



Product selection for newsboy-type products with normal demands and unequal costs

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ABSTRACT

In this paper, we consider two newsboy-type products with unequal prices and costs. Both demands are independent and follow normal distributions with unknown parameters μ and σ . We study the product selection problem which deals with comparing two products and selecting the one that has a significantly higher profitability, in which the profitability is defined to the probability of achieving a target profit under the optimal ordering policy. The statistical hypothesis testing methodology is performed to tackle this selection problem. Critical value of the test is calculated to determine the selection decision. Sample size required for a designated power and confidence level is also investigated. An application example on comparing English-teaching magazines is presented to illustrate the practicality of our approach.

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1. Introduction

The objective of inventory management is to maintain optimum levels of inventory consistent with customer demands and plant capacity. Stated simply, inventory management encompasses the principles, concepts and techniques for deciding: (1) what to order; (2) how much to order; (3) when to order; (4) where to store it; (5) when it is needed (Fogarty et al., 1991). The newsboy problem is one of the important subjects in the inventory management. It focuses on the products which have relatively short shelf-life, e.g. daily newspapers, Christmas trees, weekly/monthly magazines, milk, and seasonal products. For these products, the surplus cannot be sold in the next period and need additional cost to dispose it. If the order quantity is lower than demand, the lost sale opportunity cost should be paid. The main purpose of the newsboy problem is only to find the order quantity that maximizes (minimizes) an objective function in a single period, probabilistic demand framework. Several extensions have been proposed and solved in the literature. For example, Hadley and Whitin (1963) considered the single product and single discount. Lau and Lau (1988) and Khouja (1995, 1996) further studied the multiple products and multiple discounts, respectively. After, Khouja and Mehrez (1996) simultaneously considered multiple products and multiple discounts. Keisuke (2006) focused on the multi-period newsboy problem. Chen and Ho (2011) proposed an analysis method for the single-period

inventory problem with fuzzy demands and incremental quantity discounts. Hua et al. (in press) studied pricing and the newsboy problem with free shipping. We determine the optimal order quantity and the optimal selling price simultaneously. Tang et al. (in press) investigated the possibility and benefit of applying dynamic pricing policy in the presence of random yields in supply with deterministic/stochastic demand. This is the first attempt to bridge the two streams of study in the literature: the pricing policies in the newsboy problem and purchasing decisions in random yield environments. Among those extensions have alternative objective functions such as minimizing the expected cost, maximizing the expected profit, maximizing the expected utility, and maximizing the probability of achieving a target profit ...etc. In fact, these maximum and minimum values can be the criteria to measure the product's capacity. For example, the maximum expected profit and the maximum probability of achieving the target profit can measure the product's profitability.

For the probabilistic demand, the traditional newsboy models often assumed that the demand follows a common distribution with known parameters. However, in reality, these parameters may be unknown. Therefore, the extent of applicability of such models to managerial aspects of inventories depends on the estimation of parameters. Kevork (2010) developed the appropriate estimators for the optimal ordering quantity and the maximum expected profit. The statistical properties of the two estimators are explored for both small and large samples, analytically and through Monte-Carlo simulations. Generally speaking, the estimator will more accurate when the sample size is large. However, the cost for collecting data will increase as the sample size increases. If the sample size is not large enough, the accuracy of estimation may decrease such that the

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risk of wrong decisions increases. Therefore, it is important to determine the minimal sample size required under the tolerable risk. Most of the research also focused on distribution-free newsboy problem, in which the form of the demand distribution is not known but only the mean and variance are specified. Scarf (1958) pioneered a minimax approach, which aims to minimize the maximum cost resulting from the worst possible demand distribution. This approach can derive a simple closed-form expression for the order quantity that maximizes expected profit. Moreover, extending this distribution-free newsboy problem based on various considerations, Ouyang and Wu (1998) considered a variable lead time. Ouyang and Chang (2002) extended to consider a variable lead time with fuzzy lost sales. Alfares and Elmorra (2005) took the shortage cost into consideration. Mostard et al. (2005) analyzed the distribution-free newsboy problem with resalable returns. Liao et al. (in press) considered a linear penalty cost for lost sales in the model under customer balking, which occurs when the available inventory reaches a threshold level. Lee and Hsu (2011) developed for the decision-maker in a distribution-free newsboy problem to determine the expenditure on advertising and the order quantity.

Product selection problem: In the present market, the new products are unceasingly introduced. Although the new products are not immediately accepted by the customers, it means that a new trend is going to be popularly adopted in the market. If the managers are unable to accept the new product timely, the business may lose the competition. Therefore, the manager should observe the requirement of customers, and accept the new product at the right moment. In fact, when the new product is ordered, the old product with lower profitability may be eliminated (or curtailed) due to the spatial constraint in the warehouse. For the other case, if the capital investment in profitability improvement is implemented, one should focus on the products which have lower profitability due to the budgetary constraint. To reflect these phenomena, it is necessary to consider the product selection problem which deals with comparing all old products and selecting the one that has a significantly lower profitability. Consequently, the inventory management should add to determine the sixth parameter: (6) which to eliminate (or curtail). Note that the demands of products are usually irrelevant to each other due to the preference and identity. Therefore, the demands are independent random variables. However, few of products have certain relation. For example, two different brands of apples can be substituted each other, the milk essence is an accessory to coffee. In these cases, the random demands are dependent, and the joint probability density function of demands must be used. In order to construct the product selection problem, we preliminary make the simplifying assumption that the product demands are independent in this paper, then the dependent case will be further studied in the future research based on the assumptions and formulations of this paper.

In this paper, we consider two newsboy-type products which have unequal prices and costs. In addition, we assume that the both demands are independent, and follow the normal distributions with unknown parameters (μ and σ). The main purpose of this paper is to study the product selection problem which deals with comparing two products and selecting the one that has a significantly higher profitability. In addition, the profitability is defined to the probability of achieving a target profit under the optimal ordering policy. The rest of the paper is organized as follows. An application example on comparing English-teaching magazines with different level is introduced in the next section. In Section 3, the profitability measurement for newsboy-type product is presented. In Sections 4 and 5, the statistical hypothesis testing methodology is performed to tackle the product selection problem. The critical value of the test to determine the selection decision is calculated. Sample size required for a designated power and confidence level is also investigated. In the last section,

the English-teaching magazine selection is implemented to illustrate the practicality of our approach.

2. English-teaching magazine selection

The English-teaching magazine is one of the monthly magazines. It provides practical, interesting articles to improve English conversation skills. Radio and television programs also accompany each article and air Monday to Saturday. The publisher only provides the magazines in the beginning of each month. If the demand cannot be satisfied, the publisher must pay the lost sale opportunity cost. The surplus magazines cannot be sold in the next month, and need additional cost to dispose it. Therefore, this monthly magazine exactly belongs to newsboy-type product.

