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An Extension of the Product Acceptance Determination for One-Sided Process with Multiple Characteristics

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In statistical quality control, product acceptance determination is an important problem for producer and consumer. In practical, particularly more than one quality characteristic must be simultaneously considered to improve the product quality because of the product design. In this article, we investigate the lot sentencing problem for normally distributed process with one-sided specification and multiple characteristics. We not only provide a simple procedure to help practitioners make reliable decision easily but also tabulate the required sample size and the corresponding critical acceptance value for various producer's and consumer's risks with the capability requirements AQL (acceptable quality level) and LTPD (lot tolerance percent defective). Copyright © 2012 John Wiley & Sons, Ltd.

Keywords: critical values; multiple characteristics; one-sided specification; product acceptance determination

1. Introduction

Product acceptance determination can be used for quality assurance applications involving quality contract on product orders between producers and consumers. Product acceptance determination basically consists of a sample size for inspection and an acceptance criterion. The operating characteristic (OC) curve plots the probability of accepting the lot versus the lot fraction defective. The producer is primarily interested in insuring that good lots would be accepted, and the consumer wants to be reasonably sure that bad products would be rejected. Therefore, a product acceptance determination usually focus on two designated points, (AQL,$1-\alpha$) and (LTPD,β), on the OC curve. The symbols α and β denote the risk of producers and consumers, respectively. AQL presents the poorest level of quality for the vendor's process average, and LTPD is the poorest quality level that the consumer is willing to accept.

Pearn and Wu[1] developed a clear product acceptance determination procedure for a one-sided process with single characteristic. The results attended are very practical for industrial application but are restricted to process with only one quality characteristic. Hence, we extend the results to cases with multiple independent quality characteristics. The generalized indices C_{pu}^{T} and C_{pk}^{T} for processes with one-sided and two-sided specifications were proposed by Wu and Pearn[2] and Pearn *et al.*[3]

2. Multiple characteristics

Hsu *et al.*[4] discussed the sample size determination problem for production yield estimation with multiple quality characteristics. Then, Pearn *et al.*[5] implemented the process capability index with multiple characteristics to deal with the photolithography control in wafer fabrication. Later, Pearn and Cheng[6] proposed a process capability index for measuring production yield with multiple characteristics. Recently, Awad and Kovach[7] investigated a multiresponse optimization problem using multivariate process capability index. Then, Goethals and Cho[8] developed a target-focused index for process with multiple characteristics. Kotz and Johnson,[9] Wu *et al.*,[10] and Yum and Kim[11] provided some reviews and overviews about process capability index. Recent studies on process capability include White and Borror,[12] Spiring,[13] and Negrin *et al.*[14]

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Wu and Pearn[2] proposed the overall capability index for one-sided processes, designed as

$$C_{\mathrm{pu}}^{\mathrm{T}}=\frac{1}{3}\Phi^{-1}\left\{\prod_{j=1}^{m}\Phi\left(3C_{\mathrm{pu}j}\right)\right\}, \tag{1}$$

where $C_{\mathrm{pu}j}$ denotes the C_{pu} value of the jth characteristic for $j=1, 2, \ldots, m$, and m is the number of characteristics. $\phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. A one-to-one correspondence relationship between the index $C_{\mathrm{pu}}^{\mathrm{T}}$ and the overall process yield P can be established as

$$P=\prod_{j=1}^{m}P_j=\prod_{j=1}^{m}\Phi\left(3C_{\mathrm{pu}j}\right)=\Phi\left(3C_{\mathrm{pu}}^{\mathrm{T}}\right)=\Phi(3c) \tag{2}$$

Hence, the new index $C_{\mathrm{pu}}^{\mathrm{T}}$ provides an exact measure on the overall process yield.

