

# A computer virus spreading model based on resource limitations and interaction costs

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## ABSTRACT

Computer viruses are major threats to Internet security and privacy, therefore many researchers are addressing questions linked to virus propagation properties, spreading models, epidemic dynamics, tipping points, and control strategies. We believe that two important factors – resource limitations and costs – are being overlooked in this area due to an overemphasis on power-law connectivity distributions of scale-free networks affecting computer virus epidemic dynamics and tipping points. The study show (a) a significant epidemic tipping point does exist when resource limitations and costs are considered, with the tipping point exhibiting a lower bound; (b) when interaction costs increase or usable resources decrease, epidemic tipping points in scale-free networks grow linearly while density curves decrease linearly; (c) regardless of whether Internet user resources obey delta, uniform, or normal distributions, they retain the same epidemic dynamics and tipping points as long as the average value of those resources remains unchanged across different scale-free networks; (d) it is possible to control the spread of a computer virus in a scale-free network if resources are restricted and if costs associated with infection events are significantly increased through the use of a throttling strategy.

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## 1. Introduction

Computer viruses are major threats to Internet security and privacy leading to economic loss and psychological distress (Yuan et al., 2009; Luijff, 2012). They contain small bits of malicious and self-replicating program code that are capable of erasing software and data stored on hard disks, gathering personal and address book information from email programs, launching distributed denial of service attacks against remote servers, and disrupting network systems (Hughes and DeLong, 2007; Jose et al., 2008). The destructive capability of computer viruses is a source of concern for governments, corporations, and anti-virus professionals, and large amounts of resources are being used to identify virus propagation properties, epidemic dynamics, tipping points, and control strategies.

According to Pastor-Satorras and Vespignani, whose ideas have inspired numerous studies on epidemic dynamics and anti-virus controlling strategies (Watts, 2003; Boguñá and Pastor-

Satorras, 2002; Moreno et al., 2002), Internet-based computer viruses do not have positive epidemic tipping points (Pastor-Satorras and Vespignani, 2001a,b, 2002a,b, 2003). Accordingly, many researchers argue that regardless of spreading capability, all Internet viruses have high stability and survival probabilities (Dezső and Barabási, 2002; Liu et al., 2003; Moreno et al., 2003; Volchenkov et al., 2002). However, upon closer inspection, the large majority of epidemic studies are based on the assumption that daily Internet-based interactions and communication processes are cost-free. While this assumption is suitable for studying simple scenarios consisting of malicious scripts spread by email attachments sent to large numbers of recipients, it loses accuracy in situations where viruses are spread via attachments sent to small numbers of recipients (e.g., phishing emails), peer-to-peer digital resource sharing, free uploads/downloads of large Internet files, or via multimedia messaging services. The purpose of this study is to take a more detailed look at daily Internet-based interaction and communication process limitations among network users rather than the power-law connectivity distribution properties of scale-free communication networks that are at the center of most research efforts.

The structure of this paper is as follows: in the next section we will describe five network resource properties associated with daily Internet-based interaction and communication process limitations

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among network users. These properties will be used to propose an agent-based computer virus spreading model for simulating and analyzing how resource limitations and interaction costs influence the epidemic dynamics and tipping points of computer viruses in scale-free communication networks. In Section 4 we will discuss our main findings: (a) a significant epidemic tipping point does exist when resource limitations and costs are considered, with the tipping point exhibiting a lower bound; (b) when interaction costs increase or usable resources decrease, epidemic tipping points in scale-free networks grow linearly while density curves decrease linearly; (c) regardless of whether Internet user resources obey delta, uniform, or normal distributions, they retain the same epidemic dynamics and tipping points as long as the average value of those resources remains unchanged across different scale-free networks; (d) it is possible to control the spread of a computer virus in a scale-free network if resources are restricted and if costs associated with infection events are significantly increased through the use of a throttling strategy.

## 2. Agent-based computer virus spreading model

The agent-consumable resources in our proposed model have five properties:

- they are finite (e.g., daily CPU/network usage time and communication bandwidth for file uploads/downloads);
- they can be temporarily exhausted (e.g., one gigabyte download capacity per day);
- they are non-reproducible;
- they can recover or regenerate;
- consumption of one kind can entail consumption of other kinds, thus reducing the total amount of available resources (e.g., large attachments require large amounts of upload/download time and communication bandwidth).

Based on these properties, we designed the core algorithm of our model to reflect how resource limitations and interaction costs influence the epidemic dynamics and tipping points of computer viruses in communication networks:

```

Generate a Complex Network  $G(N, E)$  for Epidemic Simulations
Initialize Agent Set  $N$ 
for loop  $l$  to Time Step Limit do
  for each Agent  $a_i \in$  Agent Set  $N$  do
     $a_i.R \leftarrow$  Resource( $a_i$ )
  next
  for each Agent  $a_j \in$  Agent Set  $N$  do
    while ( $a_j.R \geq a_j.c$ ) do
      Randomly Choose a Neighbor Agent  $a_j$ 
      if  $a_j.R \geq a_j.c$  then do
        Contact( $a_i, a_j$ )
      end if
    end while
  end for
end for

