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Fuzzy adaptive synchronization of time-reversed chaotic systems via a new adaptive control strategy

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ABSTRACT

A novel adaptive control strategy is proposed herein to increase the efficiency of adaptive control by combining Takagi–Sugeno (T–S) fuzzy modeling and the Ge–Yao–Chen (GYC) partial region stability theory. This approach provides two major contributions: (1) increased synchronization efficiency, especially for parameters tracing and (2) a simpler controller design. Two simulated cases are presented for comparison: Case 1 utilizes normal adaptive synchronization, whereas Case 2 utilizes the Takagi–Sugeno (T–S) fuzzy model-based Lorenz systems to realize adaptive synchronization via the new adaptive scheme. The simulation results demonstrate the effectiveness and feasibility of our new adaptive strategy.

1. Introduction

Synchronization in chaotic dynamic systems has recently received a great deal of interest among scientists from various fields [1,22,28,29,34,37]. The phenomenon of synchronization of two chaotic systems is fundamental in science and has a wealth of applications in technology. Over the last several years, an increased number of applications of chaos synchronization have been proposed. There are many control techniques for synchronizing chaotic systems, such as linear error feedback control [16,30,39,40], impulsive control [6,17,41], backstepping control [19,31–33] and sliding mode control [4,7,20].

To the best of our knowledge, most of the methods mentioned above and many other existing synchronization methods mainly address the synchronization of two identical chaotic or hyperchaotic systems. The methods for synchronizing two different chaotic or hyperchaotic systems are far from straightforward because of the different structures and the parameter mismatch. Moreover, most of these methods are used to synchronize two systems with known structures and parameters. However, in practical situations, some or all of the system parameters are unknown. In recent years, an increasing number of applications for secure communication require the synchronization of two different hyperchaotic systems with uncertain parameters [5,23,35,38]. Thus, the synchronization of two different hyper-chaotic systems with uncertain parameters has been a subject of intense study.

For current adaptive synchronization, the traditional Lyapunov stability theorem and Barbalat lemma are used to prove that the error vector approaches zero as time approaches infinity; however, why these estimated parameters approach these

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uncertain values remains an open question [2,14,15,36,42]. In [9,10], Ge, Yu and Chen proposed the pragmatical asymptotically stability theorem and an assumption of equal probability for ergodic initial conditions to strictly prove that these estimated parameters approach uncertain values.

Fuzzy logic [3,21,43–46] has received much attention from control theorists as a powerful tool for nonlinear control. Among the various types of fuzzy methods, the Takagi–Sugeno fuzzy system is widely used as a tool for the design and analysis of fuzzy control systems [8,12,25,26,47]. Thus, in this paper, we use this powerful tool in our new strategy: fuzzy modeling and a new adaptive scheme. Adaptive synchronization using this approach has four advantages: (1) the new Lyapunov function is a simple linear homogeneous state function; (2) lower-order, linear and simple controllers can be obtained; (3) fewer simulation errors occur; and (4) adaptive synchronization is achieved in much less time.

The layout of the rest of this manuscript is as follows. In Section 2, a new adaptive synchronization scheme is presented. In Section 3, the time-reversed Lorenz system is introduced. In Sections 4 and 5, two simulation cases are provided for comparison and discussion. Conclusions are provided in Section 6.

2. New adaptive synchronization scheme

There are two identical nonlinear dynamical systems: the master system controls the slave system. The master system is described as

$$\dot{x} = Ax + f(x, B) \tag{2-1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ denotes a state vector, A is an $n \times n$ uncertain constant coefficient matrix, f is a nonlinear vector function, and B is a vector of uncertain constant coefficients in f.

The slave system is described as

$$\dot{y} = \widehat{A}y + f(y, \widehat{B}) + u(t) \tag{2-2}$$

where $y = [y_1, y_2, \ldots, y_n]^T \in R^n$ denotes a state vector, $\widehat{A}$ is an $n \times n$ estimated coefficient matrix, $\widehat{B}$ is a vector of estimated coefficients in f, and $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T \in R^n$ is a control input vector.

Our goal is to design a controller $u(t)$ so that the state vector of the chaotic system (2-1) asymptotically approaches the state vector of the master system (2-2).

The chaos synchronization can be accomplished if the limit of the error vector $e(t) = [e_1, e_2, \ldots, e_n]^T$ approaches zero:

$$\lim_{t\to\infty} e = 0 \tag{2-3}$$

where

$$e = x - y + K \tag{2-4}$$

where K is a positive constant, in which the error dynamics occur in the first quadrant of state space of e [9,10].

From Eq. (2-4), we have

$$\dot{e} = \dot{x} - \dot{y} \tag{2-5}$$

$$\dot{e} = Ax - \widehat{A}y + f(x, B) - f(y, \widehat{B}) - u(t) \tag{2-6}$$

A Lyapnuov function $V(e, \widetilde{A}, \widetilde{B})$ is chosen as a positive definite function in the first quadrant of the state space of e, $\widetilde{A}$ and $\widetilde{B}$ [9,10].

We have

$$\dot{V}(e, \widetilde{A}, \widetilde{B}) = e + \widetilde{A} + \widetilde{B} \tag{2-7}$$

where $\widetilde{A} = A - \widehat{A}$, $\widetilde{B} = B - \widehat{B}$. $\widetilde{A}$ and $\widetilde{B}$ are column matrices with elements that include all the elements of matrices $\widehat{A}$ and $\widehat{B}$, respectively.

The derivatives for any solution of the differential equation system consisting of Eq. (2-6) and the update parameter differential equations for $\widetilde{A}$ and $\widetilde{B}$ are

$$\dot{V}(e, \widetilde{A}, \widetilde{B}) = [Ax - \widehat{A}y + Bf(x) - \widehat{B}f(y) - u(t)] + \dot{\widetilde{A}} + \dot{\widetilde{B}} \tag{2-8}$$

where $u(t)$, $\dot{\widetilde{A}}$, and $\dot{\widetilde{B}}$ are chosen so that $\dot{V} = Ce$, C is a diagonal negative definite matrix, and $\dot{V}$ is a negative semi-definite function of e with parameter differences $\widetilde{A}$ and $\widetilde{B}$. For adaptive control of chaotic motion [23,24], the traditional Lyapunov stability theorem and Babalat lemma are used to prove that the error vector approaches zero as time approaches infinity. However, why the estimated or given parameters also approach the uncertain or goal parameters remains an open question. The pragmatical asymptotical stability theorem can answer this question.

3. Time-reversed Lorenz system

The classical Lorenz equation [18] derived by Lorenz is described as

$$\begin{cases} \frac{dx_1(t)}{dt} = a(x_2(t) - x_1(t)) \\ \frac{dx_2(t)}{dt} = cx_1(t) - x_1(t)x_3(t) - x_2(t) \\ \frac{dx_3(t)}{dt} = x_1(t)x_2(t) - bx_3(t) \end{cases} \tag{3-1}$$

when the initial condition $(x_{10}, x_{20}, x_{30}) = (-0.1, 0.2, 0.3)$ and the parameters $a = 10$, $b = 8/3$ and $c = 28$, chaos occurs in the Lorenz system. The chaotic behavior of Eq. (3-1) is shown in Fig. 1.

The classical Lorenz system has been studied in detail and frequently used for simulations [13,24,27,36,38]. However, the time-reversed Lorenz system has yet to be studied. Thus, in [11], we use positive parameters (*P*-parameters) for the original Lorenz system and negative parameters (*N*-parameters) for the time-reversed Lorenz system and provide a complete report for the time-reversed Lorenz system.

