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Technical Note

Stochastic analysis of stream–groundwater interaction subject to temporally correlated recharge

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SUMMARY

In this article, the coupled equations describing the transient groundwater flow and stream flow fluctuations are solved based on the Fourier–Stieltjes integral representation, where the lateral inflow term in the stream flow perturbation equation connects the process of groundwater flow to that of stream flow. The closed-form solutions describing the variability in groundwater flow specific discharge and stream flow discharge are developed to quantify the influence of the variability in temporally correlated recharge in an unconfined aquifer on the stream flow discharge. We found from the closed-form solutions that the temporal correlation scale and the variance of recharge fields influence the variability in groundwater flow specific discharge and the stream flow discharge positively. In addition, the hydraulic diffusivity coefficient takes the role of reducing the variability in stream flow discharge.

1. Introduction

The groundwater levels in aquifers are often fluctuating directly in response to the temporal natural replenishment from precipitation. In many areas, the groundwater serves as a major source, contributing water to the surface water. As such, the dynamic flow behavior of the surface-aquifer system is affected by the recharge which represents the process of the precipitation infiltrating into the aquifer. Consequently, it is of particular importance to understand the role of the recharge in influencing the stream flow behavior for the management of the surface flow supply. Motivated by that, this work is devoted to quantifying the influence of the temporal variation in recharge rate on the stream flow.

The interaction of stream flow and groundwater flow is affected by the complex natural events, such as the temporal fluctuations in recharge or the spatial variations in recharge and aquifer properties. The fluctuations in recharge and aquifer properties are usually stochastic in nature. The evaluation of the uncertainty involved in the processes of stream flow and groundwater flow interaction is therefore performed using the stochastic methodology. The stochastic analysis of the dynamic responses of the groundwater system to its recharge has been carried out in a number of articles (e.g., Duffy and Gelhar, 1985, 1986; Gelhar, 1974, 1993; Gelhar and Wilson, 1974; Li and Graham, 1999; Zhang and Li, 2005, 2006; Zhang and Yang, 2010), where the phreatic aquifer is connected to a stream. In their analyses, the surface water body was considered as a boundary of specified hydraulic head in the groundwater flow domain. The focus of their investigation has not been placed on the quantification of the response of surface water to the variability in the recharge process, which is the task undertaken in this wok. This paper deals with the same stream-aquifer system subject to temporally correlated recharge, however the stream flow equation (the diffusion wave equation) is included in the analysis. To achieve the goal, we adopt the nonstationary spectral approach to solve the coupled stochastic perturbation equations governing the transient groundwater flow and stream flow fluctuations. The results of this study should be of value to researchers interested in problems of stream-aquifer management from the probabilistic point of view.

2. Problem formulation

We concern the case where the transient groundwater recharge from precipitation first drains into the water table of an unconfined aquifer and then discharges from the aquifer to the surface water. See Fig. 1 for a schematic of the stream-aquifer system. As such, the flow system considered in the study is the combination of a stream flow and an unconfined groundwater flow models. Assume that the stream is connected to the unconfined aquifer in such a way that

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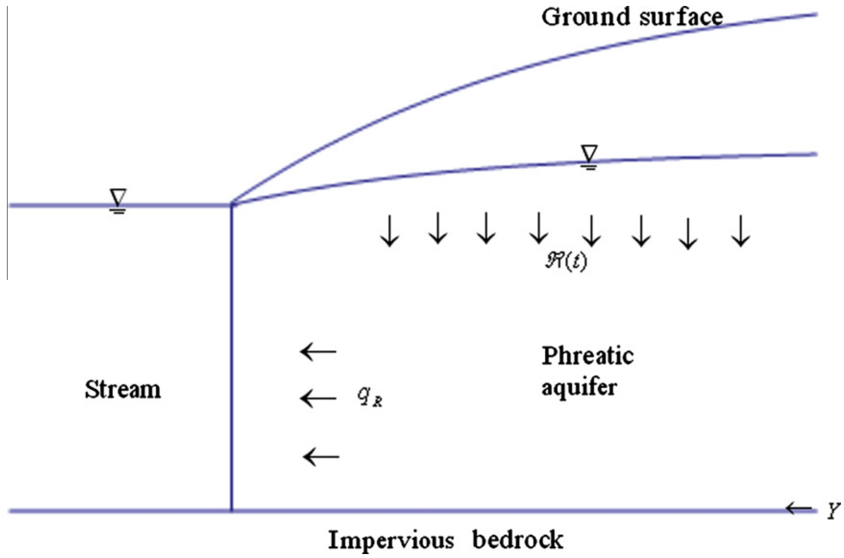