```

```

end
next
comment  $0.0 \leq$  random value  $r < 1.0$ 
for each Agent  $a_i \in$  Agent Set  $N$  do
  if  $a_i.NowState$  is Infected then do
    if random value  $r <$  recovery rate  $\gamma$  then do
       $a_i.NextState \leftarrow$  Susceptible
    end if
  end if
next
comment from time  $t$  to time  $(t + 1)$ 
for each Agent  $a_i \in$  Agent Set  $N$  do
   $a_i.NowState \leftarrow a_i.NextState$ 
next
next

procedure Contact (Agent  $a_x$ , Agent  $a_y$ ) is
  if  $a_x.NowState$  is Infected and  $a_y.NowState$  is Susceptible then do
    if random value  $r <$  infection rate  $\nu$  then do
       $a_y.NextState \leftarrow$  Infected
    end if
  end if
  if  $a_x.NowState =$  Susceptible and  $a_y.NowState =$  Infected then do
    if random value  $r <$  infection rate  $\nu$  then do
       $a_x.NextState \leftarrow$  Infected
    end if
  end if
   $a_x.R \leftarrow a_x.R - a_x.c$ 
   $a_y.R \leftarrow a_y.R - a_y.c$ 
return

procedure Generate an Ordered Network is
  for each Agent  $a_i \in$  Agent Set  $N$  do
    Connect  $a_i$  to  $z$  Nearest Neighbor by a Neighborhood Structure
  next
return

procedure Generate a Watts and Strogatz' [17] Small-World Network is
  comment Step 1: Create a fundamental network framework
  Generate an Ordered Network  $N$ 
  comment Step 2: Rewire Edges as long-range shortcuts
  for each Edge  $e_{i,j} \in$  Edges(Network  $N$ ) do
    if Random Value  $r \leq$  Probability  $p$  then do
      label Generate shortcut:
      Agent  $a_x \leftarrow$  Choose an Agent by Random
      if ( $a_x = a_i$ ) or ( $a_x = a_j$ ) or isLinked( $a_i, a_x$ ) then do
        goto label Generate shortcut
      end if
      Remove Edge  $e_{i,j}$  between Agent  $a_i$  and  $a_j$  from Network
      if isBroken(Network  $N$ ) then do
        Cancel the Previous Remove Operator
        goto label Generate shortcut
      end if
      Add a Shortcut between Agent  $a_i$  and  $a_x$ 
    end if
  next
return

procedure Generate a Barabási and Albert' [18] Scale-Free Network is
  for each Agent  $a_i \in$  Agent Set  $N$  do
    Assign 0 to Vertex Degrees Ratio of Agent  $a_i$ 
  next
  for loop  $l$  to  $z_0$  do
    Agent  $a_x \leftarrow$  Choose an Agent by random
    Assign 1 to Vertex Degrees Ratio of Agent  $a_x$ 
  next
  for each Agent  $a_i \in$  Agent Set  $N$  do
    for loop  $l$  to  $z$  do
      label Generate shortcut:
      Agent  $a_x \leftarrow$  Choose an Agent by Probability  $P(a_x)$ 
      if ( $a_i = a_x$ ) or isLinked( $a_i, a_x$ ) then do
        goto label Generate shortcut
      end if
      Add a Shortcut between Agent  $a_i$  and  $a_x$ 
      Add 1 to Vertex Degrees Ratio of Agent  $a_x$ 
      Add 1 to Vertex Degrees Ratio of Agent  $a_i$ 
    next
  next
return

function  $P$  (Agent  $a_x$ ) return real
  declare variable total as integer
  total  $\leftarrow$  0
  for each Agent  $a_i \in$  Agent Set  $N$  do
    Add Vertex Degrees Ratio of Agent  $a_i$  to total
  next
   $P \leftarrow$  Vertex Degrees Ratio of Agent  $a_x$  / total
return

```

According to experimental requirements, a specific complex network  $G(N, E)$  must be built (either Watts and Strogatz's small-world (Watts and Strogatz, 1998) or Barabási and Albert's scale-free (Barabási and Albert, 1999)), consisting of  $n = |N|$  agents (agent set  $N = \{a_1, a_2, \dots, a_n\}$ , with  $a_i$  representing a computer in a communication network  $G$ ) and  $m = |E|$  links (indicating interactions and contacts between two computers, with those having direct connections labeled "neighbors"). Each agent can have one of two possible epidemiological statuses: *susceptible* or *infected*. Two *infected* agent characteristics are (a) it is *infected* at time  $(t - 1)$ , and (b) it is

capable of infecting others. A *susceptible* agent is vulnerable to a computer virus but has not yet been *infected*. Only a small number of agents are designated as *infected* at the beginning of each simulation run; all others are designated as *susceptible*. At the beginning of each time step, usable resources  $a_i \cdot R$  for each agent  $a_i$  are reset to  $Resource(a_i)$ , where  $0 \leq a_i \cdot R \leq R_{max}$ , meaning that all agents are either renewed and/or receive supplemental resources. The statistical distribution of usable resources can be delta (fixed value  $r_{constant}$ ), uniform, normal, or power-law, as long as the average  $\langle r \rangle$  value of agent resources satisfies  $\langle r \rangle = r_{constant}$ .

