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## Signature of Schwinger's pair creation rate via radiation generated in graphene by strong electric current

B. Rosenstein,<sup>1,2,3,\*</sup> M. Lewkowicz,<sup>3</sup> and H.C. Kao<sup>4</sup>

<sup>1</sup>*Electrophysics Department, National Chiao Tung University, Hsinchu 30050, Taiwan, R. O. C.*

<sup>2</sup>*National Center for Theoretical Sciences, Hsinchu 30043, Taiwan, R. O. C.*

<sup>3</sup>*Physics Department, Ariel University Center of Samaria, Ariel 40700, Israel*

<sup>4</sup>*Physics Department, National Taiwan Normal University, Taipei 11677, Taiwan, R. O. C.*

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Electron - hole pairs are copiously created by an applied electric field near the Dirac point in graphene or similar 2D electronic systems. It was shown recently that for sufficiently large electric fields  $E$  and ballistic times the I-V characteristics become strongly nonlinear due to Schwinger's pair creation rate, proportional to  $E^{3/2}$ . Since there is no energy gap the radiation from the pairs' annihilation is enhanced. The spectrum of radiation is calculated and exhibits a maximum at  $\omega = \sqrt{eEv_g/\hbar}$ . The angular and polarization dependence of the emitted photons with respect to the graphene sheet is quite distinctive. For very large currents the recombination rate becomes so large that it leads to the second Ohmic regime due to radiation friction.

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### INTRODUCTION

Electronic mobility in graphene, especially one suspended on leads, is extremely large [1] so that a graphene sheet is one of the purest electronic systems. The relaxation time of charge carriers due to scattering off impurities, phonons, riplons, etc., in suspended graphene samples of submicron length is so large that the transport is ballistic [2, 3]. The ballistic flight time in these samples can be estimated as  $t_{bal} = L/v_g$ , where  $v_g \simeq 10^6 m/s$  is the graphene velocity characterizing the massless "ultra - relativistic" spectrum of graphene near Dirac points,  $\varepsilon_k = v_g |\mathbf{k}|$ , and  $L$  is the length of the sample that can exceed several  $\mu m$  [4, 5]. The extraordinary physics appears right at the Dirac point at which the density of states vanishes. In particular, at this point graphene exhibits a quasi - Ohmic behaviour,  $\mathbf{J} = \sigma \mathbf{E}$ , even in the purely ballistic regime.

A physical picture of this "resistivity" without either charge carriers or dissipation is as follows [6]. The electric field creates electron - hole excitations in the vicinity of the Dirac points similar to the Landau-Zener tunneling effect in narrow gap semiconductors or electron - positron pair creation in Quantum Electrodynamics first studied by Schwinger [7] (later referred to as LZS). Importantly, in graphene the energy gap is zero, thus the pair creation is possible at zero temperature and arbitrary small  $\mathbf{E}$ , even within linear response. Although the absolute value of the quasiparticle velocity  $v_g$  cannot be altered by the electric field due to the "ultra - relativistic" dispersion relation, the orientation of the velocity can be influenced by the applied field. The electric current,  $e\mathbf{v}$ , proportional to the projection of the velocity  $\mathbf{v}$  onto the direction of the electric field is increased by the field. These two sources of current, namely creation of moving charges by the electric field (polarization) and their re-

orientation (acceleration) are responsible for the creation of a stable current.

Agreement over the qualitative explanation notwithstanding, determination of the value of the minimal DC conductivity at Dirac point in the limit of zero temperature had undergone a period of experimental and theoretical uncertainty. After the value in graphene on substrate was measured to be about  $\sigma = 4e^2/h$  [8], it was shown in experiments on suspended samples [2] that the zero temperature limit was not achieved and in fact that these early samples had too many charged "puddles", so that they represented an average around the neutrality or the Dirac point. The value in early-on suspended samples [2] was half of that and most recently settled at the "dynamical"  $\sigma_2 = \frac{\pi}{2} \frac{e^2}{h}$  in best samples at 2K temperature [4]. Theoretically several different values appeared. The value  $\sigma_1 = \frac{4}{\pi} \frac{e^2}{h}$  had been considered as the "standard" one for several years [9] and appeared as a zero disorder limit in many calculations like the self consistent harmonic approximation, although different regularizations within the Kubo formalism resulted in different values [10].

