], the following differential equations are obtained:

$$\begin{cases} \dot{x}_1 = -2a_1\sqrt{1-x_1^2}\sin x_2 \\ \dot{x}_2 = -w_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1-x_1^2}} \cos x_2 \\ \dot{x}_3 = -2a_2\sqrt{1-x_3^2}\sin x_4 \\ \dot{x}_4 = -w_2(x_3 - x_1) + 2a_2 \frac{x_3}{\sqrt{1-x_3^2}} \cos x_4 \end{cases} \tag{3-1}$$

where x_1 and x_3 are polarizations, x_2 and x_4 are quantum phase displacements, a_1 and a_2 are proportional to the interdot energy inside each cell, and ω_1 and ω_2 are parameters that weigh effects on the cell of the difference of the polarization of neighboring cells, like the cloning templates in traditional CNNs. When $a_1 = 6.8$, $a_2 = 4.3$, $w_1 = 4.7$, $w_2 = 3.9$, and the initial states are chosen as (0.1, 0.5, 0.1, 0.5), the nanosystem is chaotic, as shown in Fig. 1.

By using the new fuzzy model, the Quantum-CNN system can be linearized as simple linear equations. The steps (TFS) of fuzzy modeling are shown as follows.

Step 1(T)—Transforming: Assume that

1) $\sqrt{1-x_1^2}\sin x_2 \in [-Z_1, Z_1]$ and $Z_1 > 0$;
2) $\cos x_2/\sqrt{1-x_1^2} \in [-Z_2, Z_2]$ and $Z_2 > 0$;
3) $\sqrt{1-x_3^2}\sin x_4 \in [-Z_3, Z_3]$ and $Z_3 > 0$;
4) $\cos x_4/\sqrt{1-x_3^2} \in [-Z_4, Z_4]$ and $Z_4 > 0$.

then each equation in (3-1) can be exactly represented by the new fuzzy model, respectively, as follows:

for $\dot{x}_1 = -2a_1\sqrt{1-x_1^2}\sin x_2$, we have

$$\text{Rule 1: IF } \sqrt{1-x_1^2}\sin x_2 \text{ is } M_{11}, \quad \text{THEN } \dot{x}_1 = -2a_1Z_1 \tag{3-2}$$

$$\text{Rule 2: IF } \sqrt{1-x_1^2}\sin x_2 \text{ is } M_{12}, \quad \text{THEN } \dot{x}_1 = 2a_1Z_1 \tag{3-3}$$

for $\dot{x}_2 = -w_1(x_1 - x_3) + 2a_1(x_1/\sqrt{1-x_1^2})\cos x_2$, we have

$$\text{Rule 1: IF } \cos x_2/\sqrt{1-x_1^2} \text{ is } M_{21},$$
$$\text{THEN } \dot{x}_2 = -w_1(x_1 - x_3) + 2a_1x_1Z_2 \tag{3-4}$$

$$\text{Rule 2: IF } \cos x_2/\sqrt{1-x_1^2} \text{ is } M_{22},$$
$$\text{THEN } \dot{x}_2 = -w_1(x_1 - x_3) - 2a_1x_1Z_2 \tag{3-5}$$

for $\dot{x}_3 = -2a_2\sqrt{1-x_3^2}\sin x_4$, we have

$$\text{Rule 1: IF } \sqrt{1-x_3^2}\sin x_4 \text{ is } M_{31}, \quad \text{THEN } \dot{x}_3 = -2a_2Z_3 \tag{3-6}$$

$$\text{Rule 2: IF } \sqrt{1-x_3^2}\sin x_4 \text{ is } M_{32}, \quad \text{THEN } \dot{x}_3 = 2a_2Z_3 \tag{3-7}$$

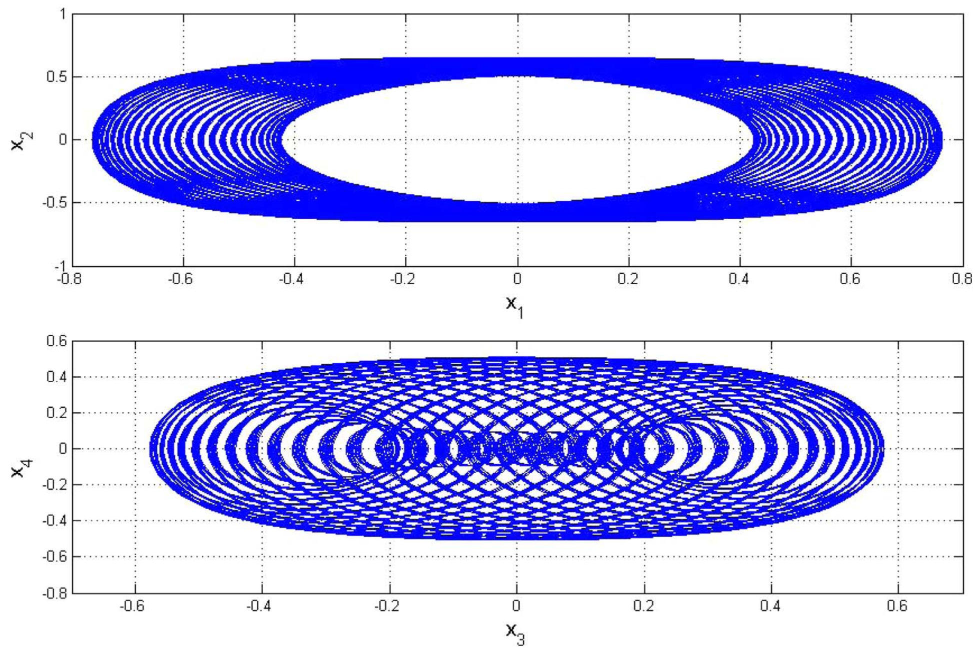

Fig. 1. Chaotic behavior of the Quantum-CNN system.

