Profitability evaluation for newsboy-type product with normally distributed demand

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Abstract: This paper considered newsboy-type products with a normally distributed demand, and defined a product's profitability as the probability of achieving the target profit under optimal ordering condition. In order to determine whether the product is already unworthy of being ordered (or manufactured) in a competitive market, we conducted a profitability evaluation which examines whether the profitability meets a designated requirement. As the parameters μ and σ are unknown, we introduced a new index (achievable capacity index; I_A) which has a simple form expression of the product's profitability, and found an unbiased and effective estimator of I_A to estimate the actual I_A. In addition, we utilised a statistical hypothesis testing methodology. The critical value of the test was calculated to determine the evaluation results. The sample size required for the designated power and confidence level was also investigated. An application example for a fresh food product was provided to illustrate the utilisation of the proposed approach. [Received 24 July 2010; Revised 13 February 2011; Accepted 02 May 2011]

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1 Introduction

The traditional 'newsboy problem' focused on short shelf-life products such as daily newspapers, monthly or weekly magazines, seasonal products, or fresh foods. For such products, surplus stock cannot be sold in the next period and requires additional costs for disposal. If the ordering or manufacturing quantity is lower than actual demand, the seller will lose possible sales. Demand for such products is unknown, and always assumed to be a random variable with a known probability distribution. Therefore, the determination of the ordering/manufacturing quantity is critical in the newsboy problem. Several extensions to the newsboy problem have been proposed and discussed in the literature. Among those extensions have alternative objective functions such as minimising the expected cost (Nahmias, 1993), maximising the expected profit (Khouja, 1995), maximising the expected utility (Ismail and Louderback, 1979; Lau, 1997), and maximising the probability of achieving a target profit (Ismail and Louderback, 1979; Shih, 1979; Lau, 1980; Sankarasubramanian and Kumaraswamy, 1983). In fact, these maximum and minimum values can be adopted to measure the product's capacity. For instance, the maximum expected profit and maximum probability of achieving the target profit can be used as a measure of the product's profitability. Unfortunately, the existing literature examining inventory management is focused only on the optimal ordering or manufacturing quantity, and does not take into account the importance of these values. On the other hand, almost all of the literature on inventory model assumed that the demand is deterministic and constant. Unfortunately, in fact, the demand is always unknown. Ertogral (2011) considered the joint economic lot sizing problem (JELP) under stochastic demand. Several studies also considered that demand is a random variable and follows some common distribution such as uniform, exponential, normal and so on, in which the parameters of these distributions are assumed to be known. However, in reality, these parameters may be unknown. Therefore, the extent of applicability of such studies to managerial aspects of inventories depends on the estimation of demand parameters. Berk et al. (2007) recognised two general approaches for demand estimation: the Frequentist and the Bayesian. Kevork (2010) developed appropriate estimators for the optimal ordering quantity and the maximum expected profit when demand is normally distributed, and investigated their statistical properties for both small and large samples, analytically and through Monte-Carlo simulations. Furthermore, most of the research focused on a distribution-free newsboy problem, in which the form of the demand distribution is not known, and only the mean and variance are specified. Scarf (1958) pioneered a 'minimax' approach, which aims to minimise the maximum cost resulting from the worst possible demand distribution. This approach can derive a simple closed-form expression for the ordering quantity, which maximises expected profit. Extending this distribution-free newsboy problem, Moon and Choi's (1995) study included balking; Ouyang and Wu (1998) considered a variable lead time; Ouyang and Chang (2002) extended to consider a variable lead time with fuzzy lost sales; Alfares and Elmorra (2005) took the shortage cost into consideration; and Mostard et al. (2005) analysed resalable returns. In their study, the ordering quantities for normal, lognormal and uniform cases are derived, and they found that the distribution-free order rule performs well when the coefficient of variation (cv) is at most 0.5, but far from optimal when the cv is large.

1.1 Profitability evaluation problem

Inventory management encompasses the principles, concepts and techniques for deciding

1. how much to order
2. when to order
3. how and where to store the order.