Next, we introduce a magazine publisher in Taipei, Taiwan, in which we provide three level of English-teaching magazines, basic, intermediate, and high. The basic and intermediate magazines are the best teaching materials to the junior and senior students, respectively. The high magazine covers a wide range of topics. Most are reprinted from international magazines providing readers with a "Window on the World". Therefore, it is most suitable for university students and business professionals. Note that these magazines cannot be substituted each other. In this paper, we consider the following two examples on comparing English-teaching magazines.

Example 1: The magazine publisher would like to know whether the profitability of intermediate magazine (Magazine II) is better than basic magazine (Magazine I). If not, the magazine publisher is going to plan a sale promotion for senior students. The price (p), purchasing cost (c), disposal cost (c_d), and shortage cost (c_s) for two magazines are presented as follows:

Magazine I: $p_1=12$ dollars/unit, $c_1=2$ dollars/unit, $c_{d1}=3$ dollars/unit, and $c_{s1}=3$ dollars/unit.

Magazine II: $p_2=15$ dollars/unit, $c_2=3$ dollars/unit, $c_{d2}=4$ dollars/unit, and $c_{s2}=5$ dollars/unit.

Example 2: The magazine publisher would like to know whether the intermediate magazine (Magazine II) is the highest profitability of three magazines. The price and costs of the high magazine (Magazine III) are presented as follows:

Magazine III: $p_3=20$ dollars/unit, $c_3=5$ dollars/unit, $c_{d3}=5$ dollars/unit, and $c_{s3}=10$ dollars/unit.

In order to match these examples, the following formulation is developed based on the above parameters. Table 1 displays the demand units in 1000 for the three magazines with sample size $n_1 = n_2 = n_3 = 100$. Due to the publisher's propriety restriction, the data, prices, and costs were modified.

3. Profitability measurement

3.1. Achievable capacity index I_A

The profitability proposed in this paper is defined to the achievable capacity of profit (i.e., the probability of achieving a target profit under an optimal ordering policy). If the price (p), related costs (c , c_d , and c_s), and target profit (k) are given, the optimal ordering quantity is set based on the demand. Therefore, the level of profitability depends on the demand mean μ and the demand standard deviation σ when the demand is normally distributed. For conveniently, we develop a new index to express the product's profitability, and we call it the "achievable capacity

Table 1

The demand units (in 1000)/month for the three magazines.

Basic magazine (Magazine I)																			
23	21	26	24	24	28	24	25	26	23	24	24	26	25	26	28	27	27	28	24
22	28	25	26	24	23	28	20	28	24	26	26	28	28	26	25	28	24	25	23
27	27	24	26	30	26	26	27	30	23	23	24	28	22	23	23	25	29	25	26
25	22	22	26	24	27	28	26	25	25	26	23	24	28	22	27	26	27	23	27
28	26	25	24	26	23	23	24	25	28	23	25	22	20	25	23	25	25	23	28
Intermediate magazine (Magazine II)																			
30	29	26	27	30	24	27	30	28	24	32	24	27	26	28	31	26	24	27	22
28	23	21	29	27	26	29	23	25	22	30	28	31	26	28	25	27	26	28	25
25	28	27	28	30	22	26	27	29	27	26	28	25	28	22	26	29	29	28	32
23	27	24	27	31	30	28	27	22	24	23	29	33	26	24	28	28	26	28	28
28	28	26	29	29	24	25	27	32	33	26	30	30	33	29	27	21	26	28	23
High magazine (Magazine III)																			
21	22	23	21	25	22	25	19	23	19	18	18	28	20	23	19	21	21	18	19
21	24	24	20	22	21	20	21	21	17	24	26	19	22	18	20	19	23	22	23
23	23	23	18	25	21	21	22	22	22	24	23	24	22	19	21	23	23	18	20
22	21	23	19	23	22	22	21	21	24	22	22	23	24	17	23	24	22	21	22
23	22	24	21	23	22	21	22	23	22	19	20	18	20	21	21	20	23	21	19

index (ACI)". It is defined as

$$I_A = \frac{\mu - T}{\sigma},$$

where T is the target demand, i.e., $T = k/(p-c)$. The numerator of I_A provides the difference between demand mean and target demand. The denominator gives demand standard deviation. Obviously, it is desirable to have a I_A as large as possible.

3.2. I_A and achievable capacity of profit (profitability)

The total profit function, Z , for the newsboy-type product depends on the demand and order quantity, and is formulated as

$$Z = \begin{cases} (p-c)D - (c_d + c)(Q-D) = (c_p + c_e)D - c_e Q, & 0 \leq D < Q, \\ (p-c)Q = c_p Q = c_p D > k, & Q = D, \\ (p-c)Q - c_s(D-Q) = -c_s D + (c_p + c_s)Q, & Q < D < \infty, \end{cases}$$

where

 c_p profit per unit ($c_p = p - c$), c_e excess cost per unit ($c_e = c_d + c$), Q product order quantity, D demand during a period, which is a random variable following the normal distribution.

Note that if $c_p Q < k$, then the profit impossibly achieve the target profit, even the demand is large enough. Therefore, the order quantity should be at least T . For any $Q \geq T$, Z increases for $0 \leq D < Q$ and decreases for $D > Q$. The target profit will be realized when D is equal to either $LAL(Q)$ or $UAL(Q)$, so the target profit will be achieved in $D \in [LAL(Q), UAL(Q)]$, where

$$LAL(Q) = \frac{c_e Q + k}{c_p + c_e} \quad \text{and} \quad UAL(Q) = \frac{(c_p + c_s)Q - k}{c_s},$$

are the lower and upper achievable limits, respectively, and both are the functions of Q . Note that if the lost sale opportunity cost is neglected (i.e., $c_s \rightarrow 0$), we have $UAL(Q) \rightarrow \infty$. Then the achievable interval becomes an one-side limit, $[LAL(Q), \infty)$. It implies that the shortage is unable to decrease the profit. Under the assumption that the demand is normally distributed, $N(\mu, \sigma^2)$, the probability of achieving the target profit is

$$\Pr[Z \geq k] = \Phi\left(\frac{UAL(Q) - \mu}{\sigma}\right) - \Phi\left(\frac{LAL(Q) - \mu}{\sigma}\right), \quad (1)$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Note that the coefficient of variation (cv) should

be lower than 0.3 for neglecting the negative tail (Lau, 1997), i.e., the probability of negative demand is $f(D < 0) = \Phi(-\mu/\sigma) = \Phi(-1/cv) < \Phi(-1/0.3) \approx 0$. However, if $cv > 0.3$, we argue that the normal distribution is not appropriate, and truncated normal distribution is more suitable for modeling the demand instead of normal distribution.