for $\dot{x}_4 = -w_2(x_3 - x_1) + 2a_2(x_3/\sqrt{1 - x_3^2}) \cos x_4$, we have

$$\text{Rule 1: IF } \cos x_4/\sqrt{1 - x_3^2} \text{ is } M_{41},$$
$$\text{THEN } \dot{x}_4 = -w_2(x_3 - x_1) + 2a_2 x_3 Z_4 \quad (3\text{-}8)$$
$$\text{Rule 2: IF } \cos x_4/\sqrt{1 - x_3^2} \text{ is } M_{42},$$
$$\text{THEN } \dot{x}_4 = -w_2(x_3 - x_1) - 2a_2 x_3 Z_4 \quad (3\text{-}9)$$

where

$$M_{11} = \frac{1}{2}\left(1 + \frac{\sqrt{1 - x_1^2}\sin x_2}{Z_1}\right)$$
$$M_{12} = \frac{1}{2}\left(1 - \frac{\sqrt{1 - x_1^2}\sin x_2}{Z_1}\right)$$
$$M_{21} = \frac{1}{2}\left(1 + \frac{\cos x_2/\sqrt{1 - x_1^2}}{Z_2}\right)$$
$$M_{22} = \frac{1}{2}\left(1 - \frac{\cos x_2/\sqrt{1 - x_1^2}}{Z_2}\right)$$
$$M_{31} = \frac{1}{2}\left(1 + \frac{\sqrt{1 - x_3^2}\sin x_4}{Z_3}\right)$$
$$M_{32} = \frac{1}{2}\left(1 - \frac{\sqrt{1 - x_3^2}\sin x_4}{Z_3}\right)$$
$$M_{41} = \frac{1}{2}\left(1 + \frac{\cos x_4/\sqrt{1 - x_3^2}}{Z_4}\right)$$
$$M_{42} = \frac{1}{2}\left(1 - \frac{\cos x_4/\sqrt{1 - x_3^2}}{Z_4}\right)$$

where M_{11}, M_{12} M_{21}, M_{22} M_{31}, M_{32} M_{41}, and M_{42} are fuzzy sets of the first to fourth equations of (3-1). $Z_1 = 1$, $Z_2 = 2$, $Z_3 = 0.5$, $Z_4 = 1.5$, $M_{11} + M_{12} = 1$, $M_{21} + M_{22} = 1$, $M_{31} + M_{32} = 1$, and $M_{41} + M_{42} = 1$.

Step 2 (F)—First: Here, (3-2), (3-4), (3-6), and (3-8) are chosen as the members of the first linear subsystem under the fuzzy rules. Then, we have the first linear subsystem as

$$\begin{cases} \dot{x}_1 = -2a_1 Z_1 \\ \dot{x}_2 = -w_1(x_1 - x_3) + 2a_1 x_1 Z_2 \\ \dot{x}_3 = -2a_2 Z_3 \\ \dot{x}_4 = -w_2(x_3 - x_1) + 2a_2 x_3 Z_4). \end{cases} \quad (3\text{-}10)$$

Step 3 (S)—Second: As we go on, (3-3), (3-5), (3-7), and (3-9) are chosen as the members of the second linear subsystem under the fuzzy rules as well, therefore, the second linear subsystem can be obtained as follows:

$$\begin{cases} \dot{x}_1 = 2a_1 Z_1 \\ \dot{x}_2 = -w_1(x_1 - x_3) - 2a_1 x_1 Z_2 \\ \dot{x}_3 = 2a_2 Z_3 \\ \dot{x}_4 = -w_2(x_3 - x_1) - 2a_2 x_3 Z_4). \end{cases} \quad (3\text{-}11)$$

The final output of the fuzzy Quantum-CNN system is inferred as follows and the chaotic behavior of fuzzy system is shown in Fig. 2:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} M_{11} & 0 & 0 & 0 \\ 0 & M_{21} & 0 & 0 \\ 0 & 0 & M_{31} & 0 \\ 0 & 0 & 0 & M_{41} \end{bmatrix} \times \begin{bmatrix} -2a_1 Z_1 \\ -w_1(x_1 - x_3) + 2a_1 x_1 Z_2 \\ -2a_2 Z_3 \\ -w_2(x_3 - x_1) + 2a_2 x_3 Z_4 \end{bmatrix}$$

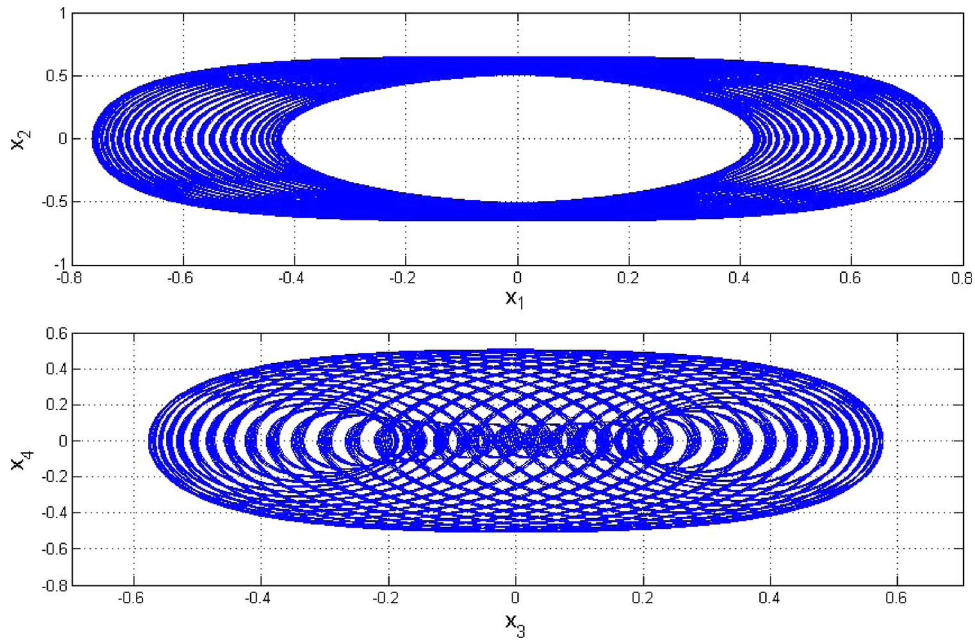

Fig. 2. Chaotic behavior of the new fuzzy Quantum-CNN system.

$$+\begin{bmatrix} M_{12} & 0 & 0 & 0 \\ 0 & M_{22} & 0 & 0 \\ 0 & 0 & M_{32} & 0 \\ 0 & 0 & 0 & M_{42} \end{bmatrix} \times \begin{bmatrix} 2a_1 Z_1 \\ -w_1(x_1 - x_3) - 2a_1 x_1 Z_2 \\ 2a_2 Z_3 \\ -w_2(x_3 - x_1) - 2a_2 x_3 Z_4 \end{bmatrix}. \quad (3\text{-}12)$$

Equation (3-12) can be rewritten as a simple mathematical expression as follows:

$$\dot{X}(t) = \sum_{i=1}^{2} \Psi_i \left(A_i X(t) + \widetilde{b}_i \right) \quad (3\text{-}13)$$

where Ψ_i are diagonal matrices described as follows:

$$\text{dia}(\Psi_1) = [\, M_{11} \quad M_{21} \quad M_{31} \quad M_{41} \,]$$
$$\text{dia}(\Psi_2) = [\, M_{12} \quad M_{22} \quad M_{32} \quad M_{42} \,]$$
$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -w_1 + 2a_1 Z_2 & 0 & w_1 & 0 \\ 0 & 0 & 0 & 0 \\ w_2 & 0 & -w_2 + 2a_2 Z_4 & 0 \end{bmatrix}$$
$$\widetilde{b}_1 = \begin{bmatrix} -2a_1 Z_1 \\ 0 \\ -2a_2 Z_3 \\ 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -w_1 - 2a_1 Z_2 & 0 & w_1 & 0 \\ 0 & 0 & 0 & 0 \\ w_2 & 0 & -w_2 - 2a_2 Z_4 & 0 \end{bmatrix}$$
$$\widetilde{b}_2 = \begin{bmatrix} 2a_1 Z_1 \\ 0 \\ 2a_2 Z_3 \\ 0 \end{bmatrix}.$$

Via using the new fuzzy model, the number of fuzzy rules in the fuzzy Quantum-CNN system can be reduced from 2^4 to 2×4, and only two linear subsystems can express such complex chaotic behaviors. The simulation results are perfectly the same as that of the original chaotic behavior of the Quantum-CNN system.