Fig. 1. Schematic representation of the stream-aquifer system.

water level in the stream is at the same elevation as the water table of the unconfined aquifer at the interface. In addition, the hydraulic gradient in the subsurface flow field is modeled perpendicular to the stream flow direction.

In this paper, the stream flow is formulated based on the Saint–Venant equations (e.g., Chow et al., 1988). Representing the balance of mass and momentum along the stream, the Saint–Venant equations provide a framework for studying the behavior of gradually varied, one-dimensional, unsteady surface flow. In practical applications, the exclusion of the inertial terms (the local and convective accelerations) in the momentum equation simplifies the system equations to the diffusion wave equation (e.g., Fan and Li, 2006; Gottardi and Venutelli, 2008; Moussa, 1996; Sivapalan et al., 1997; Sulis et al., 2010) as

$$\frac{\partial Q}{\partial t} = D\left(\frac{\partial^2 Q}{\partial X^2} - \frac{\partial q_R}{\partial X}\right) - U\left(\frac{\partial Q}{\partial X} - q_R\right) \quad (1)$$

where Q is the stream flow discharge, D and U are wave diffusivity and celerity, respectively, and q_R is the lateral inflow rate (per unit stream length).

In the following, it is assumed that the groundwater flow in the unconfined aquifer near a stream is toward the stream. The stream therefore gains water from the adjacent aquifer. As such, the lateral inflow term in the stream flow equation is denoted as a source term representing the groundwater flow toward the nearby stream. That is, the following analysis is not applicable to the case of losing stream, where the water infiltrates from the stream into the aquifer. In addition, we are only concerned the case of uniformly distributed lateral inflow along the stream.

Eq. (1) is highly nonlinear due to dependence of the diffusivity and celerity coefficients on the stream flow discharge. However, it may be linearized in perturbation form based on the steady uniform reference values of the discharge and flow cross-sectional area as (e.g., Lal, 2001; Moramarco et al., 1999; Yen and Tsai, 2001)

$$\frac{\partial Q'}{\partial t} = D_0\frac{\partial^2 Q'}{\partial X^2} - U_0\left(\frac{\partial Q'}{\partial X} - q\right) \quad (2)$$

where $Q' = Q - Q_0$, $q = q_R - q_0$, and Q_0 and q_0 represent the steady uniform initial values. According to Yen and Tsai (2001), for a prismatic rectangular channel

$$U_0 = \left(1 + \frac{\alpha}{2}\frac{B}{B + 2Y_0}\right)V_0 \quad (3)$$

$$D_0 = \frac{1}{2}\left[1 - \left(1 - 2\frac{U_0}{V_0} + \frac{U_0^2}{V_0^2}\right)F_0^2\right]\frac{V_0 Y_0}{S_0} \quad (4)$$

where B is the channel bottom width, V_0 and Y_0 are the steady uniform initial flow velocity and depth, respectively, S_0 is the channel bed slope, $F_0 = V_0/(gY_h)^{0.5}$, Y_h is the hydraulic depth, $\alpha = 1$ for Chezy's equation and $\alpha = 4/3$ for Manning's equation. Note that due to the assumption of uniformly distributed lateral inflow, the term $\partial q/\partial X$ can be removed from (2).

