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Maximum stiffness design of laminated composite plates via a constrained global optimization approach

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The optimal lamination arrangements of laminated composite plates with maximum stiffness subject to side constraints are investigated via a constrained multi-start global optimization approach. In the optimal design process, the deformation analysis of laminated composite plates is accomplished by utilizing a shear deformable laminated composite finite element and the optimal design problem, which has been converted into an unconstrained minimization problem via the general augmented Lagrangian method, is solved by utilizing the proposed unconstrained multi-start global optimization technique to determine the optimal fiber angles and layer group thicknesses of the laminated composite plates for attaining maximum stiffness and simultaneously satisfying the imposed side constraints. The feasibility of the proposed constrained multi-start global optimization algorithm is validated by means of a simple but representative example and its applications are demonstrated by means of a number of examples on the maximum stiffness design of symmetrically laminated composite plates. The effects of length-to-thickness ratio, aspect ratio, and number of layer groups upon the optimum fiber angles and layer group thicknesses of the plates are investigated.

INTRODUCTION

In recent years, the increasing use of laminated composite materials in the construction of mechanical, aerospace, marine and automotive structures has made the design of laminated composite structures an important research topic. The common objective in the optimal design of laminated composite structures is to design layer orientations, layer thicknesses or number of layers which will give the minimum weight of the structure and satisfy the imposed constraints. A selected list of some of the literature published in this area is given in the references.[1–7] Recently, a number of researchers have studied the optimal design of laminated composite plates for attaining better behavioral performances.[8–12] Although a substantial amount of effort has been devoted to this area, as indicated by the extensive literature published on the subject, a vast proportion of the published work is limited to simple plates consisting of a very few layers. As is well known, laminated composite plates may be composed of many layers of different orientations and even a relatively simple composite plate may possess many design variables. The increase in the number of design variables when coupled with the highly nonlinear way in which strains and deflections vary with changes in fiber orientation can result in great difficulties in obtaining convergence to a local minimum when conventional optimization techniques used by the previous researchers are employed. Furthermore, it appears to be extremely expensive if not intractable to find the global optimum using the conventional optimization techniques. For these reasons the aforementioned works on the optimal design of laminated composite plates were restricted to fairly simple cases and the results have therefore found only limited applications in practical design. It is obvious that if a broader application of optimal design in laminated composite structures is desired, more efficient and reliable global optimization techniques will be required.

Recently, Kam & Snyman[13] have proposed an unconstrained global optimization technique for the design of fiber angles of laminated composite plates with maximum stiffness. The unconstrained global optimization method has also been successfully used in designing laminated composite plates for maximum buckling strength or vibration frequency.[14–15] In this paper, the previously proposed unconstrained global optimization method is extended to treat layer group thicknesses as design variables and include side constraints. In the optimal design process, the finite element analysis of the laminated plates is accomplished using a shear deformable finite element and the original constrained optimization problem is converted into an unconstrained one via the general augmented Lagrangian approach.[16] The converted unconstrained optimization problem is then solved using the previous unconstrained global optimization algorithm to determine the optimal layups of the laminated composite plates. The feasibility of the present optimization algorithm is first validated by means of a simple but representative example. A number of examples on the maximum stiffness design of symmetrically laminated composite plates subject to thickness constraints are given to illustrate the applications of the proposed method and study the effects of length-to-thickness ratio, aspect ratio, and number of layer groups upon the optimal solution of the plates.

FINITE ELEMENT ANALYSIS OF LAMINATED COMPOSITE PLATES

Consider a rectangular plate of area $a \times b$ and uniform thickness h subject to transverse load as shown in Fig. 1. The plate is composed of a finite number of layer groups in which each layer group contains several orthotropic layers of same fiber angle and uniform thickness. The x and y coordinates of the plate are taken in the midplane of the plate. The displacement field is assumed to be of the form

$$\begin{aligned} u_1(x,y,z)&=u_0(x,y)+z\cdot\psi_x(x,y)\\ u_2(x,y,z)&=v_0(x,y)+z\cdot\psi_y(x,y)\\ u_3(x,y,z)&=w(x,y) \end{aligned} \tag{1}$$

where u_1, u_2, u_3 are displacements in the x, y, z directions, respectively, and u_0, v_0, w the associated midplane displacements; ψ_x and ψ_y are shear rotations.

