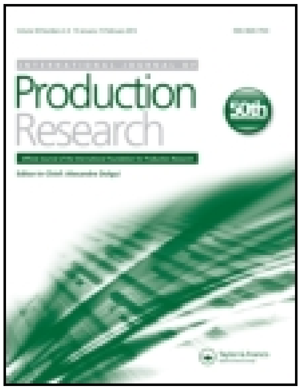

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In this paper, we solve the optimal sequencing, lot-sizing and scheduling decisions for several products manufactured through several firms in a serial-type supply chain so as to minimise the sum of setup and inventory holding costs while meeting given demand from customers. We propose a three-phase heuristic to solve this NP-hard problem using a time-varying lot- sizing approach. First, based on the theoretical results, we obtain candidate sets of the production frequencies and cycle time using a junction-point heuristic. Next, we determine the production sequences for each firm using a bin-packing method. Finally, we obtain the production times of the products for each firm in the supply chain system by iteratively solving a set of linear simultaneous equations which were derived from the constraints. Then, we choose the best solution among the candidate solutions. Based on the numerical experiments, we show that the proposed three-phase heuristic efficiently obtains feasible solutions with excellent quality which is much better than the upper-bound solutions from the common cycle approach.

Keywords: serial-type supply chain; economic lot scheduling problem; inventory control; time-varying lot-sizing policy; scheduling

1. Introduction

In a flow shop production system, items have to go through several facilities before they become finished products. Similarly, from the viewpoint of a serial-type supply chain, items must experience the required manufacturing operations by visiting a series of firms in the supply chain. Figure 1 demonstrates an example of the routing and transfer operations of items in a serial-type supply chain.

Those enterprise groups, that establish 'fully vertical-integrated' supply chains, have to face the challenge in the production planning and scheduling from the integration among the firms in such a serial-type supply chain system (though enjoying significant advantage from the economy of huge scale). For example, Foxconn Technology Group, applying its so-called 'single-whip' principle, forms a serial-type supply chain system, that includes 15 factories, in Longhua Science & Technology Park, Shenzhen City of China. (Note: Foxconn Technology Group is a Taiwan-registered corporation listed among Forbes' top 50 enterprises in 2010, and its notable products include the Amazon Kindle, iPad, iPhone, PlayStation 3, Xbox 360 and Wii. Foxconn is the largest exporter in Greater China.) In such a unique serial-type supply chain system, all the firms mutually share complete information with total confidence since each one belongs to Foxconn. The supply chain managers must pay attention to the determination of the lot sizes of different items and the generation of a feasible production schedule. Since the whole lot of a particular item on one facility must be finished before it is transferred to the next facility, and a transferred lot has to wait if another lot occupies the next facility, it is even more difficult to generate a production schedule for a serial-type supply chain than the single facility system.

Mass customisation is the new paradigm that replaces mass production, which is no longer suitable for modern commercial environment of growing product variety and opportunities for e-commerce. From this point of view, a time-varying lot-sizing (*TVLS*) policy is more realistic for today's turbulent markets than a constant lot-sizing policy. Therefore, we propose a heuristic for solving the optimal lot-sizing and scheduling problem in a serial-type supply chain system using a time-varying lot-sizing policy in this paper.

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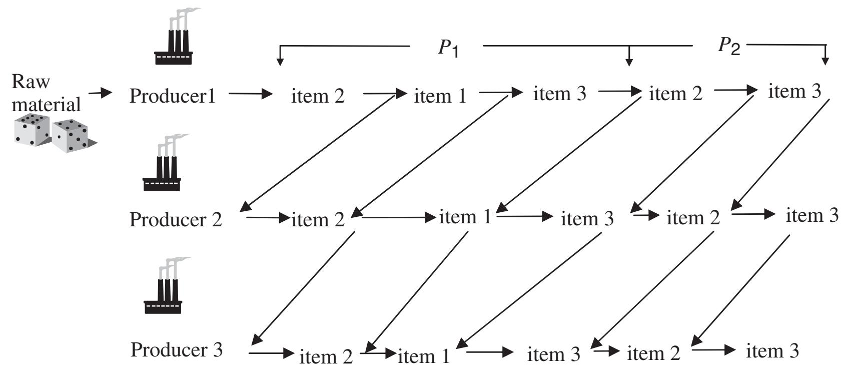

Figure 1. The routing and transfer operations of items in a serial-type supply chain.

2. Literature review

Considering the setup cost and setup time incurred when the machine switches from one product to the next, the key issue is to select a production sequence and a batch size for each product run. Since this problem is frequently encountered in many industries, it has attracted the attention of many researchers over 40 years. Basically, there are three types of approaches for solving the economic lot scheduling problem (ELSP): the common cycle approach, the basic period approach, and the time-varying lot-sizing approach. Since the latter allows different lot sizes for any given product during a cyclic schedule and it usually gives better solutions than the previous two approaches, numerous papers have addressed this problem.

Maxwell (1964) first provided the time-varying lot sizes approach that relaxes the restriction of equally spaced production lots, i.e. the approach allows the lot sizes of each product to be different within a cycle time. Provided that there is enough time for setups, Dobson (1987) developed a heuristic to show that the production order in a cycle can be converted into a feasible production schedule in which the production lots are not necessarily equal. An extended study allowing the set-up time to be sequence dependent was presented by Dobson in 1992. Gallego and Roundy (1992) study the ELSP with backorders under the TVLS approach. Raza and Akgunduz (2008) consider the time-varying lot size approach to solve the *ELSP,* which deals with the production assignment of several different products on a given single production facility. A computational study of the existing solution algorithms, Dobson's heuristic, hybrid genetic algorithm, neighborhood search heuristics, Tabu search and a proposed simulated annealing algorithm were presented and compared.

This presentation is different from most literatures introduced above since it solves the optimal scheduling problem in a serial-type supply chain system (with multi-facility) by time-varying lot-sizing policy. Considering this problem is closely related to the ELSP, we provide some background on the ELSP before introducing the mathematical model for the time-varying lot-sizing policy. The ELSP concerns with lot-sizing and scheduling decisions in a single production facility for n products so as to minimise the average total costs while meeting the given demands of each product. In the literature, most of the solution approaches for solving the ELSP employed a so-called 'basic period-based cyclic schedule' which uses a basic period as the base for production planning and scheduling. One may refer to Elmaghraby (1978), Lopez and Kingsman (1991) and Yao (1999) for the problem definition and assumptions of the ELSP. The problem formulations for the ELSP that uses basic periods can be classified as either the 'basic period' (BP) approach or the 'extended basic period' (EBP) approach. The most commonly used approach to deal with this problem is the common cycle (CC) approach where a lot of each product is produced each cycle. The CC and BP approaches can be viewed as specific cases of the EBP approach. The cost of the CC approach can be regarded as the upper bound of the cost for the ELSP.