The time-reversed Lorenz system can be described as follows:

$$\begin{cases} \frac{dx_1(-t)}{d(-t)} = a(x_2(-t) - x_1(-t)) \\ \frac{dx_2(-t)}{d(-t)} = cx_1(-t) - x_1(-t)x_3(-t) - x_2(-t) \\ \frac{dx_3(-t)}{d(-t)} = x_1(-t)x_2(-t) - bx_3(-t) \end{cases} \tag{3-2}$$

From the left-hand sides of Eq. (3-2), the derivatives use the back-time. When the initial condition $(x_{10}, x_{20}, x_{30}) = (-0.1, 0.2, 0.3)$ and the parameters $a = -10$, $b = -8/3$ and $c = -28$ (*N*-parameters [45]), the chaotic behavior of Eq. (3-2) occurs, as shown in Fig. 2. Furthermore, the dynamic behaviors of the time-reversed Lorenz systems with different parameter signs are provided in Table 1.

4. Simulation results

In this section, two cases are presented for comparison. In Case 1, an adaptive synchronization with a traditional adaptive method is provided. In Case 2, an adaptive synchronization with the new strategy is presented to synchronize two chaotic systems. The time-reversed Lorenz system is the slave system, and the original Lorenz system is the master system. These two systems are described in the equations shown below:

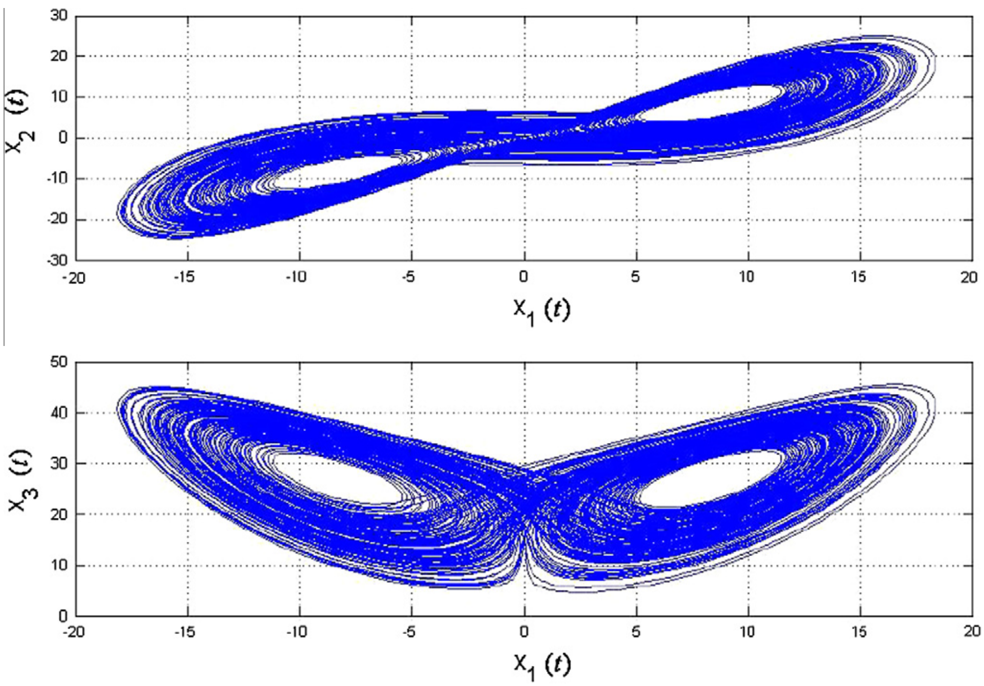

Fig. 1. Projections of the phase portrait of a chaotic contemporary Lorenz system with *P*-parameters $a = 10$, $b = 8/3$ and $c = 28$.

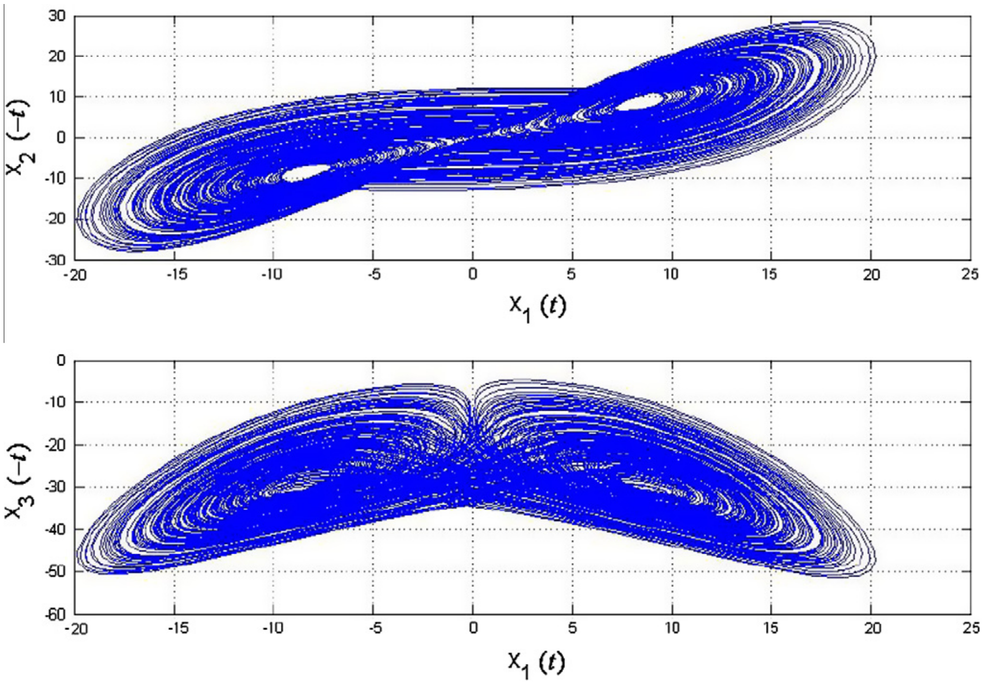

Fig. 2. Projections of the phase portrait of a chaotic time-reversed Lorenz system with *N*-parameters $a = -10$, $b = -8/3$ and $c = -28$.

Table 1
Dynamic behaviors of time reversed Lorenz system for different signs of parameters.

a	b	c	States
–	+	+	Approach to infinite
+	–	+	Approach to infinite
+	+	–	Periodic
–	–	+	Approach to infinite
–	+	–	Approach to infinite
–	–	–	Chaos and periodic

Master Lorenz system:

$$\begin{cases} \frac{dx_1(t)}{dt} = a(x_2(t) - x_1(t)) \\ \frac{dx_2(t)}{dt} = cx_1(t) - x_1(t)x_3(t) - x_2(t) \\ \frac{dx_3(t)}{dt} = x_1(t)x_2(t) - bx_3(t) \end{cases} \tag{4-1}$$

Slave time-reversed Lorenz system:

$$\begin{cases} \frac{dy_1(-t)}{d(-t)} = -\hat{a}(y_2(-t) - y_1(-t)) + u_1 \\ \frac{dy_2(-t)}{d(-t)} = -(\hat{c}y_1(-t) - y_1(-t)y_3(-t) - y_2(-t)) + u_2 \\ \frac{dy_3(-t)}{d(-t)} = -(y_1(-t)y_2(-t) - \hat{b}y_3(-t)) + u_3 \end{cases} \tag{4-2}$$

where $x_i(t)$ includes the states of the variables of the master system and $y_i(-t)$ includes the states for the slave system. Parameters a, b and c are positive uncertain parameters of the master system. $\hat{a}$, $\hat{b}$ and $\hat{c}$ are estimated parameters. u_1, u_2 and u_3 are nonlinear controllers that synchronize the slave Lorenz system with master system, i.e.

$$\lim_{t\to\infty} \mathbf{e} = 0 \tag{4-3}$$

where the error vector $\mathbf{e} = [e_1(t)e_2(t)e_3(t)]$.

Case 1: Adaptive synchronization with the traditional method.