The plate constitutive equations are written as

$$\begin{bmatrix} N_1\\ N_2\\ Q_y\\ Q_x\\ N_6\\ M_1\\ M_2\\ M_6 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & A_{16} & B_{11} & B_{12} & B_{16}\\ A_{12} & A_{22} & 0 & 0 & A_{26} & B_{12} & B_{22} & B_{26}\\ 0 & 0 & A_{44} & A_{45} & 0 & 0 & 0 & 0\\ 0 & 0 & A_{45} & A_{55} & 0 & 0 & 0 & 0\\ A_{16} & A_{26} & 0 & 0 & A_{66} & B_{16} & B_{26} & B_{66}\\ B_{11} & B_{12} & 0 & 0 & B_{16} & D_{11} & D_{12} & D_{16}\\ B_{12} & B_{22} & 0 & 0 & B_{26} & D_{12} & D_{22} & D_{26}\\ B_{16} & B_{26} & 0 & 0 & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \cdot \begin{bmatrix} u_{0,x}\\ v_{0,y}\\ w_{,y}+\psi_y\\ w_{,x}+\psi_x\\ u_{0,y}+v_{0,x}\\ \psi_{x,x}\\ \psi_{y,y}\\ \psi_{x,y}+\psi_{y,x} \end{bmatrix} \tag{2}$$

where N_1, $N_2,\ldots,$ M_6 are stress resultants; material components A_{ij}, B_{ij} and D_{ij} are given by

$$(A_{ij},B_{ij},D_{ij})=\int_{-h/2}^{h/2} Q_{ij}^{(m)}(1,z,z^2)\,\mathrm{d}z \qquad (i,j=1,2,6) \tag{3a}$$

and

$$A_{ij}=k_\alpha\cdot k_\beta\cdot\bar{A}_{ij},\quad \bar{A}_{ij}=\int_{-h/2}^{h/2} Q_{ij}^{(m)}\,\mathrm{d}z \qquad (i,j=4,5;\ \alpha=6-i,\ \beta=6-j) \tag{3b}$$

where Q_{ij} are material constants; the superscript m denotes layer number, and k_i are shear correction factors which can be evaluated from the exact expressions given by Whitney.[17] The derivation of the governing equations for the plate is based on the virtual work equation. The introduction of virtual displacements δu_i to the plate under static equilibrium gives the virtual work equation as[18]

$$\frac{1}{2}\,\delta\int_V (\sigma_{ij}\varepsilon_{ij})\,\mathrm{d}V-\int_{S_1} p_i\delta u_i\,\mathrm{d}S=0 \tag{4}$$

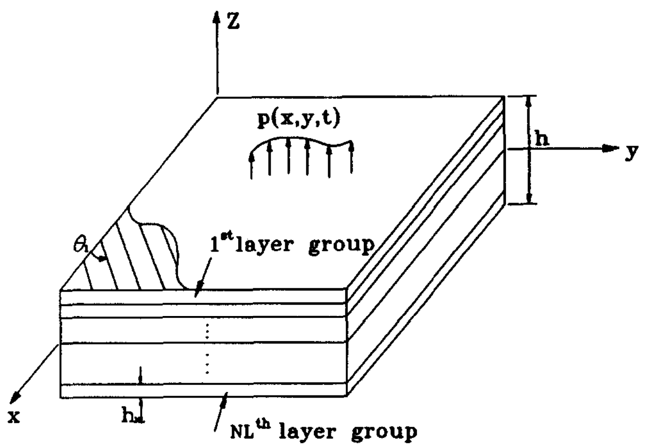


Fig. 1. Laminated composite plate.

where V is the volume and p_i surface tractions acting over the area S_1 of the plate. Herein, the finite element method developed by Kam & Chang[19] is adopted in the derivation of the load-displacement equations for the plate. The element has eight nodes and the quadratic formulation of the serendipity family with reduced integration of the 2×2 Gauss rule has been used for constructing the element stiffness matrix.

$$\mathbf{K}\Delta=\mathbf{P} \tag{5}$$

where $\mathbf{K}$, Δ, $\mathbf{P}$ are the structural stiffness matrix, displacement vector and load vector, respectively.