The solution methodologies for the ELSP that have been proposed so far may be divided into two major categories: analytical and heuristic. For a given value of basic period, the analytical approaches usually employ either dynamic programming or integer nonlinear programming model; see Bomberger (1966), Elmaghraby (1978), Axsäter (1982) for dynamic programming models and Haessler (1979) for binary integer programs. Hsu (1983) has shown that the single-facility ELSP problem is NP-hard.

To generate a production schedule for a flow shop production system (abbreviated as the FS-ELSP) is even more difficult than the single facility ELSP. The lot sizing and scheduling problems in flow shops have attracted many researchers' attention in the past. One may refer to the literatures including Ouenniche and Boctor (2001a, b), Huang and Yao (2006, 2008). Ouenniche and Boctor (2001 a, b) tested feasibility of their candidate solutions using the EBP approach, however, the limitation on the values of the multipliers $\{k_i\}$ imposed in Ouenniche and Boctor's (2001b) *two-group* heuristic could significantly affect the quality of the obtained solutions. Many problems exist in Ouenniche and Boctor's (2001a) *power-of-two* heuristic were pointed out in the study of Huang and Yao (2007a), especially on the feasibility testing of an obtained solution and the generation of a feasible production schedule. These problems may misjudge those solutions with better objective values as infeasible ones, and often lead Ouenniche and Boctor's *power-of-two* (*PoT*) heuristic to worse solutions.

In the ELSP, it is typically assumed that production and demand rates are known product-dependent constants. Research on the ELSP has focused on cyclic schedules, i.e. schedules that are repeated periodically. A basic period B is an interval of time devoted to the setup and production of a subset of (or all) the items. A solution to the problem is usually given in the form of a basic period B and a set of multipliers $\{k_i\}$, which implies that each item i is replenished after a fixed cycle time k_iB.

Moon *et al.* (2002) provided a hybrid genetic algorithm based on the TVLS approach for solving the ELSP problem of a single facility where m items are produced. They try to find a cycle length, a production sequence, production time durations, and idle time durations, so that the production sequence can be completed in the chosen cycle and demand can be fully met, and the total of inventory and set-up costs is minimised.

In this paper, we will utilise the TVLS approach for solving the optimal lot-sizing and scheduling for multiple items produced through a series of production facilities. We name it as the serial-type time-varying lot-sizing problem (or, the ST-TVLS problem). Gallego and Shaw (1997) showed that solving the ELSP for a single facility using the TVLS approach is strongly NP-hard. Our interested problem in this paper is obviously even more difficult than the conventional ELSP. The TVLS approach has two advantages (Dobson 1987): First, it can avoid a feasibility checking problem that the EBP approach meets; Second, under *PoT* policy, the TVLS approach is a quicker solution approach than the BP and EBP approaches. Dobson (1992) presented the mathematical model for the single-facility ELSP using the TVLS approach. The difficulty for the TVLS approach is how to find an optimal production sequence by minimising the average total cost. Intuitively, the TVLS approach can get better solution than the CC and BP approaches, therefore, this paper is motivated to propose a heuristic to secure the optimal lot sizing and scheduling strategies to control the inventory in serial-type supply chains by using the TVLS approach.

The rest of this paper is organised in the following manner: in Section 3, we present the ST-TVLS model. Section 4 shows our proposed three-phase heuristic. To determine the production frequencies of each member of the supply chain, we first propose a junction-point heuristic which is derived from the relaxation form of the ELSP formulation of Ouenniche and Boctor (2001a). Second, we solve the production sequence of each item by a bin-packing method. Finally, we obtain the optimal production time for all the production lot in each firm by solving a set of simultaneous equations. Section 5 gives numerical experiments and sensitivity analysis verifying that the solution quality of the proposed three-phase heuristic is much better than that obtained from the common cycle approach. Finally, Section 6 presents our concluding remarks.

3. The mathematical model

We present the mathematical model for the single-facility TVLS model and the ST-TVLS model in this section.

3.1 *The single-facility TVLS model*

Before presenting the formulation for the single-facility TVLS model, we first define the notation and introduce the assumptions used for deriving the mathematical model as follows.

The index i $(i = 1, 2, \ldots, n)$ is used in the subscripts to refer to the ith item, while the position index ℓ $(\ell = 1, \ldots, \bar{L})$ is used in the superscripts to refer to the data related to the part produced at a certain position, where $\bar{L}$ is the total number of items to be produced in cycle time T.

Production rate (units per day)	p_i	$i = 1, 2, \ldots, n$;
Demand rate (units per day)	r_i	$i = 1, 2, \ldots, n$;
Inventory holding cost ($ per unit per day)	h_i	$i = 1, 2, \ldots, n$;
Set-up cost ($)	A_i	$i = 1, 2, \ldots, n$;
Set-up time (days)	s_i	$i = 1, 2, \ldots, n$;

Production time duration for item produced at position ℓ — $t^\ell, \ell = 1, \ldots, \bar{L}$;
Idle time duration after the production of the item at position ℓ — $u^\ell, \ell = 1, \ldots, \bar{L}$;
Item produced at position ℓ — $f^\ell, \ldots, f^{\bar{L}} (f^\ell \in \{1, \ldots, n\})$

Let J_i be the position in a given sequence where item i is produced, i.e. $J_i = \{\ell \mid f^\ell = i\}$. Let L_k be the positions in a given sequence from k (where f^k is produced), up to but not including the position in the sequence where part f^k is produced again and $k = 1, \ldots, \bar{L}$.

As derived by Dobson (1992), the single-facility TVLS model can be written as

$$\inf_{\ell \in J} \min_{t \geq 0, u \geq 0, T \geq 0} \frac{1}{T} \left(\sum_{\ell=1}^{\bar{L}} \frac{1}{2} h^\ell (p^\ell - r^\ell) \left(\frac{p^\ell}{r^\ell} \right) (t^\ell)^2 + \sum_{\ell=1}^{\bar{L}} A^\ell \right). \tag{1}$$

Subjected to

$$\sum_{\ell \in J_i} p_i t^\ell = r_i T, \quad i = 1, \ldots, n \tag{2}$$

$$\sum_{\ell \in L_k} (t^\ell + s^\ell + u^\ell) = (p^k / r^k) t^k, \quad k = 1, \ldots, \bar{L} \tag{3}$$

$$\sum_{\ell=1}^{\bar{L}} (t^\ell + s^\ell + u^\ell) = T. \tag{4}$$

Constraints (2) indicate that one must allocate enough time to each item i to meet its demand over the cycle. Constraints (3) state that item i must be produced each time to last until the next time it is produced again. Constraint (4) means that the cycle time T must be the sum of production, setup, and idle times for all the items produced in the cycle. Note that, in order to find the production time easily, Moon (2002) assumed that there are no idle times for a given production sequence. They called this method a quick-and-dirty heuristic and claimed that this approximation works very well for a highly loaded facility. However, when considering a flow-shop production system (or a serial-type supply chain system), the idle time can no longer be neglected (Huang and Yao 2007a). We will have further discussion on this issue later.