The dynamical approach to transport was applied to the tight binding model of graphene [11] to resolve this "regularization ambiguity". It consists of considering the ballistic evolution of the current density in time after a sudden or gradual switching on of the electric field. The result within linear response is that the current settles very fast, on the microscopic time scale of  $t_\gamma = \hbar/\gamma \simeq 0.24 fs$  ( $\gamma$  being the hopping energy), on the value of  $J = \sigma_2 E$ . The value is identical to the one obtained (at nonzero temperatures) for the AC conductivity [12]. The two contributions, polarization and attenuation are comparable in strength and combine to produce a constant total current. However a deeper analysis of the "quasi - Ohmic" graphene system beyond the leading order in

perturbation theory in electric field revealed [13] that on the time scale

$$t_{nl} = \sqrt{\frac{\hbar}{eEv_g}}, \quad (1)$$

the linear response breaks down. For larger times the quasi - Ohmic behavior no longer holds. This is in contrast to dissipative systems, in which the linear response limit can be taken directly at infinite time. This perhaps is the origin of the "regularization" ambiguities in graphene, since large time and small field limits are different. The time scale on which nonlinear effects become dominant is not always very large; for example, in experiments dedicated to breakdown of Quantum Hall effect [14] in which  $E = 10^4 V/m$ , nonlinearity sets in at  $t_{nl} = 0.3ps$ , that is of order ballistics time for  $L = 0.3\mu m$ . Graphene flakes under larger fields of order  $2 \cdot 10^6 V/m$  have been studied very recently (at room temperature) in specially designed high current density experiments [5]. In this case the nonlinear time is only  $20fs$ , much lower than the ballistic time  $t_{bal} = 2ps$  for  $L = 2\mu m$ . Analytic and numerical solutions of the tight binding model[13], as well as of the Dirac model describing the physics near the Dirac point demonstrated[13, 15] that at  $t_{nl}$  the electron - hole pairs creation becomes dominant and is well described by an adaptation of the well - known (non-analytic in  $\mathbf{E}$ ) Schwinger electron - positron pair creation rate

$$\frac{d}{dt}N_p = \frac{3^{3/4}}{2^{9/2}v_g^{1/2}} \left(\frac{eE}{\hbar}\right)^{3/2}. \quad (2)$$

The difference with the original derivation [7] in the context of particle physics is that the fermions are 2+1 dimensional and "massless", thus magnifying the effect. The polarization current is  $J(t) = 2ev_g N(t)$  and therefore Schwinger's creation rate leads to a linear increase with time[13]:

$$J(t) = \sigma_2 \left(\frac{\sqrt{3}}{2}E\right)^{3/2} \left(\frac{ev_g}{\hbar}\right)^{1/2} t. \quad (3)$$

The physics of pair creation is highly non-perturbative and non-linear in nature and therefore, instead of the linear response, Schwinger found an exact formula using functional methods. The rate can be intuitively understood using the much simpler instanton approach originally proposed in the context of particle physics [16] (extended later to low dimensions [17]), but is known in fact in condensed matter physics as the Landau - Zener tunneling probability [5, 15, 18]. In particle physics it is extremely difficult to observe Schwinger's creation rate and it would be interesting to establish experimentally this dynamical phase in low dimensional condensed matter physics featuring the massless Dirac quasiparticle spectrum like graphene or novel materials sharing with it

the massless Dirac spectrum like topological insulators or tuned semiconductor heterojunctions [? ]. Of course transport phenomena at rather large fields always have a background related to possible influence of leads, disorder and thermal effects like local heating, etc.