3. Statistical properties of $\hat{C}_{\mathrm{pu}}^{\mathrm{T}}$

To handle the issue for cases with multiple quality characteristics, we applied the similar technique used by Pearn *et al.*[3] to derive the asymptotic distribution for the natural estimator of $C_{\mathrm{pu}}^{\mathrm{T}}$, which is defined as

$$\hat{C}_{\mathrm{pu}}^{\mathrm{T}}=\frac{1}{3}\Phi^{-1}\left\{\prod_{j=1}^{m}\Phi\left(3\hat{C}_{\mathrm{pu}j}\right)\right\}=\frac{1}{3}\Phi^{-1}\left\{\prod_{j=1}^{m}\Phi\left(\frac{\mathrm{USL}-\bar{X}_j}{S_j}\right)\right\} \tag{3}$$

where $\bar{X}_j$ and S_j denote the sample mean and the sample variance of jth characteristic. Using the Taylor expansion technique for m-variate and taking the first order, the asymptotic distribution of $\hat{C}_{\mathrm{pu}}^{\mathrm{T}}$ is

$$\hat{C}_{\mathrm{pu}}^{\mathrm{T}}\approx N\left(C_{\mathrm{pu}}^{\mathrm{T}},\ \frac{1}{9n\left(\phi\left(3\,C_{\mathrm{pu}}^{\mathrm{T}}\right)\right)^2}\sum_{j=1}^{m}\left(a_j{}^2+b_j{}^2\right)\right) \tag{4}$$

where

$$a_j=\prod_{\substack{i=1\\ i\neq j}}^{m}\Phi\left(3C_{\mathrm{pu}i}\right)\phi\left(3C_{\mathrm{pu}j}\right)\text{ and }b_j=\frac{3C_{\mathrm{pu}j}}{\sqrt{2}}a_j,\ j=1,\ldots,m$$

$\phi(\cdot)$ represents the probability density function of the standard normal distribution. Later, Pearn *et al.*[3] also showed that the conservative lower confidence bound can be obtained by setting $C_{\mathrm{pu}1}=C_{\mathrm{pu}}^{\mathrm{T}}$ and others $C_{\mathrm{pu}j}=\infty$ $(j=2,3,\ldots,m)$. The approximate 100 $(1-\alpha)$% lower confidence bound for $C_{\mathrm{pu}}^{\mathrm{T}}$ can be expressed as

$$C_{\mathrm{pu}}^{\mathrm{T}\ \mathrm{LB}}=\frac{2\hat{C}_{\mathrm{pu}}^{\mathrm{T}}-\sqrt{\frac{4z_\alpha^2}{9n}+\frac{2z_\alpha^2}{n}\hat{C}_{\mathrm{pu}}^{\mathrm{T}\ 2}-\frac{2z_\alpha^4}{9n^2}}}{2-z_\alpha^2/n} \tag{5}$$

Therefore, we would derive the reliable critical value c_0 with such parameter setting. At this time, the approximate sampling distribution of $\hat{C}_{\mathrm{pu}}^{\mathrm{T}}$ can be rewritten as a simpler form, that is,

$$\hat{C}_{\mathrm{pu}}^{\mathrm{T}}\approx N\left(\mathrm{C}_{\mathrm{pu}}^{\mathrm{T}},\frac{1}{9n}+\frac{\left(C_{\mathrm{pu}}^{\mathrm{T}}\right)^2}{2n}\right). \tag{6}$$

4. Product acceptance determination

The $C_{\mathrm{pu}}^{\mathrm{T}}$ index can be used as a quality benchmark for acceptance of a product lot. Let (AQL,$1-\alpha$) and (LTPD,β) be the two points on the OC curve of interest. Two conditions are considered:

Pr{Reject the lot| $P\geq$ AQL} = Pr{Reject the lot| $C_{\mathrm{pu}}^{\mathrm{T}}\geq C_{\mathrm{AQL}}\}\leq\alpha$
Pr{Accepting the lot| $P\leq$ LTPD} = Pr{Accepting the lot| $C_{\mathrm{pu}}^{\mathrm{T}}\leq C_{\mathrm{LTPD}}\}\leq\beta$

That is, for the protection of producers, the probability of rejecting acceptable lots is less than α. At the same time, for the protection of consumers, the probability of accepting unqualified (unacceptable) lots is less than β. Therefore, our object is solving the following two nonlinear simultaneous equations and then obtaining the required inspection sample size n and critical acceptance value c_0 of $\hat{C}^{\mathrm{T}}_{\mathrm{pu}}$

$$\alpha \geq \int_0^{c_0} \frac{\sqrt{n}}{\sqrt{\pi\left(\frac{2}{9}+C_{\mathrm{AQL}}{}^2\right)}} \exp\left[-\frac{n\left(x-C_{\mathrm{AQL}}\right)^2}{\left(\frac{2}{9}+C_{\mathrm{AQL}}{}^2\right)}\right] \mathrm{d}x \tag{7}$$