3.2 *The ST-TVLS model*

In this section, we present the formulation for the ST-TVLS model for a serial-type supply chain system with m firms. Since more than one facility/firm is involved in the manufacturing process, we have to re-define the notation as follows. The index j ($j = 1, 2, \ldots, m$) is used to denote the jth firm. The problem can be viewed as one that decides cycle length (days) T_j, production sequences, production times and idle times through a serial-type supply chain so as to minimise the sum of setup and inventory holding costs while a given demand is fulfilled. The setup costs A_i of item i are the sum over all the facilities, $i = 1, 2, \ldots, n$. For each finished item in position ℓ, the demand rate is denoted as r^ℓ, and the process time (on facility m) is denoted as $t^{\ell m}$. For item in position ℓ on facility j, the production rate is $p^{\ell j}$, and the holding cost rate (per unit per unit time) for the work-in-process (WIP) is $h^{\ell j}$. To indicate the production sequence in a production schedule, we denote $d^{\ell j}$ as the starting time of item in position ℓ on facility j.

We note that the production planning and scheduling in a serial-type supply chain system share some common characteristics with the flow-shop production system. We indicate some important features that should be taken into account in the formulation of the ST-TVLS model. As stated in Ouenniche and Boctor's (2001a) study, when dealing with the ELSP in flow shops, no products can be transferred to the next facility before its production lot is finished at the current facility at each facility. The importance of the delayed time in the feasibility testing of a solution for the ELSP in flow shops and its effect on the total cost are thoroughly discussed by Huang and Yao (2007b). Since no facility can process more than one product at a time, and a lot is not transferred to the next facility until the entire lot is processed at the current facility, a waiting time could exist during the lot transfer between two successive facilities, which will increase total cost accordingly.

Inspired by the models of Dobson (1992) and Ouenniche and Boctor (2001a), we present the ST-TVLS model as follows.

$$\text{Min } Z = \frac{1}{m}\sum_{j=1}^{m}\left\{\frac{1}{T_j}\sum_{\ell=1}^{\bar{L}} A^{\ell}\right\} + \sum_{\ell=1}^{\bar{L}}\left\{h^{\ell m}\frac{t^{\ell m} r^{\ell}}{2}\left(1-\frac{r^{\ell}}{p^{\ell m}}\right) + \frac{(r^{\ell})^2}{2}\sum_{j=2}^{m} t^{\ell j} h^{\ell,j-1}\left(\frac{1}{p^{\ell j}}-\frac{1}{p^{\ell,j-1}}\right)\right\} + \sum_{j=2}^{m}\sum_{\ell=1}^{\bar{L}} h^{\ell,j-1} r^{\ell}\,(d^{\ell j}-d^{\ell,j-1}). \tag{5}$$

Subjected to

$$\sum_{\ell \in J_i} p^{\ell j} t^{\ell j} = r^{\ell} T_j, \quad i = 1,\ldots,n, j = 1,\ldots,m \tag{6}$$

$$\sum_{\ell \in L_k} (t^{\ell j} + s^{\ell j} + u^{\ell j}) = (p^{kj}/r^{kj}) t^{kj}, \quad k = 1,\ldots,\bar{L}, j = 1,\ldots,m \tag{7}$$

$$\sum_{\ell=1}^{\bar{L}} (t^{\ell j} + s^{\ell j} + u^{\ell j}) = T_j, \quad j = 1,\ldots,m. \tag{8}$$

4. The proposed three-phase heuristic

We propose a three-phase heuristic for solving the ST-TVLS model in this section. We first give an overview of the three-phase heuristic as follows. In the first phase, we obtain candidate sets of the production frequencies and cycle time using a junction-point heuristic. The second phase determines the production sequences for each firm using a bin-packing method. Finally, in the third phase, we obtain the production times of the products for each firm in the supply chain system by iteratively solving a set of linear simultaneous equations which were derived from the constraints. Then, we choose the best solution among the candidate solutions. Figure 2 displays a flowchart for the proposed three-phase heuristic.

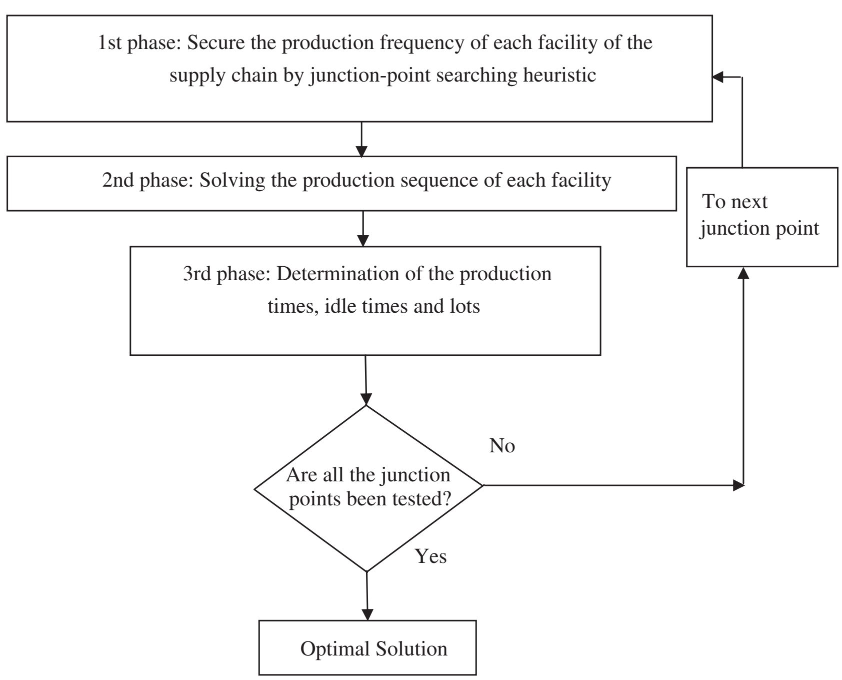

Figure 2. The proposed three-phase heuristic for solving the ST-TVLS problem.

We will discuss the details of each phase in the following subsections.

4.1 *Phase one: search for the production frequency*

In this phase, we determine the production frequencies by solving a relaxation formulation of Ouenniche and Boctor's (2001a) model which was solved under the *PoT* policy, i.e. the cycle time of each item T_i is a *PoT* integer multiplier of a basic period B ($T_i = k_i B$ where $k_i = 2^{\eta_i}$, η_i is a non-negative integer). When using a *PoT* policy, one should examine the production schedule of a global cycle of KB, where $K = \mathrm{lcm}\{k_i\} = \max\{k_i\}$. Note that if we did not use *PoT* policy, the value of $\mathrm{lcm}\{k_i\}$ could be very large in some extreme cases where there are prime numbers in the set $\{k_i\}$. Therefore, we confine the scope of this study under *PoT* policy.