In order to derive the achievable capacity of profit, we must find the optimal order quantity that maximizes $\Pr[Z \geq k]$ (i.e., the optimal ordering policy). We take the first-order of $\Pr[Z \geq k]$ with respect to Q , and obtain

$$\frac{d\Pr[Z \geq k]}{dQ} = \frac{1}{\sqrt{2\pi}\sigma} \left[\frac{c_p + c_s}{c_s} e^{-(1/2)((UAL(Q) - \mu)/\sigma)^2} - \frac{c_e}{c_p + c_e} e^{-(1/2)((LAL(Q) - \mu)/\sigma)^2} \right].$$

It is well known that the necessary condition for Q to be optimal must satisfy the equation $d\Pr[Z \geq k]/dQ = 0$, which implies

$$\mu = \frac{UAL(Q) + LAL(Q)}{2} - \frac{\omega\sigma^2}{UAL(Q) - LAL(Q)}, \quad (2)$$

where $\omega = \ln[1 + c_p A/c_s c_e] > 0$ and $A = c_p + c_e + c_s$. Solving Eq. (2), the optimal order quantity, Q^* , can be obtained as follows:

$$Q^* = T + \frac{c_s(c_p + c_e)(c_p\mu - k)}{c_p(c_p A + 2c_e c_s)} + \sqrt{\left[\frac{c_s(c_p + c_e)(c_p\mu - k)}{c_p(c_p A + 2c_e c_s)} \right]^2 + \frac{2c_s^2(c_p + c_e)^2\omega\sigma^2}{c_p A(c_p A + 2c_e c_s)}} > T. \quad (3)$$

In addition, the sufficient condition is also calculated as follows:

$$\frac{d^2\Pr[Z \geq k]}{dQ^2} \Big|_{Q = Q^*} = -\frac{(c_p + c_s)e^{-(1/2)((UAL(Q^*) - \mu)/\sigma)^2}}{\sqrt{2\pi}\sigma^3 c_s^2 (c_p + c_e)} \times \left\{ \frac{[UAL(Q^*) - LAL(Q^*)](c_p A + 2c_e c_s)}{2} + \frac{c_p A \omega \sigma^2}{UAL(Q^*) - LAL(Q^*)} \right\} < 0.$$

We can conclude that the stationary point Q^* is a global maximum. By using Eq. (2) and substituting Eq. (3) into Eq. (1), the achievable capacity of profit, AC , can be obtained as follows:

$$AC = \Phi\left(G + \frac{\omega}{2G}\right) - \Phi\left(-G + \frac{\omega}{2G}\right), \quad (4)$$

where

$$G = \frac{UAL(Q^*) - LAL(Q^*)}{2\sigma} = M\left(\frac{\mu - T}{\sigma}\right) + \sqrt{M^2\left(\frac{\mu - T}{\sigma}\right)^2 + M\omega} \\ = MI_A + \sqrt{M^2 I_A^2 + M\omega} > 0$$

and

$$M = \frac{c_p A}{2(c_p A + 2c_e c_s)} > 0.$$

It is easy to see that $AC(I_A)$ is a function of I_A . Taking the first-order derivative of $AC(I_A)$ with respect to I_A , we obtain

$$\frac{dAC(I_A)}{dI_A} = \frac{MG}{\sqrt{2\pi} \sqrt{M^2 + M\omega}} \left[e^\omega + 1 + \frac{\omega}{2G^2} (e^\omega - 1) \right] e^{-(1/2)(G + \omega/2G)^2} > 0.$$

As a result, $AC(I_A)$ is a strictly increasing function of I_A . If the value of I_A increases (decreases), AC becomes higher (lower). Therefore, we can express the achievable capacity of profit (profitability) according to the value of I_A .

3.3. Estimation of I_A

Since the μ and σ are unknown, the historical data of the demand ought to be collected to estimate the actual I_A . First, the natural estimator \hat{I}_A is considered. If a sample of size n is given as $\{x_1, x_2, \dots, x_n\}$, the natural estimator \hat{I}_A is obtained by replacing the μ and σ by their estimators $\bar{x} = \sum_{i=1}^n x_i/n$ and $s = [\sum_{i=1}^n (x_i - \bar{x})^2 / (n-1)]^{1/2}$, i.e.,

$$\hat{I}_A = \frac{\bar{x} - T}{s}.$$

Furthermore, we rewrite the natural estimator \hat{I}_A , and obtain

$$\hat{I}_A = \frac{\bar{x} - T}{s} = \frac{1}{\sqrt{n}} \times \frac{\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} + \left(\frac{\mu - T}{\sigma/\sqrt{n}} \right)}{\sqrt{\frac{(n-1)s^2}{\sigma^2}/(n-1)}} = \frac{1}{\sqrt{n}} \times \frac{Z + \sqrt{n}I_A}{\sqrt{\frac{\chi^2(n-1)}{n-1}}} = \frac{1}{\sqrt{n}} \times t_{n-1}(\theta). \quad (5)$$

From the Eq. (5), the estimator \hat{I}_A is distributed as $n^{-1/2}t_{n-1}(\theta)$, where $t_{n-1}(\theta)$ is a non-central t random variable with $n-1$ degree of freedom and the non-centrality parameter $\theta = \sqrt{n}I_A$. However, since $E(\hat{I}_A) = [(n-1)/2]^{1/2} \Gamma[(n-2)/2]/\Gamma[(n-1)/2]I_A \neq I_A$, the estimator \hat{I}_A is biased. To tackle this problem, we add the correction factor $b = [2/n-1]^{1/2} \Gamma[(n-1)/2]/\Gamma[(n-2)/2]$ to \hat{I}_A . Then we obtain unbiased estimator $b\hat{I}_A$ which we denote as \tilde{I}_A . Since $b < 1$ ($n > 2$), $Var(\tilde{I}_A) < Var(\hat{I}_A)$. The estimator \tilde{I}_A is based only on the complete and sufficient statistics (\bar{x}, s^2) , consequently \tilde{I}_A is the uniformly minimum variance unbiased estimator (UMVUE) of I_A .