Conceptually, the groundwater flow in the phreatic aquifer is modeled in one dimensional horizontal plane perpendicular to the stream flow direction (e.g., Gelhar, 1974; Gelhar, 1993; Zhang and Li, 2005, 2006). Considering that the saturated thickness of the aquifer changes a relative small amount, the governing perturbation equation for groundwater flow fluctuations in the phreatic aquifer under the Dupit assumption of horizontal flow can be simplified further to (e.g., Gelhar, 1993)

$$S_y\frac{\partial \phi}{\partial t} = T\frac{\partial^2 \phi}{\partial Y^2} + \Re(t) \quad (5)$$

where ϕ is the hydraulic head, Y is the spatial coordinate along the groundwater flow direction, S_y is the phreatic aquifer storativity, T is the transmissivity, and $\Re$ is the evenly distributed transient recharge rate.

If the recharge in (5) is treated as a random process in time and characterized by its mean and covariance, this will lead the dependent variable (output) of (5), the hydraulic head, to be a random function. In turn, the lateral inflow, $q_R = -T\partial\phi/\partial Y$, introduced on the right-hand side of (1), will lead (1) and (2) to be stochastic stream flow equations. As such, we define the expected values of ϕ and R and their fluctuations around the averages as

$$H = \langle \phi(Y,t)\rangle \quad h = \phi - H \quad (6)$$

$$\bar{\Re} = \langle \Re(t)\rangle \quad r = \Re - \bar{\Re} \quad (7)$$

where $\langle\rangle$ denotes the expected value operator, $\langle h\rangle = 0$, and $\langle r\rangle = 0$. Introducing (6) and (7) into (5), and taking the expectation of it leads to the equation for the mean groundwater flow. By subtracting the mean flow equation from (5), one gets the first-order equation for perturbations in groundwater flow

$$S_y\frac{\partial h}{\partial t}=T\frac{\partial^2 h}{\partial Y^2}+r(t) \tag{8}$$

Note that (2) is linked to (8) by the interaction term q in (2), which may be expressed by

$$q=-T\frac{\partial h}{\partial Y} \tag{9}$$

The coupled stream-aquifer perturbation equations provide a basic framework for allowing us to develop the expressions quantifying the variability of discharge fluctuations in the stream in terms of the variability of recharge fluctuations in an unconfined aquifer.

3. Solutions of aquifer-stream perturbation equations

Analytical solutions to the coupled perturbation Eqs. (2) and (8) for groundwater flow and stream flow will be found based on the Fourier–Stieltjes integral representation approach (or spectral approach), where the flow domains in the proposed models are considered to be finite. One of the major advantages of spectral approach lies in the fact that it produces tractable closed-form results.

3.1. Groundwater flow model

It is reasonable to assume that the fluctuations in recharge, r (a source term in (8)), is a second-order stationary process in time (e.g., Gelhar, 1993; Gelhar and Wilson, 1974; Zhang and Li, 2006). Therefore, it can be expressed in terms of time Fourier–Stieltjes representation as (Gelhar, 1993)

$$r(t)=\int_{-\infty}^{\infty}\exp(i\omega t)dZ_r(\omega) \tag{10}$$

where ω is the frequency and dZ_r is the Fourier amplitude of the fluctuations. The hydraulic head perturbed quantities in (8) is, however, presented by the Fourier–Stieltjes representation of a non-stationary process (Li and McLaughlin, 1991) resulting from the effect of a finite flow domain as

$$h(Y,t)=\int_{-\infty}^{\infty}\Theta_{hq}(Y,t,\omega)dZ_r(\omega) \tag{11}$$

where $\Theta_{hq}(Y,t,\omega)$ is an unknown transfer function to be determined. Introducing (10) and (11) into (8) admits

$$\frac{\partial\Theta_{hq}}{\partial t}=\frac{T}{S_y}\frac{\partial^2\Theta_{hq}}{\partial Y^2}+\frac{\exp(i\omega t)}{S_y} \tag{12}$$

We focus only on the case where the boundary and initial conditions are deterministic. As such, the stochastic boundary and initial conditions associated with (12) are in the forms of

$$\Theta_{hq}(0,t)=0 \tag{13a}$$

$$\Theta_{hq}(\ell,t)=0 \tag{13b}$$

$$\Theta_{hq}(Y,0)=0 \tag{13c}$$

where ℓ is the length of the aquifer.