Our phase-one heuristic aims at determining the optimal value of the basic period B, the set of optimal replenishment frequencies, a feasible assignment with its corresponding production sequence, and the starting times so as to minimise the average total costs. It obtains an approximation solution of the production frequencies by relaxing Ouenniche and Boctor's (2001a) model. More specifically, by substituting the term $d^{\ell j} - d^{\ell, j-1} = t^{\ell, j-1}$ into Equation (5), the objective function of the relaxed problem is reorganised as

$$(R)\, Z(k_1, k_2, \ldots, k_n, B) = \sum_{i=1}^{n} TC_i(k_i, B) = \sum_{i=1}^{n} \left(\frac{C_{1i}}{k_i B} + C_{2i} k_i B \right) \tag{9}$$

where

$$C_{1i} = A_i$$

$$C_{2i} = h_i \frac{r_i}{2} \left\{ 1 - \frac{r_i}{p_{im}} \right\} + \frac{r_i^2}{2} \sum_{j=2}^{m} h_{i,j-1} \left\{ \frac{1}{p_{ij}} + \frac{1}{p_{i,j-1}} \right\}$$

Since the terms on the right-side of (9) are notably separable, we are motivated to study the properties of $TC_i(k_i, B)$ so as to establish foundation for our new heuristic. The following heuristic focuses on the multipliers k_i because the solution of ST-TVLS model does not include the search for the basic period. Let us define a new function $TC_i^*(B)$ by taking the optimal value of k_i at any value $B' > 0$ for the function $TC_i(k_i, B)$ as follows.

$$TC_i^*(B') = \min_{k_i \in N^+} \left\{ TC_i(k_i^*(B), B) \middle| \forall B = B' \geq 0 \right\}. \tag{10}$$

That the function $TC_i^*(B)$ is a piece-wise convex with respect to B can be easily proved by referring to Huang and Yao's papers (2005, 2006). Consequently, the problem (R) can be re-written by

$$(R_1) \quad \Gamma(B) = \inf_{B>0} \left\{ \sum_{i=1}^{n} TC_i^*(B) \right\}. \tag{11}$$

where the function $\Gamma(B)$ is the optimal objective function value of problem (R_1) with respect to B. Since the $\Gamma(B)$ function is the sum of a convex function and n piece-wise convex functions, it is obvious piece-wise convex with respect to B. The junction point for piece-wise convex function $TC_i^*(B)$ can be defined as a particular value of B where two consecutive convex curves $TC_i(k_i, B)$ and $TC_i(2k_i, B)$ concatenate (Huang and Yao 2006). Importantly, such a junction point provides us with the information on at 'what value of B' where one should change its multiplier value from k_i to $2k_i$ so as to secure the optimal value for the $TC_i^*(B)$ function. A closed form to locate the junction points can be derived by letting the difference between $TC_i(k_i, B)$ and $TC_i(2k_i, B)$ to be zero. Then we have the junction point for the multipliers k_i and $2k_i$ of product i as

$$\delta_i(k_i) = \frac{1}{k_i} \sqrt{\frac{C_{1i}}{2C_{2i}}} \tag{12}$$

The following inequality (13) holds for the junction points.

$$\delta_i(v) < \cdots < \delta_i(k+1) < \delta_i(k) \cdots < \delta_i(2) < \delta_i(1) \tag{13}$$

where v is an (unknown) upper bound on the value of k. Theorem 1 is an immediate result from Equations (12) and (13).

Theorem 1: *With regard to a junction point w of the $TC_i^*(B)$ function, suppose that k_L^* and k_R^* are the optimal maintenance frequencies for the convex curves on the left side (i.e. $B \leq w$) and the right side (i.e. $B > w$) of w, respectively, then $k_L^* = 2k_R^*$.*

As shown in the studies of Yao (1999, 2005) and Huang (2006), all the junction points for each facility will be inherited by the $\Gamma(B)$ function. In other words, if w is a junction point for a product i, w must also show as a junction point on the piece-wise convex curve of the $\Gamma(B)$ function. The following corollary is a by-product of theorem 1, and it provides an easier way to obtain the optimal maintenance frequency $k_i^*(B) \in N^+$ for the $TC_i^*(B)$ function for any given $B > 0$.

Corollary 1: *For any given $B > 0$, an optimal value of $k_i^*(B) \in N^+$ for the $TC_i^*(B)$ function is given by*

$$k_i^*(B) = \begin{cases} 1 & B > \sqrt{C_{1i}/C_{2i}} \\ 2^p, & \sqrt{C_{1i}/C_{2i}}/2^p < B \leq \sqrt{C_{1i}/C_{2i}}/2^{p-1};\ p \in N^+ \end{cases}. \tag{14}$$

$K_{PoT}(B)(=\{k_i^*(B)\}_{i=1}^n)$ is denoted as the set of optimal production multipliers of item i at time B. Recall that the $TC_i^*(B)$ function is piece-wise convex with respect to B, these theoretical results encourage us to solve the problem (R) by searching along the B-axis. To design a search algorithm that obtains an optimal solution for the problem (R), one need to define the search range by a lower and an upper bound on the B-axis, which are denoted by B_L and B_U, respectively. The best local minimum in $[B_L, B_U]$ must be no worst than any solution outside this searching range. One may refer to Huang and Yao's paper (2008) for the derivation of lower and an upper bound. Under PoT policy, we may serve $B_{cc}/2$ as the lower bound, i.e. $B_{\min} = B_{cc}/2$. While, the upper bound on the search range can be obtained by the common cycle (CC) approach in which it requires that $k_i = 1$ for all i, i.e. all of the products share a common production cycle. We set

$$B_{cc} = \sqrt{\sum_{i=1}^{n} C_{1i} \Big/ \sum_{i=1}^{n} C_{2i}} \tag{15}$$

For any given vector of $\mathbf{k}$, one may locate its local minimum, $\breve{B}(\mathrm{k})$, by taking the first derivative of the objective function $\Gamma(B)$ and equating it to zero.

$$\breve{B}(k_i) = \sqrt{\left(\sum_{i=1}^{n} C_{1i}/k_i\right) \Big/ \left(\sum_{i=1}^{n} C_{2i}k_i\right)} \tag{16}$$

It is obvious that $\breve{B}(k) \leq B_{cc}$ since $k_i \geq 1$ for all i. Therefore, there exists no local minimum for $B > B_{cc}$; that is, there exist no local minima for $B > B_{cc}$ for the $\Gamma(B)$ function.

By the rationale discussed above, the proposed search algorithm for our phase-one heuristic is summarised as follows.