The error vector $\mathbf{e} = [e_1(t)e_2(t)e_3(t)]$ and

$$\begin{cases} e_1(t) = x_1(t) - y_1(-t) \\ e_2(t) = x_2(t) - y_2(-t) \\ e_3(t) = x_3(t) - y_3(-t) \end{cases} \tag{4-4}$$

From Eq. (4-4), we have the following error dynamics:

$$\begin{cases} \frac{de_1(t)}{dt} = \frac{dx_1(t)}{dt} - \frac{dy_1(-t)}{dt} = \frac{dx_1(t)}{dt} + \frac{dy_1(-t)}{d(-t)} \\ \frac{de_2(t)}{dt} = \frac{dx_2(t)}{dt} - \frac{dy_2(-t)}{dt} = \frac{dx_2(t)}{dt} + \frac{dy_2(-t)}{d(-t)} \\ \frac{de_3(t)}{dt} = \frac{dx_3(t)}{dt} - \frac{dy_3(-t)}{dt} = \frac{dx_3(t)}{dt} + \frac{dy_3(-t)}{d(-t)} \end{cases}$$

$$\begin{aligned} \dot{e}_1 &= a(x_2 - x_1) + (-\hat{a}(y_2 - y_1) + u_1) \\ \dot{e}_2 &= cx_1 - x_1x_3 - x_2 + (-(\hat{c}y_1 - y_1y_3 - y_2) + u_2) \\ \dot{e}_3 &= x_1x_2 - bx_3 + (-(y_1y_2 - \hat{b}y_3) + u_3) \end{aligned} \tag{4-5}$$

The two systems will be synchronized for any initial condition with the appropriate controllers and update laws for the estimated parameters. Thus, the following controllers and update laws are designed using the pragmatical asymptotical stability theorem as follows:

The Lyapunov function is selected as

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2\right) \tag{4-6}$$

where $\tilde{a} = a - \hat{a},\ \tilde{b} = b - \hat{b}$ and $\tilde{c} = c - \hat{c}$.

The time derivative of this function is

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + \tilde{a}\dot{\tilde{a}} + \tilde{b}\dot{\tilde{b}} + \tilde{c}\dot{\tilde{c}} \\ &= e_1(a(x_2 - x_1) + (-\hat{a}(y_2 - y_1) + u_1)) + e_2(cx_1 - x_1x_3 - x_2 + (-(\hat{c}y_1 - y_1y_3 - y_2) + u_2)) + e_3(x_1x_2 - bx_3 \\ &\quad + (-(y_1y_2 - \hat{b}y_3) + u_3)) + \dot{\tilde{a}}(a - \hat{a}) + \dot{\tilde{b}}(b - \hat{b}) + \dot{\tilde{c}}(c - \hat{c}) \end{aligned} \tag{4-7}$$

The update laws for the uncertain parameters are

$$\begin{cases} \dot{\tilde{a}} = -\dot{\hat{a}} = -(x_2 - x_1)e_1 + \tilde{a}e_1 \\ \dot{\tilde{c}} = -\dot{\hat{c}} = -(x_1)e_2 + \tilde{c}e_2 \\ \dot{\tilde{b}} = -\dot{\hat{b}} = (x_3)e_3 + \tilde{b}e_3 \end{cases} \tag{4-8}$$

From Eqs. (4-7 and (4-8), the appropriate controllers can be designed as

$$\begin{cases} u_1 = -\hat{a}(x_2 - x_1 - y_2 + y_1) - \tilde{a}^2 - e_1 \\ u_2 = -\hat{c}(x_1 - y_1) + x_1x_3 + x_2 + y_1y_3 + y_2 - \tilde{c}^2 - e_2 \\ u_3 = \hat{b}(x_3 - y_3) - x_1x_2 - y_1y_2 - \tilde{b}^2 - e_3 \end{cases} \tag{4-9}$$

We obtain

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 < 0 \tag{4-10}$$

which is a negative semi-definite function of $[e_1,e_2,e_3]$, $\hat{a},\ \hat{b}$ and $\hat{c}$. The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain a common origin for the error dynamics (4-5) and the parameter dynamics (4-8) are asymptotically stable. From the pragmatical asymptotically stability theorem [26,27], D is a 6-manifold ($n = 6$) and the number of error state variables is $p = 3$. When $e_1 = e_2 = e_3 = 0$ and $\hat{a},\ \hat{b}$, and $\hat{c}$ have arbitrary values, $\dot{V} = 0$; thus, X has three dimensions, $m = n - p = 6 - 3 = 3$, and $m + 1 < n$ is satisfied. According to the pragmatical asymptotically stability theorem, the error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point is pragmatically asymptotically stable. From the equal probability assumption, the equilibrium point is actually asymptotically stable. The simulation results are shown in Figs. 3–5.

Case 2: Adaptive synchronization with the new adaptive strategy.

To achieve simple and linear controllers, the master and slave system should be transferred into a fuzzy set.

Fuzzy modeling of the Lorenz system:

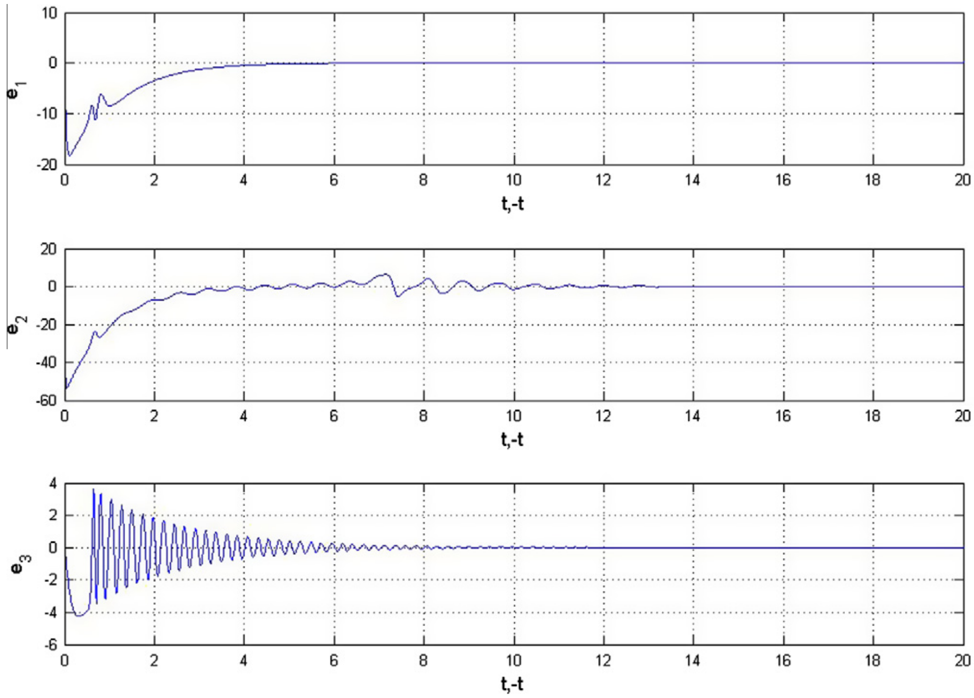

Fig. 3. Time histories of the errors for Case 1.