Model 2—New Fuzzy Model of the Qi System: The four-order autonomous Qi system [41] is given as follows:

$$\begin{cases} \dot{y}_1 = a_3(y_2 - y_1) + y_2 y_3 y_4 \\ \dot{y}_2 = b_3(y_1 + y_2) - y_1 y_3 y_4 \\ \dot{y}_3 = -c_3 y_3 + y_1 y_2 y_4 \\ \dot{y}_4 = -d_3 y_4 + y_1 y_2 y_3 \end{cases} \quad (3\text{-}14)$$

where y_1, y_2, y_3, and y_4 are the state variables of the system, and a_3, b_3, c_3, and d_3 are all positive real parameters. This Qi system exhibits a complex dynamical behavior, including the familiar period-doubling route to chaos as well as hopf bifurcations [34]. For the system parameters, i.e., $a_3 = 35$, $b_3 = 10$, $c_3 = 1$, $d_3 = 10$, and initial conditions $(y_{10}, y_{20}, y_{30}, y_{40}) = (2, 5, 2, 5)$, the Qi model exhibits chaotic motion, as shown in Fig. 3.

A. New Fuzzy Model

Step 1 (T)—Transforming: Assume that

1) $y_3 y_4 \in [-Z_5, Z_5]$ and $Z_5 > 0$;
2) $y_1 y_2 \in [-Z_6, Z_6]$ and $Z_6 > 0$.

Then, we have the following T–S fuzzy rules:

Rule 1 : IF $y_3 y_4$ is N_{11},

$$\text{THEN } \begin{aligned} \dot{y}_1 &= a_3(y_2 - y_1) + (c + Z_5)y_2 \\ \dot{y}_2 &= b_3(y_1 + y_2) - (c + Z_5)y_1 \end{aligned} \quad (3\text{-}15)$$

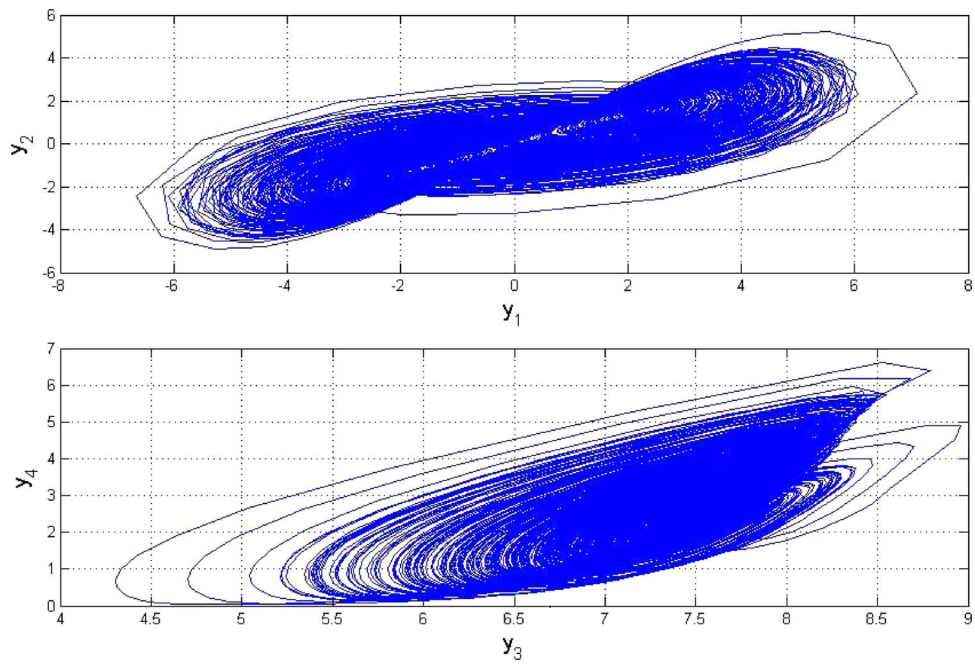

Fig. 3. Chaotic behavior of the Qi system.

Rule 2 : IF y_3y_4 is N_{12},

$$\text{THEN} \begin{array}{l} \dot{y}_1 = a_3(y_2 - y_1) + (c - Z_5)Z_5y_2 \\ \dot{y}_2 = b_3(y_1 + y_2) - (c - Z_5)y_1 \end{array} \tag{3-16}$$

Rule 1 : IF y_1y_2 is N_{21},

$$\text{THEN} \begin{array}{l} \dot{y}_3 = -c_3y_3 + Z_6y_4 \\ \dot{y}_4 = -d_3y_4 + Z_6y_3 \end{array} \tag{3-17}$$

Rule 2 : IF y_1y_2 is N_{22},

$$\text{THEN} \begin{array}{l} \dot{y}_3 = -c_3y_3 - Z_6y_4 \\ \dot{y}_4 = -d_3y_4 - Z_6y_3 \end{array} \tag{3-18}$$

where

$$N_{11} = \frac{1}{2}\left(1 + \frac{y_3y_4 - c}{Z5}\right) \quad N_{12} = \frac{1}{2}\left(1 - \frac{y_3y_4 - c}{Z5}\right)$$

$$N_{21} = \frac{1}{2}\left(1 + \frac{y_1y_2}{Z6}\right) \quad N_{22} = \frac{1}{2}\left(1 - \frac{y_1y_2}{Z6}\right)$$

where N_{11}, N_{12}, N_{21}, and N_{22} are fuzzy sets of the first to fourth equations of (3-14), and $N_{11} + N_{12} = 1$, $N_{21} + N_{22} = 1$, $c = 20$, $Z_5 = 80$, and $Z_6 = 50$.