At present, in many inventory control systems, new products are continually introduced. However, if a new product is recommended, the ordering quantity of an existing product will be curtailed due to the spatial restriction in the warehouse. An existing product may be substituted completely by a new product if the existing product does not have a good capacity. Therefore, product capacity evaluation is a practical problem occurring frequently in inventory control. Before exploring product capacity evaluation, one should select an appropriate criterion for measuring a product's capacity. To the best of our knowledge, criteria such as profitability, quality, reputation, fashion, and performance express a product's capacity, especially, the profitability is a common criterion. For example, Trubint et al. (2006) adopted profitability, quality of service and urban construction as the criteria for finding optimal retail outlet locations. Steers (1975) measured organisational effectiveness with profitability and market share. With regards to profitability evaluation, Pekka and Jukka (2002) investigated profitability evaluation for intelligent transport system (ITS) investments. Unfortunately, the profitability evaluation problem has not been explored in inventory management. In our paper, we focused on newsboy-type products, with a normally distributed demand. Our main purpose is to study the profitability evaluation problem, which deals with examining whether a product's profitability meets a designated requirement. Profitability is defined as the probability of achieving a target profit under an optimal ordering condition. In order to make the problem more relevant and applicable in practice, we assumed that the demand mean and demand standard deviation are unknown. We used a statistical hypothesis testing methodology to examine this evaluation problem. The remainder of the paper is organised as follows. In the next section, the notation and assumptions related to this study are presented. In Section 3, we examine the profitability measurement. In this section, by using the relationship between demand properties (μ and σ) and target demand, we attempt to develop a new index [achievable capacity index (ACI); I_A] which has a simple form expression of the product's profitability. We then explore the relationship between the profitability and the value of I_A. In Section 4, we find an unbiased and effective estimator of I_A to estimate the actual I_A, and implement the profitability evaluation by using a statistical hypothesis testing methodology. The critical value of the test is calculated to determine the evaluation results. The sample size required for a designated power and confidence level is also investigated. In the last two sections, the profitability evaluation for a fresh food product is applied to illustrate the utilisation of our approach, and finally concluding remarks are provided.

2 Notation and assumptions

We consider a newsboy-type product with normally distributed demand. The surplus stock and unsatisfied demand must pay the disposal cost and opportunity cost, respectively. In addition, we define the profitability as the probability of achieving the target profit under an optimal ordering condition, in which the target profit is predetermined according to the product property and the sales experience. The main aim of this paper is to investigate the profitability evaluation problem which deals with examining whether the product's profitability meets designated requirement. The evaluation results can determine whether the existing product is already unworthy of being ordered (or manufactured) in the competitive market.

2.1 Notation

For convenience, the notations used in this study are as below:

p	selling price per unit
c	purchasing/manufacturing cost per unit
c_p	net profit per unit (i.e., $c_p = p - c$)
c_d	disposal cost for a surplus product
c_e	excess cost per unit (i.e., $c_e = c_d + c$)
c_s	shortage cost per unit (i.e., the lost sale opportunity cost)
k	target profit
T	target demand
Q	ordering quantity
D	demand during a period, which is a random variable
$f(\cdot)$	probability density function of D
Z	profit during a period
$\mathbb{P}$	profitability for the newsboy-type product
I_A	achievable capacity index.

2.2 Assumptions

The following assumptions are used throughout this paper:

1. consider the newsboy-type product with normally distributed demand, $N(\mu, \sigma)$
2. to make the problem more relevant, the parameters μ and σ are unknown, but satisfied that $cv = \mu/\sigma < 0.3$ for neglecting the negative tail, i.e., $f(D < 0) = \Phi(-\mu/\sigma) = \Phi(-1/cv) < \Phi(-1/0.3) \approx 0$
3. the target demand is the minimal demand required for satisfying the target profit, i.e., $T = k/(p - c) = k/c_p$

4 in order to possibly achieve the target profit, the ordering quantity must be greater than or equal to target demand, i.e., $Q \geqslant T$.

3 Profitability measurement

3.1 ACI I_A

If the related parameters (p, c, c_d, c_s, and T) and optimal ordering quantity are given, the level of profitability depends on the demand mean μ and the demand standard deviation σ. Therefore, we develop a new index to express the product's profitability, and so-called 'ACI'. It is defined as:

$$I_A = \frac{\mu - T}{\sigma}.$$

The numerator of I_A provides the difference between demand mean and target demand. The denominator gives demand standard deviation. Obviously, it is desirable to have a I_A as large as possible.

3.2 Profitability and I_A

Based on Sankarasubramanian and Kumaraswamy (1983), the profit function, Z, depends on the demand and ordering quantity, and is formulated as follows:

$$Z = \begin{cases} pD - c_d(Q - D) - cQ = (c_p + c_e)D - c_e Q, & 0 \leqslant D \leqslant Q, \\ pQ - c_s(D - Q) - cQ = -c_s D + (c_p + c_s)Q, & Q < D < \infty. \end{cases}$$

Note that if the surplus products can be salvaged, the value of c_d is negative and redefine into salvage price. For any $Q \geqslant T$, Z is strictly increasing in $D \in [0, Q]$ and strictly decreasing in $D \in [Q, \infty)$, and has a maximum at point $D = Q$. The maximum value of Z is larger than or equal to k, i.e., $Z = pD - cQ = c_p D = c_p Q \geqslant c_p T = k$. The target profit will be realised when D is equal to either $LAL(Q)$ or $UAL(Q)$, so the target profit will be achieved in $D \in [LAL(Q), UAL(Q)]$, where

$$LAL(Q) = \frac{c_e Q + k}{c_p + c_e} \quad \text{and} \quad UAL(Q) = \frac{(c_p + c_s)Q - k}{c_s}$$

are the lower and upper achievable limits, respectively, and both are the functions of Q. Note that since $Q \geq T$ presented in the Section 2.2, assumption (4),

$$UAL(Q) - LAL(Q) = \frac{c_p(c_p + c_e + c_s)}{c_s(c_p + c_e)} \times (Q - T) \geq 0.$$