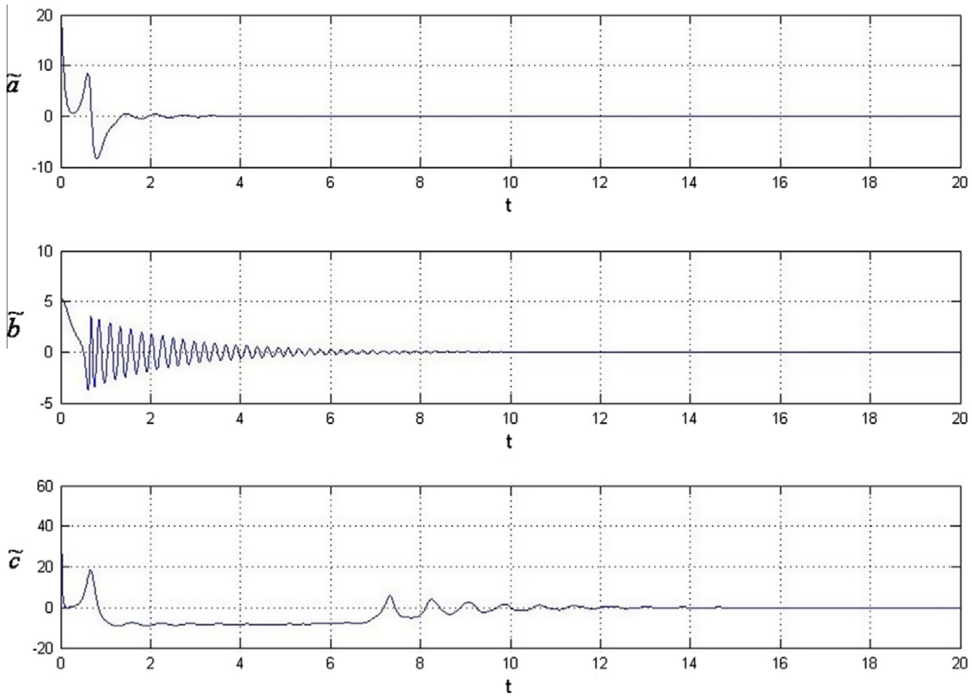

Fig. 4. Time histories of the parametric errors for Case 1.

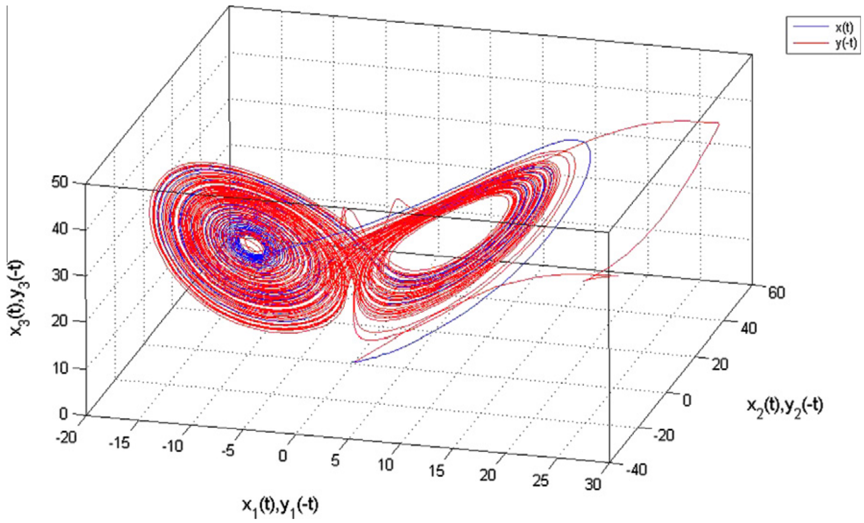

Fig. 5. Phase portraits of the synchronization for Case 1.

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = cx_1 - x_1x_3 - x_2 \\ \dot{x}_3 = x_1x_2 - bx_3 \end{cases} \tag{4-11}$$

Assuming that $x_1 \in [-d, d]$ and $d > 0$, the Lorenz system can be exactly represented with a T–S fuzzy model as follows:

$$\text{Rule 1}: \text{ IF } x \text{ is } M_1, \text{ THEN } \dot{X}(t) = A_1X(t) \tag{4 - 12}$$

$$\text{Rule 2}: \text{ IF } x \text{ is } M_2, \text{ THEN } \dot{X}(t) = A_2X(t) \tag{4-13}$$

where

$$X = [x_1, x_2, x_3]^T$$

$$A_1 = \begin{bmatrix} -a & a & 0 \\ c & -1 & -d \\ 0 & d & -b \end{bmatrix}, \quad A_s = \begin{bmatrix} -a & a & 0 \\ c & -1 & d \\ 0 & -d & -b \end{bmatrix}$$

$$M_1(x) = \frac{1}{2}\left(1 + \frac{x_1}{d}\right), \quad M_2(x) = \frac{1}{2}\left(1 - \frac{x_1}{d}\right)$$

and $d = 20$. M_1 and M_2 are fuzzy sets of the Lorenz system. We call (4-12) the first liner subsystem and (4-13) the second liner subsystem under the fuzzy rule. The final output of the fuzzy Lorenz system is inferred as follows, and the chaotic behavior is shown in Fig. 6.

$$\dot{X}(t) = \sum_{i=1}^{2} h_iA_iX(t) \tag{4-14}$$

where

$$h_1 = \frac{M_1}{M_1 + M_2}, \quad h_2 = \frac{M_2}{M_1 + M_2}$$

Because $M_1 + M_2 = 1$, Eq. (4-21) can be described as follows:

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} M_1 \\ M_1 \\ M_1 \end{bmatrix}^T \begin{bmatrix} a(x_2 - x_1) \\ cx_1 - dx_3 - x_2 \\ dx_2 - bx_3 \end{bmatrix} + \begin{bmatrix} M_2 \\ M_2 \\ M_2 \end{bmatrix}^T \begin{bmatrix} a(x_2 - x_1) \\ cx_1 + dx_3 - x_2 \\ -dx_2 - bx_3 \end{bmatrix} \tag{4-15}$$

Fuzzy modeling of the time-reversed Lorenz system:

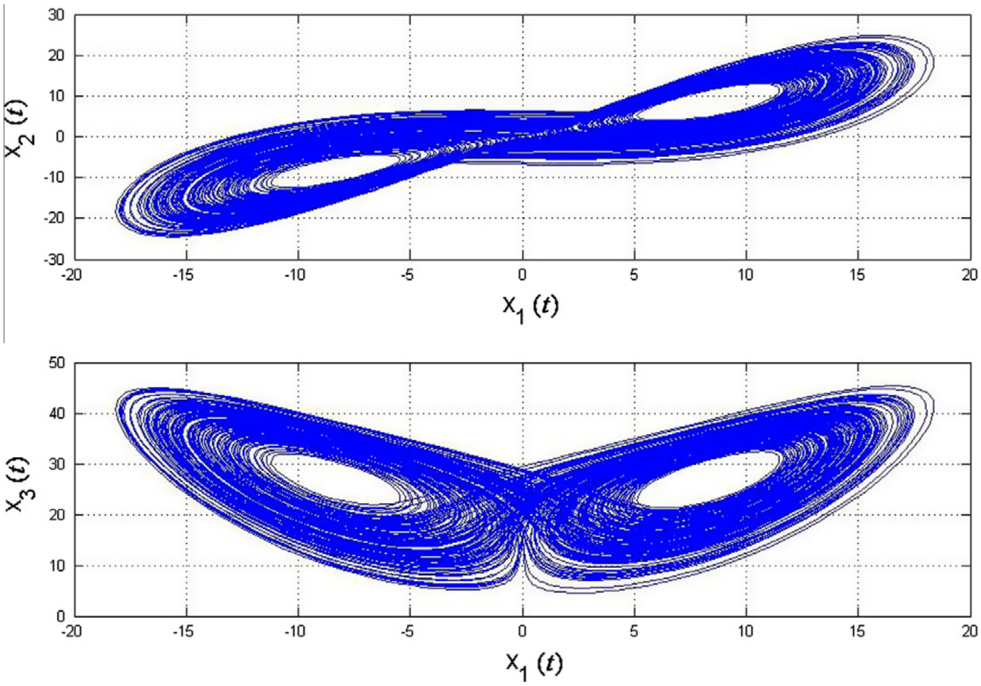

Fig. 6. Projections of the phase portrait of a fuzzy chaotic Lorenz system with *P*-parameters $a = 10$, $b = 8/3$ and $c = 28$.