Step 1: The initialisation.

(a) Obtain the common cycle period B_{cc} by Equation (15). Let $\bar{\ell} = 0$ and locate the smallest junction point that is greater than (or equal to) B_{cc}, and define it as $w_0 = \min\{\delta_i(k_i) : \delta_i(k_i) > B_{cc}\}$. If w_0 does not exist, let $\bar{\ell} = 1$, $w_0 = B_{cc}, K_1 = \{1, \ldots, 1\}, B_1 = B_{cc}$, and calculate the cost $\Gamma^*(\breve{K}_1, B_{cc})$.

(b) Obtain the multipliers by Equation (14). Calculate $\alpha = \arg\max_i\{\delta_i(k_i) < w_0\}$ and let $w_1 = \delta_\alpha(k_\alpha), j = 1$. If the local minimum locates within $[w_1, w_0]$, i.e., $\breve{B}(K_{PoT}(B_{cc})) \in [w_1, w_0]$, let $\bar{\ell} = 1$, $\breve{K}_{\bar{\ell}} = K_{PoT}(B_{cc})$, $\breve{B}_{\bar{\ell}} = \breve{B}(\breve{K}_{\bar{\ell}})$ and compute $\Gamma^*(\breve{K}_{\bar{\ell}}, \breve{B}_{\bar{\ell}})$.

Step 2: The search procedure.

(a) Calculate the next set of multipliers $K_{PoT}(w_j)$ by $K_{PoT}(w_j) \equiv (K_{PoT}(w_{j-1}) \backslash \{k_\alpha\}) \cup \{2k_\alpha\}$.

(b) Find $\alpha = \arg\max_i\{\delta_i(k_i) < w_j\}$ and let $w_j = \delta_\alpha(k_\alpha)$.

(c) If $\breve{B}(K_{PoT}(B_{cc})) \in [w_{j+1}, w_j]$, let $\bar{\ell} = \bar{\ell} + 1$, $\breve{K}_{\bar{\ell}} = K_{PoT}(w_j)$, $\breve{B}_{\bar{\ell}} = \breve{B}(\breve{K}_{\bar{\ell}})$ and compute $\Gamma^*(\breve{K}_{\bar{\ell}}, \breve{B}_{\bar{\ell}})$.

Step 3: Let $j = j + 1$. If $w_j < \breve{B}_1/2$, then go to Step 4, otherwise go back to Step 2.

Step 4: Secure a candidate of the multipliers $(K^*_{PoT}, B^*_{PoT}) = \arg\min_{\bar{\ell}}\{\Gamma^*(\breve{K}_{\bar{\ell}}, \breve{B}_{\bar{\ell}})\}$. Once the optimal multipliers are secured (the basic period can be neglected), one can obtain the production frequency **y,** which is the inverse of the multipliers.

4.2 *Phase two: find the production sequence*

The production sequence can be determined by using the bin-packing heuristic which has been suggested by many researchers (Doll and Whybark 1973, Dobson 1987). Once the production frequency **y** is obtained from phase one, the bin-packing heuristic attempts to spread them out as evenly as possible. We create b bins in which $b = \max_i\{k_i\}$. For each product i, the production time duration v_i is estimated by $v_i = s_i + (r_i t/p_i k_i)$. The lots is assumed to be equally spaced, that is, we do the bin-packing with b bins and y_i items of height v_i for all i. Although, the value of t is not determined yet at this moment, the value of $r_i/p_i k_i$ can be used as a reference value for comparison since $s_i \ll (r_i t/p_i k_i)$. The products are ordered lexicographically $(y_{i,}\ v_i)$ by using a variation of the longest processing time (LPT) rule. The products are ordered by frequency y_i first, then by $r_i/p_i k_i$, among products with identical frequencies. By minimising the maximum height of the bin, production sequence **f** can be determined (one may refer to Dobson (1987) for details).

4.3 *Phase three: determine the production times, idle times and total cost*

In the third phase, we first secure the position set L_k $(k = 1, \ldots, \bar{L})$. Then, we obtain the production time $t^{\ell 1}, \ell = 1, \ldots, \bar{L}$ of the first firm in the supply chain by solving a system of linear equations induced from constraints (7). The cycle time T can be obtained from either constraint (6) or (8). Next, we will calculate the production times for all the downstream firms.

Note that the waiting time $u^{\ell j}(\ell = 1, \ldots, \bar{L}, j = 2, \ldots, m)$ during the batch transfer between two neighbouring facilities (i.e. to calculate the values of $d^{\ell,j} - d^{\ell,j-1}$) needs to be taken accounts, although there is no feasibility problem for a ST-TVLS problem. The value of $d^{\ell j}$ for all facilities in sequence can be obtained by the following steps.

For the first firm in the supply chain system $(j = 1)$, the starting time can be determined as follows:

$$d^{11} = s^1,$$

$$d^{\ell 1} = d^{\ell-1,1} + t^{\ell-1,1} + s^{\ell}, \quad \ell = 2, \ldots, \bar{L}. \tag{17}$$

The starting times for the following firms $(j > 1)$ are as follows:

$$d^{1j} = d^{1,j-1} + t^{1,j-1} + s^1,$$

$$d^{kj} = \max(d^{k-1,j} + t^{k-1,j} + s^k, d^{k,j-1} + t^{k,j-1} + s^k) \quad k = 2, \ldots, \bar{L}. \tag{18}$$

The production times $t^{\ell 1}, \ell = 1, \ldots, \bar{L}$ of the first firm in the supply chain system are calculated by solving a system of linear equations induced from constraints (7).

$$(p^{kj}/r^k - 1)t^{kj} - \sum_{\ell \in L_k/k} t^{\ell j} = \sum_{\ell \in L_k} (s^{\ell j} + u^{\ell j}), \quad k = 1, \ldots, \bar{L}, \quad j = 2, \ldots, m. \tag{19}$$

This equation assures that enough product i must be produced each time to last until the next time it is produced again. Since two sets of unknowns are to be determined, the processing times t^{kj} and idle times u^{kj}, an iterative process is necessary because the simultaneous equations are not enough for solving all the unknowns at a time. One may first calculate an initial value of t^{kj}, and accordingly secure a set of waiting time u^{kj}. The final values of t^{kj} and u^{kj} are determined through an iterative process until their values no longer vary.

The details of our phase-three heuristics are summarised as follows.

Step 1: Calculate $L_k, k = 1, \ldots, \bar{L}$.

Step 2: Determine the processing time of the finished goods (i.e. the production time of the final facility of the supply chain, t_1^{km}) by solving the simultaneous equations which is reorganised from Equation (19) as follows

$$(1 - p^{kj}/r^k)t^{kj} + \sum_{\ell \in L_k/k} t^{\ell j} = -\sum_{\ell \in L_k} (s^{\ell j} + u^{\ell j}), \quad k = 1, \ldots, \bar{L}.$$

Step 3: Determine the production time of the rest of facilities by $t^{kj} = p^{km} t^{km} / p^{kj}$, which assure the continuity of the production lot transferring between different facilities.

