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An extended fuzzy measure on competitiveness correlation based on WCY 2011

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ABSTRACT

The fuzzy measure can highlight important information in analyzing component features, patterns, and trends. However, fuzzy densities and interaction effects are usually unknown or uncertain for implications thus making the fuzzy measure limited in applications. This research proposes an extended fuzzy measure to derive the conditional fuzzy densities from dominance-based rough set approach (DRSA), multiply preferences and the derived densities into utilities, fulfill fuzzy measure identification, and empower the fuzzy measure to aggregate utilities. For illustration, the extended fuzzy measure is applied on World Competitiveness Yearbook 2011 to imply policy-making information for Greece, Italy, Portugal, and Spain.

1. Introduction

National competitiveness plays an important role as an aggregation power of a nation to enhance its people's lives and cope with worldwide challenges [1–3]. The fuzzy measure can highlight component information in analyzing features [4,5], patterns [6,7], and multi-criteria decision making (MCDM) [8–11]. However, applying the fuzzy measure to analyze competitiveness has difficulties. First, the fuzzy densities based on outcome probabilities are not designed for '*if...then...*' implications [12,13]. Substituting the fuzzy densities by conditional probabilities makes the fuzzy measure identification hard because the aggregation boundaries for the conditional probabilities and the outcome probabilities might be different. Second, the fuzzy measure cannot identify the interaction effects of compound components, composed of preferences and densities [14–16]. There are two reasons for this identification problem. One is that the fuzzy measure is designed for a single type of components. The other is because the interaction effects might have mixed types and cause ambiguity in estimating the competitiveness. The three typical interaction types are additive, sub-additive, and super-additive effects. The additive type is that interaction effect is similar to the expected effects. It is desired by users due to ease of assuming the components to be independent. The sub-additive interaction, however, yields some substitution effects and reduces the expectation of components independence, while the super-additive interaction yields additional effects than the expected effects.

With the aforementioned problems, key challenges for analyzing competitiveness are summarized as the followings:

- *World Competitiveness Yearbook* (WCY) is the most well-known annual report of national competitiveness [2]. It presents preferences for national performance with criteria values. However, it neither assumes criteria weights for grouping nations nor provides competitiveness features for decision making.
- Dominance-based rough set approach (DRSA) can provide preference features however it cannot handle analysis on components aggregation. Contrarily, the fuzzy measure can aggregate densities while it cannot identify densities for '*if...then*' implications.

To overcome the above challenges, an Extended Fuzzy Measure (EFM), as shown in Fig. 1, is designed. It makes the fuzzy measurements possible in the information system of DRSA. Firstly, EFM associates criteria to a given class by DRSA to derive the Conditional Fuzzy Densities (CFD) for '*if...then*' implications. CFDs are used to replace the fuzzy densities of the fuzzy measure and substitute the weights of the utility functions. The integration between the fuzzy measure and utility theory thus becomes possible. Secondly, EFM multiplies a preference and a CFD into a utility and aggregate utilities into a multiplicative utility function [17,18]. In

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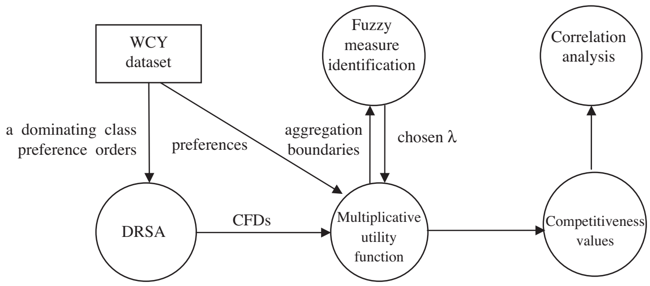

Fig. 1. The design of the extended fuzzy measure (EFM).

this research the multiplicative utility function is proved to perform aggregation as well as the fuzzy measure. Thirdly, EFM fulfills fuzzy measure identification on the aggregated utilities and chooses a resulted λ to provide competitiveness values. Finally, EFM analyzes correlations of the competitiveness values among four factors. Users can easily read and understand relationship among economic performance, government efficiency, business efficiency, and infrastructure. The terms in Fig. 1 alternatively includes a dominating class and preference orders to generate CFD.

This paper has two main parts. The first is the implementation of EFM. The second is a case study about the application of EFM which focuses on Greece, Italy, Portugal, and Spain. The remainder of this paper is organized as follows: Section 2 reviews DRSA and fuzzy measure, Section 3 presents EFM by the multiplicative utility function, Section 4 addresses results of an EFM application, Section 5 presents discussions on EFM and the case study, and finally concluding remarks are stated to close the paper.

2. Literature review

To date, the International Institute for Management Development (IMD) annually publishes the most well-known report, *World Competitiveness Yearbook* (WCY), which ranks and analyzes how a nation's environment can create and develop sustainable enterprises. Its reports are used as the competitiveness data in this research. To get inside of the competitiveness features DRSA is applied, which is reviewed below.

DRSA is a powerful technique of relational structure and has been successfully applied in many fields [19–29]. In classification application, it can be used to induce objects assigned to $Cl_t^{\geqslant}$ (the upward union of classes which includes objects ranked at least tth) or to $Cl_t^{<}$ (the downward union of classes which includes objects ranked less than tth), where Cl is a cluster set containing preference-ordered classes Cl_t, $t \in T$ and $T = \{1, 2, \ldots, n\}$. The formulations for the above statement can be expressed as $Cl = \{Cl_1, \ldots, Cl_t, \ldots, Cl_n\}$, $Cl_1 = \{y \in U{:}y$ is ranked in the top position$\}$, $Cl_2 = \{y \in U{:}y$ is ranked in the second position$\}, \ldots,$ and $Cl_n = \{y \in U{:}y$ is ranked in the bottom position$\}$ where U is a set with decision makers' preference orders and n is the number of preference-ordered classes. For all s, $t \in T$ and $s \geqslant t$ (rank of $s \geqslant$ rank of t), every object in Cl_s is preferred to be at least as good as any of object in Cl_t. The upward union is constructed as $Cl_t^{\geqslant} = \cup_{s \geqslant t} Cl_s$ for $s \geqslant t$; inversely, the downward union as $Cl_t^{<} = \cup_{s<t} Cl_s$ for $s < t$.