Agents randomly interact with multiple neighbors during each time step, with usable resources and costs consumed in every interaction. Each agent  $a_i$  interacts with a randomly selected neighbor agent  $a_j$ . Regardless of the interaction result, agents  $a_i$  and  $a_j$  expend interaction costs  $a_i \cdot c$ ,  $a_j \cdot c$ , where  $0 \leq a_i \cdot c \leq a_i \cdot R$  and  $0 \leq a_j \cdot c \leq a_j \cdot R$ , and resources decrease accordingly. If  $a_i \cdot R < a_i \cdot c$  after an interaction, that agent cannot interact with other neighbors; otherwise, agents continue to randomly select other neighboring agents for interactions until their resources are depleted.

When an *infected* agent  $a_i$  and adjacent *susceptible* agent  $a_j$  interact, whether or not  $a_j$  is *infected* by  $a_i$  is determined by infection rate  $\nu$ , and agent  $a_i$  recovery and return to susceptibility is determined by recovery rate  $\gamma$ . Spreading rate  $\lambda$  is defined as  $\nu/\gamma$ ; generally,  $\gamma = 1$  and  $\lambda = \nu$ . We defined  $\rho(t)$  as the density of *infected* agents present at time step  $t$ ; when time step  $t$  becomes infinitely large,  $\rho$  represents a steady infected density.

Our proposed model is written as a general-purpose and extendable Java application that is suitable for detailed simulation experiments and classroom demonstrations of computer immunization and anti-virus control strategies. For source code and binary executables for specific research requirements, please contact the corresponding author.

### 3. Model analysis

Our proposed model is expressed as

$$\frac{d\rho_k(t)}{dt} = -\rho_k(t) + \lambda S_k [1 - \rho_k(t)] \theta[\{\rho_k(t)\}] \quad (2)$$

where  $S_k$  is the minimum value for the ratio between an agent's resources ( $R$ ) in relation to interaction costs ( $c$ ) and its connectivity ( $k$ ). With the exception of  $S_k$ , the symbols used here are consistent with those used by Pastor-Satorras and Vespignani in their discussions of spreading dynamics.  $\rho_k(t) \ll 1$  is the probability that a node with  $k$  links is *infected* at time  $t \geq 0$  (neglecting the higher order).  $\lambda$  is a pre-determined constant representing the spreading capability of specific computer viruses, defined as the ratio between the rates at which healthy agents in a population become *infected* and *infected* agents recover. The term  $\{\rho_k(t)\}$  denotes the set containing all  $\rho_k(t)$  for all positive  $k$ , as well as the alternative representation  $\{\rho_1(t), \rho_2(t), \rho_3(t), \dots\}$ . Accordingly  $\theta[\{\rho_k(t)\}]$  is the probability that any given agent will be linked to an *infected* agent. According to Pastor-Satorras and Vespignani, this probability is proportional to the infection rate, and can therefore be reduced to  $\theta(\lambda)$ .

In Eq. (3) we define  $\rho_k$  as the steady state of  $\rho_k(t)$  by solving the stationary condition  $d\rho_k(t)/dt = 0$ . Substituting  $\theta(\lambda)$  in that equation:

$$\theta = \frac{1}{\langle k \rangle} \sum_k k P(k) \frac{\lambda S_k \theta}{1 + \lambda S_k \theta} \quad (3)$$

As shown, a trivial solution is  $\theta = 0$ . Next, inequality (4) is derived based on the possibility that the right-hand side of Eq. (3) has a non-singular solution:

$$\left. \frac{d}{d\theta} \left( \frac{1}{\langle k \rangle} \sum_k k P(k) \frac{\lambda S_k \theta}{1 + \lambda S_k \theta} \right) \right|_{\theta=0} \geq 1. \quad (4)$$

Without using a concave function as an alternative proof, we show that Eq. (4) is a contradiction. Assuming that Eq. (4) does not hold, it should be expressed as

$$\left. \frac{d}{d\theta} \left( \frac{1}{\langle k \rangle} \sum_k k P(k) \frac{\lambda S_k \theta}{1 + \lambda S_k \theta} \right) \right|_{\theta=0} < 1. \quad (5)$$

After defining

$$F(\theta) = \theta - \frac{1}{\langle k \rangle} \sum_k k P(k) \frac{\lambda S_k \theta}{1 + \lambda S_k \theta} \quad (6)$$

we observe that a trivial solution for  $F(0) = 0$  is  $\theta = 0$ . Next, note that the first derivative of  $F(\theta)$  at 0 with respect to  $\theta$  is larger than 0:

$$\left. \frac{d}{d\theta} F(\theta) \right|_{\theta=0} = 1 - \left. \frac{d}{d\theta} \left( \frac{1}{\langle k \rangle} \sum_k k P(k) \frac{\lambda S_k \theta}{1 + \lambda S_k \theta} \right) \right|_{\theta=0} > 0 \quad (7)$$

However, this implies that non-trivial solutions for  $F(\theta) = 0$  do not exist for any  $\theta > 0$ , which contradicts inequality (5). We therefore obtained  $\lambda_c = \langle k \rangle / \sum_k k P(k) S_k$  as a conclusion regarding epidemic tipping points. By deriving the above conclusion in advance, we obtained a separate conclusion for the lower epidemic tipping point boundary,  $\lambda_c \geq 1 / ((R/c)^2 / \langle k \rangle + R/c)$  (as  $\langle k \rangle \rightarrow \infty$ ,  $\lambda_c$  is at minimum equal to  $c/R$ ), which also implies that resources and interaction costs significantly affect epidemic tipping point values.