Step 4: If it is the first time iteration, go to Step 5. Otherwise, check the convergence of the iteration by examining the value of t^{kj} with its previous value for all the facilities of the supply chain. If all the production values keep the same, which means that the production times are determined. After the calculation of the total production time of each facility T_j and the total cost of the supply chain $\Gamma_\eta(K_\eta, B_{\eta j})$, one should go back to phase one to test next junction point. If all the junction points have been investigated, go to Step 6.

Step 5: Calculate the starting time d^{kj} and waiting time u^{kj}, $j = 1, \ldots, m-1$, $k = 1, \ldots, \bar{L}$, from the first facility to the last one. Go back to Step 2.

Step 6: The optimal solution can be determined by $\Gamma^*(K_\eta^*, B_{\eta j}^*) = \arg\min_\eta \{\Gamma_\eta(K_\eta, B_{\eta j})\}$.

5. Numerical experiments

In this section, we first use an example to demonstrate the implementation of the proposed three-phase heuristic. Then, we conduct random experiments. The third part presents sensitivity analysis on the effect of the setup time, the setup cost and the holding cost.

5.1 *A demonstrative example*

Here, we take an example with five facilities and 10 products, with its data set shown in Table 1, to demonstrate the implementation of the proposed three-phase heuristic.

Note that only one set of multiplier (that is, one junction point) is chosen as a candidate of the production frequency each time in the first phase. And then, the corresponding production sequence, the idle time and the average total cost are determined through phase two and three. After the first round, we go back to phase one to choose another candidate of the production frequency and to go round and begin again until we finish the searching of all the junction points. Among all these tests, the set of multiplier with the minimum cost will be the optimal solution.

Before starting a searching process, the feasibility criteria for each facility in the supply chain system, $\sum_{i=1}^{n} r_i/p_i < 1$, should be test. The feasible criteria for each facility are fulfilled, as listed in Table 2.

Next, we present the implementation details of the proposed three-phase heuristic.

Phase 1: Search for the production frequency.

(1) Obtain the upper bound on the search range by Equation (15), we have $B_{cc} = 11.62$. Set $\eta = 1$. Calculate the junction points of all the products, as shown in Table 3.
(2) We have the first junction point locate at $w_1 = 11.62$.
(3) Obtain the production frequency of each product, that is, the multipliers $K_{PoT}(\delta_i) = \{1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$.

Phase 2: Find the production sequence of the products by using our bin-packing heuristic. In this example, the products are manufactured in the fifth firm using the following production sequence: $(f_1, f_2, \ldots, f_{\bar{L}}) = (9, 4, 3, 8, 7, 1, 5, 6, 2, 10)$.

Phase 3: Determine the production times, idle times and total cost.

Table 1. The data set of the demonstrative example.

Product i	Producer j	Demand rate r_i	Production rate p_{ij}	Setup time τ_{ij}	Sum of setup cost A_i	Inventory cost h_{ij}
1	1	376.6	7638.4	0.24	66.7	0.0040
	2		5365.6	0.16		0.0021
	3		6845.9	0.34		0.0031
	4		6433.3	0.19		0.0047
	5		8624.5	0.30		0.0047
2	1	212.4	5496.3	0.24	85.7	0.0047
	2		6791.2	0.23		0.0043
	3		9796.8	0.30		0.0012
	4		7224.8	0.26		0.0021
	5		10737.5	0.31		0.0031
3	1	589.6	9306.2	0.34	71.4	0.0028
	2		10741.1	0.18		0.0043
	3		5698.3	0.22		0.0042
	4		9550.8	0.18		0.0018
	5		7467.9	0.29		0.0022
4	1	502.5	5210.5	0.21	95.9	0.0046
	2		10292.9	0.22		0.0039
	3		6947.7	0.22		0.0040
	4		6236.3	0.21		0.0013
	5		5672.9	0.34		0.0010
5	1	411.1	5927.7	0.21	148.9	0.0013
	2		8317.9	0.18		0.0040
	3		6238.8	0.21		0.0018
	4		9326.3	0.32		0.0022
	5		10236.4	0.24		0.0011
6	1	201.7	10610.1	0.23	138.4	0.0040
	2		6374.9	0.28		0.0023
	3		6642.7	0.27		0.0038
	4		5615.2	0.25		0.0017
	5		7995.2	0.33		0.0021
7	1	450.7	10944.6	0.26	92.0	0.0030
	2		7356.9	0.27		0.0039
	3		9654.0	0.18		0.0014
	4		8184.9	0.30		0.0022
	5		9617.9	0.35		0.0050
8	1	602.1	6232.9	0.31	141.7	0.0014
	2		5673.1	0.25		0.0012
	3		10450.4	0.20		0.0039
	4		6344.8	0.23		0.0042
	5		7805.6	0.22		0.0019
9	1	733.7	7317.0	0.16	134.3	0.0029
	2		6295.0	0.22		0.0022
	3		8531.6	0.23		0.0027
	4		5342.8	0.33		0.0046
	5		5137.4	0.30		0.0013
10	1	191.1	9664.2	0.16	130.9	0.0015
	2		6122.8	0.23		0.0028
	3		9836.8	0.33		0.0022
	4		6128.3	0.15		0.0034
	5		10248.3	0.26		0.0012

Table 2. Feasibility testing.

Firm	$\sum_{i=1}^{n} r_i/p_i$
1	0.59
2	0.60
3	0.56
4	0.63
5	0.58

Table 3. The junction points.

i	1	2	3	4	5	6	7
δ_1	0.08044	0.16883	0.33766	0.67532	1.35064	2.70128	5.40255
δ_2	0.16187	0.32374	0.64748	1.29495	2.58990	5.17980	10.35960
δ_3	0.08613	0.17226	0.34452	0.68904	1.37807	2.75615	5.51229
δ_4	0.12711	0.25423	0.50846	1.01691	2.03383	4.06765	8.13530
δ_5	0.20745	0.41490	0.82979	1.65959	3.31918	6.63836	13.27671
δ_6	0.24690	0.49380	0.98760	1.97520	3.95041	7.90081	15.80163
δ_7	0.09221	0.18443	0.36885	0.73771	1.47542	2.95084	5.90167
δ_8	0.12720	0.25440	0.50880	1.01761	2.03521	4.07042	8.14084
δ_9	0.10465	0.20930	0.41861	0.83722	1.67444	3.34888	6.69775
δ_{10}	0.31582	0.63164	1.26327	2.52655	5.05310	10.10620	20.21239

Table 4. The production time of the firms.