Step 2 (F)—First: Here, (3-15) and (3-17) are chosen as the members of the first liner subsystem under the fuzzy rules. Then, we have the first linear subsystem as

$$\begin{cases} \dot{y}_1 = a_3(y_2 - y_1) + (c + Z_5)y_2 \\ \dot{y}_2 = b_3(y_1 + y_2) - (c + Z_5)y_1 \\ \dot{y}_3 = -c_3y_3 + Z_6y_4 \\ \dot{y}_4 = -d_3y_4 + Z_6y_3. \end{cases} \tag{3-19}$$

Step 3 (S)—Second: As we go on, (3-16) and (3-18) are chosen as the members of the second linear subsystem under the fuzzy rules as well, therefore, the second linear subsystem can be obtained as follows:

$$\begin{cases} \dot{y}_1 = a_3(y_2 - y_1) + (c - Z_5)y_2 \\ \dot{y}_2 = b_3(y_1 + y_2) - (c - Z_5)y_1 \\ \dot{y}_3 = -c_3y_3 - Z_6y_4 \\ \dot{y}_4 = -d_3y_4 - Z_6y_3. \end{cases} \tag{3-20}$$

The final output of the fuzzy Qi system is inferred as follows, and the chaotic behavior of fuzzy system is shown in Fig. 4:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} = \begin{bmatrix} N_{11} & 0 & 0 & 0 \\ 0 & N_{11} & 0 & 0 \\ 0 & 0 & N_{21} & 0 \\ 0 & 0 & 0 & N_{21} \end{bmatrix}^T \times \begin{bmatrix} a_3(y_2 - y_1) + (c + Z_5)y_2 \\ b_3(y_1 + y_2) - (c + Z_5)y_1 \\ -c_3y_3 + Z_6y_4 \\ -d_3y_4 + Z_6y_3 \end{bmatrix} + \begin{bmatrix} N_{12} & 0 & 0 & 0 \\ 0 & N_{12} & 0 & 0 \\ 0 & 0 & N_{22} & 0 \\ 0 & 0 & 0 & N_{22} \end{bmatrix}^T \times \begin{bmatrix} a_3(y_2 - y_1) + (c - Z_5)Z_5y_2 \\ b_3(y_1 + y_2) - (c - Z_5)y_1 \\ -c_3y_3 - Z_6y_4 \\ -d_3y_4 - Z_6y_3 \end{bmatrix}. \tag{3-21}$$

Equation (3-21) can be rewritten as a simple mathematical expression as follows:

$$\dot{Y}(t) = \sum_{i=1}^{2} \Gamma_i \left(C_iY(t) + \widetilde{c}_i\right) \tag{3-22}$$

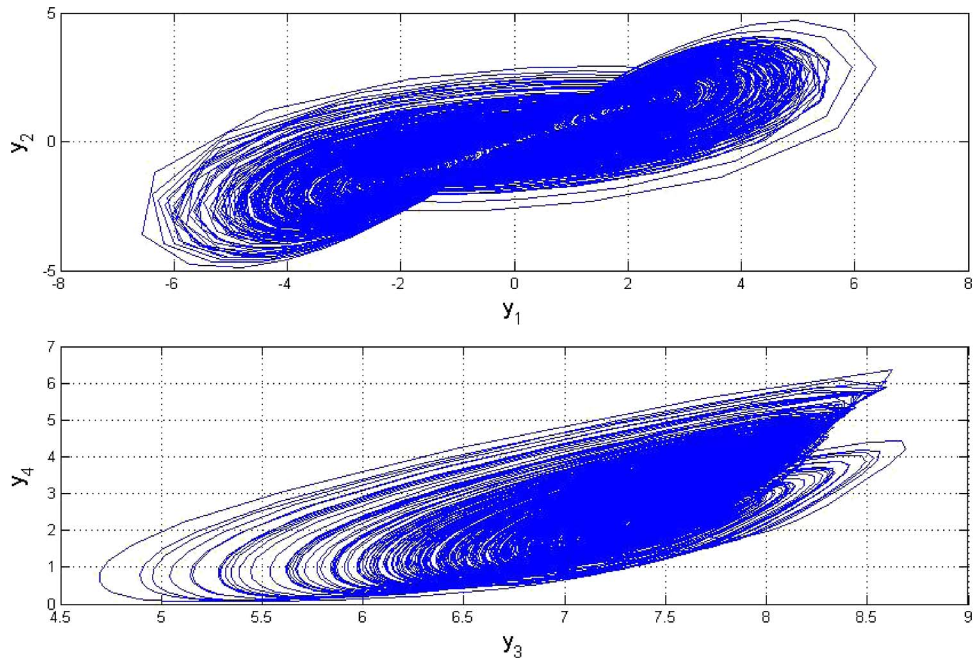

Fig. 4. Chaotic behavior of the new fuzzy Qi system.

where

$$\mathrm{dia}(\Gamma_1) = [\,N_{11} \quad N_{11} \quad N_{21} \quad N_{21}\,]$$
$$\mathrm{dia}(\Gamma_2) = [\,N_{12} \quad N_{12} \quad N_{22} \quad N_{22}\,]$$
$$C_1 = \begin{bmatrix} -a_3 & c+Z_5+a_3 & 0 & 0 \\ -c-Z_5+b_3 & b_3 & 0 & 0 \\ 0 & 0 & -c_3 & Z_6 \\ 0 & 0 & Z_6 & -d_3 \end{bmatrix}$$
$$\widetilde{c}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$C_2 = \begin{bmatrix} -a_3 & c-Z_5+a_3 & 0 & 0 \\ -c+Z_5+b_3 & b_3 & 0 & 0 \\ 0 & 0 & -c_3 & -Z_6 \\ 0 & 0 & -Z_6 & -d_3 \end{bmatrix}$$
$$\widetilde{c}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Via using the new fuzzy model, two linear subsystems are enough to express such complex chaotic behaviors. The simulation results are perfectly the same as that of the original chaotic behavior of the Qi system.

IV. Fuzzy Synchronization Scheme

In this section, we are going to derive the new fuzzy synchronization scheme based on our new fuzzy model of synchronizing two totally different fuzzy chaotic systems. Given the following fuzzy systems as the master and slave systems: master system

$$\dot{X}(t) = \sum_{i=1}^{2} \Psi_i \left(A_i X(t) + \widetilde{b}_i \right) \tag{4-1}$$

slave system

$$\dot{Y}(t) = \sum_{i=1}^{2} \Gamma_i \left(C_i Y(t) + \widetilde{c}_i \right) + BU(t). \tag{4-2}$$

Equations (4-1) and (4-2) represent the two different chaotic systems, and (4-2) has control input $U(t)$. Define the error signal as $e(t) = X(t) - Y(t)$. Then, we have

$$\begin{aligned} \dot{e}(t) &= \dot{X}(t) - \dot{Y}(t) \\ &= \sum_{i=1}^{2} \Psi_i \left(A_i X(t) + \widetilde{b}_i \right) - \sum_{i=1}^{2} \Gamma_i \left(C_i Y(t) + \widetilde{c}_i \right) - BU(t). \end{aligned} \tag{4-3}$$