$$\beta \geq \int_{c_0}^{\infty} \frac{\sqrt{n}}{\sqrt{\pi\left(\frac{2}{9}+C_{\mathrm{LTPD}}{}^2\right)}} \exp\left[-\frac{n\left(x-C_{\mathrm{LTPD}}\right)^2}{\left(\frac{2}{9}+C_{\mathrm{LTPD}}{}^2\right)}\right] \mathrm{d}x \tag{8}$$

where C_{AQL} and C_{LTPD} represent the capability requirements corresponding to the AQL and the LTPD on the basis of the $C^{\mathrm{T}}_{\mathrm{pu}}$ index, respectively. Although the two previously mentioned equations are satisfied, the product acceptance determination procedure would judge the lots under controllable risks. To solve Equations (7) and (8), we let

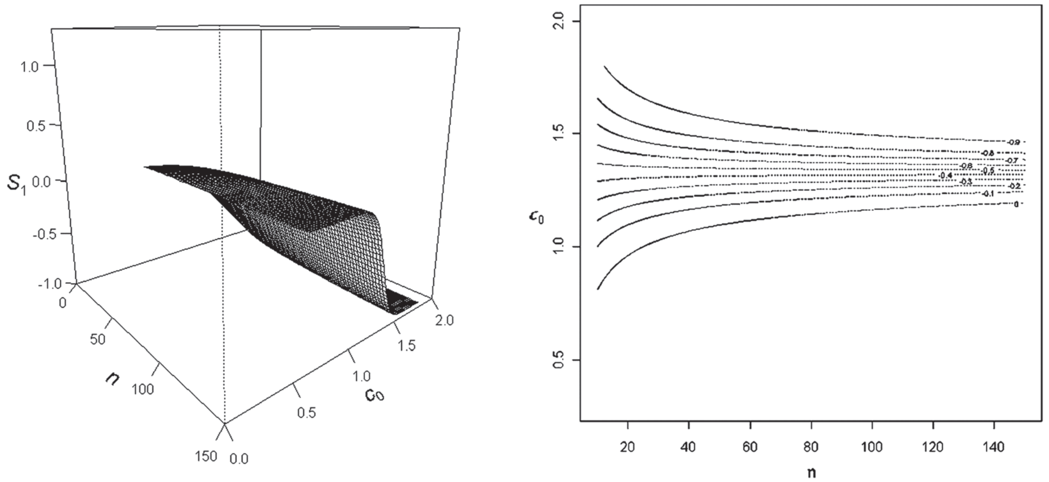
$$S_1(n, c_0) = \int_{c_0}^{\infty} \frac{\sqrt{n}}{\sqrt{\pi\left(\frac{2}{9}+C_{\mathrm{AQL}}{}^2\right)}} \exp\left[-\frac{n\left(x-C_{\mathrm{AQL}}\right)^2}{\left(\frac{2}{9}+C_{\mathrm{AQL}}{}^2\right)}\right] dx - (1-\alpha) \tag{9}$$

$$S_2(n, c_0) = \int_{c_0}^{\infty} \frac{\sqrt{n}}{\sqrt{\pi\left(\frac{2}{9}+C_{\mathrm{LTPD}}{}^2\right)}} \exp\left[-\frac{n\left(x-C_{\mathrm{LTPD}}\right)^2}{\left(\frac{2}{9}+C_{\mathrm{LTPD}}{}^2\right)}\right] dx - \beta \tag{10}$$

For $C_{\mathrm{AQL}} = 1.33$ and $C_{\mathrm{LTPD}} = 1.00$, Figures 1a and 1b and Figures 2a and 2b display the surface and the contour plots of Equations (9) and (10), respectively, with $\alpha = \beta = 0.05$. Then, Figure 3a presents the two surfaces simultaneously, and Figure 3b shows the two contour plots simultaneously.