The determination of the solution to (12) and (13) can be done through the method of eigenfunction expansions (e.g., Farlow, 1993; Haberman, 1998)

$$\Theta_{hq}=\frac{2}{\pi S_y}\sum_{n=1}^{\infty}\frac{1-\cos(n\pi)}{n}\sin\left(n\pi\frac{Y}{\ell}\right)\frac{\exp(i\omega t)-\exp(-\beta n^2 t)}{\beta n^2+i\omega} \tag{14a}$$

where $\beta=\pi^2T/(\ell^2S_y)$. Gelhar and Wilson (1974) and Gelhar (1993) defined a characteristic time scale t_c, referred to as the hydraulic response time, as $t_c=S_Y\ell^2/(3T)$. It may be represented as a critical time for groundwater flow moving into a stream following a recharge event. The focus of this study is placed on the case where $t\gg t_c$, i.e., $\beta t\gg\beta t_c=\pi^2/3$. Under the condition that $\beta t\gg\pi^2/3$, the summation in (14a) converges rapidly (e.g., Haberman 1998) and is therefore approximated as

$$\Theta_{hq}\approx\frac{4}{\pi S_y}\sin(\pi\xi_Y)\frac{\exp(i\omega t)-\exp(-\beta t)}{\beta+i\omega} \tag{14b}$$

where $\xi_Y=Y/\ell$. This leads (11) and (9) to

$$h(Y,t)=\frac{4}{\pi S_y}\sin(\pi\xi_Y)\int_{-\infty}^{\infty}\frac{\exp(i\omega t)-\exp(-\beta t)}{\beta+i\omega}dZ_r(\omega) \tag{15}$$

$$q=-T\frac{\partial h}{\partial Y}=4\frac{T}{S_y\ell}\cos(\pi\xi_Y)\int_{-\infty}^{\infty}\frac{\exp(-\beta t)-\exp(i\omega t)}{\beta+i\omega}dZ_r(\omega) \tag{16}$$

It follows from the use of representation theorem for q that

$$\sigma_q^2=\langle qq^*\rangle=\frac{16}{\pi^4}\beta^2\ell^2\cos^2(\pi\xi_Y)\times\int_{-\infty}^{\infty}\frac{1-2\exp(-\beta t)\cos(\omega t)+\exp(-2\beta t)}{\beta^2+\ell^2}S_{rr}(\omega)d\omega \tag{17}$$

where σ_q^2 is the variance of groundwater flow specific discharge (inflow for the stream flow model), the asterisk stands for the operation of complex conjugation and $S_{rr}(\omega)$ is the spectrum of recharge perturbations.

3.2. Surface flow model

We proceed to solve the stream flow discharge perturbation Eq. (2) in frequency domain. Making use of the Fourier–Stieltjes representation for stream flow discharge fluctuations

$$Q'(X,t)=\int_{-\infty}^{\infty}\Theta_{Qq}(X,t,\omega)dZ_r(\omega) \tag{18}$$

and replacing the source term in (2) by (16) leads (2) to

$$\frac{\partial\Theta_{Qq}}{\partial t}=D_0\frac{\partial^2\Theta_{Qq}}{\partial X^2}-U_0\frac{\partial\Theta_{Qq}}{\partial X}+4\frac{U_0T}{S_y\ell}\cos(\pi\xi_Y)\times\frac{\exp(-\beta t)-\exp(i\omega t)}{\beta+i\omega} \tag{19}$$

where $\Theta_{Qq}(X,t,\omega)$ is an unknown transfer function. Similar to the groundwater flow model, deterministic boundary and initial conditions are proposed to the stream flow model, which imply that

$$\Theta_{Qq}(0,t)=0 \tag{20a}$$

$$\Theta_{Qq}(L,t)=0 \tag{20b}$$

$$\Theta_{Qq}(X,0)=0 \tag{20c}$$

where L is the length of the channel.