3.4. Distribution of estimator \tilde{I}_A

We first define $R = \tilde{I}_A = b(\bar{x} - T)/s = Y/V$, where $Y = b(\bar{x} - T)/\sigma$ and $V = \sqrt{s^2/\sigma^2}$. Since $D \sim N(\mu, \sigma^2)$, we have $Y \sim N(b(\mu - T)/\sigma, b^2/n)$. In addition, it is well known that the random variable $(n-1)s^2/\sigma^2$ follows the chi-squared distribution with $n-1$ degree of freedom, we then have $V^2 = s^2/\sigma^2 \sim \Gamma((n-1)/2, 2/(n-1))$. By using the technique of change-of-variable, the probability density function of V is derived as follows:

$$f_V(v) = \frac{2v^{n-2}}{\Gamma(\frac{n-1}{2}) \left(\frac{2}{n-1}\right)^{(n-1)/2}} \exp\left(-\frac{n-1}{2}v^2\right), \quad v > 0.$$

Because Y and V are independent continuous random variables, the probability density function of R can be obtained by the *Jacobian approach*,

$$f_R(r) = \int_0^\infty f_Y(vr)f_V(v)|v| \, dv = \frac{\sqrt{2n} \left(\frac{n-1}{2}\right)^{(n-1)/2}}{b\sqrt{\pi} \Gamma(\frac{n-1}{2})} \int_0^\infty v^{n-1} \exp\left\{-\frac{1}{2} \left[\frac{(vr - bI_A)^2}{\frac{b^2}{n}} + (n-1)v^2\right]\right\} \, dv, \quad -\infty < r < \infty.$$

4. Development of the exact method

To compare the two newsboy-type products with unequal prices and costs (Product I: $c_{p1} = p_1 - c_1$, $c_{e1} = c_{d1} + c_1$, c_{s1} ; Product II: $c_{p2} = p_2 - c_2$, $c_{e2} = c_{d2} + c_2$, c_{s2}), we consider the hypothesis

testing for comparing the two AC values,

$$H_0 : AC_2 - AC_1 \leq h \quad \text{vs} \quad H_1 : AC_2 - AC_1 > h,$$

where $0 \leq h < 1$ is a designated outperformance. If $h = 0$, the test is only to determine whether the Product II has a significantly better profitability than the Product I. However, the statistical properties of the estimator of AC are difficult to describe. Even, it is impossible to define the unbiased estimator of AC . From the last section, we have proven that the achievable capacity index I_A can express the achievable capacity of profit (profitability). Therefore, we adopt the indices I_{A1} and I_{A2} to present the profitability of Products I and II, respectively. First, we assume that two products' profitability are equal, i.e., $AC_1(I_{A1}) = AC_2(I_{A2})$. Because $AC_1(I_{A1})$ and $AC_2(I_{A2})$ are monotonically increasing functions of $I_{A1} \in (-\infty, \infty)$ and $I_{A2} \in (-\infty, \infty)$, respectively, and their ranges are $(0, 1)$. For any $I_{A2} \in (-\infty, \infty)$, there exists an unique $I_{A1} \in (-\infty, \infty)$ such that $AC_1(I_{A1}) = AC_2(I_{A2})$ holds, and vice versa. Then we can show that $I_{A1} = AC_1^{-1}(AC_2(I_{A2}))$ and $I_{A2} = AC_2^{-1}(AC_1(I_{A1}))$, where AC_1^{-1} and AC_2^{-1} are the inverse functions of AC_1 and AC_2 , respectively. Therefore, if the value of I_{A2} is ϕ , the corresponding value of I_{A1} is $AC_1^{-1}(AC_2(\phi))$. From the above results, we can adopt the following hypothesis testing for comparing two I_A values:

$$H_0 : I_{A2}^c - I_{A1} \leq \delta \quad \text{vs} \quad H_1 : I_{A2}^c - I_{A1} > \delta,$$

where $I_{A2}^c = AC_1^{-1}(AC_2(I_{A2}))$ and $\delta \geq 0$ is a designated outperformance. Note that if $\delta = 0$, the test is only to determine whether the Product II has a significantly better profitability than the Product I.

Before implementing this test, we should first derive the cumulative distribution function (CDF) and probability density function (PDF) of the test statistic $W = I_{A2}^c - I_{A1}$. If the sample sizes of Products I and II are n_1 and n_2 , the PDF of the estimators $\tilde{I}_{A1} = R_1$ and $\tilde{I}_{A2} = R_2$ is

$$f_{R_1}(r_1) = \frac{\sqrt{2n_1} \left(\frac{n_1-1}{2}\right)^{(n_1-1)/2}}{b_1 \sqrt{\pi} \Gamma\left(\frac{n_1-1}{2}\right)} \times \int_0^\infty v_1^{n_1-1} \exp\left\{-\frac{1}{2} \left[\frac{(v_1 r_1 - b_1 I_{A1})^2}{\frac{b_1^2}{n_1}} + (n_1-1)v_1^2\right]\right\} dv_1$$

and

$$f_{R_2}(r_2) = \frac{\sqrt{2n_2} \left(\frac{n_2-1}{2}\right)^{(n_2-1)/2}}{b_2 \sqrt{\pi} \Gamma\left(\frac{n_2-1}{2}\right)} \times \int_0^\infty v_2^{n_2-1} \exp\left\{-\frac{1}{2} \left[\frac{(v_2 r_2 - b_2 I_{A2})^2}{\frac{b_2^2}{n_2}} + (n_2-1)v_2^2\right]\right\} dv_2,$$

where $b_1 = [2/n_1 - 1]^{1/2} \Gamma[(n_1-1)/2]/\Gamma[(n_1-2)/2]$, $b_2 = [2/n_2 - 1]^{1/2} \Gamma[(n_2-1)/2]/\Gamma[(n_2-2)/2]$, $-\infty < r_1 < \infty$, and $-\infty < r_2 < \infty$. Since $I_{A2}^c = AC_1^{-1}(AC_2(I_{A2}))$, we set $R_2^c = I_{A2}^c = AC_1^{-1}(AC_2(I_{A2})) = AC_1^{-1}(AC_2(R_2))$, and derive the CDF of R_2^c as follows:

$$F_{R_2^c}(r_2^c) = P(R_2^c \leq r_2^c) = P(AC_1^{-1}(AC_2(R_2)) \leq r_2^c) = P(R_2 \leq AC_2^{-1}(AC_1(r_2^c))) = F_{R_2}(AC_2^{-1}(AC_1(r_2^c))) = \int_{-\infty}^{AC_2^{-1}(AC_1(r_2^c))} f_{R_2}(r_2) \, dr_2,$$

where $-\infty < r_2^c < \infty$. Under the assumptions that two products are independent, we can easily obtain the CDF of $W = R_2^c - R_1$, i.e.,

$$F_W(w) = P(W \leq w) = P(R_2^c - R_1 \leq w) = \int_{-\infty}^w P(R_2^c - R_1 \leq w | R_1 = r_1) f_{R_1}(r_1) \, dr_1 = \int_{-\infty}^w P(R_2^c \leq w + r_1) f_{R_1}(r_1) \, dr_1 = \int_{-\infty}^w F_{R_2^c}(w + r_1) f_{R_1}(r_1) \, dr_1$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{AC_2^{-1}(AC_1(w+r_1))} f_{R_1}(r_1) f_{R_2}(r_2) dr_2 dr_1,$$

where $-\infty < w < \infty$. Taking the first-order derivative of $F_W(w)$ with respect to w , the PDF of w can be obtained as follows:

$$f_W(w) = \int_{-\infty}^{\infty} f_{R_1}(r_1) f_{R_2}(AC_2^{-1}(AC_1(w+r_1))) \times \frac{dAC_2^{-1}(AC_1(w+r_1))}{dw} dr_1.$$