The fuzzy controllers are designed as follows:

$$U(t) = u_1(t) + u_2(t) \tag{4-4}$$

where

$$u_1(t) = \sum_{i=1}^{2} \Psi_i F_i X(t) - \sum_{i=1}^{2} \Gamma_i P_i Y(t)$$
$$u_2(t) = \sum_{i=1}^{2} \Psi_i \widetilde{b}_i - \sum_{i=1}^{2} \Gamma_i \widetilde{c}_i$$

such that $\|e(t)\| \to 0$ as $t \to \infty$. The design is to determine the feedback gains F_i and P_i. By substituting $U(t)$ into (4-3), we obtain

$$\dot{e}(t)=\sum_{i=1}^{2}\Psi_i\left\{(A_i-BF_i)X(t)\right\}-\sum_{i=1}^{2}\Gamma_i\left\{(C_i-BP_i)Y(t)\right\}. \tag{4-5}$$

Theorem: The zero solution of error system in (4-5) is asymptotically stable, and the slave system in (4-2) can synchronize the master system in (4-1) under the fuzzy controller in (4-4), if the following conditions can be satisfied:

$$G=(A_1-BF_1)=(A_i-F_i)=(C_i-BP_i)<0, \qquad i=1,2. \tag{4-6}$$

Proof: The errors in (4-5) can be exactly linearized via the fuzzy controllers in (4-4) if there exist feedback gains F_i and P_i such that

$$(A_1-BF_1)=(A_2-BF_2)=(C_1-BP_1)=(C_2-BP_2)<0. \tag{4-7}$$

Then, the overall control system is linearized as

$$\dot{e}(t)=Ge(t) \tag{4-8}$$

where $G=(A_1-BF_1)=(A_2-BF_2)=(C_1-BP_1)=(C_2-BP_2)<0$.

Consequently, the zero solution of error system (4-8) linearized via the fuzzy controller (4-4) is asymptotically stable.■

V. Simulation Results

There are two examples in this section to investigate the effectiveness and feasibility of our new fuzzy model. In example 1, the fuzzy Quantum-CNN system is chosen as the master system to be synchronized by the fuzzy Qi system. In example 2, to show the potentiality of our new model, we further use the T–S fuzzy Lorenz system to be the master system. The new fuzzy Qi system in this case is still considered as the slave system to synchronize the T–S fuzzy Lorenz system.

Example 1—Synchronization of the Quantum-CNN System and Qi System: The fuzzy Quantum-CNN system in (3-12) is chosen as the master system, and the fuzzy slave system, i.e., Qi system, with fuzzy controllers is given as follows:

$$\dot{Y}(t)=\sum_{i=1}^{2}\Gamma_iC_iY(t)+BU(t) \tag{5-1}$$

where Γ_i's are diagonal matrices, i.e.,

$$\begin{aligned}\text{dia}(\Gamma_1)&=[N_{11}\quad N_{11}\quad N_{21}\quad N_{21}]\\ \text{dia}(\Gamma_2)&=[N_{12}\quad N_{12}\quad N_{22}\quad N_{22}]\end{aligned}$$

$$C_1=\begin{bmatrix}-a_3 & c+Z_5+a_3 & 0 & 0\\ -c-Z_5+b_3 & b_3 & 0 & 0\\ 0 & 0 & -c_3 & Z_6\\ 0 & 0 & Z_6 & -d_3\end{bmatrix}$$

$$C_2=\begin{bmatrix}-a_3 & c-Z_5+a_3 & 0 & 0\\ -c+Z_5+b_3 & b_3 & 0 & 0\\ 0 & 0 & -c_3 & -Z_6\\ 0 & 0 & -Z_6 & -d_3\end{bmatrix}.$$

Therefore, the error and error dynamics are

$$\mathbf{e}=X(t)-Y(t) \tag{5-2}$$

$$\begin{aligned}\mathbf{e}&=\dot{X}(t)-\dot{Y}(t)\\ &=\sum_{i=1}^{2}\Psi_i\left(A_iX(t)+\widetilde{b}_i\right)-\sum_{i=1}^{2}\Gamma_iC_iY(t)-BU(t)\end{aligned} \tag{5-3}$$

where $\mathbf{e}=[e_1,e_2,e_3,e_4]$, $X(t)=[x_1,x_2,x_3,x_4]$, and $Y(t)=[y_1,y_2,y_3,y_4]$. B is chosen as the identity matrix, and the fuzzy controllers in (4-3) are used. Then, according to the theorem, we have $G=[A_1-BF_1]=[A_2-BF_2]=[C_1-BF_1]=[C_2-BF_2]<0$. G is chosen as

$$G=\begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{bmatrix}. \tag{5-4}$$

Thus, the feedback gains F_1, F_2, P_1, and P_2 can be determined by the following equations:

$$F_1=B^{-1}[A_1-G]=\begin{bmatrix}1.0000 & 0 & 0 & 0\\ 22.500 & 1.0000 & 4.7000 & 0\\ 0 & 0 & 1.0000 & 0\\ 3.9000 & 0 & 9.0000 & 1.0000\end{bmatrix}$$

$$F_2=B^{-1}[A_2-G]=\begin{bmatrix}1.0000 & 0 & 0 & 0\\ -31.90 & 1.0000 & 4.7000 & 0\\ 0 & 0 & 1.0000 & 0\\ 3.9 & 0 & -16.80 & 1.0000\end{bmatrix}$$

$$P_1=B^{-1}[C_1-G]=\begin{bmatrix}-34 & 135 & 0 & 0\\ -90 & 11 & 0 & 0\\ 0 & 0 & 0 & 50\\ 0 & 0 & 50 & -9\end{bmatrix}$$

$$P_2=B^{-1}[C_2-G]=\begin{bmatrix}-34 & 25 & 0 & 0\\ 70 & 11 & 0 & 0\\ 0 & 0 & 0 & -50\\ 0 & 0 & -50 & -9\end{bmatrix}. \tag{5-5}$$

The synchronization errors are shown in Fig. 5. It is clear that the zero solution of the error system is asymptotically stable.