The solution of (19) and (20) is then given as

$$\Theta_{Qq}=\frac{8}{\pi^2}\frac{\beta\ell U_0}{\beta+i\omega}\cos(\pi\xi_Y)\exp(0.5\mu\xi_X)\times\sum_{n=1}^{\infty}\frac{1-\exp(-0.5\mu)\cos(n\pi)}{n\pi+\mu^2/(4n\pi)}\times\sin(n\pi\xi_X)\left[\frac{\exp(-(\varepsilon n^2+\upsilon)t)-\exp(-\beta t)}{\beta-\alpha}+\frac{\exp[-(\varepsilon n^2+\upsilon)t]-\exp(i\omega t)}{n\varepsilon+\upsilon+i\omega}\right] \tag{21a}$$

where $\mu = U_0L/D_0$, $\xi_X = X/L$, $\varepsilon = D_0\pi^2/L^2$, $\nu = U_0^2/(4D_0)$, and $\alpha = \varepsilon + \nu$. In the case of a large βt (e.g., Haberman, 1998), the approximation of (21a) is in the form of

$$\Theta_{Qq} \approx \frac{8}{\pi}\frac{D_0U_0}{L^2\alpha}\ell\beta\left[1+\exp\left(-\frac{\mu}{2}\right)\right]\cos(\pi\xi_Y) \times \exp\left(\frac{\mu}{2}\xi_X\right)\sin(\pi\xi_X) \times \frac{1}{\beta+i\omega}\left[\frac{\exp(-\beta t)-\exp(-\alpha t)}{\alpha-\beta}-\frac{\exp(i\omega t)-\exp(-\alpha t)}{\alpha+i\omega}\right] \quad (21b)$$

where $\alpha = D_0\pi^2/L_2 + U_0^2/(4D_0)$. Substituting (21b) into (18) results in

$$Q'(X,t) = \frac{8}{\pi}\frac{D_0U_0}{L^2\alpha}\ell\beta\left[1+\exp\left(-\frac{\mu}{2}\right)\right]\cos(\pi\xi_Y)\exp\left(\frac{\mu}{2}\xi_X\right)\sin(\pi\xi_X) \int_{-\infty}^{\infty}\frac{1}{\beta+i\omega}\left[\frac{\exp(-\beta t)-\exp(-\alpha t)}{\alpha-\beta}-\frac{\exp(i\omega t)-\exp(-\alpha t)}{\alpha+i\omega}\right]dZ_r(\omega) \quad (22)$$

Using the representation theorem with (22) results in the spectrum of the stream flow discharge as

$$S_{QQ}(\omega,X) = \frac{64}{\pi^2}\frac{D_0^2U_0^2}{L^4\alpha^2}\ell^2\beta^2 \times\left[1+\exp\left(-\frac{\mu}{2}\right)\right]^2\cos^2(\pi\xi_Y)\exp(\mu\xi_X)\sin^2(\pi\xi_X) \times\frac{1}{\beta^2+\omega^2}\left[\frac{\exp(-\beta t)-\exp(-\alpha t)}{\alpha-\beta}-\frac{\exp(i\omega t)-\exp(-\alpha t)}{\alpha+i\omega}\right] \times\left[\frac{\exp(-\beta t)-\exp(-\alpha t)}{\alpha-\beta}-\frac{\exp(-i\omega t)-\exp(-\alpha t)}{\alpha-i\omega}\right]S_{rr}(\omega) \quad (23)$$

Finally, the variance of the stream flow discharge can now be obtained by integrating (23) over the frequency domain

$$\sigma_Q^2 = \int_{-\infty}^{\infty}S_{QQ}(\omega)d\omega = \frac{64}{\pi^2}\frac{D_0^2U_0^2}{L^4\alpha^2}\ell^2\beta^2 \times\left[1+\exp\left(-\frac{\mu}{2}\right)\right]^2\cos^2(\pi\xi_Y)\exp(\mu\xi_X)\sin^2(\pi\xi_X) \times\int_{-\infty}^{\infty}\frac{1}{\beta^2+\omega^2}\left[\frac{\exp(-\beta t)-\exp(-\alpha t)}{\alpha-\beta}-\frac{\exp(i\omega t)-\exp(-\alpha t)}{\alpha+i\omega}\right] \times\left[\frac{\exp(-\beta t)-\exp(-\alpha t)}{\alpha-\beta}-\frac{\exp(-i\omega t)-\exp(-\alpha t)}{\alpha-i\omega}\right]S_{rr}(\omega)d\omega \quad (24)$$