Fig. 1 plots the CDF and PDF of W for $I_{A1}=2.0, 2.5, I_{A2}^c=2.0, 2.5$ and $n_1=n_2=n=30, 50, 100, 150, 200$. From **Fig. 1**, we can see that (1) the larger the value of $I_{A2}^c - I_{A1}$, the larger the variance of W ,

(2) the PDF of W is unimodal and is rather symmetric to $I_{A2}^c - I_{A1}$ even for small sample sizes.

5. Product selection procedure

5.1. Selection determine

Assume that the minimum requirement of I_{A1} and I_{A2}^c values are E , we consider the hypothesis testing: $H_0: I_{A2}^c - I_{A1} \leq \delta$ vs $H_1: I_{A2}^c - I_{A1} > \delta$. Given a level of Type I error α (i.e., the chance of incorrectly judging $I_{A2}^c - I_{A1} \leq \delta$ as $I_{A2}^c - I_{A1} > \delta$), the decision rule

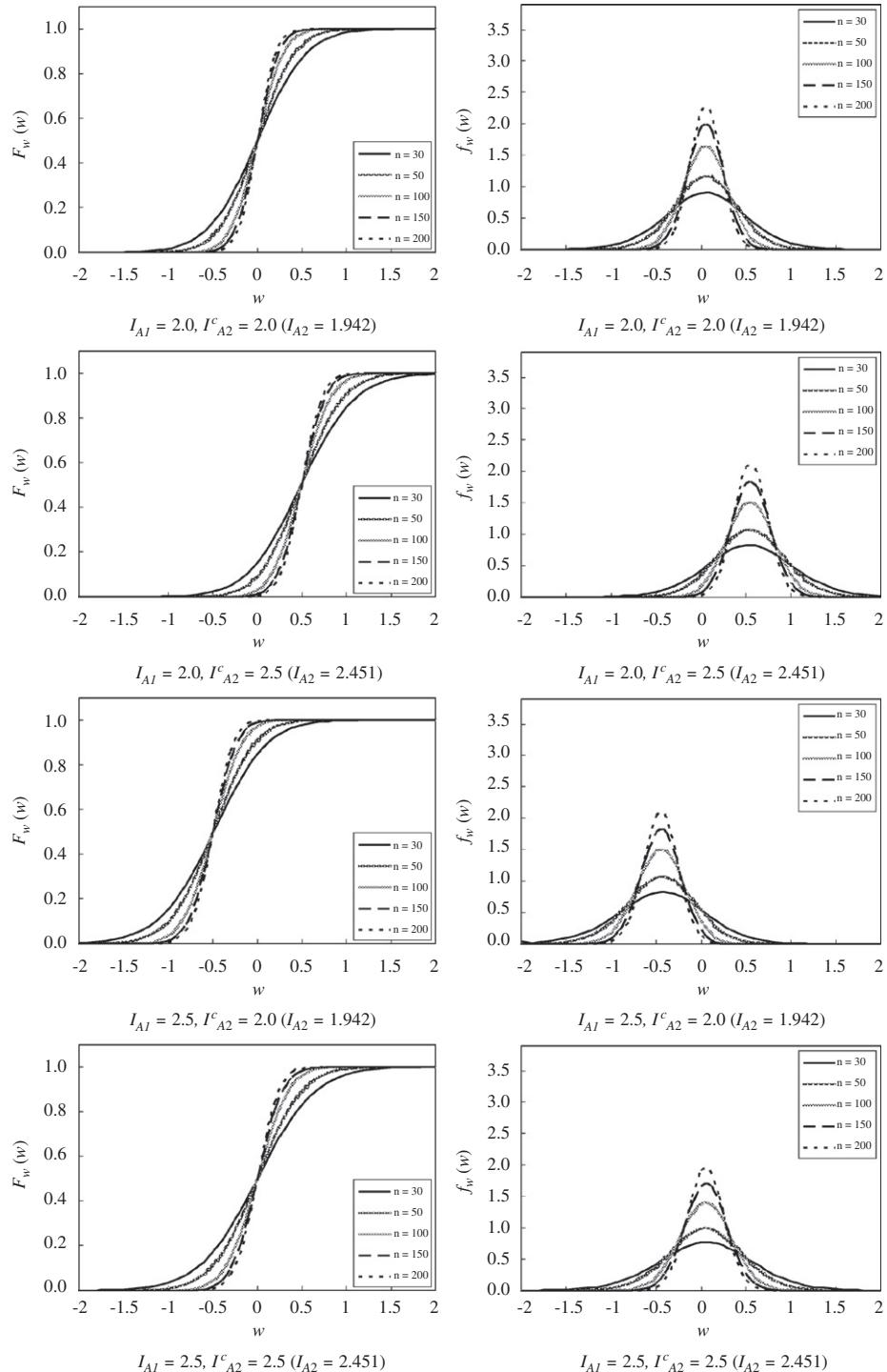


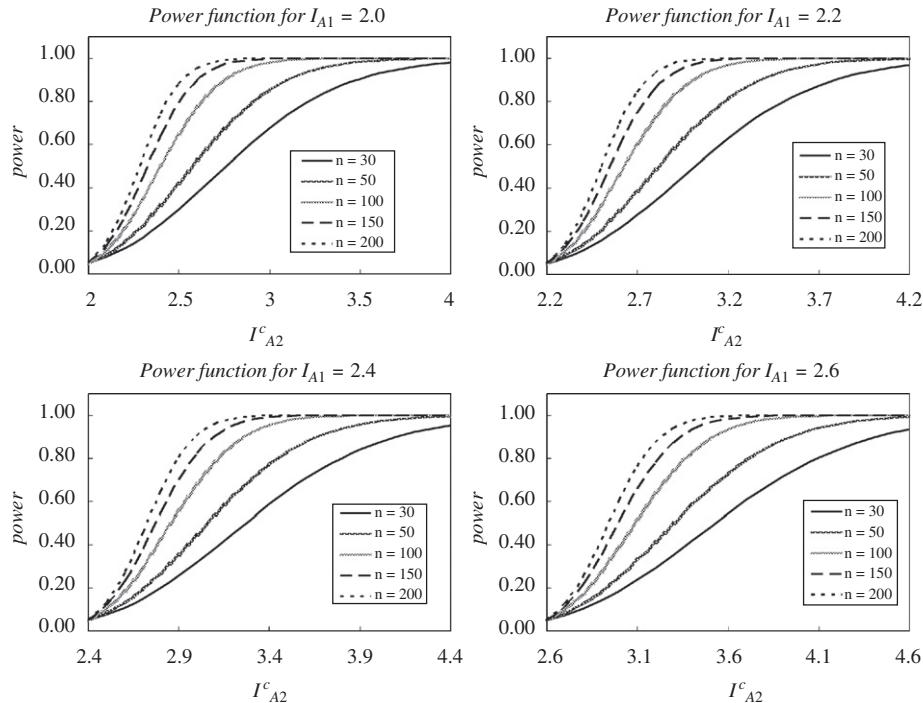
Fig. 1. CDF and PDF plots of W for sample sizes $n=30, 50, 100, 150, 200$.

Table 2Critical values for rejecting $I_{A2}^c - I_{A1} \leq \delta$ with $n=30(10)200$ and $\alpha=0.05$.

I_{A1}	n	(I_{A2}^c, I_{A2})					
		(2.0,1.908) $\delta=0.0$	(2.1,2.005) $\delta=0.1$	(2.2,2.102) $\delta=0.2$	(2.3,2.199) $\delta=0.3$	(2.4,2.296) $\delta=0.4$	(2.5,2.450) $\delta=0.5$
2.0	30	0.747	0.863	0.980	1.095	1.215	1.333
	40	0.641	0.754	0.868	0.983	1.097	1.212
	50	0.570	0.682	0.794	0.907	1.019	1.133
	60	0.519	0.629	0.740	0.851	0.963	1.074
	70	0.479	0.589	0.699	0.809	0.919	1.030
	80	0.448	0.556	0.665	0.775	0.884	0.994
	90	0.422	0.530	0.638	0.747	0.856	0.965
	100	0.399	0.507	0.615	0.723	0.832	0.940
	110	0.380	0.488	0.595	0.703	0.811	0.919
	120	0.364	0.471	0.578	0.686	0.793	0.901
	130	0.349	0.456	0.563	0.670	0.777	0.885
	140	0.336	0.443	0.550	0.656	0.763	0.871
	150	0.325	0.431	0.537	0.644	0.751	0.858
	160	0.314	0.420	0.527	0.633	0.739	0.846
	170	0.305	0.411	0.517	0.623	0.729	0.836
	180	0.296	0.402	0.508	0.614	0.720	0.826
	190	0.288	0.394	0.499	0.605	0.711	0.817
	200	0.281	0.386	0.492	0.597	0.703	0.809
(I_{A2}^c, I_{A2})							