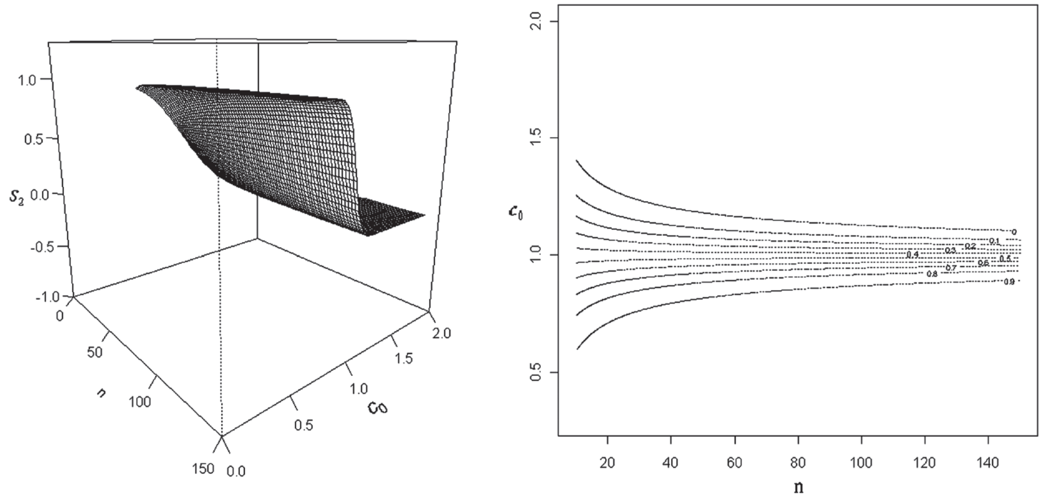
From Figure 3b, we can see that the interaction of $S_1(n, c_0)$ and $S_2(n, c_0)$ contour curves at level 0 is (n, c_0)= (79, 1.14502), which is the solution to nonlinear simultaneous Equations (9) and (10).That is, in this case, the minimum required sample size n=79, and the corresponding critical acceptance value c_0=1.14502 of the product acceptance determination on the basis of the capability index $C^{\mathrm{T}}_{\mathrm{pu}}$. For the purpose of practical applications, we performed extensive calculations to obtain the solution of Equations (9) and (10) then tabulated the critical acceptance values (c_0) and the required sample sizes (n) for the product acceptance on the basis of $C^{\mathrm{T}}_{\mathrm{pu}}$ index in Table AI.

From the results presented in Table AI, we observed that the greater the risk (α and/or β) the producer or customer could suffer, the smaller the required sample size n. This phenomenon can be interpreted intuitively: if we expect that the chance of wrongly concluding a bad process as good or good lots as bad ones is smaller, then more sample information is needed to judge the lots. Further, for fixed α-risk, β-risk, and C_{AQL}, the required sample sizes become larger when the C_{LTPD} becomes larger. This can also be explained by the same reasoning. The required sample size is smaller when the difference between C_{AQL} and C_{LTPD} is significant because it is relatively easier to reach the correct decision. For the proposed product acceptance determination procedure to be practical and convenient to use, a step-by-step algorithm is provided as follows:



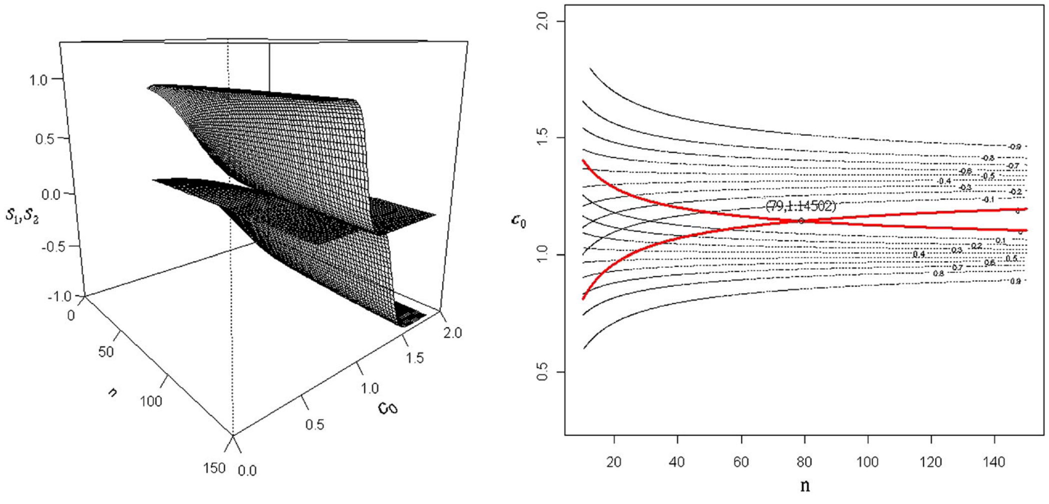
(a) (b)