Since  $\lambda_c$  represents the tipping point at which a computer virus becomes epidemic, managing its value should be a primary concern for computer scientists and anti-virus experts. In summary, the lower bound of epidemic tipping point  $\lambda_c$  decreases when interaction cost  $c$  decreases or average resource  $R$  increases. Accordingly, an agent's available resources increase when  $c/R$  decreases, thereby enhancing its ability to contact most other agents via underlying communication networks. This result supports existing knowledge about immunization and anti-virus controlling strategies: restricting a computer's resources increases the epidemic tipping point. Neglecting resources makes  $R$  infinitely large, meaning that they are inexhaustible and that the epidemic tipping point  $\lambda_c$  will continue to approach 0 as long as the average number of links is sufficiently large. Our proposed model is therefore identical to Pastor-Satorras and Vespignani's model in that a computer virus has the potential to achieve epidemic proportions even when the number of *infected* agents is very small.

Since an infection event requires sufficient resources, controlling the  $c/R$  ratio can increase the epidemic tipping point  $\lambda_c$  and decrease the steady-state density  $\rho$ . In contrast, computer viruses can spread very quickly via small email attachments distributed to a large number of recipients because they can be simultaneously transmitted to many sites. Affected areas can be very large over a short time period, with disastrous results in terms of lost data, work delays, and money (e.g., the email worm "I LOVE YOU", the worst network attack to date, spread over the Internet on 4 May 2000 and inflicted billions of dollars of damage worldwide). Initially designed to slow the spread of a computer virus, a throttling strategy (Morre et al., 2003) for containing virus infections places restrictions on uploads/downloads from remote servers (e.g., one gigabyte per day) – in other words, resources are purposefully limited in order to increase the epidemic tipping point. In one form of this strategy, many P2P services place restrictions on downloading to control the resource limitations. Another throttling strategy is charging upload/download fees for exceeding daily limitations – in other words, increasing communication costs.

4. Results and discussion

Toward the goals of determining the reliability and robustness of our results and ensuring the applicability of our conclusions to scale-free networks whose connectivity distribution probabilities satisfy  $P(k) \sim k^{-\alpha}$  where  $2 < \alpha \leq 3$ , we built 8 scale-free and 8 small-world networks (Table 1), all containing different numbers of nodes and links. All sensitivity analysis experiments<sup>1</sup> were simulated using these networks in order to determine the consistency of our results; no weakening or side effects were observed when node and link numbers were changed. Except for node and link numbers, all parameter settings for the 8 scale-free networks were identical. Scale-free network #3 was designated as our default; unless otherwise indicated, it was used to generate all results reported and discussed in this paper. According to those results, our conclusions are not limited to our proposed agent-based computer virus spreading model based on the 8 scale-free networks.

We used the first simulation experiment to show that a computer virus spreading in a scale-free network has a nonzero, positive, and significant epidemic tipping point if resources and interaction costs are taken into consideration – a conclusion that conflicts with those reported by past researchers (Pastor-Satorras and Vespignani, 2001a,b, 2002a,b, 2003; Dezsö and Barabási, 2002; Liu et al., 2003; Moreno et al., 2003; Volchenkov et al., 2002). To evaluate how node and link numbers in scale-free networks affect epidemic tipping points, all experiments were simulated using scale-free or small-world networks with different numbers of nodes and links. The value of usable resources per agent was reset to 16 units at the beginning of each time step. Daily interaction and communication process costs were designated as one unit, accounting for 6.25% of an agent’s total usable resources.

We used three types of complex networks to analyze relationships between effective spreading rate and steady density for our proposed model: small-world; scale-free without interaction costs ((Pastor-Satorras and Vespignani, 2001a,b, 2002a,b, 2003) model); scale-free with limited resources and interaction costs. As shown in Fig. 1, the 8 simulation suites generated consistent results that did not become contradictory when node and link numbers were adjusted, suggesting that our results can be applied to different scale-free networks used to simulate computer virus diffusion scenarios. The curves marked with triangles indicate that Pastor-Satorras and Vespignani’s cost-free model reached a 0 level of steady density in a continuous and smooth manner when the effective spreading rate was decreased, indicating the absence of an epidemic tipping point without interaction costs. The curves marked with squares indicate that computer viruses do have epidemic tipping points small-world homogeneous networks. In a similar manner, the curves marked with circles also indicate that computer viruses have significant epidemic tipping points in scale-free networks when resources and interaction costs are considered. According to these results, resources, interaction costs, and average vertex degree impact epidemic dynamics and tipping points in scale-free networks to a much greater degree than node and link numbers.