$$\begin{cases} \dot{y}_1 = -\hat{a}(y_2 - y_1) + u_1 \\ \dot{y}_2 = -(\hat{c}y_1 - y_1y_3 - y_2) + u_2 \\ \dot{y}_3 = -(y_1y_2 - \hat{b}y_3) + u_3 \end{cases} \tag{4-16}$$

Assuming that $y_1 \in [-e, e]$ and $e > 0$, Eq. (4-16) can be exactly represented with a T–S fuzzy model as follows:

$$\text{Rule 1}: \text{ IF } y \text{ is } N_1, \text{ THEN } \dot{Y}(t) = B_1Y(t) + U_1 \tag{4 - 17}$$

$$\text{Rule 2}: \text{ IF } y \text{ is } N_2, \text{ THEN } \dot{Y}(t) = B_2Y(t) + U_2 \tag{4-18}$$

where

$$Y = [y_1, y_2, y_3]^T$$

$$B_1 = \begin{bmatrix} \hat{a} & -\hat{a} & 0 \\ -\hat{c} & 1 & e \\ 0 & -e & \hat{b} \end{bmatrix}, \quad B_2 = \begin{bmatrix} \hat{a} & -\hat{a} & 0 \\ -\hat{c} & 1 & -e \\ 0 & e & \hat{b} \end{bmatrix}$$

$$N_1(x) = \frac{1}{2}\left(1 + \frac{y_1}{e}\right), \quad N_2(x) = \frac{1}{2}\left(1 - \frac{y_1}{e}\right)$$
$$U_1 = [u_{11}, u_{12}, u_{13}], \quad U_2 = [u_{21}, u_{22}, u_{23}]$$

and $e = 20$. M_1 and M_2 are fuzzy sets of the time-reversed Lorenz system. We call (4-17) the first liner subsystem and (4-18) the second liner subsystem under the fuzzy rule. The final output of the fuzzy time-reversed Lorenz system is inferred as follows, and the chaotic behavior is shown in Fig. 7.

$$\dot{Y}(t) = \sum_{i=1}^{2} g_iB_iY(t) \tag{4-19}$$

where

$$g_1 = \frac{N_1}{N_1 + N_2}, \quad g_2 = \frac{N_2}{N_1 + N_2}$$

Because $N_1 + N_2 = 1$, Eq. (4-19) can be described as follows:

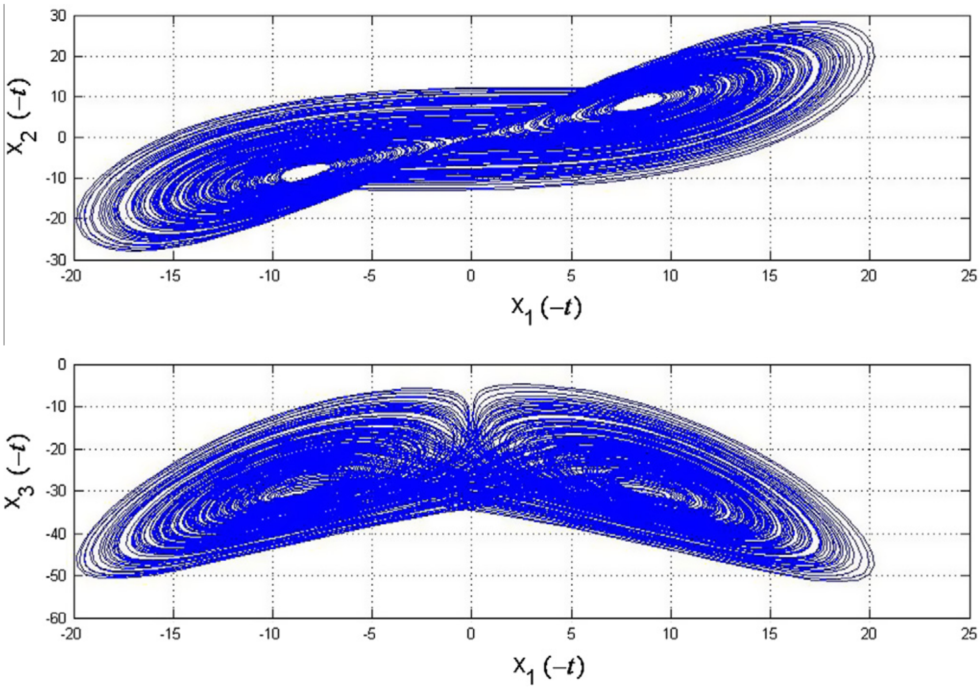

Fig. 7. Projections of the phase portrait of a fuzzy chaotic time-reversed Lorenz system with *N*-parameters $a = -10$, $b = -8/3$ and $c = -28$.

$$\dot{\mathbf{y}} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} N_1 \\ N_1 \\ N_1 \end{bmatrix}^T \begin{bmatrix} -\hat{a}(y_2 - y_1) + u_{11} \\ -(\hat{c}y_1 - ey_3 - y_2) + u_{12} \\ -(ey_2 - \hat{b}y_3) + u_{13} \end{bmatrix} + \begin{bmatrix} N_2 \\ N_2 \\ N_2 \end{bmatrix}^T \begin{bmatrix} -\hat{a}(y_2 - y_1) + u_{21} \\ -(\hat{c}y_1 + ey_3 - y_2) + u_{22} \\ -(-ey_2 - \hat{b}y_3) + u_{23} \end{bmatrix} \tag{4-20}$$

For adaptive synchronization, the fuzzy sets in Eq. (4-20) are substituted with the estimated parameters as follows:

$$\dot{\mathbf{y}} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} \widehat{N}_1 \\ \widehat{N}_1 \\ \widehat{N}_1 \end{bmatrix}^T \begin{bmatrix} -\hat{a}(y_2 - y_1) + u_{11} \\ -(\hat{c}y_1 - ey_3 - y_2) + u_{12} \\ -(ey_2 - \hat{b}y_3) + u_{13} \end{bmatrix} + \begin{bmatrix} \widehat{N}_2 \\ \widehat{N}_2 \\ \widehat{N}_2 \end{bmatrix}^T \begin{bmatrix} -\hat{a}(y_2 - y_1) + u_{21} \\ -(\hat{c}y_1 + ey_3 - y_2) + u_{22} \\ -(-ey_2 - \hat{b}y_3) + u_{23} \end{bmatrix} \tag{4-21}$$

where $\widehat{N}_1 = \hat{f} \times \frac{1}{2}\left(1 + \frac{y_1(t)}{e}\right) + \hat{g} \times S_1, \widehat{N}_2 = \hat{f} \times \frac{1}{2}\left(1 + \frac{y_1(t)}{e}\right) + \hat{g} \times S_1$

$\hat{f}$ and $\hat{g}$ are the estimated parameters and $(\hat{f}_0, \hat{g}_0) = (1, 0)$. The goal values for the estimated parameters $\hat{f}$ and $\hat{g}$ are 0 and 1. S_i, $i = 1 - 2$ are the fuzzy sets of the master system, in which $S_i = M_i$ and $i = 1 - 2$.

The synchronization flowchart is shown in Fig. 8. The synchronizing processes in Fig. 8 are divided into two steps. (1) Use the first linear subsystem of the slave system in Eq. (4-17) to trace the trajectory of the first linear subsystem of the master system in (4-12). (2) Use the second linear subsystem of the slave system in Eq. (4-18) to trace the trajectory of the first linear subsystem of the master system in (4-13).