Figure 1. (a) Surface plot of $S_1(n,c_0)$. (b) Contour plot of $S_1(n,c_0)$



(a) (b)

Figure 2. (a) Surface plot of $S_2(n,c_0)$. (b) Contour plot of $S_2(n,c_0)$



(a) (b)

Figure 3. (a) Surface plot of S_1 and S_2. (b) Contour plot of S_1 and S_2

Step 1: Decide the process capability requirements (i.e. set the values of C_{AQL} and C_{LTPD}) and choose the α-risk, the chance of wrongly concluding a capable lot as incapable, and the β-risk, the chance of wrongly concluding a bad lot as good one.
Step 2: Check Table AI to find the critical value (or acceptance criterion) and the required number of product units for inspection (n,c_0) on the basis of the given values of α-risk,β-risk,C_{AQL}, and C_{LTPD}.
Step 3: Calculate the value of $\hat{C}^T_{pu}$ (sample estimator) from these n inspected samples.
Step 4: Make decisions to accept the entire lot if the estimated $\hat{C}^T_{pu}$ value is greater than the critical value $c_0(\hat{C}^T_{pu} > c_0)$. Otherwise, reject the entire product.

5. Case study

The thin-film transistor–type LCD has been intensively developed for electric appliances such as cell phones, personal digital assistants, notebook computers, and monitors. In recent years, LCDs are applied to information equipment and visual equipment, including personal computers, as thin, light, low-power-consuming, wide-viewing angle, high color purity, and lower-power panel display. Response time (rising and falling) and brightness uniformity are essential product quality characteristics, which has significant impact to product quality.

For a 15-inch thin-film transistor LCD module, the data collected from a panel plant have three interesting characteristics: response time (rising), response time (falling), and brightness nonuniformity, with upper specification limits $USL_1 = 7$ ms, $USL_2 = 18$ ms, and

Table I. Sample data of 79 observations

Response time (rising)									
5.975	5.824	5.870	6.455	5.679	5.626	6.032	5.755	5.867	5.939
6.146	5.917	6.067	6.164	6.454	7.314	6.132	6.107	6.442	5.884
5.719	6.381	6.298	6.426	6.384	6.387	6.320	6.162	6.360	5.524
5.772	6.234	6.053	5.816	6.129	6.297	6.226	5.973	5.964	5.889
6.008	6.056	6.031	6.220	6.030	6.619	6.172	6.396	5.241	6.142
5.594	5.341	6.119	6.024	6.028	6.403	6.159	5.451	5.305	6.191
5.453	6.099	6.368	6.006	6.035	6.468	6.062	5.566	5.678	5.883
5.535	5.849	6.154	5.936	6.488	5.930	5.677	6.722	5.542	
Response time (falling)									
15.413	14.844	14.484	14.835	14.821	14.709	15.202	15.212	14.941	15.511
15.171	14.834	15.309	15.299	15.265	15.505	15.456	15.259	15.045	15.397
14.960	14.182	14.740	14.748	14.839	15.413	15.134	14.824	15.291	14.653
14.691	14.795	15.524	15.028	14.348	15.240	14.337	15.398	14.874	14.855
14.585	15.426	15.386	15.201	15.047	14.750	15.030	14.568	15.591	14.404
15.241	15.298	14.376	15.468	14.481	15.618	16.048	14.661	15.188	14.255
15.582	14.950	15.057	15.347	14.691	14.737	15.266	15.115	15.135	15.042
14.997	15.357	15.143	14.829	15.216	14.773	15.027	14.790	15.421	
Brightness nonuniformity									
12.196	13.604	12.648	13.295	13.077	13.147	12.916	13.193	13.042	12.933
12.702	12.463	12.778	13.588	13.779	13.360	12.684	12.586	12.556	13.104
12.565	12.750	13.951	12.443	12.874	12.849	13.425	13.411	12.379	12.904
13.181	12.921	12.845	12.215	12.323	12.979	12.816	13.381	12.957	13.118
13.544	12.201	12.727	12.747	13.404	13.171	12.705	12.773	12.870	13.241
12.872	13.748	12.743	12.973	12.667	13.404	13.067	12.795	13.743	12.924
12.444	12.752	13.176	12.253	12.802	14.111	12.217	12.742	13.386	12.722
12.928	12.293	13.141	13.439	12.852	13.145	13.441	13.028	13.709	