A representation of the upward union, called the dominating set, can rely on a set of criteria, P. It follows the dominance principle of requiring each chosen object at least as good as object x in all considered criteria of P. The granules of a dominating set based on P can be viewed as the granular cones in the criteria value space. Vice versa the dominated set for the downward union follows the dominance principle and has the granules in the opposite direction. These cones are named as P-dominating and P-dominated sets [26], respectively. It is said that object y P-dominates object x with respect to a criteria set P (denotation yD_Px). Given $x, y \in U$ and P, the dominance sets are formulated as.

P-dominating set: $D_P^+(x) = \{y \in U, yD_Px\}$

P-dominated set: $D_P^-(x) = \{y \in U, xD_Py\}$.

where x, $y \in Cl$, $y \succsim_q x$ for $D_P^+(x), x \succsim_q y$ for $D_P^-(x)$, and all $q \in P$. The assignment of objects into P-dominating set and $Cl_t^{\geqslant}$ has two types of consistency. One is called consistent assignment, i.e., objects can be properly assigned into $D_P^+(x)$ and $Cl_t^{\geqslant}$. The other is inconsistent assignment, i.e., objects assigned in $Cl_t^{\geqslant}$ possibly violate the dominance principle of $D_P^+(x)$. According to the dominance consistency, there are two approximations available.

$$\underline{P}(Cl_t^{\geqslant}) = \{x \in U : D_P^+(x) \subseteq Cl_t^{\geqslant}\},$$
$$\overline{P}(Cl_t^{\geqslant}) = \bigcup_{x \in Cl_t^{\geqslant}} D_P^+(x), \boldsymbol{B}np(Cl_t^{\geqslant}) = \overline{P}(Cl_t^{\geqslant}) - \underline{P}(Cl_t^{\geqslant})$$

$$\underline{P}(Cl_t^{<}) = \{x \in U, D_P^-(x) \subseteq Cl_t^{<}\},$$
$$\overline{P}(Cl_t^{<}) = \bigcup_{x \in Cl_t^{<}} D_P^-(x), \boldsymbol{B}np(Cl_t^{<}) = \overline{P}(Cl_t^{<}) - \underline{P}(Cl_t^{<})$$

where $t = 1, \ldots, n$, $\boldsymbol{B}np(Cl_t^{\geqslant})$ and $\boldsymbol{B}np(Cl_t^{<})$ are P-doubtful regions. Objects in P-doubtful regions are inconsistent. In a simple word, $\underline{P}(Cl_t^{\geqslant})$ requires that the largest union of P-dominating sets should be properly included in $Cl_t^{\geqslant}$. $\overline{P}(Cl_t^{\geqslant})$ requires that the smallest union of P-dominating sets should contain all elements of $Cl_t^{\geqslant}$ while allow some inconsistent objects.

The proper assignments can be expressed with the coverage rate defined by Pawlak [30,31] and Greco et al. [22]. There are two typical coverage rates (CR) for the upward unions $Cl_t^{\geqslant}$ and the downward union $Cl_t^{<}$, which are formulated as follows:

$$CR(Cl_t^{\geqslant}) = \frac{|\underline{P}(Cl_t^{\geqslant})|}{|Cl_t^{\geqslant}|}, \quad CR(Cl_t^{<}) = \frac{|\underline{P}(Cl_t^{<})|}{|Cl_t^{<}|}$$

The symbol CR is used to express "the probability of objects in the P-lower approximation relatively belonging to the corresponding union of decision classes." Alternatively, the accuracy rate presents the ratio of the proper assignment to the possible assignment. Two typical accuracy rates (α) are listed as:

$$\alpha(Cl_t^{\geqslant}) = \frac{|\underline{P}(Cl_t^{\geqslant})|}{|\overline{P}(Cl_t^{\geqslant})|} = \frac{|\underline{P}(Cl_t^{\geqslant})|}{|U| - |\underline{P}(Cl_{t-1}^{<})|},$$
$$\alpha(Cl_t^{<}) = \frac{|\underline{P}(Cl_t^{<})|}{|\overline{P}(Cl_t^{<})|} = \frac{|\underline{P}(Cl_t^{<})|}{|U| - |\underline{P}(Cl_{t+1}^{\geqslant})|}$$

The symbol α is used to present "a ratio of the cardinalities of P-lower approximation to those of P-upper approximation, i.e., the degree of the properly classified approximation relative to the possibly classified approximation." The relative importance of criteria in mathematics is reviewed next.

Saaty (2001) proposed that pair-wise comparisons and inductions can be formulated as ratios, and then transformed into the priority of criteria, or the criteria weights [32]. He also mentioned that the ratios represent how much more or less a criterion is as compared to another, and that its application can determine how close the criteria are. Also, he emphasized that ratio operations are independent from irrelevant alternatives. Thus the ratio scales derived from different (criteria) scales can be implemented mathematically to generate a characteristic ratio with invariance. Based on these theories, a multiplication of two ratios, the coverage and the accuracy rates, can be used to express an accuracy of the conditional probability. The fuzzy measure about the new ratio is reviewed next.

Sugeno presented the theories of fuzzy measures and fuzzy integrals as means of expressing fuzzy systems in 1974 [12]. The reviewing of the Sugeno's definitions is described below with a set, Q, a set function $g(\cdot)$ called a fuzzy measure for all subsets,$\beta(Q)$, of Q [33–37].