Our second simulation focused on relationships among epidemic tipping point, steady density curve, and the ratio of interaction costs to an agent’s usable resources. To analyze the influences of the ratio on the other two factors, we employed 10 usable resource values and assigned daily interaction and communication process costs as one unit. As shown in Fig. 2a, the epidemic tipping point significantly increased as the ratio grew. For instance, when the value of an agent’s usable resources was set at 8 units at

<sup>1</sup> In the interest of robustness, all epidemic dynamics and tipping points discussed in this paper represent average values for 30 runs.

**Table 1** Parameters for eight scale-free networks and eight small-world networks built using different numbers of nodes and average vertex degrees.

| Properties                         | Barabási and Albert (1999) scale-free networks |       |       |       |        |        |        |        | Watts and Strogatz (1998) small-world networks with rewiring rate = 0.01 |       |       |       |        |        |        |        |
|------------------------------------|--|-------|-------|-------|--------|--------|--------|--------|--|-------|-------|-------|--------|--------|--------|--------|
|                                    | SFN#1  | SFN#2 | SFN#3 | SFN#4 | SFN#5  | SFN#6  | SFN#7  | SFN#8  | SWN#1  | SWN#2 | SWN#3 | SWN#4 | SWN#5  | SWN#6  | SWN#7  | SWN#8  |
| Number of nodes                    | 1000   | 1000  | 2000  | 2000  | 10,000 | 10,000 | 20,000 | 20,000 | 1000   | 1000  | 2000  | 2000  | 10,000 | 10,000 | 20,000 | 20,000 |
| Number of edges                    | 2000   | 4000  | 4000  | 8000  | 20,000 | 40,000 | 40,000 | 80,000 | 2000   | 4000  | 4000  | 8000  | 20,000 | 40,000 | 40,000 | 80,000 |
| Average vertex degrees             | 4  | 8     | 4     | 8     | 4      | 8      | 4      | 8      | 4  | 8     | 4     | 8     | 4      | 8      | 4      | 8      |
| Exponent of power-law distribution | 2.4  | 2.4   | 2.4   | 2.4   | 2.4    | 2.4    | 2.4    | 2.4    | 0.28   | 0.37  | 0.28  | 0.37  | 0.28   | 0.37   | 0.28   | 0.37   |
| Average clustering coefficient     | 0.02   | 0.03  | 0.01  | 0.02  | 0.00   | 0.00   | 0.00   | 0.00   | 7.0  | 4.4   | 7.9   | 4.9   | 9.9    | 6.0    | 10.6   | 6.5    |
| Average degree of separation       | 4.2  | 3.2   | 4.4   | 3.4   | 5.1    | 3.9    | 5.3    | 4.1    | 0.28   | 0.37  | 0.28  | 0.37  | 0.28   | 0.37   | 0.28   | 0.37   |