Example 2—Synchronization of the Lorenz System and Qi System: In this case, the fuzzy Qi system in (5-1) is still chosen as the slave system, and we further illustrate the T–S fuzzy model of the Lorenz system to show the potentiality of our new model. The four-order Lorenz system can be described as follows:

$$\begin{cases}\dot{x}_1=a(x_2-x_1)\\ \dot{x}_2=\mathrm{cx}_1-x_1x_3-x_2\\ \dot{x}_3=x_1x_2-\mathrm{bx}_3\\ \dot{x}_4=0\end{cases} \tag{5-6}$$

where a, b, and c are the parameters. When $a=10$, $b=8/3$, $c=28$, and initial state is (0.5, 1, 5, 0), the dynamic behavior is chaotic. Assume that $x_1\in[-Z_7,Z_7]$ and $Z_7>0$. Then, the

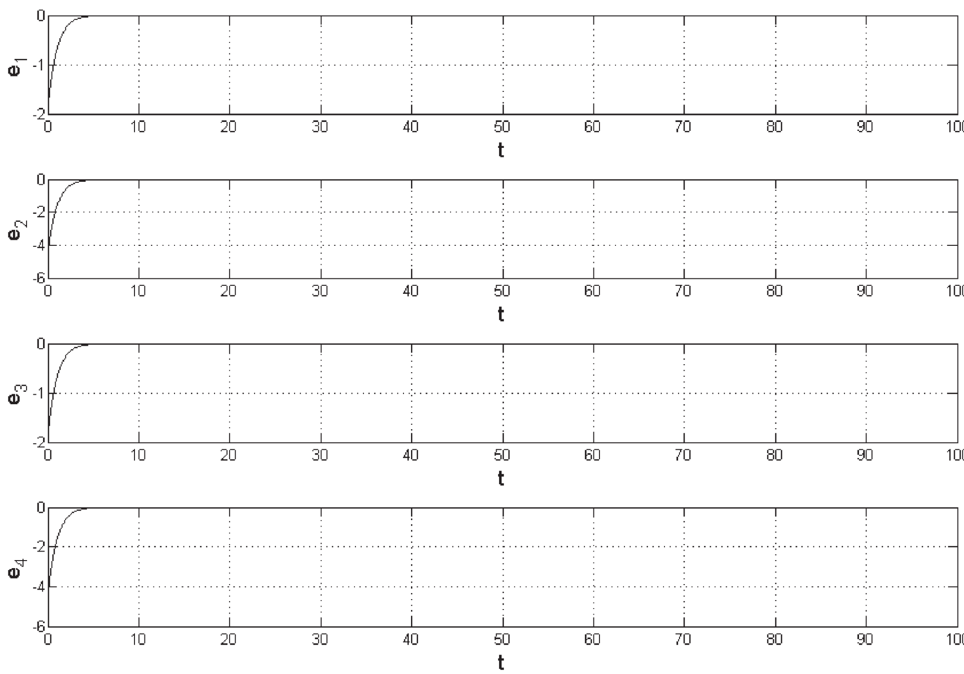

Fig. 5. Time histories of errors for case I.

Lorenz system can be exactly represented by the T–S fuzzy model as follows:

$$\text{Rule 1 : IF } x \text{ is } Q_1, \qquad \text{THEN } \dot{X}(t) = A_1 X(t), \qquad (5\text{-}7)$$
$$\text{Rule 2 : IF } x \text{ is } Q_2, \qquad \text{THEN } \dot{X}(t) = A_2 X(t) \qquad (5\text{-}8)$$

where

$$X = [x_1, x_2, x_3, x_4]^T$$
$$A_1 = \begin{bmatrix} -a & a & 0 & 0 \\ c & -1 & -Z_7 & 0 \\ 0 & Z_7 & b & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -a & a & 0 & 0 \\ c & -1 & Z_7 & 0 \\ 0 & -Z_7 & b & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$Q_1(x) = \frac{1}{2}\left(1 + \frac{x_1}{Z_7}\right)$$
$$Q_2(x) = \frac{1}{2}\left(1 - \frac{x_1}{Z_7}\right)$$

and $Z_7 = 20$. M_1 and M_2 are fuzzy sets of Lorenz system. Here, we call (3-2) the first linear subsystem under the fuzzy rule and (3-3) the second linear subsystem under the fuzzy rule. The final output of the fuzzy Lorenz system is inferred as follows, and the chaotic behavior is shown in Fig. 6:

$$\dot{X}(t) = \sum_{i=1}^{2} \Theta_i A_i X(t) \qquad (5\text{-}9)$$

where

$$\Theta_1 = \frac{Q_1}{Q_1 + Q_2} \quad \Theta_2 = \frac{Q_2}{Q_1 + Q_2}.$$

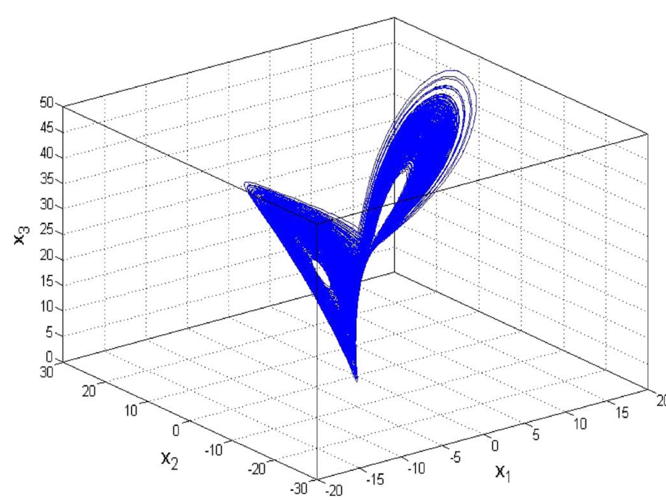

Fig. 6. Chaotic behavior of the drive fuzzy Lorenz system.

The error and error dynamics are

$$\mathbf{e} = X(t) - Y(t) \qquad (5\text{-}10)$$
$$\mathbf{e} = \dot{X}(t) - \dot{Y}(t)$$
$$= \sum_{i=1}^{2} Q_i A_i X(t) - \sum_{i=1}^{2} \Gamma_i C_i Y(t) - BU(t) \qquad (5\text{-}11)$$

where $\mathbf{e} = [e_1, e_2, e_3, e_4]$, $X(t) = [x_1, x_2, x_3, x_4]$, and $Y(t) = [y_1, y_2, y_3, y_4]$. B is chosen as identity matrix, and the fuzzy controllers in (4-3) are used. Then, according to the theorem, we have $G = [A_1 - BF_1] = [A_2 - BF_2] = [C_1 - BF_1] = [C_2 - BF_2] < 0$. G is chosen as