Property 1. *Boundaries of the fuzzy measure*
$g:\beta(Q) \to [0,1]$, $g(\varnothing) = 0$, and $g(Q) = 1$ where g is a fuzzy measure function; $g(\emptyset)$ and $g(Q)$ present boundaries of the fuzzy measurement.

Property 2. *Monotonicity of the fuzzy measure*
$\forall A, B \in \beta(Q)$, if $A \subseteq B$, then $g(A) \leqslant g(B)$ where monotonicity holds for the fuzzy measure function g.

Property 3. *Continuity of the fuzzy measure*
If $A_i \in \beta(Q)$, $1 \leqslant i < \infty$, and the sequence $\{A_i\}$ is monotonic, then $\lim_{i\to+\infty} g(A_i) = g(\lim_{i\to+\infty} A_i)$.

Property 4. *λ-fuzzy measure If $\forall A, B \in \beta(Q)$, $A \cap B = \varnothing$, and $\lambda \in (-1, +\infty)$ then $g(A \cup B) = g(A) + g(B) + \lambda g(A)g(B)$, which is used to illustrate the interaction for disjoint sets. λ-fuzzy measure can present three types of interactions: (i) supper-additive with $\lambda > 0$, (ii) additive with $\lambda = 0$, (iii) sub-additive with $\lambda < 0$. Let $Q = \{q_1, q_2, \ldots, q_m\}$. With Q being a finite set, the fuzzy measure with criteria is identified in Property 5.*

Property 5. *λ-fuzzy measure with m criteria*

$$g_\lambda(\{q_1, q_2, \ldots, q_m\}) = \sum_{j=1}^{m} g_j + \lambda \sum_{j1=1}^{m-1} \sum_{j2=j1+1}^{m} g_{j1} g_{j2} + \cdots + \lambda^{n-1} g_1 g_2 \ldots g_m$$
$$= \frac{1}{\lambda}\left[\prod_{j=1}^{m}(1 + \lambda g_j) - 1\right]$$

where $-1 < \lambda < \infty$, $j = 1, \ldots, m$, and $g_j = g_\lambda(\{q_j\})$ is defined as the fuzzy density with respect to q_j.

3. The extended fuzzy measure for competitiveness features

EFM aims to multiply CFDs and preferences into utilities and then empower the fuzzy measurement to aggregate utilities for competitiveness components. Followings are their descriptions. Section 3.1 is about the dataset. As the new proposals, Section 3.2 is the information system for EFM, and Section 3.3 is the derivation of the conditional fuzzy densities. Section 3.4 proves that the multiplicative utility function can perform as well as the fuzzy measure. Section 3.5 is about identifying the range of the interaction effects.

3.1. WCY data set

Four consolidated factors and twenty criteria are included in Table 1. A total of 58 nations are included in 2011 WCY. These four factors are focuses of our correlation analysis. The twenty criteria will be used to derive the conditional fuzzy densities.

3.2. The Extended Fuzzy Measure (EFM)

This section has eight propositions. Propositions 1 and 2 are about the information system and preference expressions. Propositions 3 and 4 include an induction rule, $q_{j,t}^{\geqslant} \to Cl_t^{\geqslant}$, by DRSA and the conditional fuzzy density. Propositions 5, 6, 8 are about extension of the fuzzy measure.

Proposition 1. *Information system $EFM = (U, Q, f, V, Cl_t^{\geqslant})$ where $U = \{k | k$ represents a nation$\}$, $Q = \{q_1, q_2, \ldots, q_m\}$, $f: U \times Q \to V, V_Q = (V_{q1}, V_{q2}, \ldots, V_{qm}), Cl_t^{\geqslant}$ is a dominating union having nations at least not less than t, and t is a rank place like 29th. This proposition transforms mathematical sets into an information system.*

Proposition 2. *Preference orders $r_{zj} \succsim r_{xj} \iff f(z, q_j) \geqslant f(x, q_j), \forall z, x \in U$ where f is a function that maps a criterion to a preference value for a nation. For instance, r_{xj} and r_{zj} are preference values of nation x and z with respect to q_j.*

Table 1
Four factors and twenty criteria of national competitiveness by WCY 2011.

Economic performance		Business efficiency	
q_1	Domestic economy	q_{11}	Productivity and efficiency
q_2	International trade	q_{12}	Labor market
q_3	International investment	q_{13}	Finance
q_4	Employment	q_{14}	Management practices
q_5	Prices	q_{15}	Attitudes and values
Government efficiency		Infrastructure	
q_6	Public finance	q_{16}	Basic infrastructure
q_7	Fiscal policy	q_{17}	Technological infrastructure
q_8	Institutional framework	q_{18}	Scientific infrastructure
q_9	Business legislation	q_{19}	Health and environment
q_{10}	Societal framework	q_{20}	Education

Proposition 3. *An induction rule $q_{j,t}^{\geqslant} \to Cl_t^{\geqslant}$ where $q_{j,t}^{\geqslant}$ is a set of nations within the top t positions with respect to q_j. This rule associates a dominating set to an upward union. It is independent to addition or removal of other criteria.*

Proposition 4. *CFD of an induction rule $g_j' = g'\left(q_{j,t}^{\geqslant} \to Cl_t^{\geqslant}\right)$ where g_j' is the conditional fuzzy density for $q_{j,t}^{\geqslant} \to Cl_t^{\geqslant}$ which is a unique value to present the degree that q_j supports nations to compete the top t positions. Technically, it is an accurate coverage rate, $0 \leqslant g_j' \leqslant 1$. Its derivation is described in Model I.*

Proposition 5. *Multiplying a CFD and a preference into a utility $u_k(q_j) = g_j' r_{kj}$ where r_{kj} represents a preference for nation k with respect to q_j, $0 \leqslant r_{kj} \leqslant 100$, and $u_k(q_j)$ is a function.*