4. Closed-form expressions for the coupled models' variances

Recharge to the aquifer is a certain percentage of precipitation that crosses the water table and becomes part of the groundwater flow system. Therefore, the temporal patterns of recharge are influenced by precipitation variations under different time scales, such as the short-scale erratic distribution of precipitation and the large-scale inter-annual and seasonal precipitation. The spectral analysis of precipitation data could provide a basis for detecting the temporal patterns of variability in the precipitation time series (e.g., De Michele and Bernardara, 2005; García et al. 2002; Joshi and Pandey, 2011; Longobardi and Villani, 2010; Rodriguez-Puebla et al., 1998). On the other hand, the processes converting the precipitation to the recharge to the aquifer are highly variable. In most situations, the rate of recharge to the aquifer is not measured directly and is very difficult to quantify. However, it might be inferred indirectly as being some function of measured precipitation (e.g., Gelhar, 1993).

We quantify the variability in groundwater flow specific discharge and stream flow discharge for the case where the spectrum of the recharge fluctuations is in the form (Gelhar, 1993; Zhang and Li, 2006)

$$S_{rr}(\omega) = \frac{\sigma_r^2\lambda}{\pi(1+\lambda^2\omega^2)} \quad (25)$$

where σ_r^2 is the variance and λ the temporal correlation scale of the recharge process, respectively.

With (25), the integration of (17) in ω domain produces

$$\sigma_q^2 = \frac{16}{\pi^4}\sigma_r^2\ell^2\eta^2\cos^2(\pi\xi_Y)\left[\frac{1-\exp(-2\tau)}{\eta(1+\eta)} -2\exp(-\tau)\frac{\exp(-\tau)/\eta-\exp(-\tau/\eta)}{1-\eta^2}\right] \quad (26)$$

where $\eta = \beta\lambda$ and $\tau = \beta t$. The variance of stream flow discharge is obtained from (24) to (25) in the form

$$\sigma_Q^2 = \frac{64}{\pi^2}\sigma_r^2\frac{U_0^4\ell^2}{\mu^2\beta^4L^2}\frac{\eta^2}{\rho^2(1-\rho^2)}\cos^2(\pi\xi_Y)[1+\exp(-0.5\mu)]^2\times\sin^2(\pi\xi_X)\exp(\mu\xi_X) \times\left\{[\exp(-\tau)-\exp(-\rho\tau)]^2\frac{1+\rho}{1-\rho}\frac{1}{\eta(1+\eta)} +2\frac{\exp(-\tau)-\exp(-\rho t)}{\eta(1-\rho)} \times\left[\frac{2\exp(-\rho t)-(1+\rho\eta)\exp(-\tau/\eta)}{1-\rho^2\eta^2} -\frac{(1+\rho)\exp(-\tau)-(1+\rho\eta)\exp(-\tau/\eta)}{1-\eta^2} -\exp(-\rho\tau)\left(\frac{1}{1+\rho\eta}-\frac{\rho}{1+\eta}\right)\right] +[1+\exp(-2\rho\tau)]\left[\frac{1}{\rho\eta(1+\rho\eta)}-\frac{1}{\eta(1+\eta)}\right] -2\exp(-\rho\tau)\left[\frac{\exp(-\rho\tau)-\rho\eta\exp(-\tau/\eta)}{\rho\eta(1-\rho^2\eta^2)}-\frac{\exp(-\tau)-\eta\exp(-\tau/\eta)}{\eta(1-\eta^2)}\right]\right\} \quad (27)$$

where $\rho = \alpha/\beta$.

The graphical presentation of the variance of groundwater flow specific discharge in (26) as a function of the temporal correlation scale of recharge fields is illustrated in Fig. 2. As indicated in the figure, the temporal correlation scale affects the variation in specific discharge positively. This is because a larger correlation scale introduces a larger temporal consistency of fluctuations in the groundwater specific discharge above or below the mean specific discharge, and consequently, producing a grater variance of the groundwater flow specific discharge.