Step 1: The error and error dynamics in the first linear subsystem are

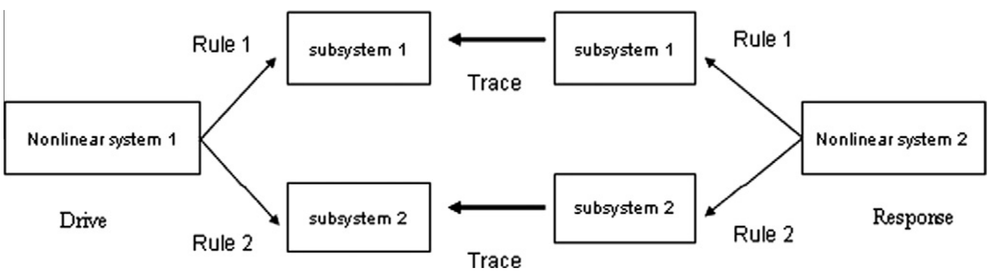

Fig. 8. The flowchart of Synchronization.

$$\begin{cases} e_1(t) = x_1(t) - y_1(-t) + K \\ e_2(t) = x_2(t) - y_2(-t) + K \\ e_3(t) = x_3(t) - y_3(-t) + K \end{cases} \tag{4-22}$$

where K is a constant (200). From (4-22), the following error dynamics are obtained:

$$\begin{cases} \frac{de_1(t)}{dt} = \frac{dx_1(t)}{dt} - \frac{dy_1(-t)}{dt} = \frac{dx_1(t)}{dt} + \frac{dy_1(-t)}{d(-t)} \\ \frac{de_2(t)}{dt} = \frac{dx_2(t)}{dt} - \frac{dy_2(-t)}{dt} = \frac{dx_2(t)}{dt} + \frac{dy_2(-t)}{d(-t)} \\ \frac{de_3(t)}{dt} = \frac{dx_3(t)}{dt} - \frac{dy_3(-t)}{dt} = \frac{dx_3(t)}{dt} + \frac{dy_3(-t)}{d(-t)} \end{cases}$$

$$\begin{cases} \dot{e}_1(t) = a(x_2 - x_1) - (-\hat{a}(y_2 - y_1) + u_{11}) \\ \dot{e}_2(t) = cx_1 - dx_3 - x_2 - (-(\hat{c}y_1 - ey_3 - y_2) + u_{12}) \\ \dot{e}_3(t) = dx_2 - bx_3 - (-(ey_2 - \hat{b}y_3) + u_{13}) \end{cases} \tag{4-23}$$

The two systems will be synchronized for any initial condition with the appropriate controllers and update laws for the estimated parameters. Thus, the following controllers and update laws are designed with our new method as follows.

The Lyapunov function is selected as

$$V = e_1 + e_2 + e_3 + \tilde{a} + \tilde{b} + \tilde{c} + \frac{1}{2}(\tilde{f}^2 + \tilde{g}^2) \tag{4-24}$$

where $\tilde{a} = a - \hat{a},\ \tilde{b} = b - \hat{b},\ \tilde{c} = c - \hat{c},\ \tilde{f} = f - \hat{f}$ and $\tilde{g} = g - \hat{g}$. a–c are positive uncertain parameters, and $\hat{a},\ \hat{b}$ and $\hat{c}$ are estimated parameters with negative initial values, where $(f,g) = (0,1)$.

The time derivative of the function is

$$\begin{aligned} \dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{\tilde{a}} + \dot{\tilde{b}} + \dot{\tilde{c}} + \tilde{f}\dot{\tilde{f}} + \tilde{g}\dot{\tilde{g}} \\ &= (a(x_2 - x_1) - (-\hat{a}(y_2 - y_1) + u_{11})) + (cx_1 - dx_3 - x_2 - (-(\hat{c}y_1 - ey_3 - y_2) + u_{12})) + (dx_2 - bx_3 - (-(ey_2 \\ &\quad - \hat{b}y_3) + u_{13})) + \dot{\tilde{a}} + \dot{\tilde{b}} + \dot{\tilde{c}} + \dot{\tilde{f}}(f - \hat{f}) + \dot{\tilde{g}}(g - \hat{g}) \end{aligned} \tag{4-25}$$

The update laws for the uncertain parameters are

$$\begin{cases} \dot{\tilde{a}} = -\dot{\hat{a}} = -(x_2 - x_1)\tilde{a} - \tilde{a}e_1 \\ \dot{\tilde{b}} = -\dot{\hat{b}} = (x_3)\tilde{b} - \tilde{b}e_3 \\ \dot{\tilde{c}} = -\dot{\hat{c}} = -(x_1)\tilde{c} - \tilde{c}e_2 \\ \dot{\tilde{f}} = -\dot{\hat{f}} = -\tilde{f}e_1 \\ \dot{\tilde{g}} = -\dot{\hat{g}} = -\tilde{g}e_2 \end{cases} \tag{4-26}$$

From Eqs. (4-25) and (4-26), the appropriate controllers can be designed as:

$$\begin{cases} u_{11} = \hat{a}(x_2 - x_1 + y_2 - y_1) \\ u_{12} = \hat{c}(x_1 + y_1) - dx_3 - x_2 - ey_3 - y_2 \\ u_{13} = dx_2 + ey_2 - \hat{b}(x_3 + y_3) \end{cases} \tag{4-27}$$

We obtain

$$\dot{V} = -(\tilde{a} + \tilde{f}^2)e_1 - (\tilde{b} + \tilde{g}^2)e_2 - \tilde{c}e_3 < 0 \tag{4-28}$$

which is a negative semi-definite function of $e_1,\ e_2,\ e_3,\ \tilde{a},\ \tilde{b},\ \tilde{c},\ \tilde{f}$ and $\tilde{g}$. The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain a common origin of the error dynamics (4-23), and the parameter dynamics (4-26) are asymptotically stable. From the pragmatical asymptotically stability theorem [26,27], D is an 8-manifold ($n = 8$) and the number of error state variables is $p = 3$. When $e_1 = e_2 = e_3 = 0$ and $\hat{a},\ \hat{b},\ \hat{c},\ \hat{f}$ and $\hat{g}$ have arbitrary values, $\dot{V} = 0$; thus, X has five dimensions $m = n - p = 8 - 3 = 5$ and $m + 1 < n$ is satisfied. According to the pragmatical asymptotically stability theorem, the error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point is pragmatically asymptotically stable. From the equal probability assumption, this point is actually asymptotically stable.

Step 2: The error and error dynamics in the second linear subsystem are

$$\begin{cases} e_1(t) = x_1(t) - y_1(-t) + K \\ e_2(t) = x_2(t) - y_2(-t) + K \\ e_3(t) = x_3(t) - y_3(-t) + K \end{cases} \tag{4-29}$$

where K is a constant (200). From Eq. (4-29), the following error dynamics are obtained:

$$\begin{cases} \frac{de_1(t)}{dt} = \frac{dx_1(t)}{dt} - \frac{dy_1(-t)}{dt} = \frac{dx_1(t)}{dt} + \frac{dy_1(-t)}{d(-t)} \\ \frac{de_2(t)}{dt} = \frac{dx_2(t)}{dt} - \frac{dy_2(-t)}{dt} = \frac{dx_2(t)}{dt} + \frac{dy_2(-t)}{d(-t)} \\ \frac{de_3(t)}{dt} = \frac{dx_3(t)}{dt} - \frac{dy_3(-t)}{dt} = \frac{dx_3(t)}{dt} + \frac{dy_3(-t)}{d(-t)} \end{cases}$$

$$\begin{cases} \dot{e}_1(t) = a(x_2 - x_1) - (-\hat{a}(y_2 - y_1) + u_{21}) \\ \dot{e}_2(t) = cx_1 + dx_3 - x_2 - (-(\hat{c}y_1 + ey_3 - y_2) + u_{22}) \\ \dot{e}_3(t) = -dx_2 - bx_3 - (-(-ey_2 - \hat{b}y_3) + u_{23}) \end{cases} \tag{4-30}$$

The two systems will be synchronized for any initial condition with the appropriate controllers and update laws for the estimated parameters. Thus, the following controllers and update laws are designed with the new method as follows.