$$G = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \qquad (5\text{-}12)$$

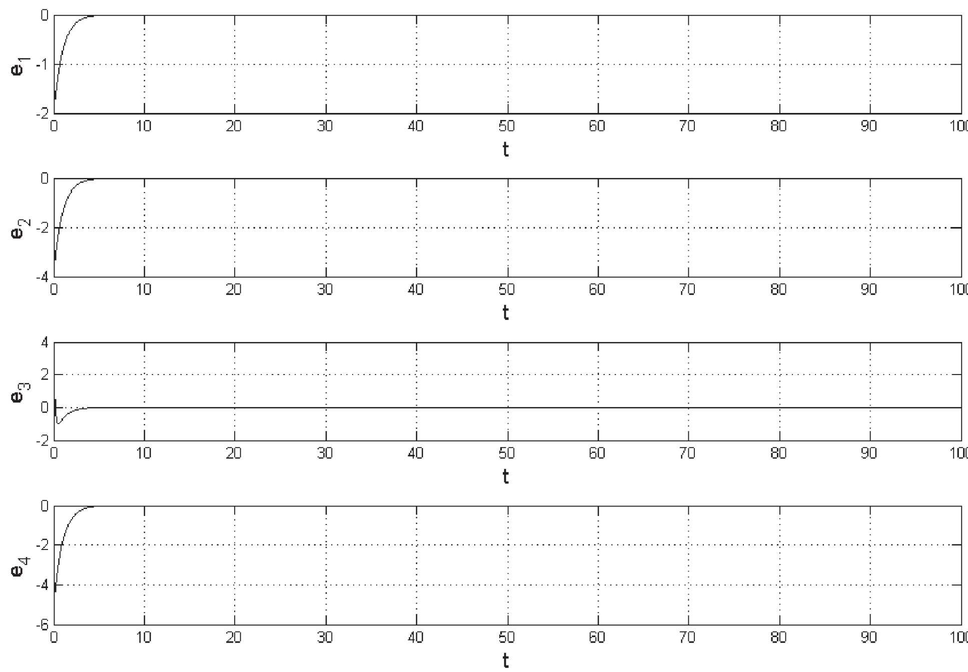

Fig. 7. Time histories of errors for case I I.

Thus, the feedback gains F_1, F_2, P_1, and P_2 can be determined by the following equations:

$$F_1 = B^{-1}[A_1 - G] = \begin{bmatrix} -9 & 10 & 0 & 0 \\ 28 & 0 & -20 & 0 \\ 0 & 20 & 11/3 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}$$

$$F_1 = B^{-1}[A_1 - G] = \begin{bmatrix} -9 & 10 & 0 & 0 \\ 28 & 0 & 20 & 0 \\ 0 & -20 & 11/3 & 0 \\ 0 & 0 & 0 & 1.0 \end{bmatrix}$$

$$P_1 = B^{-1}[C_1 - G] = \begin{bmatrix} -34 & 135 & 0 & 0 \\ -90 & 11 & 0 & 0 \\ 0 & 0 & 0 & 50 \\ 0 & 0 & 50 & -9 \end{bmatrix}$$

$$P_2 = B^{-1}[C_2 - G] = \begin{bmatrix} -34 & 25 & 0 & 0 \\ 70 & 11 & 0 & 0 \\ 0 & 0 & 0 & -50 \\ 0 & 0 & -50 & -9 \end{bmatrix}. \quad (5\text{-}13)$$

The synchronization errors are shown in Fig. 7. It is clear that the zero solution of the error system is asymptotically stable.

VI. Discussion

In this section, we further discuss and demonstrate three main contributions of our new model via comparing with the classical T–S fuzzy model. First, the T–S fuzzy model is given for modeling two complicated chaotic systems in (3-1) and (3-14) by the heavy and inefficient modeling process. Finally, fuzzy synchronization via the new and old models is also compared.

1) Only two linear subsystems are needed, and 2) the numbers of fuzzy rules can be reduced from 2^N to $2 \times N$ (where N is the number of nonlinear terms). In this case, the T–S fuzzy model is used for representing local linear models of the Quantum-CNN nanosystem in (3-1) and the Qi system in (3-14).

For the T–S fuzzy Quantum-CNN system: The system (3-1) can be exactly represented in a region of interest as follows:

$$\text{Rule } i: \text{IF } \sqrt{1-x_1^2}\sin x_2 \text{ is } M_{i1} \text{ and } \cos x_2 \Big/ \sqrt{1-x_1^2} \text{ is } M_{i2}$$
$$\text{and } \sqrt{1-x_3^2}\sin x_4 \text{ is } M_{i3} \text{ and } \cos x_4 \Big/ \sqrt{1-x_3^2} \text{ is } M_{i4}$$
$$\text{THEN } \dot{X}(t) = A_i X(t) + \widetilde{b}_i, \qquad i = 1, 2, \ldots, 16 \quad (6\text{-}1)$$

and via using the fuzzifier, the final output of the fuzzy system is equivalently inferred as

$$\dot{X}(t) = \sum_{i=1}^{16} \Psi_i \left(A_i X(t) + \widetilde{b}_i \right),$$

$$\text{where } \Psi_i = \frac{w_i}{\sum_{j=1}^{16} w_i} \text{ and } w_i = \prod_{j=1}^{4} M_{ij}. \quad (6\text{-}2)$$

Ψ_i can be regarded as the normalized weight of the IF–THEN rules and satisfies $0 \le \Psi_i \le 1$ and $\sum_{i=1}^{16} \Psi_i = 1$. The simulation results in MATLAB are shown in Fig. 8. Comparing (3-13) with (6-2), which are listed in Table I, there are 16 fuzzy rules, 16 linear subsystems, and 64 equations in (6-2); however, there are only two linear subsystems, eight fuzzy rules, and eight equations in (3-1). It is clear that the numbers of fuzzy rules are hugely reduced from 2^N to $2 \times N$ ($N = 4$ in this case), and the numbers of linear subsystems and equations in the

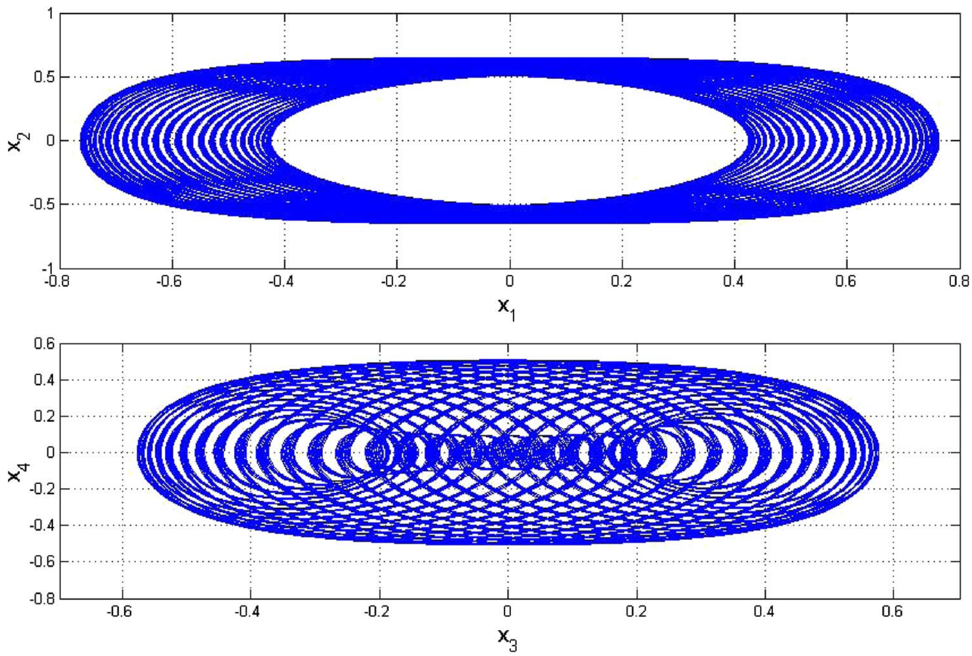

Fig. 8. Chaotic behavior of the T–S fuzzy Quantum-CNN system.