2.2		(2.2,2.102) $\delta=0.0$	(2.3,2.199) $\delta=0.1$	(2.4,2.296) $\delta=0.2$	(2.5,2.393) $\delta=0.3$	(2.6,2.490) $\delta=0.4$	(2.7,2.587) $\delta=0.5$
	30	0.801	0.918	1.035	1.153	1.271	1.388
	40	0.686	0.800	0.915	1.029	1.144	1.260
	50	0.611	0.723	0.835	0.948	1.061	1.174
	60	0.555	0.666	0.777	0.889	1.001	1.112
	70	0.513	0.623	0.733	0.843	0.954	1.065
	80	0.479	0.588	0.697	0.807	0.917	1.027
	90	0.451	0.559	0.669	0.777	0.886	0.996
	100	0.427	0.535	0.643	0.752	0.861	0.969
	110	0.407	0.514	0.622	0.730	0.839	0.947
	120	0.389	0.496	0.604	0.712	0.819	0.927
	130	0.374	0.481	0.588	0.695	0.803	0.910
	140	0.360	0.467	0.573	0.680	0.788	0.895
	150	0.348	0.454	0.561	0.667	0.774	0.881
	160	0.336	0.443	0.549	0.655	0.762	0.869
	170	0.326	0.432	0.538	0.645	0.751	0.858
	180	0.317	0.423	0.529	0.635	0.741	0.847
	190	0.308	0.414	0.520	0.626	0.732	0.838
	200	0.300	0.406	0.512	0.617	0.723	0.829
(I_{A2}^c, I_{A2})							
2.4		(2.4,2.296) $\delta=0.0$	(2.5,2.393) $\delta=0.1$	(2.6,2.490) $\delta=0.2$	(2.7,2.587) $\delta=0.3$	(2.8,2.683) $\delta=0.4$	(2.9,2.780) $\delta=0.5$
	30	0.855	0.972	1.090	1.208	1.326	1.445
	40	0.733	0.848	0.962	1.077	1.192	1.308
	50	0.652	0.764	0.877	0.990	1.103	1.217
	60	0.593	0.704	0.816	0.927	1.039	1.151
	70	0.548	0.655	0.766	0.879	0.990	1.101
	80	0.511	0.621	0.730	0.840	0.950	1.060
	90	0.481	0.590	0.699	0.808	0.917	1.027
	100	0.456	0.564	0.673	0.781	0.890	0.999
	110	0.434	0.542	0.650	0.758	0.867	0.975
	120	0.415	0.523	0.630	0.738	0.846	0.954
	130	0.399	0.506	0.613	0.721	0.828	0.936
	140	0.384	0.491	0.598	0.705	0.812	0.920
	150	0.371	0.477	0.582	0.691	0.798	0.905
	160	0.359	0.465	0.572	0.678	0.785	0.892
	170	0.348	0.454	0.560	0.667	0.773	0.880
	180	0.338	0.444	0.550	0.656	0.763	0.869
	190	0.329	0.435	0.541	0.647	0.753	0.859
	200	0.321	0.426	0.532	0.638	0.744	0.850
(I_{A2}^c, I_{A2})							
2.6		(2.6,2.490) $\delta=0.0$	(2.7,2.587) $\delta=0.1$	(2.8,2.683) $\delta=0.2$	(2.9,2.780) $\delta=0.3$	(3.0,2.877) $\delta=0.4$	(3.1,3.058) $\delta=0.5$
	30	0.911	1.029	1.147	1.265	1.383	1.502
	40	0.781	0.896	1.010	1.126	1.241	1.357

Table 2 (continued)