Eq. (27) reveals a linear relationship between the variability in stream flow discharge and the variability in recharge in the unconfined aquifer. The linear relationship between σ_Q^2 and σ_r^2 in (27) is the outcome of the assumption of linearization (or the first-order perturbation assumption) in perturbed forms of stream flow discharge Q and recharge rate $\Re$. The influence of the temporal correlation scale of recharge fields on the stream flow discharge field is demonstrated based on (27) in Fig. 3a. The figure indicates that a larger temporal correlation scale produces a higher variance of stream flow discharge. Notice from (2) that the fluctuations in the groundwater flow specific discharge acts as the source of the fluctuations in the stream flow discharge field, the outputs of (2). The enhanced variability in the surface flow discharge caused by a larger temporal correlation scale is the contribution of the larger variability in stream flow discharge (see Fig. 2). As indicated in Fig. 3b, the hydraulic diffusivity D_0 plays a role in attenuating the variability in stream flow discharge.

The stream-aquifer interaction may be considered as a system that converts recharge (input) for an aquifer into stream discharge (output). Climate change affects the amount of recharge for aquifers through changes in spatiotemporal distribution of precipitation and evapotranspiration. Therefore, acquiring time-series analysis of input data, such as temporal distribution of precipitation and evapotranspiration, and the information on hydraulic properties of an aquifer, the stochastic approach presented here provides a basic framework for assessing hydro-climatic change

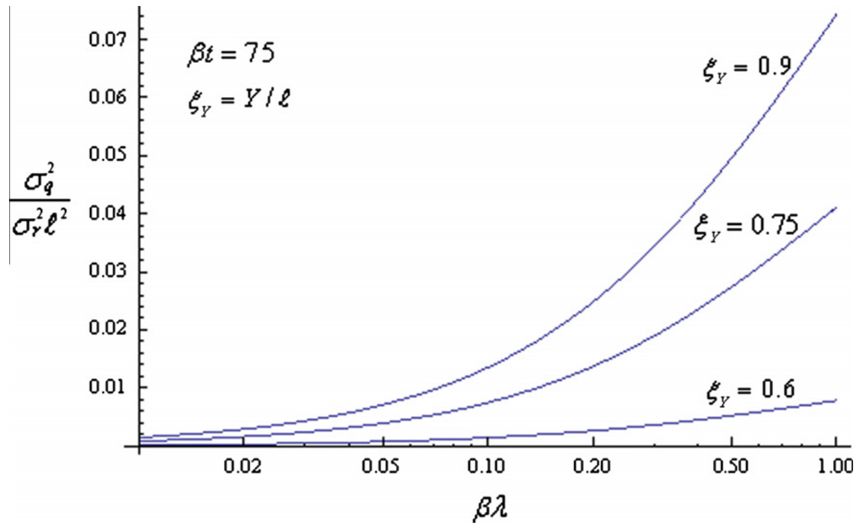

Fig. 2. Dimensionless variance of groundwater flow specific discharge as a function of temporal correlation scale of groundwater recharge process.