The Lyapunov function is

$$V = e_1 + e_2 + e_3 + \tilde{a} + \tilde{b} + \tilde{c} + \frac{1}{2}(\tilde{f}^2 + \tilde{g}^2) \tag{4-31}$$

where $\tilde{a} = a - \hat{a},\ \tilde{b} = b - \hat{b},\ \tilde{c} = c - \hat{c},\ \tilde{f} = f - \hat{f}$ and $\tilde{g} = g - \hat{g}$. a–c are positive uncertain parameters, and $\hat{a},\ \hat{b}$ and $\hat{c}$ are estimated parameters with negative initial values, where $(f,g) = (0,1)$.

The time derivative of this function is

$$\begin{aligned} \dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{\tilde{a}} + \dot{\tilde{b}} + \dot{\tilde{c}} + \tilde{f}\dot{\tilde{f}} + \tilde{g}\dot{\tilde{g}} \\ &= (a(x_2 - x_1) - (-\hat{a}(y_2 - y_1) + u_1)) + (cx_1 + dx_3 - x_2 - (-(\hat{c}y_1 + ey_3 - y_2) + u_2)) + (-dx_2 - bx_3 - (-(-ey_2 \\ &\quad - \hat{b}y_3) + u_3)) + \dot{\tilde{a}} + \dot{\tilde{b}} + \dot{\tilde{c}} + \dot{\tilde{f}}(f - \hat{f}) + \dot{\tilde{g}}(g - \hat{g}) \end{aligned} \tag{4-32}$$

The update laws for the uncertain parameters are

$$\begin{cases} \dot{\tilde{a}} = -\dot{\hat{a}} = -(x_2 - x_1)\tilde{a} - \tilde{a}e_1 \\ \dot{\tilde{b}} = -\dot{\hat{b}} = (x_3)\tilde{b} - \tilde{b}e_3 \\ \dot{\tilde{c}} = -\dot{\hat{c}} = -(x_1)\tilde{c} - \tilde{c}e_2 \\ \dot{\tilde{f}} = -\dot{\hat{f}} = -\tilde{f}e_1 \\ \dot{\tilde{g}} = -\dot{\hat{g}} = -\tilde{g}e_2 \end{cases} \tag{4-33}$$

From Eqs. (4-32) and (4-33), the appropriate controllers can be designed as

$$\begin{cases} u_{21} = \hat{a}(x_2 - x_1 + y_2 - y_1) \\ u_{22} = \hat{c}(x_1 + y_1) + dx_3 - x_2 + ey_3 - y_2 \\ u_{23} = -dx_2 - ey_2 - \hat{b}(x_3 + y_3) \end{cases} \tag{4-34}$$

We obtain

$$\dot{V} = -(\tilde{a} + \tilde{f}^2)e_1 - (\tilde{b} + \tilde{g}^2)e_2 - \tilde{c}e_3 < 0 \tag{4-35}$$

which is a negative semi-definite function of $e_1,\ e_2,\ e_3,\ \tilde{a},\ \tilde{b},\ \tilde{c},\ \tilde{f}$ and $\tilde{g}$. The Lyapunov asymptotical stability theorem is not satisfied. We cannot obtain the common origin of the error dynamics (4-30) and the parameter dynamics (4-33) are asymptotically stable. From the pragmatical asymptotically stability theorem, D is an 8-manifold system ($n = 8$) and the number of error state variables is $p = 3$. When $e_1 = e_2 = e_3 = 0$ and $\hat{a},\ \hat{b},\ \hat{c},\ \hat{f}$ and $\hat{g}$ have arbitrary values, $\dot{V} = 0$; thus, X has five dimensions $m = n - p = 8 - 3 = 5$ and $m + 1 < n$ is satisfied. According to the pragmatical asymptotically stability theorem, the error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point is pragmatically asymptotically stable. From the equal probability assumption, this point is actually asymptotically stable. After steps 1 and 2, the two linear subsystems of the slave system can be synchronized to the two linear subsystems of the master system. This synchronization shows that chaos synchronization for these two fuzzy chaotic systems can be achieved. The simulation results are shown in Figs. 9–11, where $\widetilde{N}_1 = M_1 - \widehat{N}_1$ and $\widetilde{N}_2 = M_2 - \widehat{N}_2$.

5. Discussion

In this section, the numerical simulation results in Cases 1 and 2 are compared.

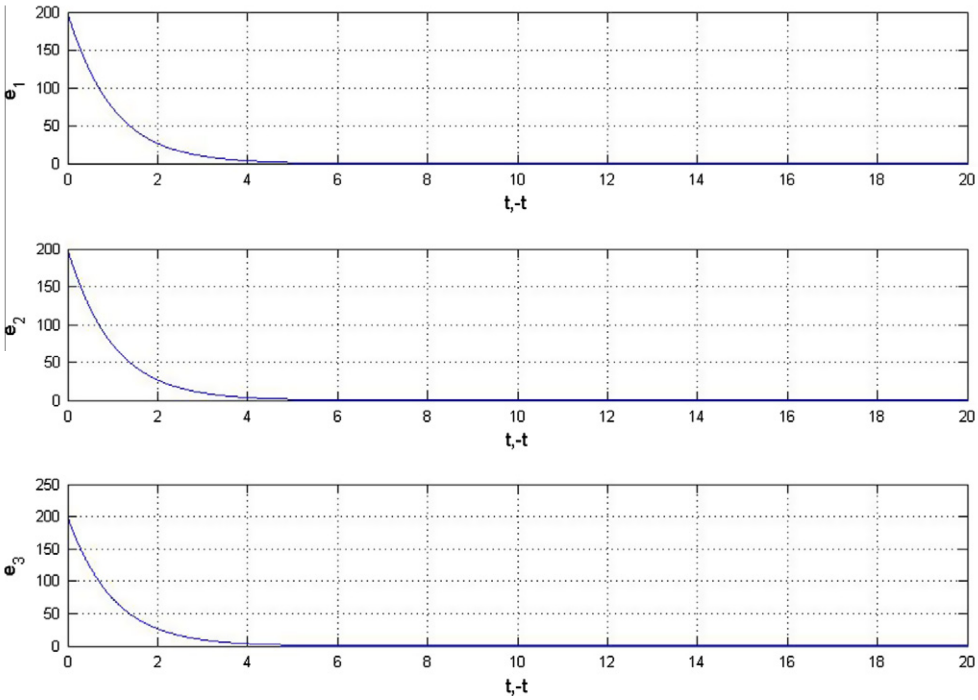

Fig. 9. Time histories of the errors for Case 2.