Proposition 6. *Boundaries of competitiveness values (aggregated utilities) $u_k(\emptyset) = 0$ and $u_k(Q) \leqslant 100$ are assumed by WCY-IMD.*

Proposition 7. *λ-fuzzy measure of EFM $u_k(q_j, q_i) = u_k(q_j) + u_k(q_i) +$ - $_{i}) = u_k(q_j) + u_k(q_i) + \lambda u_k(q_j)u_k(q_i)$ where $j \neq i$, $\forall q_j, q_i \in Q$, λ is an interaction degree when utilities are aggregated in the extended fuzzy measure, $\lambda \geqslant -1$, and $\lambda \in \Re$.*

Proposition 8. *EFM function with m criteria $u_k(Q) = u_k(q_1, q_2, \ldots, q_m) = \frac{1}{\lambda}\left(\prod_{j=1}^{m}[1 + \lambda u_k(q_j)] - 1\right) = \frac{1}{\lambda}\left(\prod_{j=1}^{m}[1 + \lambda g_j' r_{kj}] \quad -1\right)$ is an extended fuzzy measure function for nation k by considering Proposition 5. This is derived in Section 3.4.*

3.3. Deriving the Conditional Fuzzy Densities (CFD)

In our design CFD plays one role as a CFD in Proposition 4 and the other role as a weight in Proposition 5. Its dual roles can integrate the fuzzy measure and the utility functions. Fig. 2 presents the concept of Proposition 4, $g_j' = g'\left(q_{j,t}^{\geqslant} \to Cl_t^{\geqslant}\right)$, where $q_{j,t}^{\geqslant}$ is a dominating set, $\left(q_{j,t}^{\geqslant} = \bigcup_{s \geqslant t} q_{j,s}\right)$, containing nations ranked in at least t^{th} with respect to criterion q_j. g_j' is the optimal production of the coverage, $CR(Cl_t^{\geqslant})$, and the accuracy rates, $\alpha(Cl_t^{\geqslant})$. To solve g_j', the boundaries of the lower and upper approximations are defined. The object x is defined for the boundary of $\underline{P}(Cl_t^{\geqslant})$ which contains objects at least as good as x with respect to criterion q_j. The object z is defined for the boundary of $\overline{P}(Cl_t^{\geqslant})$ which contains objects at least as good as z with respect to criterion q_j. These two boundary objects are presented as slash lines in the middle part of Fig. 2. For implementation, the optimization technique could be used to search the optimal CFD and find out the positions of x and z.

The derivation of Proposition 4 is implemented mathematically in *Model I*.

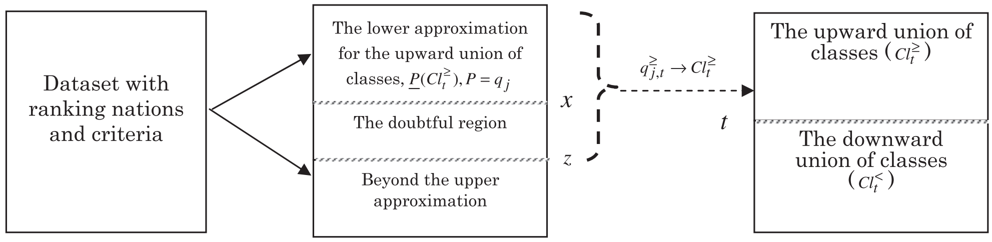

Fig. 2. The DRSA model.

Model I: Deriving g'_j

$$MAX\ g'_j = CR(Cl_t^{\geqslant}) \times \alpha(Cl_t^{\geqslant})$$

$$s.t.\ \underline{P}(Cl_t^{\geqslant}) = D_P^{+}(x),\ \ \overline{P}(Cl_t^{\geqslant}) = D_P^{+}(z), P = q_j$$
rank of x with respect to $P \geqslant$ rank of z with respect to $P \geqslant$ rank of t
$$CR(Cl_t^{\geqslant}) = \frac{|\underline{P}(Cl_t^{\geqslant})|}{|Cl_t^{\geqslant}|},\ \ \alpha(Cl_t^{\geqslant}) = \frac{|\underline{P}(Cl_t^{\geqslant})|}{|\overline{P}(Cl_t^{\geqslant})|}$$

where x and z are boundaries of Fig. 2, and CR and α are the coverage and accuracy rates. This model has been successfully programmed in *Lingo* 12.

3.4. The proof of the extended fuzzy measure function, $u_k(Q)$

The proof of Propositions 7 and 8 is presented here. In the proof statements the preference value of nation k with respect to q_j is presented as r_{kj} and each criterion is expressed as $q_j(r_{1j}, r_{2j}, \ldots, r_{kj}, \ldots, r_{58j})$.

Proof. $u_k(Q) = \frac{1}{\lambda}\left(\prod_{j=1}^{m}[1 + \lambda u_k(q_j)] - 1\right) = \frac{1}{\lambda}\left(\prod_{j=1}^{m}[1 + \lambda g'_j r_{kj}] - 1\right)$ where $\lambda \neq 0$, λ is the interaction degree, and m is the number of criteria. □

Assumptions:

1. $u_k(Q) = u_k(q_1, q_2, \ldots, q_m)$ is treated as a utility function and an extended fuzzy measure function for nation k.
2. $u_k\left(q_j^0\right) = 0$, where q_j^0 has the lowest utility with respect to criterion q_j.
3. $u_k\left(q_j^*\right) \leqslant 100$, where q_j^* has the optimum of utility with respect to criterion q_j.
4. $u_k(q_1^*, q_2^*, \ldots, q_m^*) \leqslant 100$ is limited by the competitiveness scale of WCY dataset.
5. g'_j represents the conditional fuzzy density of q_j for $Cl_t^{\geqslant}$ and $0 \leqslant g'_j \leqslant 1$.
6. $u_k(q_j) = u_k\left(q_1^0, q_2^0, \ldots, q_{j-1}^0, q_j, q_{j+1}^0, \ldots, q_m^0\right) = g'_j r_{kj}$ represents a utility of nation k with respect to q_j. $u_k(q_j)$ is independent from other criteria same as $q_{j,t}^{\geqslant} \rightarrow Cl_t^{\geqslant}$ independent from the other rules.
7. $\bar{u}_k(q_j) = u_k(q_1, q_2, \ldots, q_{j-1}, q_j^0, q_{j+1}, \ldots, q_m) = u_k(\bar{q}_j)$ is the complement of $u_k(q_j)$, where $\{q_1, q_2, \ldots, q_{j-1}, q_j^0, q_{j+1}, \ldots, q_m\} = \{\bar{q}_j\}$.