Than we can clear that $UAL(Q) \geq LAL(Q)$. Under the assumption that the demand is normally distributed, the probability of achieving the target profit is:

$$Pr[Z \geqslant k] = \Phi\left(\frac{UAL(Q) - \mu}{\sigma}\right) - \Phi\left(\frac{LAL(Q) - \mu}{\sigma}\right), \tag{1}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

Before calculating the profitability, we first find the optimal ordering quantity that maximises $Pr[Z \geqslant k]$. We take the first-order of $Pr[Z \geqslant k]$ with respect to Q, and obtain

$$\frac{\mathrm{d}Pr[Z \geqslant k]}{\mathrm{d}Q} = \frac{1}{\sqrt{2\pi}\sigma}\left[\frac{c_p + c_s}{c_s} e^{-\frac{1}{2}\left(\frac{UAL(Q)-\mu}{\sigma}\right)^2} - \frac{c_e}{c_p + c_e} e^{-\frac{1}{2}\left(\frac{LAL(Q)-\mu}{\sigma}\right)^2}\right].$$

It is well known that the necessary condition for Q to be optimal must satisfy the equation $\mathrm{d}Pr[Z \geqslant k]/\mathrm{d}Q = 0$, which implies

$$\mu = \frac{UAL(Q) + LAL(Q)}{2} - \frac{\omega\sigma^2}{UAL(Q) - LAL(Q)}, \tag{2}$$

where $\omega = \ln\left[1 + c_p A / c_s c_e\right] > 0$ and $A = c_p + c_e + c_s$. For $Q \geqslant T$, the unique optimal ordering quantity can be obtained by solving Eq. (2), i.e.,

$$\begin{aligned} Q^* = T &+ \tfrac{c_s(c_p+c_e)(c_p\mu-k)}{c_p(c_pA+2c_ec_s)} \\ &+ \sqrt{\left[\tfrac{c_s(c_p+c_e)(c_p\mu-k)}{c_p(c_pA+2c_ec_s)}\right]^2 + \tfrac{2c_s^2(c_p+c_e)^2\omega\sigma^2}{c_pA(c_pA+2c_ec_s)}} > T. \end{aligned} \tag{3}$$

In addition, the sufficient condition is also calculated as follows:

$$\begin{aligned} \left.\frac{\mathrm{d}^2 Pr[Z \geqslant k]}{\mathrm{d}Q^2}\right|_{Q=Q^*} = &- \frac{(c_p + c_s)e^{-\frac{1}{2}\left(\frac{UAL(Q^*)-\mu}{\sigma}\right)^2}}{\sqrt{2\pi}\sigma^3 {c_s}^2 (c_p + c_e)} \\ &\times \left\{ \frac{\left[UAL(Q^*) - LAL(Q^*)\right](c_pA + 2c_ec_s)}{2} \right. \\ &\left. + \frac{c_pA\omega\sigma^2}{UAL(Q^*) - LAL(Q^*)} \right\} < 0. \end{aligned}$$

We can conclude that the stationary point Q^* is a global maximum. By using equation (2) and substituting equation (3) into equation (1), the profitability, $\mathbb{P}$, can be obtained as follows:

$$\mathbb{P} = \Phi\left(G + \frac{\omega}{2G}\right) - \Phi\left(-G + \frac{\omega}{2G}\right), \tag{4}$$

where

$$\begin{aligned} G = \frac{UAL(Q^*) - LAL(Q^*)}{2\sigma} &= M\left(\frac{\mu - T}{\sigma}\right) + \sqrt{M^2\left(\frac{\mu - T}{\sigma}\right)^2 + M\omega} \\ &= MI_A + \sqrt{M^2 {I_A}^2 + M\omega} > 0, \end{aligned}$$

and

$$M = \frac{c_pA}{2(c_pA + 2c_ec_s)} > 0.$$

It is easy to see that $\mathbb{P}$ is a function of I_A. Taking the first-order derivative of $\mathbb{P}(I_A)$ with respect to I_A, we obtain

$$\frac{\mathrm{d}\mathbb{P}(I_A)}{\mathrm{d}I_A} = \frac{MG}{\sqrt{2\pi}\sqrt{MI_A{}^2 + M\omega}}\left[e^{\omega} + 1 + \frac{\omega}{2G^2}\left(e^{\omega} - 1\right)\right] e^{-\frac{1}{2}\left(G + \frac{\omega}{2G}\right)^2} > 0.$$