In this note we draw attention to a direct and unintrusive signature of the dynamical phase of LZS pair creation in a graphene sheet subject to an applied electric field. It is demonstrated that the flux of photons radiated from the surface of the sample is characterized by the creation rate since the photons are emitted via electron - hole pair annihilation and therefore proportional to  $E^{3/2}$ , a hallmark of Schwinger's process. In addition, the frequency, direction and polarization characteristics of the radiation generated by the electric field calculated here all bear footprints of the pair creation dynamics. The details of the calculations are given in [19].

## ELECTRON - HOLE RECOMBINATION RATE INTO PHOTONS.

### Amplitude for emission of a single photon

The electrons and their electromagnetic interaction with photons are approximately described near a Dirac point by the Weyl Hamiltonian:

$$H = \int d^3r \psi^\dagger \left[ -\frac{\hbar^2}{2m} (\partial_z + i\frac{e}{\hbar c}A_z)^2 + V_{conf}(z) \right] \psi. \quad (4)$$

Here  $\psi$  is the two component spinor second quantized field and  $(\mathbf{A}, A_z)$  is the vector potential (bold letters describe vectors in the graphene plane, while  $z$  is the perpendicular direction). Electrons (charge  $-e$ ) and holes (charge  $e$ ) in the graphene sheet are confined in this model to the  $z = 0$  plane by a potential  $V_{conf}$  (small shape changes can be neglected for our purposes). The only requirement from this potential is that it is strong enough to "freeze" the motion along the  $z$  direction. In the single graphene sheet one has two left handed chirality Weyl fermions described by the above Hamiltonian in which  $\sigma$  denotes the in - plane Pauli matrices and two right handed Weyl fermions represented by  $\sigma^\dagger$ .

We consider the emission of a photon with wave vector  $(\mathbf{k}, k_z)$  and frequency  $\omega = c\sqrt{\mathbf{k}^2 + k_z^2}$ , described by a linearly polarized plane wave,

$$\mathbf{A}_{ph} = \frac{2E_0}{\omega} \mathbf{e}^{(\lambda)} \sin(\mathbf{k} \cdot \mathbf{r} + k_z z - \omega t), \quad (5)$$

whereas the DC applied field is  $\mathbf{A}_{ext} = (0, -cEt)$ . For regularization we make use of a finite box  $L \times L \times L_z$ , so that momenta are discrete and the single photon's electric field is  $E_0^2 = \hbar\omega / (L^2 L_z)$ . The unit vectors

$$\begin{aligned} \mathbf{e}^{(1)} &= (-\sin \varphi, \cos \varphi); e_z^{(1)} = 0; \\ \mathbf{e}^{(2)} &= -\cos \theta (\cos \varphi, \sin \varphi); e_z^{(2)} = \sin \theta, \end{aligned} \quad (6)$$

describe polarizations that are conveniently chosen similarly to a recent calculation of electromagnetic emission due to thermal fluctuations [20]. The vectors  $\mathbf{e}^{(1)}$  and  $\mathbf{e}^{(2)}$  represent the "in plane" and the "out of plane" polarizations, respectively. The electron and the hole wave functions are  $\frac{1}{\sqrt{2L}}e^{i\mathbf{p}\cdot\mathbf{r}}u(\mathbf{p})\psi_n(z)$  and  $\frac{1}{\sqrt{2L}}e^{i\mathbf{p}'\cdot\mathbf{r}}v(\mathbf{p}')\psi_n(z')$ , with spinors defined by

$$u(\mathbf{p}) = \begin{pmatrix} 1 \\ -ie^{i\phi} \end{pmatrix}; \quad v(\mathbf{p}') = \begin{pmatrix} 1 \\ ie^{i\phi'} \end{pmatrix}, \quad (7)$$

with  $\mathbf{p} + \frac{e}{\hbar c}\mathbf{A}_{ext} = p(\cos\phi, \sin\phi)$ .  $\psi_n(z)$  are wave functions of the confinement. The interaction with a photon at time  $t$  happens when the momentum is minimally shifted due to the DC field. The Golden rule photon emission rate (for an "initial" electron with momentum  $\mathbf{p}$  and a "final" hole  $\mathbf{p}'$  and a photon of polarization  $\lambda$  and momentum  $(\mathbf{k}, k_z)$ ) is