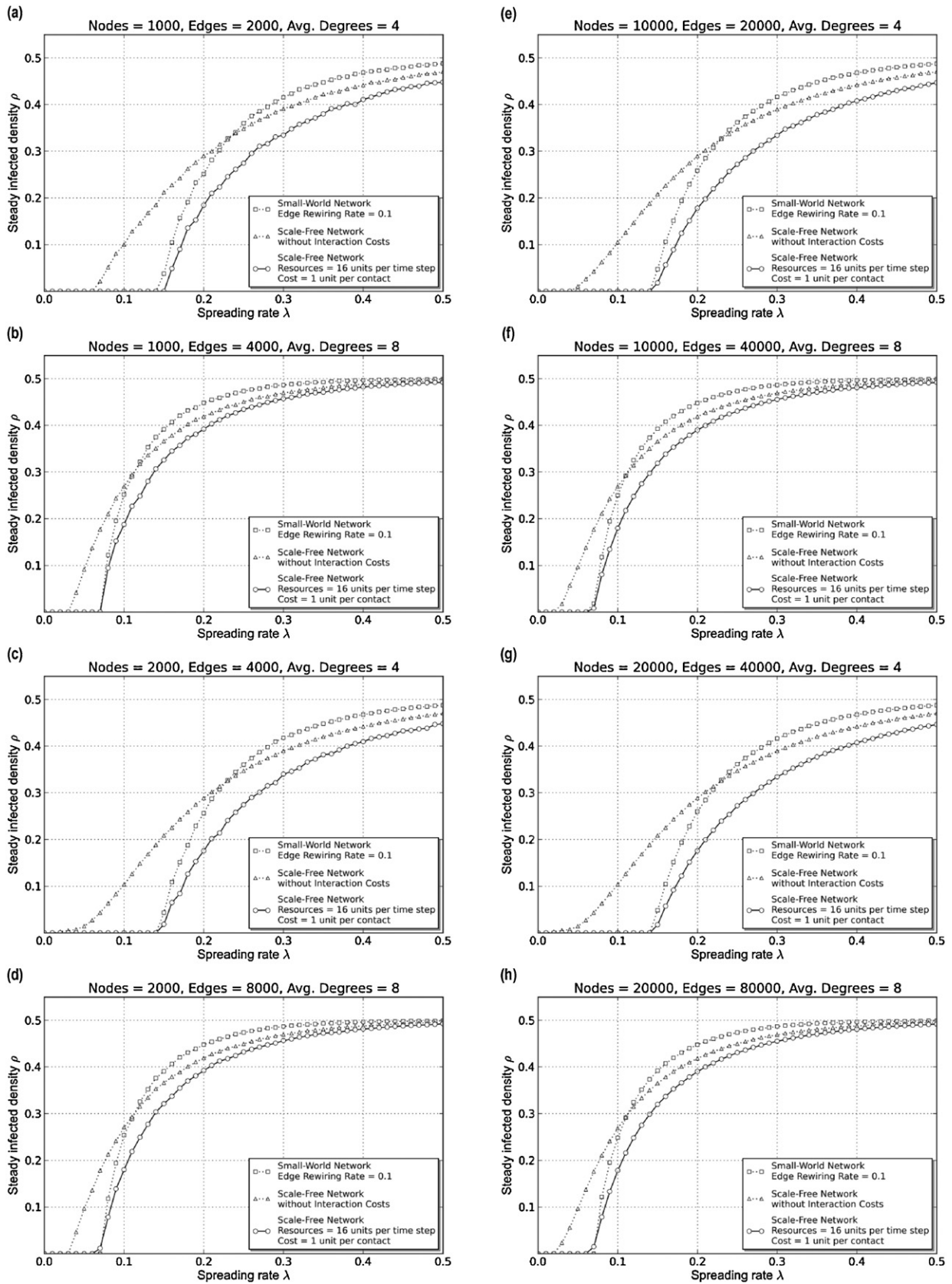


Fig. 1. Relationship between effective spreading rate  $\lambda$  and steady density  $\rho$  according to three types of complex networks: small-world, scale-free without costs, and scale-free with resource limitations and interaction costs.

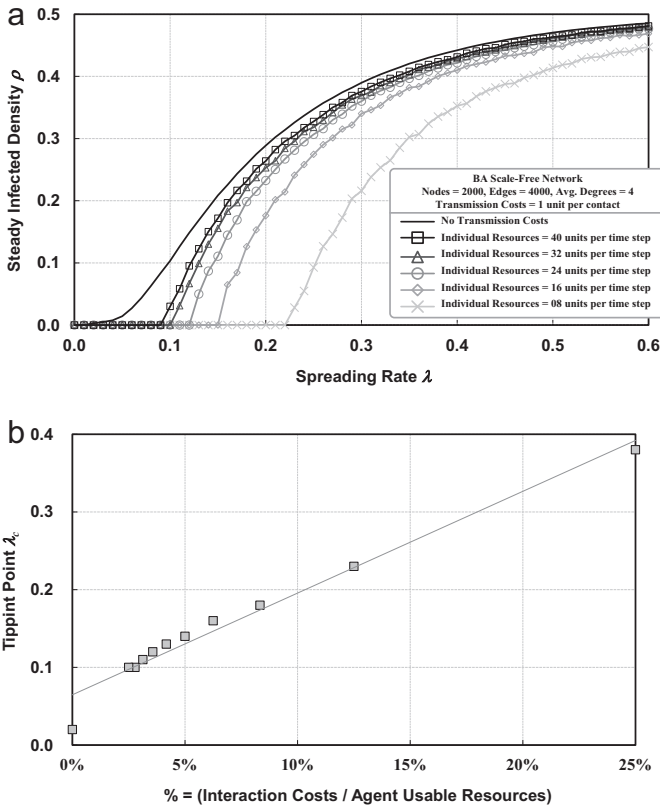


Fig. 2. Simulation results for scale-free network #3. (a) The amount of an agent's resources affected epidemic steady density curves and tipping points. (b) Linear relationship between the ratio of interaction costs to an agent's resources and epidemic tipping point.

the beginning of each time step, the epidemic tipping point was approximately 0.22 – significantly larger than for a small-world network with the same number of nodes and links and same average vertex degree. The opposite was also true: when the value of an

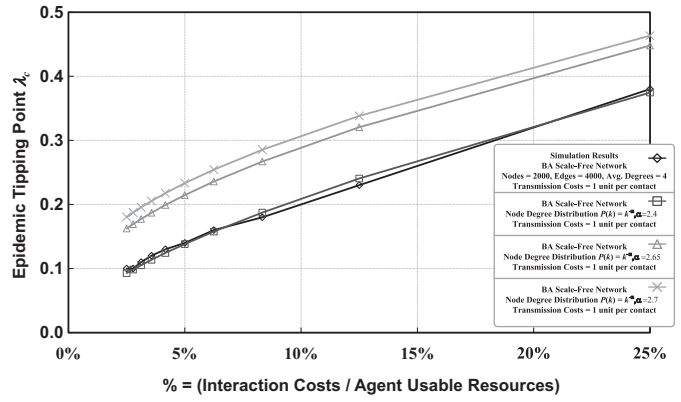


Fig. 3. As a function of the  $c/R$  (interaction costs/agent resources) ratio in scale-free networks, epidemic tipping point  $\lambda_c$  was used to analyze results from a simulation and three mathematical analyses.

agent's usable resources was set at 40 units at the beginning of each time step, the shape of the density curve was very close to that of the scale-free network without interaction costs (Fig. 2a, solid line); in addition, the epidemic tipping point decreased to 0.09. As shown in Fig. 2b, we observed (a) a linear correlation between the epidemic tipping point and the ratio, and (b) that the density curve grew at a slower rate as the ratio increased (Fig. 2a) – that is, the ratio and density exhibited a negative linear correlation when the effective spreading rate exceeded the epidemic tipping point. According to these results, when interaction costs increased or agent resources decreased, the epidemic tipping point of a computer virus spread via the Internet grew linearly, and density curve shrank linearly.