(I_{A2}^c, I_{A2})													
		$(2.6, 2.490)$ $\delta = 0.0$		$(2.7, 2.587)$ $\delta = 0.1$		$(2.8, 2.683)$ $\delta = 0.2$		$(2.9, 2.780)$ $\delta = 0.3$		$(3.0, 2.877)$ $\delta = 0.4$		$(3.1, 3.058)$ $\delta = 0.5$	
50	0.694	0.807	0.920	1.033	1.147	1.260							
60	0.631	0.743	0.854	0.966	1.078	1.190							
70	0.583	0.693	0.804	0.915	1.026	1.137							
80	0.544	0.654	0.764	0.873	0.984	1.094							
90	0.512	0.621	0.730	0.840	0.949	1.059							
100	0.485	0.594	0.702	0.811	0.920	1.029							
110	0.462	0.570	0.678	0.787	0.895	1.004							
120	0.442	0.550	0.658	0.765	0.873	0.982							
130	0.425	0.532	0.639	0.747	0.854	0.962							
140	0.409	0.516	0.623	0.730	0.838	0.945							
150	0.395	0.501	0.608	0.715	0.822	0.930							
160	0.382	0.487	0.595	0.702	0.809	0.916							
170	0.370	0.477	0.583	0.690	0.796	0.903							
180	0.360	0.466	0.572	0.678	0.785	0.891							
190	0.350	0.456	0.562	0.668	0.774	0.881							
200	0.341	0.447	0.553	0.659	0.765	0.871							

**Fig. 2.** Power curves for $I_{A1}=2.0, 2.2, 2.4, 2.6$ with sample sizes $n=30, 50, 100, 150, 200$.

is to reject H_0 if the testing statistic $W \geq c_0$, where c_0 is the critical value that satisfies

$$\Pr\{W \geq c_0 | H_0 : I_{A2}^c - I_{A1} \leq \delta, n_1, n_2, I_{A1} \geq E \text{ and } I_{A2}^c \geq E\} \leq \alpha.$$

For all combinations of $(I_{A1}, I_{A2}^c, I_{A2})$ under H_0 , the maximal critical value occurs at $I_{A1} = E$ and $I_{A2}^c = E + \delta$, and the larger the α , the smaller the critical value. Thus, we calculate the critical value c_0 with the probability

$$\Pr\{W \geq c_0 | I_{A1} = E, I_{A2}^c = E + \delta, I_{A2} = AC_2^{-1}(AC_1(E + \delta)), n_1, n_2\} = \alpha.$$

If the test rejects the null hypothesis H_0 , then there is sufficient information to conclude that Product II is significantly better than Product I by a magnitude of δ . **Table 2** shows some critical values for some minimum level requirement $E=2.0, 2.2, 2.4, 2.6$, the magnitude $\delta=0.0(0.1)0.5$ of the difference between the two products, $n_1 = n_2 = n = 30(10)200$ and $\alpha = 0.05$.

Discussion: If more than two products are considered, the multiple comparison test can be adopted to tackle product selection problem. Assume that k products are compared, we implement $m = C_2^k$ tests to decide the one which has the highest profitability, i.e.,

$$H_{0i} : I_A^c a - I_{Ab} \leq 0 \quad \text{vs} \quad H_{1i} : I_A^c a - I_{Ab} > 0,$$

where $i=1, 2, \dots, m$; $a, b = 1, 2, \dots, k$ and $a \neq b$. By imitating the method of the **Bonferroni test (1936)**, the level of significance α is adjusted by the number of comparisons m to correct for Type I error inflation. If the p -value of the test is less than or equal α/m , the test rejects the null hypothesis, then there is sufficient information to conclude that Product a is significantly better than Product b . After integrating conclusion of these tests, we can find the profitability order, then the highest profitability is decided.

5.2. Required sample size

In last subsection, the product selection procedure is developed for given α risk, the probability of incorrectly judging H_0 as H_1 , which does not take into account the β risk (Type II error: the probability of incorrectly judging H_1 as H_0). When the sample sizes and the α risk are defined, the power of test, $1-\beta$, can be calculated. Fig. 2 plots the power of the test for $I_{A1} = 2.0(0.2)2.6$ vs various values of I_{A2}^c , $n_1 = n_2 = n = 30, 50, 100, 150, 200$, and $\alpha = 0.05$. It can be seen that the larger the sample size, the larger the power of test, and consequently, the smaller the β risk.

To reduce the β risk and at the same time maintain the α risk at the required level, one could increase the sample sizes. By calculating the power for a specific value of I_{A2}^c , we may obtain the

minimal sample size required for designated power and α risk. The required sample size can be calculated by recursive search method with the following two probability equations:

$$Pr\{W \geq c_0 | H_0 : I_{A2}^c \leq I_{A1}, n_1, n_2, I_{A1} \geq E \text{ and } I_{A2}^c \geq E\} \leq \alpha$$

and

$$Pr\{W \geq c_0 | H_1 : I_{A2}^c > I_{A1}, n_1, n_2, I_{A1} \geq E \text{ and } I_{A2}^c \geq E\} \geq 1-\beta.$$

Table 3 shows the sample sizes required for various designated selection powers $1-\beta = 0.90, 0.95, 0.975, 0.99$, the minimal level requirement $E = 2.0(0.2)2.6$, and the magnitude of difference $I_{A2}^c - I_{A1} = 0.5(0.1)1.0$.

6. Magazine selection implementations

The English-teaching magazines in the publisher have a minimal requirement of profitability. The minimal requirement of the I_A values for three magazines is $I_{A1} = I_{A2}^c = I_{A3}^c = 2.0$, and the target profit for three magazines is $T = 200,000$ dollars/month. We first use the Kolmogorov-Smirnov test for the demand data from Table 1 to confirm if the data is normally distributed. A test result in p -value > 0.05 , which means that data is normally distributed. Histograms of the data are shown in Fig. 3. Now, we consider two examples presented in Section 2 as follows:

Example 1: To determine if the Magazine II's profitability is higher than Magazine I, we perform the hypothesis testing: $H_0 : I_{A2}^c - I_{A1} \leq 0$ vs $H_1 : I_{A2}^c - I_{A1} > 0$. For the demand data of the two magazines displayed in Table 1, we calculate the sample means, sample standard deviations and the sample estimators for both magazines, and obtain that $\bar{x}_1 = 25.18$, $\bar{x}_2 = 27.01$, $s_1 = 2.124$, $s_2 = 2.751$, $\tilde{I}_{A1} = 2.420$, $\tilde{I}_{A2} = 3.731$, $\tilde{I}_{A2}^c = 3.480$, and thus $W = 1.059$. If $\alpha = 0.05$, from Table 2, the critical value for $n_1 = n_2 = n = 100$, $I_{A1} = 2.0$ (the minimum requirement of I_A), $\delta = 0$ is 0.399. Since the test statistic $W = 1.059 > 0.399$, we therefore conclude that the Magazine II's profitability is higher than Magazine I with 95% confidence level. We also calculate the critical value for $\delta = 0.56, 0.57, 0.58, 0.59, 0.60, 0.61$ with $n_1 = n_2 = n = 100$, $I_{A1} = 2.0$. The decision of the hypotheses is shown in Table 4. Based on the testing results, we can conclude that the Magazine II's profitability is higher than Magazine I by a magnitude of 0.60, i.e., $I_{A2}^c > I_{A1} + 0.60$.