TABLE I
COMPARING FUZZY MODELING OF THE QUANTUM-CNN SYSTEM VIA THE NEW AND THE T–S FUZZY MODELS

Comparing terms	New	T-S fuzzy model
Numbers of fuzzy rules	8	16
Numbers of linear subsystems	2	16
Numbers of linear equations	8	64

fuzzy system are also reduced form 16 and 64 to 2 and 8. Above all, the simulation results are great as well as the old one.

For the T–S fuzzy Qi system: The system (3-14) can be exactly represented in a region of interest as follows:

Rule i : IF y_3y_4 is N_{i1} and y_1y_2 is N_{i2}
THEN $\dot{Y}(t) = C_iY(t) + \widetilde{c}_i, \qquad i = 1, 2, \ldots, 4$ (6-3)

and via using the fuzzifier, the final output of the fuzzy system is equivalently inferred as

$$\dot{Y}(t) = \sum_{i=1}^{4} \Gamma_i \left(C_iY(t) + \widetilde{c}_i\right)$$

$$\text{where } \Gamma_i = \frac{w_i}{\sum_{j=1}^{4} w_i} \text{ and } w_i = \prod_{j=1}^{2} N_{ij}. \quad (6\text{-}4)$$

Γ_i can be regarded as the normalized weight of the IF–THEN rules and satisfies $0 \leq \Gamma_i \leq 1$ and $\sum_{i=1}^{4} \Gamma_i = 1$. The simulation results in MATLAB are shown in Fig. 9, and the comparison of the new and the T–S fuzzy models is also given in Table II.

2) Fuzzy synchronization of two different chaotic systems with different numbers of nonlinear terms can be achieved with only two sets of gain K. Via the T–S fuzzy model, different chaotic systems may be transformed into different fuzzy chaotic systems with different numbers of subsystems. However, when it comes to synchronization, employing the idea of PDC to design the fuzzy control law for stabilization of the error dynamics is an inefficient and complicated work. This is due to the mismatch of the number of subsystems. For example, if the T–S fuzzy model of Quantum-CNN and Qi systems in (6-2) and (6-4) are used in example 1 illustrated in Section V, the error and error dynamics would be

$$\mathbf{e} = X(t) - Y(t) \quad (6\text{-}5)$$

$$\begin{aligned}\mathbf{e} &= \dot{X}(t) - \dot{Y}(t) \\ &= \sum_{i=1}^{16} \Psi_i \left(A_iX(t) + \widetilde{b}_i\right) - \sum_{j=1}^{4} \Gamma_j \left(C_jY(t) + \widetilde{c}_j\right) - BU(t)\end{aligned} \quad (6\text{-}6)$$

where $\mathbf{e} = [e_1, e_2, e_3, e_4]$, $X(t) = [x_1, x_2, x_3, x_4]$, and $Y(t) = [y_1, y_2, y_3, y_4]$. B is chosen as the identity matrix, and the fuzzy controllers in (4-3) are used. Then, according to the theorem, we have

$$G = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = [A_i - BF_i] = [C_j - BP_j] < 0$$

$$i = 1, 2, \ldots, 16; \qquad j = 1, 2, \ldots, 4.$$

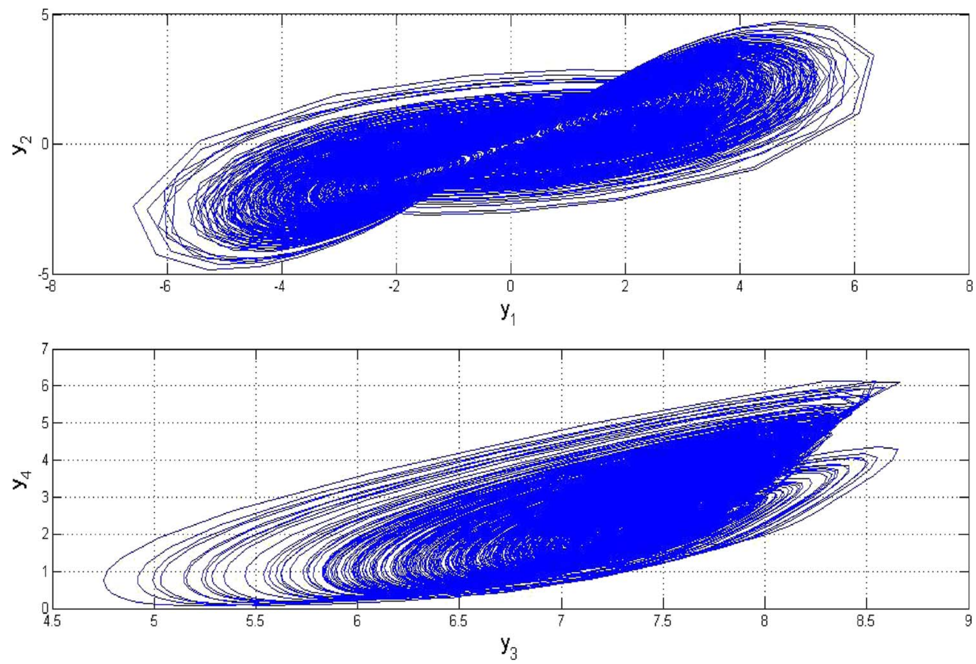

Fig. 9. Chaotic behavior of the T–S fuzzy Qi system.

TABLE II
COMPARING FUZZY MODELING OF THE QI SYSTEM VIA THE NEW AND THE T–S FUZZY MODELS

Comparing terms	New	T-S fuzzy model
Numbers of fuzzy rules	4	4
Numbers of linear subsystems	2	4
Numbers of linear equations	8	16

Twenty sets of gain K are made up to achieve the fuzzy synchronization of the two fuzzy systems. However, through our new fuzzy model, only two sets of gain K are needed to achieve the goal of fuzzy synchronization.

VII. CONCLUSION

In this paper, two different and complicated chaotic systems, namely, Quantum-CNN system and Qi system, have been successfully and efficiently simulated and synchronized via our new fuzzy model. Through the new idea, not only a complicated nonlinear system can be linearized to a simple form—linear coupling of only two linear subsystems and the numbers of fuzzy rules can be reduced from 2^N to $2 \times N$, but also a new theorem of PDC and LMI-based method can be applied to synchronize two different fuzzy systems, where only two sets of gain K are needed. We give a new field of sight on fuzzy modeling of a complicated nonlinear system and a simpler and more efficient path to achieve synchronization of the two different systems.

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