Table II. The calculated statistic of three characteristics

	$\bar{x}_i$	S_i	$\hat{C}_{puj}$
Response time (rising)	6.037239	0.348145	0.921801
Response time (falling)	15.03144	0.368657	2.68412
Brightness nonuniformity	12.9726	0.429825	1.572267

$USL_3 = 15\%$. In contract, C_{AQL} and C_{LTPD} are set to 1.33 (which is equivalent to no more than 33 ppm fraction of defectives for the product) and 1.00 (which is equivalent to 1349 ppm fraction of defectives for the product), with α-risk = 0.05 and β-risk = 0.05. Therefore, we find the acceptance critical values and inspected sample sizes of product acceptance determination $(n,c_0) = (79, 1.1450)$, as shown in Table AI.

The 79 required data of product items for inspection are taken from the process randomly, using microscope visually for inspection, and the observed measurements are displayed in Table I. On the basis of these data, sample mean, sample standard deviation, and C_{pu} indices for the three characteristics are listed in Table II. Therefore, the consumer would "reject" the entire products because the sample estimator from the inspection, 0.9218, is smaller than the critical acceptance value, 1.1450. We developed an effective product acceptance determination procedure on the basis of process capability index C_{pu}^{T} to deal with lot sentencing problem with very low fraction of defectives and multiple characteristics. The required sample size and the corresponding critical acceptance value for various producer's and consumer's risks with the capability requirements AQL and LTPD are also tabulated.

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Appendix A

Table A1. Required sample sizes (n) and critical acceptance values (c_0) for various α-risk and β-risk with selected C_{AQL} and C_{LTPD}