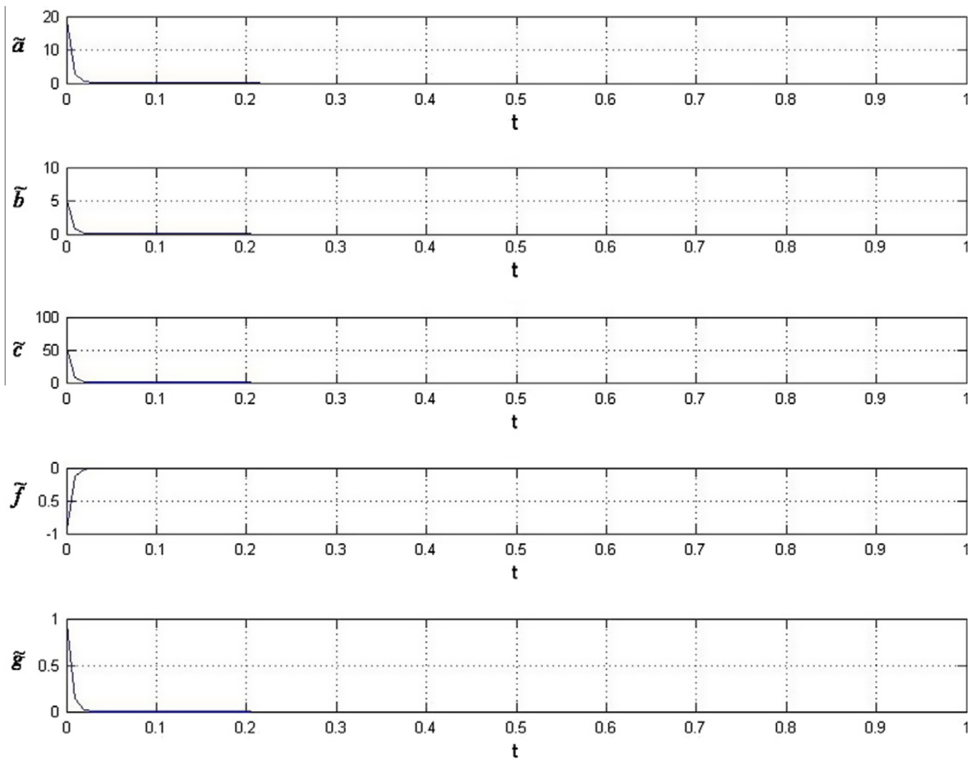

Fig. 10. Time histories of the parametric errors for Case 2.

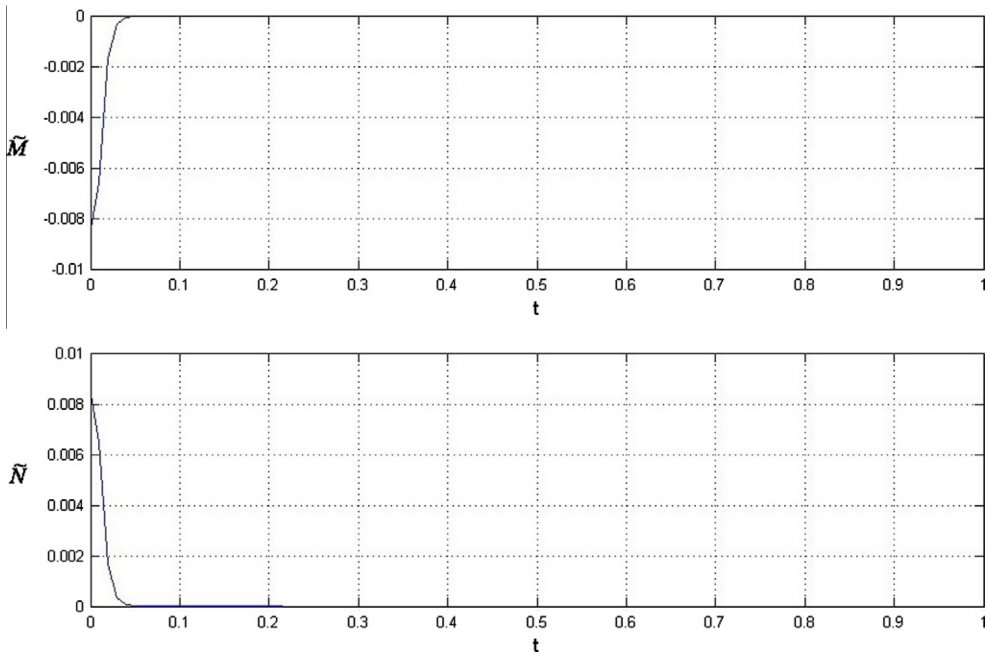

Fig. 11. Time histories of the fuzzy set errors for Case 2.

Table 2
Comparison between errors data at 19.96 s, 19.97 s, 19.98 s, 19.99 s and 20.00 s after the action of controllers.

Time	Errors for Case 2	Errors for Case 1
	e_1	e_1
19.96 s	0.00000043256	0.00011282000
19.97 s	0.00000042825	0.00010565000
19.98 s	0.00000042399	0.00009795500
19.99 s	0.00000041977	0.00008996100
20.00 s	0.00000041560	0.00008187800
	e_2	e_2
19.96 s	0.00000043267	−0.00210000000
19.97 s	0.00000042836	−0.00110000000
19.98 s	0.00000042410	−0.00020560000
19.99 s	0.00000041988	0.00061179000
20.00 s	0.00000041570	0.00130000000
	e_3	e_3
19.96 s	0.00000043378	0.00009716000
19.97 s	0.00000042946	0.00001637300
19.98 s	0.00000042519	−0.00006358000
19.99 s	0.00000042096	−0.00013330000
20.00 s	0.00000041677	−0.00018590000

In Case 1: From Figs. 3 and 4, all of the errors converge after 14 s and the parameter errors converge after 16 s. In contrast, the numerical data in Tables 2 and 3 show that $e_1 \approx 8.1 \times 10^{-5}$, $e_2 \approx 1.3 \times 10^{-3}$, and $e_3 \approx 1.8 \times 10^{-4}$ when the time approaches 20.00 s and $\tilde{a} \approx 0.00079843$, $\tilde{b} \approx 0.02310000$, and $\tilde{c} \approx 0.39130000$ when the time approaches 10.00 s.

In Case 2: Figs. 9 and 10 show that all of the errors converge at approximate 6–8 s and the parameter errors converge in 0.1 s. Additionally, the numerical data in Tables 2 and 3 show that $e_1 \approx 4.1560 \times 10^{-7}$, $e_2 \approx 4.1570 \times 10^{-7}$, and $e_3 \approx 4.1677 \times 10^{-7}$ when the time approaches 20.00 s and $\tilde{a} = \tilde{b} = \tilde{c} = 0$ when the time approaches 10.00 s.

Thus, as shown by the comparisons of the simulation results, the new adaptive scheme is effective and powerful. This scheme largely increases the convergence speed to the goal values and reduces the simulation errors. Additionally, the controllers, which are derived from the Lyapunov function, are linear.

Table 3
Comparison between parametric errors at 9.96 s, 9.97 s, 9.98 s, 9.99 s and 10.00 s after the action of controllers.

Time	Errors for Case 2	Errors for Case 1
	$\tilde{a}$	$\tilde{a}$
9.96 s	0	0.00075110
9.97 s	0	0.00075450
9.98 s	0	0.00076372
9.99 s	0	0.00077850
10.00 s	0	0.00079843
	$\tilde{b}$	$\tilde{b}$
9.96 s	0	−0.01620000
9.97 s	0	−0.00630000
9.98 s	0	0.00430000
9.99 s	0	0.01440000
10.00 s	0	0.02310000
	$\tilde{c}$	$\tilde{c}$
9.96 s	0	1.10510000
9.97 s	0	0.93560000
9.98 s	0	0.75750000
9.99 s	0	0.57490000
10.00 s	0	0.39130000

6. Conclusions

An adaptive control scheme is effective and suitable for the synchronization of two chaotic systems with different structures and parameter mismatches. Most other methods only synchronize two systems with known structures and parameters. However, in practical situations, some or all of the system parameters are unknown. The increasing numbers of applications of chaos synchronization for secure communication have made this topic more important. In this study, a new, effective and powerful scheme to achieve the adaptive synchronization of two nonlinear systems with mismatched parameters is proposed. This new scheme has two main elements: (1) for the T–S Fuzzy model, complicated and nonlinear systems can be linearized into several linear systems and the linear controllers can be obtained and (2) for the partial region stability theorem, a new Lyapunov function can be directly chosen as a simple linear homogeneous state function. Simulation results show that the state error parameters approach zero more precisely and efficiently when the synchronization and controllers are simple and linear. The new scheme in this study is an efficient and feasible tool for synchronization and is not limited to adaptive applications. Various types of applications should be studied with this scheme to improve performance, such as sliding mode control or backstepping control.

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