	Production time t_1^{kj}									
Firm j	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	$k=9$	$k=10$
1	0.70	0.68	0.45	0.68	0.29	0.35	0.49	0.13	0.27	0.14
2	0.82	0.34	0.39	0.75	0.43	0.49	0.35	0.22	0.22	0.22
3	0.60	0.51	0.73	0.41	0.33	0.39	0.46	0.21	0.15	0.14
4	0.97	0.57	0.43	0.67	0.39	0.41	0.31	0.25	0.21	0.22
5	1.00	0.62	0.56	0.54	0.33	0.31	0.28	0.18	0.14	0.13

Step 1: Calculate $L_k, k = 1, \ldots, \bar{L}$ as:

$$\begin{aligned}
L_1 &= (1, 2, 3, 4, 5, 6, 7, 8, 9, 10);\\
L_2 &= (2, 3, 4, 5, 6, 7, 8, 9, 10, 1);\\
L_3 &= (3, 4, 5, 6, 7, 8, 9, 10, 1, 2);\\
L_4 &= (4, 5, 6, 7, 8, 9, 10, 1, 2, 3);\\
L_5 &= (5, 6, 7, 8, 9, 10, 1, 2, 3, 4);\\
L_6 &= (6, 7, 8, 9, 10, 1, 2, 3, 4, 5);\\
L_7 &= (7, 8, 9, 10, 1, 2, 3, 4, 5, 6);\\
L_8 &= (8, 9, 10, 1, 2, 3, 4, 5, 6, 7);\\
L_9 &= (9, 10, 1, 2, 3, 4, 5, 6, 7, 8);\\
L_{10} &= (10, 1, 2, 3, 4, 5, 6, 7, 8, 9).
\end{aligned}$$

Set $\lambda = 1$ (use λ as the record of the number of iteration).

Step 2: Utilise the linear simultaneous equations,

$$(1 - p^{kj}/r^k)t^{kj} + \sum_{\ell \in L_k/k} t^{\ell j} = -\sum_{\ell \in L_k} (s^{\ell j} + u^{\ell j}), \quad k = 1, \ldots, \bar{L},$$

to determine the processing time of the finished products, that is t_1^{km}. We have the set of the processing time of the fifth firm as $\{1.00, 0.62, 0.56, 0.54, 0.33, 0.31, 0.28, 0.18, 0.14, 0.13\}$.

Step 3: Determine the production times of the other firms by $t_\lambda^{kj} = p^{km} t_\lambda^{km}/p^{kj}$. The results are shown in Table 4.

Step 4: Check the convergence of the iteration. Since it is the first time of iteration, $\lambda = 1$, go to Step 5.

Step 5: Calculate the starting time d^{kj} and waiting time u^{kj}, $j = 1, \ldots, m-1$, $k = 1, \ldots, \bar{L}$, from the first firm to the last one, as shown in Tables 5 and 6, respectively.

Table 5. The starting time of each firm.

j	d^{1j}	d^{2j}	d^{3j}	d^{4j}	d^{5j}	d^{6j}	d^{7j}	d^{8j}	d^{9j}	d^{10j}
1	0.16	1.07	2.09	2.85	3.79	4.32	4.87	5.59	5.96	6.40
2	1.08	2.12	2.72	3.78	4.79	5.38	6.06	6.69	7.14	7.59
3	2.13	2.96	3.69	4.72	5.40	6.22	6.81	7.55	8.06	8.54
4	3.07	4.24	4.99	5.66	6.62	7.20	7.93	8.49	9.00	9.36
5	4.33	5.68	6.59	7.37	8.26	8.89	9.44	10.05	10.53	10.93

Table 6. The waiting time of each firm.

j	u^{1j}	u^{2j}	u^{3j}	u^{4j}	u^{5j}	u^{6j}	u^{7j}	u^{8j}	u^{9j}	u^{10j}
1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.07	0.42	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.11	0.10	0.15	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 7. Production time of each firm.

	Production time t_1^{kj}									
Firm j	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$	$k=8$	$k=9$	$k=10$
1	0.70	0.68	0.45	0.68	0.29	0.35	0.49	0.13	0.27	0.14
2	0.82	0.34	0.39	0.75	0.43	0.49	0.35	0.22	0.22	0.22
3	0.60	0.51	0.73	0.41	0.33	0.39	0.46	0.21	0.15	0.14
4	0.97	0.57	0.43	0.67	0.39	0.41	0.31	0.25	0.21	0.22
5	1.00	0.62	0.56	0.54	0.33	0.31	0.28	0.18	0.14	0.13

Set $\lambda = \lambda + 1 = 2$ and continue the iteration. Go back to Step 3 of Phase 3, we have, at the second time, the production time of the finished product as $\{1.00, 0.62, 0.56, 0.54, 0.33, 0.31, 0.28, 0.18, 0.14, 0.13\}$. We update the production times and waiting times of each firm, and summarise the information in Table 7.

Step 4: Check the convergence of the iteration.

(a) Since the values of all the production lots keep the same, this iteration converges, and the production times are thus determined.
(b) We obtain the total production time of each firm T_j as $T_1 = 6.536$, $T_2 = 6.942$, $T_3 = 6.776$, $T_4 = 6.840$ and $T_5 = 7.031$.
(c) Calculate the total cost of the supply chain system $\Gamma_1(K_1, B_{1j}) = \$220.83$.
(d) Proceed with the next junction point starting from phase 1 again at the next iteration.

Next iteration: Set $\eta = \eta + 1 = 2$.

(1) Since $w_2 = 10.36 > B_{cc}/2$, we continue the investigation by checking the next junction point.
(2) The value $\alpha = \arg\max_i\{\delta_i(k_i) < w_\eta\} = 2$ means that the multiplier of product number two needs to be updated, i.e. $w_2 = \delta_2(k_2)$.
(3) By utilising $K_{PoT}(w_2) \equiv (K_{PoT}(w_1)\backslash\{k_2\}) \cup 2k_2\}$, we have $K_{PoT}(w_2) = \{1, 2, 1, 1, 1, 1, 1, 1, 1,\}$.
(4) Following the same procedures, we have $\Gamma_2(K_2, B_{2j}) = \$162.25$.

According to our results, the proposed three-phase heuristic investigated only 7 junction points before termination, and the information on their locations and corresponding costs are summarised in Table 8.

Table 8. The searching process.

w_η	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	$\Gamma_\eta(K_\eta, B_{\eta j})$
11.620	1	1	1	1	1	1	1	1	1	1	220.83
10.360	1	2	1	1	1	1	1	1	1	1	282.74
8.140	1	2	1	1	1	1	1	2	1	1	233.87
8.135	1	2	1	2	1	1	1	2	1	1	256.73
6.700	1	2	1	2	1	1	1	2	2	1	188.75
5.900	1	2	1	2	1	1	2	2	2	1	178.69
5.510	1	2	2	2	1	1	2	2	2	1	162.25

As shown in Table 8, the optimal solution can be determined by $(K_\eta^*, B_{\eta j}^*) = \arg\min_\eta\{\Gamma_\eta(K_\mu, B_{\eta j})\}$. The minimum total cost is located in the last iteration with the value of \$162.25. In this example, the optimal cost (\$162.25) is significantly less than that from the common cycle (\$220.83), which is well known as an upper-bound solution, by 26.5%.