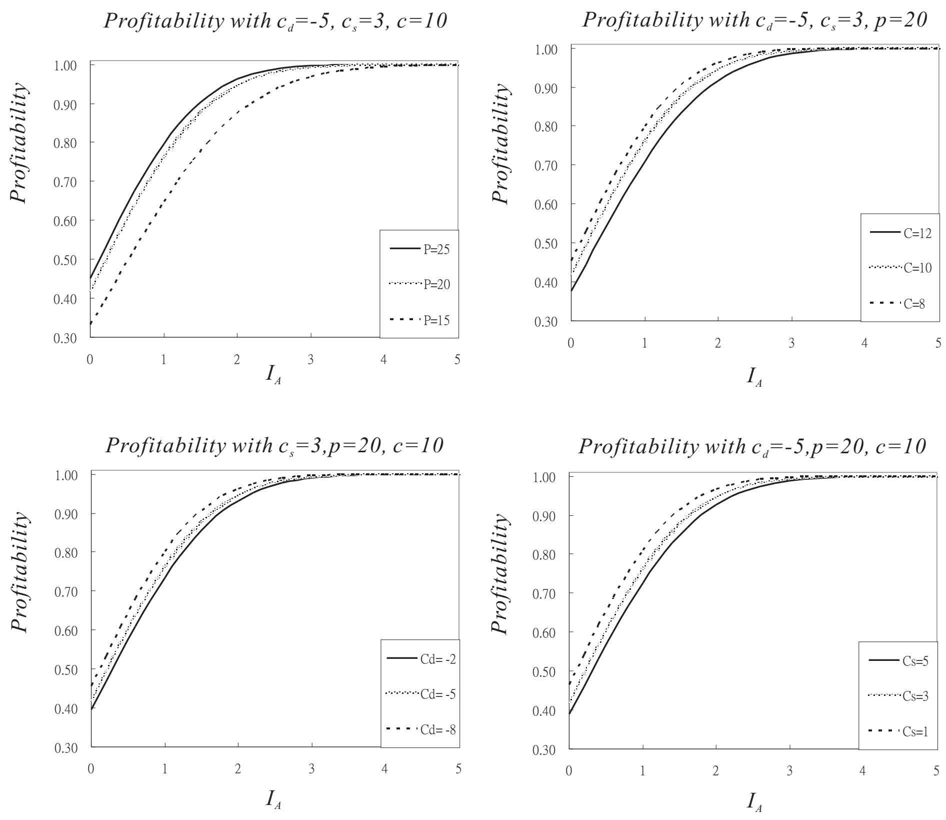
As a result, $\mathbb{P}(I_A)$ is a strictly increasing function of I_A. Therefore, we can express the product's profitability according to the value of I_A. Based on the parameters $p = 20$, $c = 10$, $c_d = -5$, and $c_s = 3$, Figure 1 plots the profitability versus various values of I_A for the effects of changes in the parameters p, c, c_d, and c_s. From Figure 1, the following observations can be made:

1 With increase in the value of I_A, the product's profitability increases. Obviously, it is desirable to have a I_A as large as possible.

2 When the value of parameter p increases, the product's profitability increases. It implies that if the customers can satisfy the price changes, the product's profitability is going to be increased when the selling price increases.

3 The product's profitability decreases as c, c_d, and c_s increase. If the purchasing (or manufacturing) cost per unit, disposal cost for a surplus product and shortage cost per unit could be reduced effectively, the product's profitability could be improved.

Figure 1 Profitability versus various values of I_A for the effects of changes p, c, c_d, and c_s



4 Profitability evaluation problem

4.1 Estimation of I_A

The historical data of the demand ought to be collected in order to estimate the actual I_A due to unknown μ and σ. First, the natural estimator $\hat{I}_A$ is considered. If a sample of size n is given as $\{x_1, x_2, ..., x_n\}$, the natural estimator $\hat{I}_A$ is obtained by replacing the μ and σ by their estimators $\overline{x} = \sum_{i=1}^{n} x_i/n$ and $s = \left[\sum_{i=1}^{n}(x_i - \overline{x})^2/(n-1)\right]^{1/2}$, i.e.,

$$\hat{I}_A = \frac{\bar{x} - T}{s}.$$

Furthermore, we rewrite the natural estimator $\hat{I}_A$, and obtain

$$\begin{aligned} \hat{I}_A &= \tfrac{\bar{x}-T}{s} \\ &= \tfrac{1}{\sqrt{n}} \times \frac{\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} + \left(\frac{\mu-T}{\sigma/\sqrt{n}}\right)}{\sqrt{\frac{(n-1)s^2}{\sigma^2}/(n-1)}} \\ &= \tfrac{1}{\sqrt{n}} \times \frac{Z+\sqrt{n}I_A}{\sqrt{\frac{\chi^2(n-1)}{n-1}}} \\ &= \tfrac{1}{\sqrt{n}} \times t_{n-1}(\theta). \end{aligned} \tag{5}$$

Therefore, the estimator $\hat{I}_A$ is distributed as $n^{-1/2}t_{n-1}(\theta)$, where $t_{n-1}(\theta)$ is a non-central t random variable with $n-1$ degree of freedom and the non-centrality parameter $\theta = \sqrt{n}I_A$. Because of $E(\hat{I}_A) = [(n-1)/2]^{1/2}$ $\Gamma[(n-2)/2]/\Gamma[(n-1)/2]I_A \neq I_A$, the estimator $\hat{I}_A$ is biased. To tackle this problem, we add the correction factor $b = [2/(n-1)]^{1/2}\Gamma[(n-1)/2]/\Gamma[(n-2)/2]$ to $\hat{I}_A$. Then we obtain unbiased estimator $b\hat{I}_A$ which we denote as $\tilde{I}_A$. Since $b < 1 (n > 2)$, $Var(\tilde{I}_A) < Var(\hat{I}_A)$. The estimator $\tilde{I}_A$ is based only on the complete and sufficient statistics $(\bar{x}, s^2)$, consequently $\tilde{I}_A$ is the uniformly minimum variance unbiased estimator (UMVUE) of I_A.