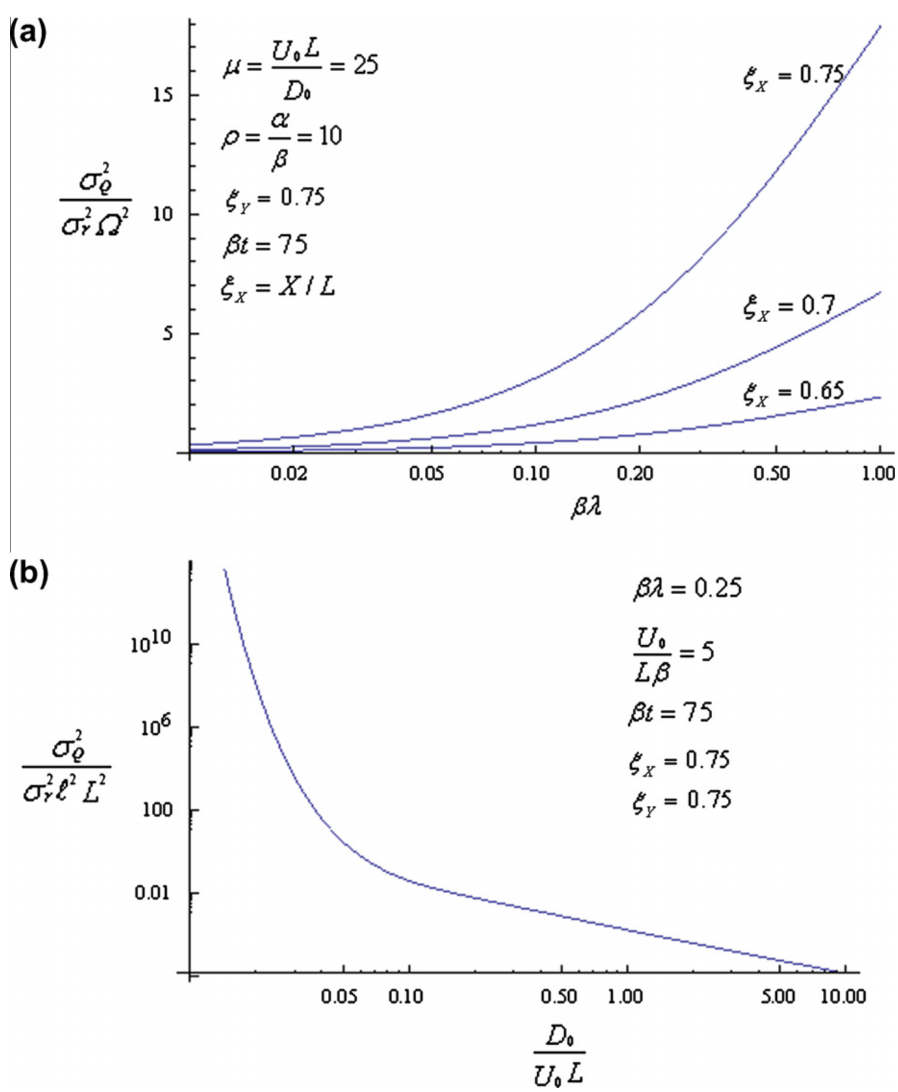

Fig. 3. (a) Dimensionless variance of surface flow discharge as a function of temporal correlation scale of groundwater recharge process, where $\Omega^2 = \frac{U_0^4 \ell^2}{\mu^2 L^2 \beta^4}$. (b) Dimensionless variance of surface flow discharge as a function of hydraulic diffusivity.

impact on the variation in stream discharge. On the other hand, by using the time series of stream discharge fluctuations and the theoretical result developed here, it is possible to identify the recharge parameters. Those are useful for characterizing the temporally correlated precipitation responding to the climate change.

The variances for groundwater and stream flow discharge fields developed in this work can be used to quantify the anticipated model error (or reliability of the model) in applying the classical deterministic models. For the purposes of stream-aquifer management, it is more reasonable to consider, say, the stream flow discharge plus two standard deviations (square root of variance) rather than to draw conclusions from only the discharge solution of the classical deterministic stream flow discharge model. In addition, the discharge variance relationship might be used in an inverse sense. If the discharge variance is determined from the variation of the discharge around a smooth mean solution, this relationship could be used to estimate the parameters of the input process. As illustrated in the figures, the discharge variances are sensitive to the form and temporal correlation scale of the covariance used for the recharge field. Consequently, identification of the appropriate covariance type based on the analytical results and flow data provide important information for the understanding of the characteristic of correlated recharge time series.

5. Conclusions

An integrated stream-aquifer flow system with temporally correlated recharge in the unconfined aquifer is analyzed stochastically. The lateral inflow term in the stream flow equation provides the mechanism coupling the groundwater flow process to the stream flow process. The use of perturbation-based nonstationary spectral perturbation techniques leads to analytical solutions for the variance of the dependent variables, the groundwater flow specific discharge and stream flow discharge. The above stochastic analysis yields important results by means of those analytical solutions. The correlation scale is found crucial to enhance the variability in groundwater flow specific discharge and stream flow discharge. In addition, the variance of surface flow discharge decreases with the increasing coefficient of hydraulic diffusivity.

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