By following the utility theory in trading preferences, we start deducing statements as below.

Let $i \neq j$, $c_j(q_j)$ and $c_i(q_i)$ are scale factors [23].

$$u_k(q_j, q_i) = u_k(q_j) + c_j(q_j)u_k(q_i) \tag{1}$$
$$= u_k(q_i) + c_i(q_i)u_k(q_j) \tag{2}$$

According to (1) and (2), $\frac{c_j(q_j)-1}{u_k(q_j)} = \frac{c_i(q_i)-1}{u_k(q_i)} = \lambda$ can be assumed and the utility for a criterion can be found in (3).

$$\text{So}, c_j(q_j) = 1 + \lambda u_k(q_j) \tag{3}$$

By substituting (3) into (1) and (2), we obtain Proposition 7 as (4).

$$u_k(q_j, q_i) = u_k(q_j) + u_k(q_i) + \lambda u_k(q_j)u_k(q_i) \tag{4}$$

Furthermore, a utility function with a full range of criteria can be formulated as (5).

$$\begin{aligned} u_k(Q) &= u_k(q_1) + c_1(q_1)u_k(q_2, q_3, \ldots, q_m) \\ &= u_k(q_1) + c_1(q_1)[u_k(q_2) + c_2(q_2)u_k(q_3, q_4, \ldots, q_m)] \\ &= u_k(q_1) + c_1(q_1)u_k(q_2) + c_1(q_1)c_2(q_2)u_k(q_3) + \cdots + \cdots \\ &\quad + c_1(q_1)\cdots c_{m-1}(q_{m-1})u_k(q_m) \end{aligned} \tag{5}$$

By substituting (3) into (5), we obtain the extended fuzzy measure function (6) for all criteria.

$$u_k(Q) = u_k(q_1) + \sum_{i=2}^{m} u_k(q_i)\prod_{j=1}^{i-1}[1 + \lambda u_k(q_j)] \tag{6}$$

The expression (6) has two cases below.

Case (i): $\lambda = 0$

$$u_k(Q) = \sum_{j=1}^{m} u_k(q_j) = \sum_{j=1}^{m} g'_j r_{kj} \tag{7}$$

Case (ii): $\lambda \neq 0$

$$1 + \lambda u_k(Q) = \prod_{j=1}^{m}(1 + \lambda u_k(q_j)) \text{ due to}$$

$$\begin{aligned} 1 + \lambda u_k(Q) &= 1 + \lambda u_k(q_1) + \sum_{i=2}^{m}\prod_{j=1}^{i-1}[1 + \lambda u_k(q_j)]\lambda u_k(q_i) \\ &= [1 + \lambda u_k(q_1)][1 + \lambda u_k(q_2)] + \sum_{i=3}^{m}\prod_{j=1}^{2}[1 + \lambda u_k(q_j)]\lambda u_k(q_i) \\ &= \prod_{j=1}^{2}[1 + \lambda u_k(q_j)] + \sum_{i=3}^{m}\prod_{j=1}^{2}[1 + \lambda u_k(q_j)]\lambda u_k(q_i) \\ &= \prod_{j=1}^{3}[1 + \lambda u_k(q_j)] + \sum_{i=4}^{m}\prod_{j=1}^{3}[1 + \lambda u_k(q_j)]\lambda u_k(q_i) \\ &= \prod_{j=1}^{m}[1 + \lambda u_k(q_j)] \quad \text{where } \lambda \neq 0 \text{ due to } u_k(q_j) \\ &= u_k\left(q_1^0, q_2^0, \cdots, q_{j-1}^0, q_j, q_{j+1}^0, \cdots, q_m^0\right). \end{aligned}$$

So, the extended fuzzy measure function for Q is reformulated as (8).

$$\begin{aligned} 1 + \lambda u_k(Q) &= \prod_{j=1}^{m}[1 + \lambda u_k(q_j)] \Longleftrightarrow u_k(Q) \\ &= \frac{1}{\lambda}\left(\prod_{j=1}^{m}[1 + \lambda u_k(q_j)] - 1\right) \end{aligned} \tag{8}$$

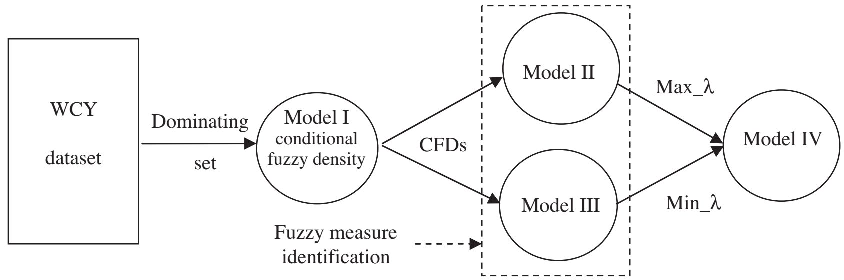

Fig. 3. Fuzzy measure identification of EFM.