5.2 *Random experiments*

We randomly generate the instances for our numerical experiments by picking the parameters values from the following uniform distribution functions: the demand rate from UNIF [140–740], the production rate from UNIF [5000–11,000], the setup time from UNIF [0.15–0.35], setup cost in UNIF [60–150], holding cost in UNIF [0.001–0.005]. We have a total of 16 combinations of the number of firms (m) and the number of items (n) with $m = 3, 5, 7, 10$ and $n = 5, 10, 15, 20$, and tested 250 randomly generated instances for each combination.

Recall that the solution from the common cycle approach may serve as an upper bound for the ST-TVLS model. Therefore, we compare the solution from the proposed three-phase heuristic with that from the common cycle approach. Also, we define a performance measure CR, which is the percentage of cost reduction, by

$$CR = (\Gamma_{CC} - \Gamma^*)/\Gamma_{CC} \cdot 100\% \tag{20}$$

We carried out our numerical experiments on a PC (Pentium 4 CPU 3.2 GHz with 1 G RAM). After solving the instances, we summarised the statistics of CR for all the combinations in Table 9.

We have two interesting observations from Table 9:

(1) As the number of firms (m) increases, the average cost reduction increases. It is evident that our three-phase heuristic can obtain a much better schedule than the common cycle approach, and the advantage turns more significant for those larger size serial-type supply chains.
(2) Note that we solve all the instances (i.e. a total of 4000 instances) in 12.29 seconds. The average run time for those $m = 10$ instances is only 0.6 seconds longer than those $m = 3$ instances. Therefore, our numerical results show that the proposed three-phase heuristic is able to effectively solve solution with excellent quality.

5.3 *Sensitivity analysis*

Here, we conduct sensitivity analysis of the setup time, the setup cost and the holding cost on the optimal solution quality. First, we set up the baseline ranges for the setup time, the setup cost and the holding cost as [0.15–0.35], UNIF [60–150], and [0.001–0.005], respectively. Then, in our sensitivity analysis, we change the range of a particular parameter by multiplying its baseline range by the factors of 1, 2, 3, 5 and 10, respectively. For example, as the baseline range of the setup time multiplies by factor 2, it becomes UNIF [0.30–0.70]. Using the combination of 10 firms and 10 products as our base, we summarise the results of our sensitivity analysis in Tables 10.

We would like have some discussions on our sensitivity analysis as follows:

(1) The higher the holding cost, the smaller the cost reduction. In other words, the common cycle approach is apt to obtain a close-to-optimal solution as the holding cost is high. One may observe a similar trend in the setup time, and it is even more sensitive than the holding cost.

Table 9. The cost reduction for different number of firms and items.

Number of firms m	Number of items n	Cost reduction (percentage) Min. CR (%)	Max. CR (%)	Avg. CR (%)
3	5	0	28.5	8.3
	10	0	23.5	9.2
	15	0	19.4	4.8
	20	0	14.8	1.2
5	5	0	30.9	12.2
	10	0	27.8	12.5
	15	0	20.5	4.7
	20	0	17.2	1.2
7	5	0	39.0	15.0
	10	0	32.1	14.5
	15	0	18.7	3.8
	20	0	16.0	1.0
10	5	0	43.7	19.5
	10	0	39.2	16.4
	15	0	19.5	3.7
	20	0	12.9	0.9

Table 10. The sensitivity analysis on the three parameters.

Parameters	Factor	Cost reduction (percentage) Min. CR (%)	Max. CR (%)	Avg. CR (%)
Setup time	1	0.0	39.2	16.4
	3	0.0	6.0	0.3
	5	0.0	0.2	0.0
	7	0.0	0.0	0.0
	10	0.0	0.0	0.0
Setup cost	1	0.0	39.2	16.4
	3	2.2	44.7	26.0
	5	3.8	48.0	29.3
	7	5.7	47.2	30.4
	10	12.1	55.2	32.3
Holding cost	1	0.0	39.2	16.4
	3	0.0	18.7	5.7
	5	0.0	14.2	2.5
	7	0.0	7.1	0.5
	10	0.0	5.4	0.2

Table 11. The relation between cost reduction and setup cost.

Number of firms m	Number of items n	Average cost reduction (percentage) Setup cost UNIF [6–15]	Setup cost UNIF [60–150]	Setup cost UNIF [600–1500]
3	5	4.1	8.3	8.8
	10	1.5	9.2	13.6
	15	0.2	4.8	15.2
	20	0.1	1.2	10.8
5	5	3.5	12.2	14.8
	10	0.9	12.5	20.4
	15	0.2	4.7	19.9
	20	0.0	1.2	14.3
7	5	3.3	15.0	19.5
	10	0.4	14.5	26.8
	15	0.1	3.8	22.6
	20	0.1	1.0	15.8
10	5	3.1	19.5	25.3
	10	0.2	16.4	32.3
	15	0.1	3.7	26.3
	20	0.0	0.9	16.8

(2) The average CR decreases as the number of items increases. It is because the value of $\sum_{i=1}^{n} r_i/p_i$ increases when the number of items increases, and the common cycle approach becomes favourable in such a case.

Finally, Table 11 presents the sensitivity analysis of the setup cost on the values of CR. One may observe that the average CR increases with the values of the setup cost.

6. Concluding remarks

In this paper, we solve the optimal sequencing, lot-sizing and scheduling decisions for several products manufactured through several firms in a serial-type supply chain so as to minimise the average of total costs. Using a time-varying lot-sizing policy, we formulate the *ST-TVLS* model for the concerned problem. We propose a three-phase heuristic that obtains the production frequencies of each product, find the production sequence and determine the production times as well as idle times for all the production lots in the three phases. Based on our numerical experiments, we demonstrate that the proposed three-phase heuristic not only is effective, but also obtains solutions much better than the upper-bound solutions from the common cycle approach.

Some topics may serve as possible extensions of this study in the future. One could extend the decision making scenario of the lot-sizing and scheduling problem to a supply chain system with a more general structure, e.g. an assembly-type supply chain, etc. As one may easily figure that the co-ordination among the scheduling of the production lots from branches of the supply chain system could become a problem with severe challenge. Another one could be the integration of the lot-sizing and scheduling problem in a serial-type supply chain system with the transportation operations. The key issue could be the trade-off between the effect of shipment consolidation in transportation and reducing number of setups in production and the economies of scale in holding inventory.

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