A comparison of results from our mathematical model and second simulation is presented as Fig. 3. We used several probability degrees for  $P(k) \sim k^{-\alpha}$  and found that at an  $\alpha$  of 2.7 or 2.65, the values for both curves exceeded those derived from the simulation experiment. The two curves matched at an  $\alpha$  of 2.4.

The motivation for the third simulation was to investigate the effects of the statistical distribution of an agent's usable resources on the epidemic dynamics and tipping points of computer viruses

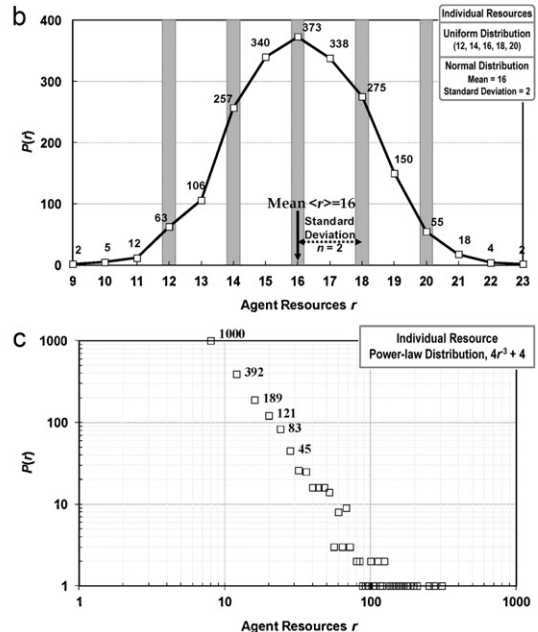
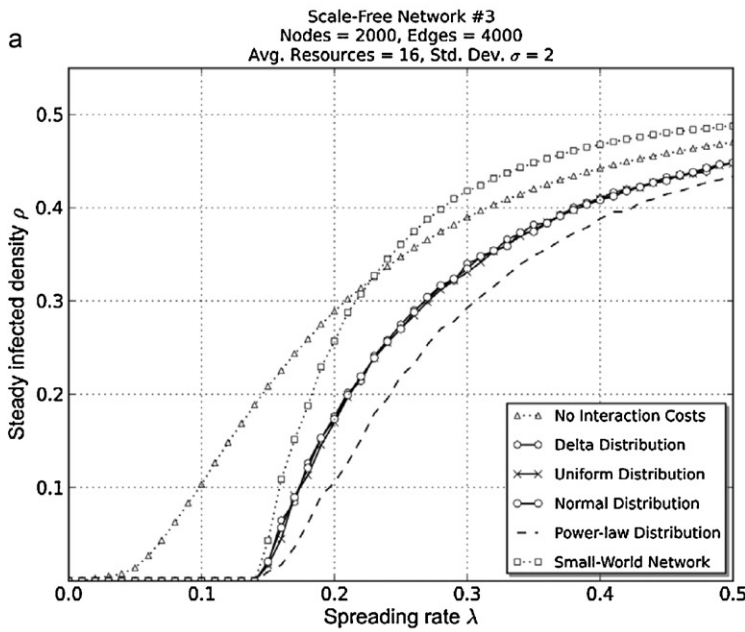
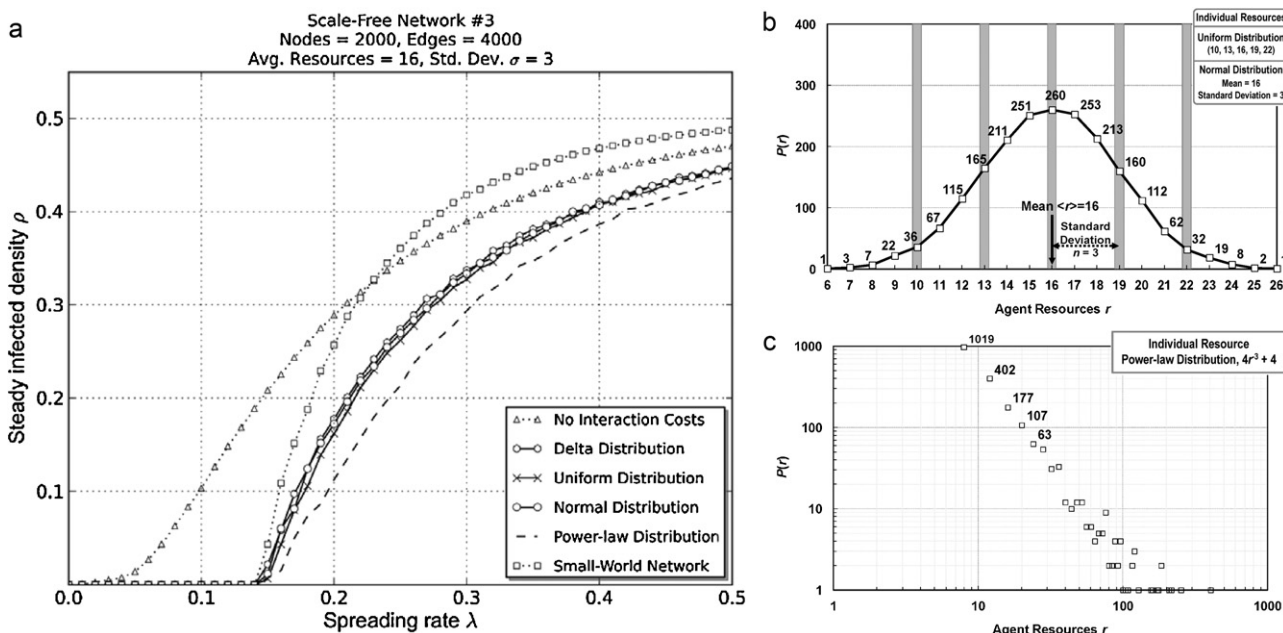


