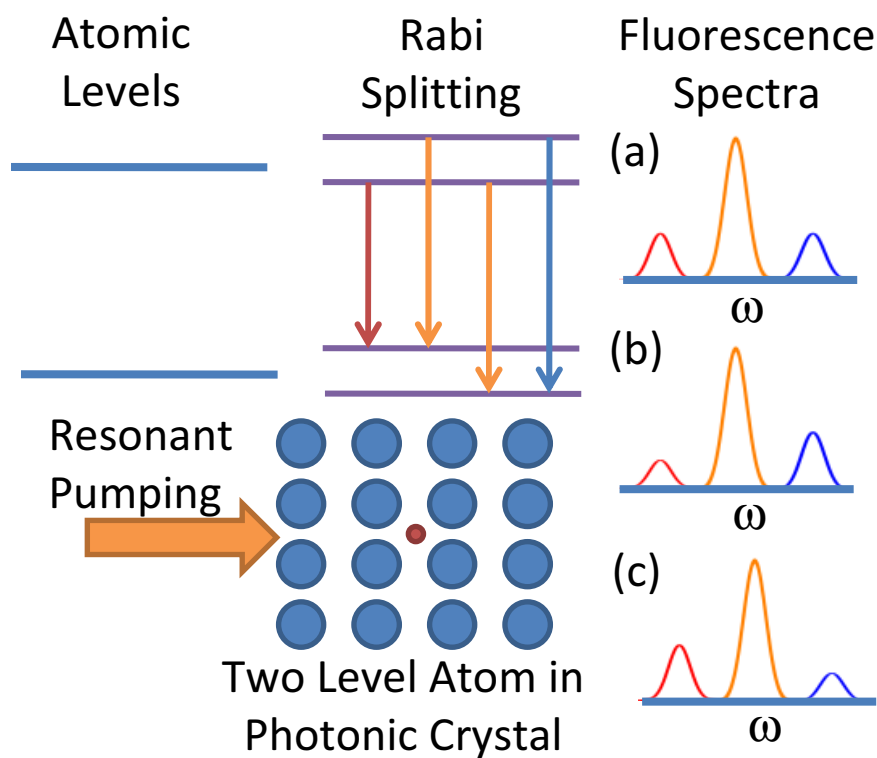


## EXPERT OPINION

## Resonance fluorescence spectra near a photonic bandgap

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Optical radiation properties of a single atom have been among the most popular research problems in quantum optics since the early years. In contrast to classical intuition, the quantum theory asserts that the optical radiation properties are not wholly determined by the intrinsic characteristics of the atom but also depend on the electromagnetic mode structure seen by the atom even when these modes are empty (i.e., vacuum states). The emission frequency will be slightly modified through the interaction with these vacuum states (the Lamb shift), and the spontaneous emission rate can be enhanced when the atom is placed inside a resonant cavity (the Purcell effect). These are the two well-known examples dated back to the 1940s. Another fundamental example is the resonance fluorescence from a two-level atom. If the two-level atom is excited on resonance by an intense optical field, the fluorescence spectrum can exhibit three peaks (the Mollow triplet), which was theoretically predicted in 1969 [1] and experimentally observed in 1975 by using the sodium atoms [2]. The coupling of the optical pump field with the two atomic levels produces Rabi oscillation in the time domain or equivalently the energy level splitting in the frequency domain. From such a dressed atom picture, it is easy to understand that the down transition from the upper two sub-levels to the lower two sub-levels will exhibit three transmission frequencies since the frequency splitting of



**Figure 1** Mollow triplet from resonance fluorescence. The Rabi splitting caused by light-atom coupling explains the appearance of three transmission peaks. In free space, the Mollow triplet is symmetric as

the two levels are semi-classically the same. This has been illustrated in Fig. 1. Determination of the fluorescence spectral shape requires more theoretical efforts. Similar to the Purcell effect case, the atom is coupled to a reservoir of photon vacuum modes besides the pumping field. The atomic oscillation is damped by such reservoir-coupling

in (a). Theories have predicted that the Mollow triplet will become asymmetric when the two-level atom is placed inside a photonic crystal and is near a photonic bandgap. They are illustrated in (b) or (c).

and the three transmission lines are thus broadened to be with certain line-widths due to the entering reservoir noises.