4.2 Distribution of estimator $\tilde{I}_A$

We first define $R = \tilde{I}_A = b(\overline{x} - T)/s = Y/V$, where $Y = b(\overline{x} - T)/\sigma$ and $V = \sqrt{s^2/\sigma^2}$. Since $D \sim N(\mu, \sigma^2)$, we have $Y \sim N(b(\mu - T)/\sigma, b^2/n)$. In addition, it is well known that the random variable $(n-1)s^2/\sigma^2$ follows the chi-squared distribution with $n-1$ degree of freedom, we then have $V^2 = s^2/\sigma^2 \sim \Gamma((n-1)/2, 2/(n-1))$. By using the technique of change-of-variable, the probability density function of V is derived as follows:

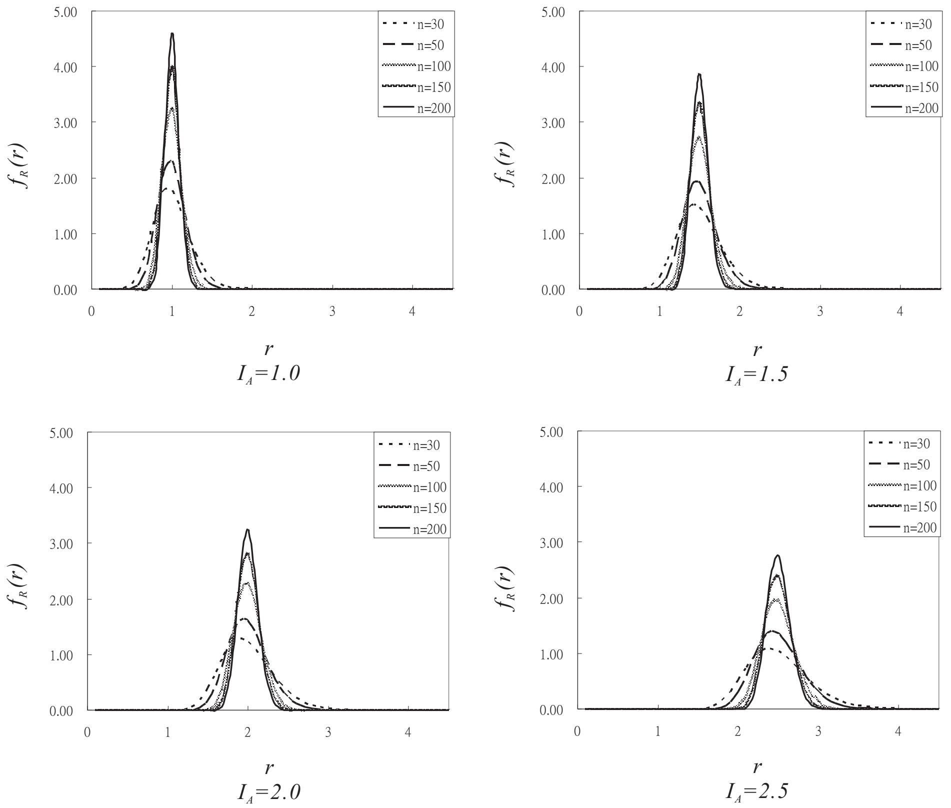
$$f_V(v) = \frac{2v^{n-2}}{\Gamma\left(\frac{n-1}{2}\right)\left(\frac{2}{n-1}\right)^{\frac{n-1}{2}}} exp\left(-\frac{n-1}{2}v^2\right), \qquad v > 0.$$

Because Y and V are independent continuous random variables, the probability density function of R can be obtained by the *Jacobian approach*, i.e.,

$$
\begin{aligned}
f_R(r) &= \int_0^{\infty} f_Y(vr) f_V(v) \mid v \mid dv \\
&= \frac{\sqrt{2n}\left(\frac{n-1}{2}\right)^{\frac{n-1}{2}}}{b\sqrt{\pi}\Gamma\left(\frac{n-1}{2}\right)} \int_0^{\infty} v^{n-1} exp\left\{ -\frac{1}{2}\left[\frac{(vr-bI_A)^2}{\frac{b^2}{n}} + (n-1)v^2\right]\right\} dv, \\
&-\infty < r < \infty.
\end{aligned}
$$

Figure 2 plots the probability density function of R, I_A =1.0, 1.5, 2.0, 2.5, and $n = 30$, 50, 100, 150, 200 (from bottom to top in plots). From Figure 2, we can see that (1) the larger the value of I_A, the larger the variance of $R = \tilde{I}_A$, (2) the distribution of R is unimodal and is rather symmetric to I_A even for small sample sizes.