		$C_{AQL} = 1.33$		$C_{AQL} = 1.50$		$C_{AQL} = 1.50$		$C_{AQL} = 1.67$		$C_{AQL} = 1.67$		$C_{AQL} = 2.00$	
		$C_{LTPD} = 1.00$		$C_{LTPD} = 1.00$		$C_{LTPD} = 1.33$		$C_{LTPD} = 1.33$		$C_{LTPD} = 1.50$		$C_{LTPD} = 1.67$	
α	β	n	C_0	n	C_0	n	C_0	n	C_0	n	C_0	n	C_0
0.01	0.01	159	1.1455	79	1.2070	839	1.4102	233	1.4824	1034	1.5811	360	1.8213
	0.02	142	1.1348	71	1.1914	749	1.4053	211	1.4727	920	1.5757	322	1.8105
	0.03	134	1.1289	67	1.1797	692	1.4014	195	1.4648	849	1.5718	298	1.8037
	0.04	127	1.1230	64	1.1758	652	1.3984	184	1.4590	800	1.5688	284	1.7988
	0.05	121	1.1172	61	1.1680	621	1.3960	178	1.4551	763	1.5664	270	1.7930
	0.06	116	1.1133	59	1.1621	596	1.3936	170	1.4492	735	1.5645	258	1.7891
	0.07	112	1.1094	57	1.1563	572	1.3916	164	1.4453	702	1.5620	249	1.7852
	0.08	109	1.1055	55	1.1504	553	1.3896	158	1.4424	677	1.5601	241	1.7813
	0.09	106	1.1035	55	1.1504	542	1.3887	154	1.4395	660	1.5586	235	1.7773
	0.10	102	1.0996	53	1.1406	519	1.3862	149	1.4355	637	1.5566	228	1.7754
0.02	0.01	138	1.1553	68	1.2227	737	1.4158	204	1.4932	906	1.5859	317	1.8320
	0.02	124	1.1455	61	1.2070	654	1.4102	181	1.4824	805	1.5811	280	1.8213
	0.03	115	1.1387	57	1.1953	601	1.4067	169	1.4756	739	1.5771	258	1.8140
	0.04	109	1.1328	54	1.1875	565	1.4038	159	1.4697	694	1.5742	244	1.8086
	0.05	103	1.1270	52	1.1797	538	1.4014	152	1.4648	659	1.5713	232	1.8037
	0.06	99	1.1230	50	1.1758	512	1.3989	146	1.4609	629	1.5693	221	1.7988
	0.07	95	1.1191	48	1.1699	491	1.3965	139	1.4551	605	1.5674	213	1.7949
	0.08	92	1.1152	46	1.1621	472	1.3945	134	1.4512	579	1.5649	205	1.7910
	0.09	89	1.1123	45	1.1592	458	1.3931	131	1.4492	562	1.5635	198	1.7871
	0.10	86	1.1084	44	1.1553	441	1.3911	127	1.4453	541	1.5615	192	1.7842
0.05	0.01	111	1.1738	54	1.2500	597	1.4250	164	1.5122	735	1.5955	253	1.8496
	0.02	97	1.1631	48	1.2358	523	1.4199	144	1.5015	643	1.5903	222	1.8394
	0.03	90	1.1567	44	1.2236	476	1.4160	132	1.4941	586	1.5864	203	1.8320
	0.04	84	1.1504	41	1.2139	444	1.4131	123	1.4878	546	1.5835	190	1.8262
	0.05	79	1.1450	41	1.2139	418	1.4104	116	1.4824	513	1.5808	180	1.8213
	0.06	76	1.1416	39	1.2031	396	1.4080	112	1.4790	487	1.5784	170	1.8164
	0.07	73	1.1377	37	1.1953	378	1.4058	106	1.4736	464	1.5762	163	1.8125
	0.08	70	1.1328	35	1.1904	362	1.4038	102	1.4697	445	1.5742	157	1.8091
	0.09	67	1.1289	34	1.1855	348	1.4019	98	1.4658	428	1.5723	150	1.8047
	0.10	65	1.1260	33	1.1797	335	1.3999	96	1.4629	411	1.5703	145	1.8013
0.06	0.01	105	1.1785	52	1.2598	570	1.4275	155	1.5166	702	1.5979	242	1.8545
	0.02	92	1.1680	46	1.2441	495	1.4222	136	1.5059	609	1.5925	210	1.8440
	0.03	85	1.1616	42	1.2324	451	1.4185	124	1.4985	554	1.5889	193	1.8369
	0.04	79	1.1553	39	1.2227	420	1.4155	116	1.4927	516	1.5859	180	1.8311

(*Continues*)