Interestingly, analytic formula for the resonance fluorescence spectral shape can be elegantly derived for the free space case. At first sight, this is an amazing result since the problem is actually a nonlinear one.

The starting quantum operator equations for the atom-light coupling are intrinsically nonlinear, not to mention that the reservoir light modes are also continuously distributed. One can readily eliminate the reservoir modes to obtain the evolution equations (Bloch equations) for the atomic quantum operators ( $\sigma_+$ ,  $\sigma_-$ ,  $\sigma_z$ ). Here the Pauli spin operators have been used for describing the two-level atom. In these equations, the reservoir-coupling contributes terms like

$$\int_0^t G(t-t')\sigma_a(t)\sigma_b(t')dt'$$

where the function  $G(t)$  is the response function of the reservoir-coupling. If the reservoir is broadband enough such that the flat-band model can be used,  $G(t)$  can be assumed to be a  $\delta$ -function and the nonlinear reservoir-coupling terms become equal time products of the Pauli operators, which can be reduced to linear terms by utilizing the unique properties of the Pauli operators. Such a linearization procedure is the key for obtaining analytic formula. The linearized Bloch equations can then be solved by the Fourier transform directly. Otherwise the quantum regression theorem can be utilized to obtain the evolution equations for the correlation products and the optical radiation spectra can then be calculated from the solution of these linear equations [3]. If the atom is in a photonic crystal structure and near the bandgap, the strongly varying density of the photonic states will invalid the  $\delta$ -function assumption and calls for more careful theoretical treatment. One can do a better job by approximating  $\sigma_a(t)$  in terms of the operators at  $t'$  so that the equations can still be linearized with the presence of the response functions.

In this way, the non-Markovian effects caused by the colored reservoir can be partially taken into account, which hopefully can give us reasonable results.

The recent paper by Boedecker and Henkel [4] has carefully carried out the scheme outlined above based on the quantum regression theorem as well as the fluctuation dissipation theorem. In their detailed derivation, the approximation of  $\sigma_a(t)$  in terms of the operators at  $t'$  has been carried out up to the 1-st order expansion. Resonance fluorescence spectra near a photonic bandgap are calculated under both the 0-th and 1-st order approximations respectively for the comparison. Mollow triplets with unequal spectral side-peaks are theoretically predicted, as can be intuitively expected from the unequal spectral distribution of the light states. However, somewhat to my surprise, they show that the 1-st order approximation can lead to qualitative differences in the symmetry of the spectrum when compared to the 0-th order approximation, which is schematically illustrated in Fig. 1 (b) and (c). These are certainly very interesting new results, which also indicate that one should be very careful to justify the introduced approximations for studying this subtle problem.

In the literature, there have been some theoretical studies that tried to solve this non-Markovian problem by more direct approaches including the direct numerical simulation method [5]. Mollow triplets with unequal spectral side-peaks have also been obtained. Although careful comparison among these approaches seems to be not available at the current stage, it may be required eventually in order to better determine the accuracy for all the obtained theoretical results.

The squeezing spectra [6] as well as the intensity correlation properties [7] may also be worthy for further studies since in principle they can be measured experimentally and may exhibit more unique characteristics. On the experimental side, recently there have been important progresses on observing the Mollow triplet in single molecules [8] as well as in semiconductor quantum dots [9]. These experimental successes should have brought closer the possibility of performing actual experiments for observing the resonance fluorescence spectra near a photonic bandgap. Hopefully experimental verification of the current theoretical predictions can be expected in the near future.

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