Figure 2 PDF plots of R for sample sizes n = 30, 50, 100, 150, 200 (from bottom to top in plots)



4.3 Hypothesis testing with I_A and evaluation results

To judge whether the profitability meets the designated requirement, we ought to consider the hypothesis testing:

$$H_0: \ \mathbb{P} \leqslant \mathbb{C} \quad \text{versus} \quad H_1: \ \mathbb{P} > \mathbb{C},$$

where $\mathbb{C}$ is the designated requirement. However, the statistical property of the estimator of $\mathbb{P}$ is difficult to describe. Even, it is impossible to define the unbiased estimator of $\mathbb{P}$. From the last subsection, we have proven that I_A can express the profitability. Therefore, we consider the following hypothesis testing:

$$H_0: \ I_A \leqslant C \quad \text{versus} \quad H_1: \ I_A > C,$$

where C is the designated requirement of I_A. Based on the probability density function of R and a given level of Type I error α (i.e., the chance of incorrectly judging $I_A \leqslant C$ as $I_A > C$); the decision rule is to reject H_0 if the testing statistic $R \geqslant c_0$, where c_0 is the critical value that satisfies

$$\Pr\{R \geqslant c_0 \mid H_0: \ I_A \leqslant C, n\} \leqslant \alpha.$$

Since the larger the I_A value, the larger the critical value, we calculate the critical value c_0 with the condition $I_A = C$, i.e.,

$$\Pr\{R \geqslant c_0 \mid I_A = C, n\} = \alpha.$$

Table 1 shows the critical values for $I_A = 1.0(0.2)3.0$, $n = 30(10)200$ and $\alpha = 0.05$.

Table 1 Critical values for rejecting $I_A \leqslant C$ with $n = 30(10)200$ and $\alpha = 0.05$

n	$I_A = C$										
	1.0	*1.2*	*1.4*	*1.6*	*1.8*	*2.0*	*2.2*	*2.4*	*2.6*	*2.8*	*3.0*
30	1.402	1.635	1.871	2.108	2.348	2.589	2.830	3.073	3.317	3.562	3.806
40	1.343	1.571	1.801	2.032	2.265	2.500	2.735	2.971	3.208	3.445	3.683
50	1.304	1.528	1.754	1.982	2.211	2.441	2.672	2.904	3.136	3.369	3.602
60	1.276	1.497	1.721	1.946	2.172	2.399	2.627	2.855	3.085	3.314	3.544
70	1.254	1.474	1.695	1.918	2.142	2.367	2.592	2.819	3.045	3.272	3.500
80	1.237	1.455	1.675	1.896	2.118	2.341	2.565	2.789	3.014	3.239	3.465
90	1.223	1.440	1.658	1.878	2.099	2.320	2.542	2.765	2.989	3.212	3.436
100	1.211	1.427	1.644	1.863	2.082	2.303	2.524	2.745	2.967	3.189	3.412
110	1.200	1.416	1.632	1.850	2.068	2.288	2.508	2.728	2.949	3.170	3.392
120	1.191	1.406	1.622	1.839	2.056	2.275	2.494	2.713	2.933	3.153	3.374
130	1.184	1.397	1.613	1.829	2.046	2.263	2.481	2.700	2.919	3.139	3.358
140	1.177	1.390	1.604	1.820	2.036	2.253	2.471	2.689	2.907	3.125	3.344
150	1.170	1.383	1.597	1.812	2.028	2.244	2.461	2.678	2.896	3.114	3.332
160	1.165	1.377	1.591	1.805	2.020	2.236	2.452	2.669	2.886	3.103	3.321
170	1.160	1.372	1.585	1.799	2.013	2.229	2.444	2.660	2.877	3.094	3.311
180	1.155	1.367	1.579	1.793	2.007	2.222	2.437	2.653	2.869	3.085	3.301
190	1.151	1.362	1.574	1.787	2.001	2.216	2.430	2.646	2.861	3.077	3.293
200	1.147	1.358	1.570	1.782	1.996	2.210	2.424	2.639	2.854	3.070	3.285

4.4 Required sample size

In the previous subsection, the procedure is to test whether the profitability meets the designated requirement for given α risk (Type I error). But, the β risk (Type II error: the probability of incorrectly judging H_1 as H_0) is not taken into account. Once the sample size and the α risk are defined, the power of test, $1 - \beta$, can be calculated. The power of the test for C =1.0, 2.0 versus various values of I_A, $n = 30, 50, 100, 150, 200$, and $\alpha = 0.05$ is showed in Figure 3. It is seen that the larger the sample size, the larger the power of test, and consequently, the smaller the β risk. The required sample size for designated α and β risks can be calculated by recursive search method with the following two probability equations:

$$\Pr\{R \geqslant c_0 \mid H_0: \ I_A \leqslant C, n\} \leqslant \alpha, \text{ and}$$

$$\Pr\{R \geqslant c_0 \mid H_1: \ I_A > C, n\} \geqslant 1 - \beta.$$

In Table 2, we tabulate the sample sizes required for α = 0.05, designated power = 0.90, 0.95, 0.975, 0.99, designated requirement C = 1.0, 1.2, 1.4, 1.6, and difference of expected I_A and designated requirement $I_A - C$ = 0.3(0.1)1.0.