Table A1. *Continued.*

		$C_{AQL}=1.33$		$C_{AQL}=1.50$		$C_{AQL}=1.50$		$C_{AQL}=1.67$		$C_{AQL}=1.67$		$C_{AQL}=2.00$	
		$C_{LTPD}=1.00$		$C_{LTPD}=1.00$		$C_{LTPD}=1.33$		$C_{LTPD}=1.33$		$C_{LTPD}=1.50$		$C_{LTPD}=1.67$	
α	β	n	C_0	n	C_0	n	C_0	n	C_0	n	C_0	n	C_0
	0.05	75	1.1504	37	1.2148	394	1.4128	110	1.4878	485	1.5833	170	1.8262
	0.06	71	1.1455	35	1.2075	373	1.4104	104	1.4824	459	1.5808	160	1.8213
	0.07	68	1.1416	34	1.2031	356	1.4082	99	1.4780	437	1.5786	153	1.8164
	0.08	65	1.1367	33	1.1953	341	1.4063	95	1.4741	419	1.5767	147	1.8135
	0.09	63	1.1343	32	1.1934	327	1.4043	92	1.4707	402	1.5747	141	1.8096
	0.10	60	1.1294	30	1.1836	314	1.4023	89	1.4668	386	1.5728	136	1.8062
0.07	0.01	100	1.1826	48	1.2627	546	1.4297	148	1.5210	670	1.6000	230	1.8584
	0.02	88	1.1729	43	1.2495	473	1.4243	130	1.5107	582	1.5947	201	1.8486
	0.03	80	1.1650	39	1.2363	429	1.4207	118	1.5032	528	1.5911	183	1.8413
	0.04	75	1.1597	37	1.2300	399	1.4177	110	1.4971	490	1.5881	170	1.8350
	0.05	70	1.1538	35	1.2188	374	1.4150	104	1.4922	460	1.5854	160	1.8301
	0.06	67	1.1499	33	1.2139	353	1.4126	98	1.4868	434	1.5830	153	1.8262
	0.07	64	1.1455	32	1.2090	336	1.4104	94	1.4829	413	1.5808	145	1.8218
	0.08	61	1.1411	31	1.2051	322	1.4084	90	1.4790	395	1.5786	139	1.8179
	0.09	59	1.1377	30	1.2002	308	1.4063	86	1.4746	379	1.5769	133	1.8135
	0.10	57	1.1348	28	1.1895	296	1.4045	83	1.4707	364	1.5750	127	1.8096
0.08	0.01	96	1.1868	46	1.2695	523	1.4316	143	1.5254	645	1.6021	221	1.8625
	0.02	84	1.1768	41	1.2559	453	1.4265	124	1.5146	558	1.5969	192	1.8525
	0.03	76	1.1689	37	1.2422	411	1.4229	114	1.5083	506	1.5933	176	1.8457
	0.04	71	1.1631	35	1.2354	380	1.4198	105	1.5015	467	1.5901	162	1.8394
	0.05	67	1.1582	33	1.2275	356	1.4170	99	1.4966	438	1.5874	152	1.8340
	0.06	64	1.1543	31	1.2192	337	1.4148	95	1.4922	414	1.5852	144	1.8296
	0.07	61	1.1504	30	1.2109	319	1.4125	89	1.4863	394	1.5830	137	1.8254
	0.08	58	1.1455	29	1.2090	305	1.4104	85	1.4824	375	1.5808	131	1.8213
	0.09	56	1.1406	28	1.2046	292	1.4084	82	1.4795	359	1.5789	126	1.8179
	0.10	54	1.1387	27	1.1953	280	1.4065	79	1.4756	344	1.5769	122	1.8149
0.09	0.01	92	1.1904	45	1.2773	505	1.4336	137	1.5293	620	1.6039	213	1.8662
	0.02	80	1.1802	39	1.2607	435	1.4285	119	1.519	536	1.5989	185	1.8567
	0.03	73	1.1729	36	1.2510	394	1.4248	109	1.5122	485	1.5952	167	1.8491
	0.04	68	1.1675	33	1.2402	365	1.4219	100	1.5054	449	1.5923	155	1.8433
	0.05	64	1.1626	31	1.2319	341	1.4192	94	1.5000	418	1.5895	145	1.8379
	0.06	61	1.1582	30	1.2275	321	1.4167	89	1.4951	395	1.5872	137	1.8330
	0.07	58	1.1523	29	1.2227	305	1.4146	85	1.4915	375	1.5850	131	1.8296
	0.08	55	1.1494	27	1.2129	290	1.4124	81	1.4863	357	1.5828	124	1.8250
	0.09	53	1.1460	26	1.207	279	1.4106	78	1.4834	341	1.5808	119	1.8213
	0.10	51	1.1406	25	1.2017	266	1.4084	75	1.4795	327	1.5789	114	1.8174
0.10	0.01	89	1.1943	43	1.2822	486	1.4353	133	1.5332	599	1.6057	205	1.8699
	0.02	77	1.1841	37	1.2656	419	1.4302	114	1.5225	516	1.6007	177	1.8599
	0.03	70	1.1768	34	1.2554	378	1.4266	104	1.5156	467	1.5972	160	1.8525
	0.04	65	1.1709	32	1.2461	349	1.4236	96	1.5093	429	1.5940	148	1.8467
	0.05	61	1.1660	30	1.2393	326	1.4210	90	1.5042	401	1.5914	139	1.8418
	0.06	58	1.1602	28	1.2305	308	1.4187	85	1.4990	379	1.5891	131	1.8372
	0.07	55	1.1575	27	1.2256	292	1.4165	81	1.4951	359	1.5869	125	1.8330
	0.08	52	1.1523	26	1.2188	277	1.4143	77	1.4902	341	1.5847	119	1.8291
	0.09	50	1.1489	25	1.2148	265	1.4124	74	1.4871	326	1.5828	114	1.8252
	0.10	49	1.1472	24	1.2090	255	1.4106	71	1.4824	312	1.5808	109	1.8215

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