If the expected $I_{A2}^c = 0.60$ and selection power is 0.95, the sample size required is 195 as in Table 3. Since the sample sizes of two magazines are smaller than 195, the selection power for testing $H_0 : I_{A2}^c - I_{A1} \leq 0$ vs $H_1 : I_{A2}^c - I_{A1} > 0$ would be less than 0.95. In fact, the power of test for $I_{A2}^c = 2.6$ is 0.7723, that is the β risk of incorrectly accepting $I_{A2}^c \leq I_{A1}$ while actually $I_{A2}^c > I_{A1}$ is true is up to 0.1777. In order to reduce the β risk, we would suggest the manager to collect more demand data for satisfying a designated power.

Example 2: To determine if the Magazine II is the highest profitability of three magazines, we perform the following $m = C_2^3 = 3$ tests:

$$H_{01} : I_{A2}^c - I_{A1} \leq 0 \quad \text{vs} \quad H_{11} : I_{A2}^c - I_{A1} > 0,$$

$$H_{02} : I_{A3}^c - I_{A1} \leq 0 \quad \text{vs} \quad H_{12} : I_{A3}^c - I_{A1} > 0,$$

$$H_{03} : I_{A3}^c - I_{A2} \leq 0 \quad \text{vs} \quad H_{13} : I_{A3}^c - I_{A2} > 0.$$

Table 3
Sample size required for testing $H_0 : I_{A2}^c - I_{A1} \leq 0$ vs $H_1 : I_{A2}^c - I_{A1} > 0$.

I_{A1}	I_{A2}^c	Power				I_{A1}	I_{A2}^c	Power			
		0.90	0.95	0.975	0.99			0.90	0.95	0.975	0.99
2.0	2.5	216	275	332	405	2.2	2.7	246	313	379	461
	2.6	153	195	235	288		2.8	174	222	268	327
	2.7	115	146	177	216		2.9	130	166	201	246
	2.8	90	114	138	169		3.0	101	130	157	192
	2.9	72	92	112	136		3.1	82	105	126	155
	3.0	60	76	92	113		3.2	68	86	105	128
2.4	2.9	279	356	429	523	2.6	3.1	314	400	483	590
	3.0	197	251	304	371		3.2	221	283	341	417
	3.1	147	188	227	278		3.3	166	212	255	312
	3.2	115	147	177	217		3.4	129	165	196	244
	3.3	93	118	143	175		3.5	104	133	160	197
	3.4	76	97	118	144		3.6	86	109	132	162

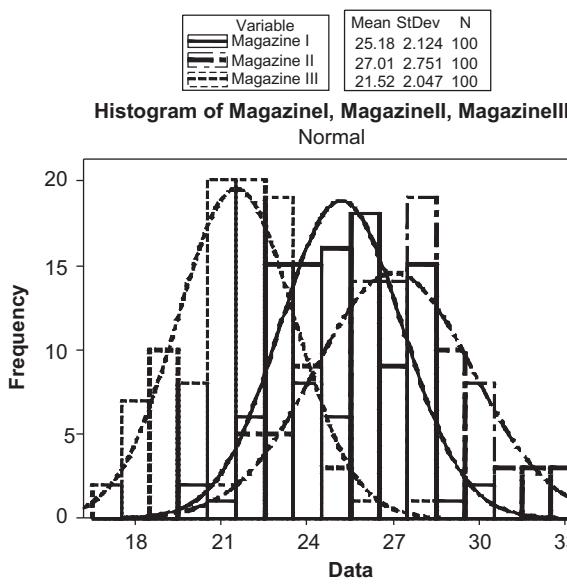


Fig. 3. Histograms of the demand data for the Magazine I, Magazine II, and Magazine III.

Table 4
Critical values and decisions of testing the two magazines.

I_{A1}	2.00	2.00	2.00	2.00	2.00	2.00
I_{A2}^c	2.56	2.57	2.58	2.59	2.60	2.61
δ	0.56	0.57	0.58	0.59	0.60	0.61
c_0	1.006	1.017	1.028	1.038	1.049	1.060 > 1.059
Decision	Reject H_0	Do not reject H_0				

If $\alpha = 0.05$, we calculate the p -value for three tests, and obtain that $\mathbf{P}_1 = 0.00002 < \alpha/m = 0.01667$, $\mathbf{P}_2 = 0.00017 < \alpha/m = 0.01667$, and $\mathbf{P}_3 = 0.78698 > \alpha/m = 0.01667$. We can conclude that Magazine II is significantly better than Magazine I (reject H_{01}), Magazine III is significantly better than Magazine I (reject H_{02}), and Magazine II is significantly better than Magazine III (accept H_{03}). Then, the Magazine II's profitability is the highest with 95% confidence level (i.e., Magazine II > Magazine III > Magazine I).

7. Conclusions

In this paper, we have investigated the product selection problem for two newsboy-type products with normal demands and unequal costs. Furthermore, we develop a new index, achievable capacity index I_A , which has a simple-form to express the product's profitability. Then, an unbiased and effective estimator of I_A to estimate the actual I_A is derived as the parameters of the demand distribution are unknown. We provided the hypothesis testing to solve this selection problem, i.e., $H_0 : I_{A2}^c - I_{A2} \leq \delta$ vs $H_1 : I_{A2}^c - I_{A1} > \delta$, where $\delta \geq 0$. Some tables are shown to determine selection decisions and sample size required under the designated risks (Types I and II errors). Note that our product selection procedure can be applied to cases with unequal sample sizes, $n_1 \neq n_2$. It is also noted that the case for two products with equal price and costs can be suited this selection procedure. Finally, a real-world application comparing English-teaching magazines is presented to illustrate the practicality of the exact approach. Some extensions of this research might be of interest. One is to investigate the imprecise demand by combining the fuzzy set concepts. The second is to consider that the demand follows truncated normal distribution as $cv > 0.3$. The last is to assume that the demands are dependent random variables.

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