Figure 3 Power curves for $C = 1.0, 2.0$, with sample sizes $n = 30, 50, 100, 150, 200$

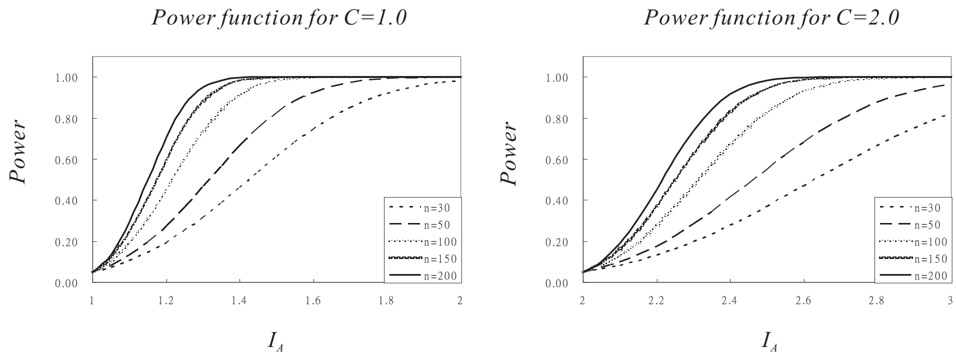


Table 2 Sample size required for testing $H_0: \ I_A \leqslant C$ versus $H_1: \ I_A > C$

C	I_A	*Power*				C	I_A	*Power*			
		0.90	*0.95*	*0.975*	*0.99*			*0.90*	*0.95*	*0.975*	*0.99*
1.0	1.3	161	201	239	288	1.2	1.5	185	231	275	331
	1.4	94	117	139	167		1.6	109	135	160	192
	1.5	63	78	91	110		1.7	73	90	106	127
	1.6	46	56	67	79		1.8	53	65	77	91
	1.7	35	43	51	60		1.9	41	50	58	69
	1.8	28	35	40	48		2.0	33	40	46	55
	1.9	24	28	33	39		2.1	27	33	38	45
	2.0	20	24	28	33		2.2	23	28	32	38
1.4	1.7	213	266	316	381	1.6	1.9	245	306	364	438
	1.8	125	155	184	221		2.0	143	178	212	254
	1.9	83	103	122	146		2.1	96	119	140	168
	2.0	61	75	88	105		2.2	69	86	101	121
	2.1	46	57	67	80		2.3	53	65	77	92
	2.2	37	45	53	63		2.4	42	52	61	72
	2.3	31	37	44	52		2.5	35	43	50	59
	2.4	26	32	37	43		2.6	30	36	42	49

5 Profitability evaluation for a fresh food

We consider a fresh food industry in Hsinchu, Taiwan, in which provides more than twenty different kinds of lunch boxes, breads, sandwiches for shopping malls and convenience stores. These fresh food products are prepared each day and have relatively short shelf-life (about one or two days). The overdue products can not be sold and need additional cost to dispose them. If the manufacturing quantity can not satisfy the order from the malls and stores, then the supplier must pay the lost sale opportunity cost. Therefore, these fresh food products exactly belong to the newsboy-type products.

Now, a new lunch box is recommended, the manufacturing quantity of the existing lunch box which has the lowest profitability should be curtailed due to the capacity constraints (manpower or machines). Note that in order to maintain fresh, the lunch boxes are prepared in the morning and the life cycle is only 12 hours. However, the supplier would like to know whether the profitability of the existing lunch box is higher than some level. If the existing lunch box is incapable, it must be replaced with the new one. The selling price of the existing lunch box is \$20 per unit, the manufacturing cost is \$10 per unit, and the target profit is \$200,000. In addition, the lost sale opportunity cost is \$3 per unit. The surplus (overdue) lunch boxes can be manufactured into fertilisers, then the salvage price is \$5 per unit. Table 3 displays the demand units in thousand for the existing lunch box with sample size n = 100. Due to the company's propertied restriction, the prices, costs, and sample data were modified.

Table 3 Sample data with 100 observations

Demand units in thousand/day									
26.56	25.51	22.00	22.60	23.20	23.37	25.44	24.64	23.16	22.70
22.37	20.87	22.20	24.14	25.34	24.26	23.24	21.90	22.67	22.83
23.02	25.50	25.46	26.60	22.66	21.24	21.42	21.95	21.62	27.57
24.11	26.89	24.64	24.10	22.03	24.59	25.36	19.40	20.70	25.93
23.72	23.33	25.22	23.31	23.19	24.86	24.96	23.89	24.49	19.60
20.81	24.78	21.12	21.14	23.96	24.29	26.07	22.57	24.85	23.65
22.60	24.94	25.72	24.27	25.40	20.84	23.05	20.45	23.24	20.56
24.24	25.36	22.09	23.43	26.36	27.38	20.56	23.52	24.95	21.51
22.20	25.31	23.83	24.23	24.31	25.97	22.03	26.13	18.99	21.51
22.17	20.44	25.18	25.50	23.82	23.50	24.54	25.45	25.91	24.20

We first use the Kolmogorov-Smirnov test for the sample data from Table 3 to confirm if the data is normally distributed. A test result in p-value >0.05, which means that data is normally distributed. Histogram of the data is shown in Figure 4.