3.5. Identifying the range of interaction degree

λ of Proposition (8) should exist in the range of Proposition 6. Fig. 3 gives an illustration.

By accepting the CFDs derived from Model I, Model II can backwardly find out the maximum boundary of λ where the top score in WCY 2011 is 100 from US. Model III can backwardly find out the minimum boundary of λ where the bottom score is 35.25 from Venezuela.

Model II: MAX λ

$$u_k(Q) \leqslant 100, \max\_\lambda = \lambda, \quad u_k(Q) = \frac{1}{\lambda}\left(\prod_{j=1}^{m}[1 + \lambda g_j' r_{kj}] - 1\right)$$

Model III: MIN λ

$$u_k(Q) \geqslant 35.25, \min\_\lambda = \lambda, \quad u_k(Q) = \frac{1}{\lambda}\left(\prod_{j=1}^{m}[1 + \lambda g_j' r_{kj}] - 1\right)$$

Model IV is designed to aggregate utilities with a resulted λ for nations' factors. Its λ choice focuses on the smallest interaction, i.e., max_λ and leaves the biggest interaction among factors as possible. The significant correlation between each two factors thus can be caught.

Model IV:

$$u_k(Q_{\text{economy}}), Q_{\text{economy}} = \{q_1, q_2, \ldots, q_5\}, \lambda = \mathbf{max}\_\lambda$$
$$u_k(Q_{\text{government}}), Q_{\text{government}} = \{q_6, q_7, \ldots, q_{10}\}, \lambda = \mathbf{max}\_\lambda$$
$$u_k(Q_{\text{business}}), Q_{\text{business}} = \{q_{11}, q_{12}, \ldots, q_{15}\}, \lambda = \mathbf{max}\_\lambda$$
$$u_k(Q_{\text{infrastructure}}), Q_{\text{infrastructure}} = \{q_{16}, q_{17}, \ldots, q_{20}\}, \lambda = \mathbf{max}\_\lambda$$

Two points make EFM different from Sugeno's. First, summation of CFD might not be restricted to one. This arises from the difference between conditional probabilities and outcome probabilities. Second, EFM has a closed interval for the interaction degree, min_$\lambda \leqslant \lambda \leqslant$ max_λ, while the traditional fuzzy measure does not.

4. Results

The conditional fuzzy densities of Proposition 4 for the upper half nations of WCY 2011 are calculated by Model I and summarized in the left part of Table 2. The results reveal some disclosures. First, none of the conditional fuzzy densities equals to 1, which means none of criteria can completely classify dominance classes. Second, the resulted λ, $-0.028 \leqslant \lambda \leqslant -0.010$, has min_$\lambda = -0.028$ and max_$\lambda = -0.010$. Obviously, only a single type of interaction exists for the aggregation. This makes EFM reliable in analysis. Third, institutional framework (q_8) plays a leading criterion in supporting nations to achieve the upper half positions with the conditional fuzzy density 0.87. Fourth, the government and business efficiencies have correlation coefficient up to 0.85 by choosing $\lambda = -0.01$, presented in the right part of Table 2. They are obtained from the fuzzily measured factors $u_k(E)$, $u_k(G)$, $u_k(B)$, and $u_k(I)$ where $E = \{q_1, q_2, q_3, q_4, q_5\}$, $G = \{q_6, q_7, q_8, q_9, q_{10}\}$, $B = \{q_{11}, q_{12}, q_{13}, q_{14}, q_{15}\}$, and $I = \{q_{16}, q_{17}, q_{18}, q_{19}, q_{20}\}$.

In a summary, this research has achieved three merits. First, the fuzzy measure was successfully extended on '*if*…,*then*…' implications. Second, the compound components (preferences and implication probabilities) were successfully multiplied into utilities for fuzzy measurements. Third, the fuzzy measure identification was successfully fulfilled for WCY 2011. These findings are further discussed in Section 5 and a case study on Greece, Italy, Portugal, and Spain is also presented.

5. Discussions and implications

This research uses EFM instead of Choquet's integral to aggregate utilities. There are two reasons for this choice. Firstly, a huge number of additions up to $n(2^m - 1)$ times have to be considered by Choquet's integral when involving m criteria and n nations. Secondly, the corresponding preferences to the conditional fuzzy densities are hard to get for Choquet's integral. In our empirical experiments, EFM processes computation as easy as a utility function.

According to Section 4, the resulted λ belongs to the interaction type (iii) below. This satisfies the requirement 'only one type of interaction effect exists for competitiveness utilities'. Three mathematical formulae related to λ are illustrated below.

(i) $\lambda > 0: \frac{1}{\lambda}\left[\prod_{j=1}^{m}(1 + \lambda u_k(q_j) - 1\right] > \sum_{j=1}^{m} u_k(q_j)$. The aggregated utility has additional effects than the expected.

(ii) $\lambda = 0: \frac{1}{\lambda}\left[\prod_{j=1}^{m}(1 + \lambda u_k(q_j) - 1\right] = \sum_{j=1}^{m} u_k(q_j)$. The aggregated utilities just fit. Neither overlapping nor additional effect than the expected.

Table 2
The resulted CFD and λ of the extended fuzzy measure for WCY 2011.

Conditional fuzzy densities								Factors correlation				
E		G		B		I			E	G	B	I
g_1'	0.65	g_6'	0.49	g_{11}'	0.74	g_{16}'	0.77	E	1	0.67	0.74	0.49
g_2'	0.51	g_7'	0.31	g_{12}'	0.52	g_{17}'	0.77	G		1	**0.85**	0.73
g_3'	0.57	g_8'	**0.87**	g_{13}'	0.77	g_{18}'	0.74	B			1	0.62
g_4'	0.52	g_9'	0.74	g_{14}'	0.83	g_{19}'	0.65	I				1
g_5'	0.34	g_{10}'	0.68	g_{15}'	0.63	g_{20}'	0.59	Chosen $\lambda = -0.01$				

Note: E: Economic, G: Government, B: Business, I: Infrastructure.