Fig. 4. Simulation results for scale-free network #3. (a) Effects of different statistical distribution types for agent resources on the epidemic steady density curves and tipping points of computer viruses spread within scale-free networks. (b) Uniform ( $n = 5, r = 2$ ) and normal (standard deviation = 2) distributions of agent resources with an average ( $r$ ) value of 16. (c) Power-law distribution (degree = 3) of agent resources.



**Fig. 5.** Simulation results for scale-free network #3. (a) Effects of different statistical distribution types for agent resources on the epidemic steady density curves and tipping points of computer viruses spread within scale-free networks. (b) Uniform ( $n=5$ ,  $r=3$ ) and normal (standard deviation = 3) distributions of agent resources with an average ( $r$ ) value of 16. (c) Power-law distribution (degrees = 3) of agent resources.

spread via the Internet. Our specific goal was to determine how different statistical distribution types and distribution parameters affect the steady density curves of viruses in contexts of limited agent resources and interaction costs.

The density curves marked with diamonds, crosses, and circles in Figs. 4a and 5a, respectively, represent delta (fixed value = 16), uniform, and normal resource distributions; parameters are shown in Figs. 4b and 5b. The results indicate nearly identical epidemic tipping points and overlapping density curves (indicating no statistically significant differences) when the average values of usable resources were equal. However, as shown in Figs. 4c and 5c, when those same resources represented a power-law distribution and no correlation existed between the total amount of an agent's usable resources and vertex degree, the resulting dashed density curve grew more slowly compared to those for the other three distribution types, even when they all shared the same epidemic tipping point.

As shown in Figs. 4 and 5, the same results emerged as long as the average usable resource values were identical. Note that density curves and epidemic tipping points were very similar across the distribution types, regardless of whether the resources had a uniform distribution with a range of 2 or 3 or a normal distribution with a standard deviation of 2 or 3 (Figs. 4b and 5b). According to the density curves shown in Figs. 4a and 5a, as long as researchers ensure that usable resources do not reflect a power-law distribution, at the beginning of each time step they can assign usable resources for each agent as the fixed average value ( $r$ ) of the statistical distribution derived from the real-world scenario being studied.

## 5. Conclusion

Research on the epidemic dynamics of computer viruses has increasingly incorporated Watts and Strogatz's (Watts and Strogatz, 1998) description of small-world networks (characterized by tightly clustered connections and short paths between node pairs) and Barabási and Albert's (Barabási and Albert, 1999) insights regarding scale-free networks marked by power-law connectivity distributions. The list of researchers using network approaches

to computer virus models and analyses also includes Kuperman and Abramson (2001), Liu et al. (2009), Meloni et al. (2012), Newman (2002, 2003), Newman and Watts (1999), Pastor-Satorras and Vespignani (Watts, 2003; Boguñá and Pastor-Satorras, 2002; Moreno et al., 2002; Pastor-Satorras and Vespignani, 2001a,b), and Watts (Pastor-Satorras and Vespignani, 2002a). All of these investigators have noted that the topological properties underlying communication networks exert considerable influence on computer virus epidemic dynamics and spreading characteristics, and support subtle analyses that non-network-directed approaches cannot. To simplify their experiments, researchers have tended to overlook resource limitations and interaction costs, both of which exert significant impacts on computer virus epidemic dynamics and tipping points. In this paper we described five characteristics of network user resources, and proposed an agent-based computer virus spreading model for investigating how resource limitations and interaction costs influence the epidemic dynamics and tipping points of computer viruses in scale-free networks. According to results from our first set of experiments, resource limitations, interaction costs, and average vertex degree are among those factors exerting significant impacts on epidemic tipping points, but node and link numbers were found to have little impact. Results from our second experimental set provide insight into how the ratio of single infection event costs to total amount of an agent's resources affects density curves and epidemic tipping points. We found that when interaction costs increased, or when the total amount of an agent's resources decreased, the epidemic tipping point of an infection event in a scale-free network grew, and density decreased at certain transmission rates. Results from our third set of experiments indicate that – regardless of delta, uniform, or normal distribution – they have nearly identical density curves and epidemic tipping points as long as average resource values remain the same across different networks.

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