$$W_{nn'}^{(\lambda)}(\mathbf{p}, \mathbf{p}', \mathbf{k}, k_z, t) = \frac{2\pi}{\hbar} \left| F_{nn'}^{(\lambda)} \right|^2 N_{\mathbf{p}}(t) N_{-\mathbf{p}'}(t) \times \delta(\hbar v_g(p + p') - \hbar\omega). \quad (8)$$

Here  $N_{\mathbf{p}}(t)$  is the density of electrons in a certain momentum range produced by the electric field  $\mathbf{E}$ , and  $N_{-\mathbf{p}'}(t)$  the density of holes (equal to that of the electrons at the opposite momentum due to particle - hole symmetry). The density calculated using the simple Landau - Zener creation rate expression for one of the flavours is [13, 15, 16]:

$$N_{\mathbf{p}}(t) = \Theta(p_y) \Theta\left(\frac{e}{\hbar}Et - p_y\right) \exp\left(-\frac{\pi\hbar v_g}{eE}p_x^2\right), \quad (9)$$

where  $\Theta$  are the Heaviside functions. The transition amplitude is given by

$$F_{nn'}^{(\lambda)} = i\frac{E_0}{\omega} \frac{ev_g}{2L^2} e^{i(v_g(p+p')-\omega)t} \mathcal{F}_{\mathbf{p},\mathbf{p}'}^{(\lambda)} \int dz e^{ik_z z} \psi_n^*(z) \psi_{n'}(z) \delta(\mathbf{p} + \mathbf{p}' - \mathbf{k}), \quad (10)$$

where matrix elements  $\mathcal{F}_{\mathbf{p},\mathbf{p}'}^{(\lambda)} \equiv v^\dagger(-\mathbf{p}') \boldsymbol{\sigma} \cdot \mathbf{e}^{(\lambda)} u(\mathbf{p})$  are

$$\begin{aligned} \left| \mathcal{F}_{\mathbf{p},\mathbf{p}'}^{(1)} \right|^2 &= 2 \left[ 1 - \cos(2\varphi - \phi - \phi') \right]; \\ \left| \mathcal{F}_{\mathbf{p},\mathbf{p}'}^{(2)} \right|^2 &= 2 \cos^2\theta \left[ 1 + \cos(2\varphi - \phi - \phi') \right]. \end{aligned} \quad (11)$$

### Spectral emittance

For tight confinement to the  $z = 0$  plane one should consider only the ground state  $n = n' = 0$ . Note that the perpendicular component of the wave vector  $k_z$  is "free" from conservation that prohibits the process in fully relativistic QED. The phase space for annihilation is very limited due to  $v_g \ll c$  and leads to important simplifications.

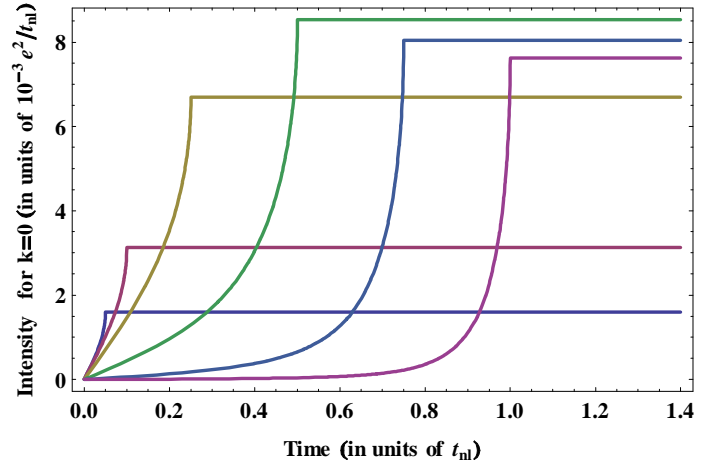


Fig.1. The spectral emittance in direction perpendicular to the graphene plane,  $\mathbf{k} = 0$ . Polarizations are summed over. The emittance (in units of  $e^2/t_{nl}$ ) for various frequencies (in units of  $t_{nl}^{-1}$ ) as function of ballistic time from  $0.1t_{nl}$  to  $1.4t_{nl}$ .