Table 3
Interaction effects of EFM and LD.

Interaction types		λ interval within the slash lines	
		EFM $-0.028 \le \lambda \le -0.010$ when $t = 29$	LD $-0.010 \le \lambda \le 0.020$ when $t = 29$
Super-additive effects	$\lambda > 0$		
Additive effects	$\lambda = 0$		Three effects mixed together
Sub-additive effects	$-1 < \lambda < 0$		

Table 4
Merits of related techniques.

	LD	DRSA	Utility theory	Fuzzy measure	EFM
Driving implication probabilities (densities)	Y	Y	N	N	Y
Non-additive aggregation	N	N	Y	Y	Y
Identifying interaction type	N	N	Y	N	Y
Total merits	1	1	2	1	3

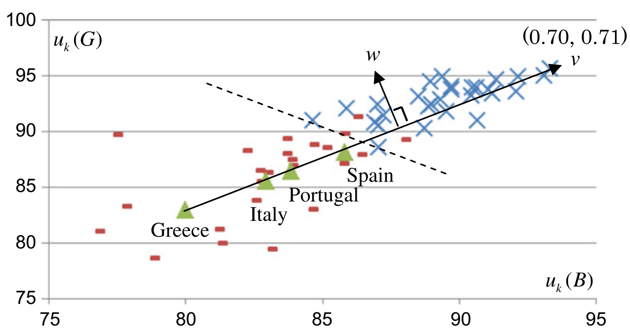

Fig. 4. The dominance pattern and correlation feature of government and business factors.

(iii) $\lambda < 0 : \frac{1}{\lambda}\left[\prod_{j=1}^{m}(1 + \lambda u_k(q_j) - 1\right] < \sum_{j=1}^{m} u_k(q_j)$. The aggregated utility has overlapping thus makes competitiveness less than the expected of independent utilities.

The maximum and minimum boundaries of λ can be used to verify the quality of the conditional fuzzy densities. To give a clear illustration about this merit, an experiment is designed to compare EFM and the linear discriminant method (LD) [38] in the mixed interaction and weighting problems. These two methods respectively use the dataset of WCY 2011 to generate a set of fuzzy densities and a set of regression weights for the upper half nations. The regression weights are lists with gray background in Table A1 of Appendix A, which is processed by a complete classification. Both adopt Model II and III to find out their intervals of λ which is presented in slash area of Table 3.

The comparison results show that EFM does not have mixed interaction problems while LD has difficulty in determining an interaction type. Moreover, LD has many weights equal to zero, which opposes the WCY's assertion that each criterion has its own influence in the competitiveness. For instance, four criteria weights of business efficiency equal to zero.

To compare merits among related techniques for aggregation analysis, driving implication probabilities (CFD or weights), non-additive aggregation, and identifying interaction type are used as comparative items in Table 4.

As we can see, EFM has all the merits. The fuzzy measure neither provides implication probabilities nor identifies interaction type. The linear discriminant method can generate regression weights however can neither identify interaction effect nor aggregate non-additively. DRSA can drive implication probabilities however cannot handle utilities aggregation and identify interaction effects. The utility theory might non-additively aggregate and identify interactions while requiring decision makers' knowledge to provide weights. EFM combines merits from DRSA, utility theory, and the fuzzy measure to highlight important information in components analysis.

Here a case study of applying EFM on WCY 2011 is conducted to highlight competitiveness features, patterns, and trends for Greece, Italy, Portugal, and Spain. A correlation feature between the government and business efficiencies is presented in Fig. 4 in which the vertical axis is scaled by $u_k(G)$, the horizontal axis is scaled by $u_k(B)$, the interaction degree is chosen $\lambda = -0.01$, and the government efficiency goes up with the increase of the business efficiency. Alternatively, Fig. 4 has two dominance patterns which are visually formed by a dashed line for classification. The upper half nations belong to the dominating (right) side and the bottom half nations belong to the dominated (left) side. Only two nations violate this rule. The scale of this pattern highlights that Spain has a bigger potential to approach the dominating positions while Greece stays away from the classification line.

Model V is designed to find out the competitiveness characteristics for the pattern based on $(u_k(B), u_k(G))$. It solves the minimum square distance from $c_g u_k(G) + c_b u_k(B) + h = 0$ (k is a nation) in which the characteristics of the pattern is presented as the solid line v, c is a coefficient vector (c_g, c_b) for $(u_k(B), u_k(G))$, and c and v perpendicular each other. The resulted values of v is (0.70, 0.71) which gives the change rates between government and business efficiencies. Currently, Greece, Italy, Portugal, and Spain have the same problem in debts. They stand close to the solid line v, signifying their government and business has tight relationship. If a proposal can lead a nation to move along the solid line v then it can help these four nations.

Model V:

$$MIN = (c_g u_k(G) + c_b u_k(B) + h)^2 k : \text{an index for nations}$$
$$s.t.\ c_g + c_b = 1 \text{ where } c_g \text{ and } c_b \text{ represent coefficients}$$
$$\text{for } u_k(G) \text{ and } u_k(B), \text{respectively.}$$
$$v \bullet c = 0 \text{ where } v = (v_g, v_b) \text{ and } c = (c_g, c_b)$$

The historical trends of government and business efficiencies in WCY 1997–2011 are presented as Fig. 5 with correlation