If the designated requirement of the I_A value is $C = 1.2$, we implement the hypothesis testing: $H_0: \ I_A \leqslant 1.2$ versus $H_1: \ I_A > 1.2$. For the data displayed in Table 3, we calculate the sample mean, sample standard deviation, and sample estimator, and obtain that $\overline{x} = 23.593$, $s = 1.882$ and $R = 1.894$. Based on Table 1, the critical value is 1.427 as $C = 1.2$, $n = 100$ and $\alpha = 0.05$. Since $R = 1.894 > 1.427 = c_0$, we conclude that I_A is more than 1.2 with 95% confidence level. Therefore, the supplier only curtails output of the the existing lunch box. Furthermore, we calculate the critical value for $C = 1.4, 1.5, 1.6, 1.61, 1.62, 1.63$ with $n = 100$. The decision of the hypotheses are shown in Table 4. Based on the testing results, we can conclude that the profitability of the existing lunch box is higher than 1.62 with 95% confidence level. Assume that the expected I_A is 1.6. We use a hypothesis testing with a designated power of 0.95, the sample size required to sample is 135 as in Table 2. In this example, the sample size is less than 135, the power for testing $H_0: \ I_A \leqslant 1.2$ versus $H_1: \ I_A > 1.2$ would be less than 0.95. In fact, the power of test for the expected $I_A = 1.6$ is 0.8766, that is the β risk is up to 0.0734. In order to reduce the β risk, we would suggest the supplier to sample for a designated power with as large sample size as in Table 2.

Figure 4 Histogram of demand data (see online version for colours)

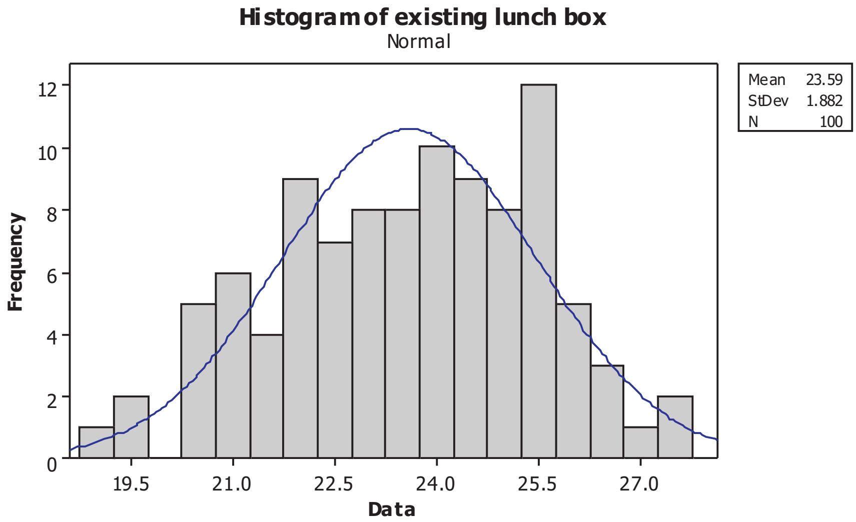


Table 4 Critical values and decisions of testing the existing lunch box

C	1.40	1.50	1.60	1.61	1.62	1.63
c_0	1.644	1.753	1.863	1.874	1.885	$1.896 > R$
Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Do not reject H_0

6 Concluding remarks

In this paper, we investigated a profitability evaluation problem which examines whether the profitability meets a designated requirement. In addition, we developed a new index which has a simple form expression of profitability. The proposed index, which we refer to as the ACI (I_A), can reduce the difficulty of effective estimation when the parameters μ and σ are unknown. For example, the unbiased and effective estimator $\tilde{I}_A$ is found effortlessly, and the distribution of estimator $\tilde{I}_A$ can be derived. By advantageously using I_A, we presented the hypothesis test to solve the evaluation problem, i.e., $H_0: \ I_A \leqslant C$ as against $H_1: \ I_A > C$, where C is the designated requirement of I_A. Some tables are shown to practitioners or managers for deciding whether the existing product is already unworthy of being ordered/manufactured under the accepted risks (Type I and Type II errors). Finally, a real-world application of a fresh food product is presented to illustrate the practicality of the exact approach. The results of our study suggest three dimensions which could be addressed by future research. The first is to investigate the imprecise demand by combining the fuzzy set concepts. Second, of interest is the truncated normal distribution to relax the assumption of $cv < 0.3$. The last is to further explore the estimating and testing I_A, based on multiple samples, using the demand data from subsamples.

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