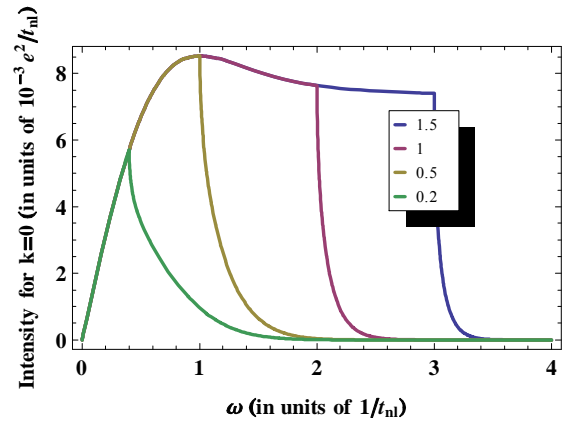


Fig.2. The emittance at various times (in units of  $t_{nl}$ ) as function of frequency.

Let us define the spectral emittance per volume of the  $k$  - space (and area of the graphene flake) as

$$\begin{aligned} \mathcal{M}^{(\lambda)}(\mathbf{k}, k_z, t) &= \frac{4\hbar\omega}{L^2} \sum_{\mathbf{p},\mathbf{p}'} \frac{d}{dk_z d\mathbf{k}} W^{(\lambda)}(\mathbf{p}, \mathbf{p}', \mathbf{k}, k_z, t) \\ &= \frac{e^2 v_g^2}{(2\pi)^4} \int d\mathbf{p} \left| \mathcal{F}_{\mathbf{p},\mathbf{k}-\mathbf{p}}^{(\lambda)} \right|^2 N_{\mathbf{p}} N_{\mathbf{k}-\mathbf{p}} \\ &\quad \times \delta(v_g(p + |\mathbf{k} - \mathbf{p}|) - \omega), \end{aligned} \quad (12)$$

where the integration over  $\mathbf{p}'$  was performed using the delta function expressing the conservation of momentum. We first study the frequency dependence of the radiation in the direction perpendicular to the graphene flake,  $\mathbf{k} = \mathbf{0}$ . Multiplying with 4 for the spin and valley degeneracy, summing over the polarizations ( $\sum_{\lambda} \left| \mathcal{F}_{\mathbf{p},\mathbf{p}}^{(\lambda)} \right|^2 = 4$ ), and integrating over  $\mathbf{p}$  one obtains, using  $\omega = ck_z$ , the spectral emittance

$$\mathcal{M}(\mathbf{k} = \mathbf{0}, \omega, t) = \frac{e^2 v_g^2 t_{nl}^2}{\pi^4} \int_{-t/t_{nl}}^0 d\bar{p} \Theta(t_{nl}\omega/2 + \bar{p}) \frac{\exp[-2\pi(t_{nl}^2\omega^2/4 - \bar{p}^2)]}{\omega(t_{nl}^2\omega^2/4 - \bar{p}^2)^{1/2}}. \quad (13)$$