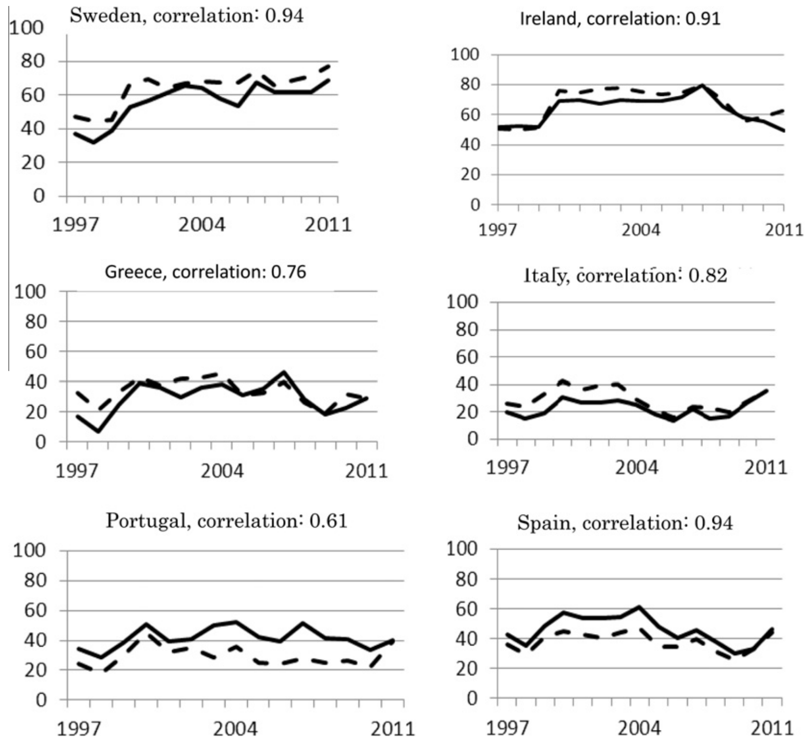

Fig. 5. The trends of government and business efficiencies, WCY 1997 ~ 2011.

Table A1
The conditional fuzzy densities of EFM and weights of LD for WCY 2011.

Economic performance			Government Efficiency			Business Efficiency			Infrastructure		
g'_1	0.65	0.01	g'_6	0.49	0.03	g'_{11}	0.74	0.00	g'_{16}	0.77	0.00
g'_2	0.51	0.02	g'_7	0.31	0.19	g'_{12}	0.52	0.00	g'_{17}	0.77	0.00
g'_3	0.57	0.05	g'_8	**0.87**	0.00	g'_{13}	0.77	0.21	g'_{18}	0.74	0.07
g'_4	0.52	0.04	g'_9	0.74	0.00	g'_{14}	0.83	0.00	g'_{19}	0.65	0.14
g'_5	0.34	0.14	g'_{10}	0.68	0.00	g'_{15}	0.63	0.00	g'_{20}	0.59	0.09

Note : the columns with gray background for LD and white background for CFD.

coefficients, Greece (0.76), Ireland (0.91), Italy (0.82), Portugal (0.61), and Spain (0.94). The vertical axis is the performance scale from 0 to 100 for government or business efficiencies. The horizontal axis extends years from 1997 to 2011. The solid lines represent the historical tracks of government efficiency and the dashed lines for the business efficiency. These historical correlations verify the effectiveness of EFM.

Usually people assume government should lead business development. For these six nations, Greece, Italy, Ireland, Portugal, and Spain were different from Sweden which has been getting better in government and business efficiencies. Further, Ireland ever performed well before the financial crisis in 2008. Her competitiveness behavior is different from the rest four nations. In order to get details inside of Greece, Italy, Portugal, and Spain the government efficiency in 2011 is analyzed in Model VI by approximating the minimum square distance to a hyper plane.

Model VI:

$$MIN = (c_6 r_{k6} + c_7 r_{k7} + c_8 r_{k8} + c_9 r_{k9} + c_{10} r_{k10} + h)^2$$
$k \in$ Greece, Italy, Portugal, Spain
$s.t.\ c_6 + c_7 + c_8 + c_9 + c_{10} = 1, c_j$ represents a coefficient for q_j, $j = 6, \ldots, 10$
$v \bullet c = 0, c = (c_6, c_7, c_8, c_9, c_{10})$ and $v = (v_6, v_7, v_8, v_9, v_{10})$

Model VI discloses that public finance (q_6), fiscal policy (q_7), institutional framework (q_8), business legislation (q_9), and societal framework (q_{10}) of these four nations with $v = (0.27, 0.28, 0.26, 0.86, -0.22)$. Its values imply that the improvement of societal framework might scarify public finance, fiscal policy, institutional framework, and business legislation, or vice versa. If an improving proposal in societal framework need not sacrifice public finance, fiscal policy, institutional framework, and business legislation then these four nations will have a chance to improve competitiveness.

In the future work, the multi-objective programming might play an important role to discover a MCDM proposal for Greece, Italy, Portugal, and Spain. Alternatively, national happiness might give another thinking to enhance people's life. Applying the happiness and competitiveness together to overcome debt crisis is a good issue in the future, too.

6. Concluding remarks

This research has achieved some merits: Successfully extending the fuzzy measure on '*if*...,*then*...' implications, empowering the fuzzy measure to aggregate utilities, fulfilling the fuzzy measure identification for the aggregated utilities, implying correlations to highlight competitiveness features, and estimating characteristics of government efficiency for Greece, Italy, Portugal, and Spain. Furthermore, the case study on WCY 2011 provides some disclosures of intelligent analysis. A competitiveness feature discloses that government and business efficiencies are highly correlated. A dominance pattern shows Greece, Italy, Portugal, and Spain belong to a less competitiveness class while Spain has a bigger potential to achieve the upper half positions. The historical trends during 1997–2011 reveal Sweden successfully overcame the global financial crisis in 2008 due to government and business efficiencies getting better. Conversely, Greece, Italy, Portugal, and Spain have been keeping their government efficiencies in the lower performance. In the future a proposal improving societal framework without sacrificing public finance, fiscal policy, institutional framework, and business legislation will help these four nations to grow stably.

Appendix A

See Table A1.

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