The spectral emittance, presented in Fig.1 for various frequencies as function of time, increases linearly for  $t \ll \omega^{-1}$ ,  $\mathcal{M}(\mathbf{k} = \mathbf{0}, \omega, t) = \frac{2e^2 t}{\pi^4 t_{nl}^2} e^{-\pi t_{nl}^2 \omega^2 / 2}$ , then rises sharply approaching a maximum at  $t = \omega t_{nl}^2 / 2$  and stabilizes at

$$\mathcal{M}(\mathbf{k} = \mathbf{0}, \omega, t \gg t_{nl}) = \frac{e^2}{\pi^3} \omega e^{-\pi t_{nl}^2 \omega^2 / 4} I_0\left(\frac{\pi t_{nl}^2 \omega^2}{4}\right), \quad (14)$$

where  $I_0$  is the modified Bessel function. The asymptotic value rises linearly with frequency,  $\pi^{-3}\omega$ , in the infrared, reaches its maximum at  $\omega = t_{nl}^{-1}$  and falls slightly to  $\frac{\sqrt{2}e^2}{\pi^4 t_{nl}}$  in the ultraviolet. In Fig.2 the emittance at various ballistic times is given as function of frequency. For each ballistic time the curve has two parts. The first follows the universal dependence given by Eq.(14). Therefore the frequency for observation of the Schwinger effect, not surprisingly, should exceed  $\omega_{\min} = \sqrt{eEv_g/\hbar}$ ,

that amounts to  $3.6THz$  for  $E = 10^4 V/m$ , and  $50THz$  for  $E = 2 \cdot 10^6 V/m$ . At a higher frequency  $\omega_{\max} = 2t/t_{nl}^2$  the emittance sharply drops. Therefore the frequency does not exceed  $2t_{bal}/t_{nl}^2$ .

### Angular and polarization distribution

Next we consider the angular and polarization dependence of the radiated power per unit area defined as the spectral intensity, Eq(12), integrated over frequencies:

$$\mathcal{L}^{(\lambda)}(\theta, \varphi, t) \equiv \int_0^\infty d\omega \frac{\omega^2}{c^3} \mathcal{M}^{(\lambda)}\left(\mathbf{k}, \frac{\omega}{c}, t\right). \quad (15)$$

Performing integrations and simplifying, utilizing the small parameter  $v \equiv v_g/c \simeq 1/300 \ll 1$ , one obtains:

$$\mathcal{L}^{(1)}(\varphi, t) = \frac{e^4 v^4 E^2}{2^{5/2} \pi^4 c \hbar^2} \left( \frac{t}{4\pi t_{nl}} \cos^2 \varphi + \frac{t^3}{3t_{nl}^3} \sin^2 \varphi \right); \quad (16)$$

$$\mathcal{L}^{(2)}(\theta, \varphi, t) = \frac{e^4 v^4 E^2}{2^{5/2} \pi^4 c \hbar^2} \cos^2 \theta \left( \frac{t}{4\pi t_{nl}} \sin^2 \varphi + \frac{t^3}{3t_{nl}^3} \cos^2 \varphi \right). \quad (17)$$

The radiant flux from a flake of a  $\mu m \times \mu m$  size is  $4.7 \cdot 10^{-21} W$ , for  $E = 10^4 V/m$  corresponding to the emission rate of just 10 photons per second, yet for the high current samples[5] with  $E = 2 \cdot 10^6 V/m$  of the same area one gets a more significant output: the radiant flux is  $1.3 \cdot 10^{-17} W$ , corresponding to the emission rate of  $3 \cdot 10^4$  photons per second.

### CONCLUSIONS

Electron - hole pairs are copiously created via Landau-Zener-Schwinger mechanism near the Dirac points in graphene or similar 2D electronic systems by an applied electric field, provided the available ballistic time exceeds  $t_{nl}$ , Eq.(1). The recombination into photons produces a characteristic signal proportional to  $E^{3/2}$  at frequencies of order  $t_{nl}^{-1}$  that enables unintrusive and unambiguous experimental observation of the Schwinger phenomenon.

The angular and polarization dependence of the emitted photons with respect to the graphene sheet was calculated. At very high currents and sufficiently long ballistic times the recombination process becomes greatly enhanced by the electron - hole attraction and the radiation becomes an effective channel of dissipation, the radiation friction. Hopefully Schwinger's pair creation rate formula can be directly tested using novel condensed matter materials endowed with relativistic fermion spectra.

\* Electronic address: [vortexbar@yahoo.com](mailto:vortexbar@yahoo.com)

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