When the nodal displacements of the plate have been solved from eqn (5), the strain energy U stored in the plate is computed as

$$U=\frac{1}{2}\ \Delta^t\mathbf{K}\Delta. \tag{6}$$

The rates of change of strain energy with respect to design variables X_i are

$$\frac{\partial U}{\partial X_i}=-\frac{1}{2}\ \Delta^t\left(\frac{\partial \mathbf{K}}{\partial X_i}\right)\Delta. \tag{7}$$

MAXIMUM STIFFNESS DESIGN

The objective of the present optimal design of a laminated composite plate with given plate thickness and number of layer groups NL subjected to thickness constraints is the selection of fiber angles and thicknesses of the layer groups which gives the maximum stiffness of the plate. It is noted that the minimization of plate strain energy is equivalent to the maximization of plate stiffness.[8] In mathematical form, the optimal design problem is stated as

$$\begin{aligned}&\text{minimize}\quad U=U(h,\theta)\\&\text{subject to}\quad 0^\circ\leq\theta_i\leq 180^\circ\\&\qquad\qquad h_i\geq 0\quad i=1,\ldots,NL\\&\left[H=\sum_{i=1}^{NL}h_i-h=0\right]\end{aligned} \tag{8}$$

where $h=(h_1,h_2,\ldots,h_{NL})^t$, $\theta=(\theta_1,\theta_2,\ldots,\theta_{NL})^t$ are the vectors of layer group thicknesses and fiber angles, respectively.

The solution of the above constrained optimization problem using the conventional optimization techniques[16] can only yield a local minimum, not to mention the difficulty that may be encountered in enforcing convergence of the solution. Herein, a constrained multi-start global optimization method is presented for solving the above optimal design problem for attaining the global optimum solution. The above problem of eqn (8) is first converted into an unconstrained optimization problem by creating the following general augmented Lagrangian[16]

$$\begin{aligned}\Psi(\theta,h,\lambda,r_p)=U(\theta,h)+\sum_{j=1}^{NL}\ [\lambda_jx_j+r_px_j^2]\\+[\lambda_{NL+1}H+r_pH^2]\end{aligned} \tag{9}$$

with

$$\begin{aligned}&x_j=\max\left[g_j(h_j),\ \frac{-\lambda_j}{2r_p}\right]\\&g_j(h_j)=-h_j\leq 0\quad j=1,\ldots,NL\end{aligned} \tag{10}$$

where λ_j, r_p are multipliers.

The update formulae for the multipliers λ_j and r_p are

$$\begin{aligned}&\lambda_j^{n+1}=\lambda_j^n+2r_p^nx_j^n\quad j=1,\ldots,NL+1\\&r_p^{n+1}=\begin{cases}\gamma r_p^n & \text{if } r_p^{n+1}<r_p^{max}\\ r_p^{max} & \text{if } r_p^{n+1}\geq r_p^{max}\end{cases}\end{aligned} \tag{11}$$

where the superscript n denotes iteration number; γ is a constant; r_p^{max} is the maximum value of r_p. The initial values of the multipliers and the values of the parameters (γ, r_p^{max}) are chosen as

$$\begin{aligned}&\lambda_j^\circ=1{\cdot}0\quad j=1,\ldots,NL+1\\&r_p^\circ=0{\cdot}4\\&\gamma=1{\cdot}25\end{aligned} \tag{12}$$

$r_p^{max}=100$

The maximum stiffness design of the laminated composite plate has thus become the solution of the following unconstrained optimization problem

$$\text{Minimize} \quad \Psi(\theta,h,\lambda,r_p) \quad \text{with respect to } \theta \text{ and } h$$
$$\text{subject to} \quad 0°\le\theta_i\le 180° \quad i=1,\ldots,NL \tag{13}$$

The above unconstrained optimization problem can be solved straightforwardly by using the previously proposed unconstrained multi-start global optimization algorithm.[13,20] The basic idea of the unconstrained multi-start global optimization method is to solve the problem of unconstrained minimization of a differentiable objective function $F(\mathbf{y})$, $\mathbf{y}\in\mathbf{Y}\subset R^n$ and $F\subset C^1$, with several local minima $\hat{\mathbf{F}}_j$ and corresponding local minimizers $\hat{\mathbf{y}}_j$. It is noted that, for example, in the optimal design of laminated composite plates, $\mathbf{y}$ and $F(\mathbf{y})$ become $[\theta,h]$ and $\Psi(\theta,h)$, respectively. In the global minimization process, a series of starting points are selected at random from the region of interest and a local minimization algorithm is used from each starting point. The search trajectories used by the local minimization algorithm are derived from the equation of motion of a particle of unit mass in an n-dimensional conservative force field, where the potential energy of the particle is represented by $F(\mathbf{y}(t))$. In such a field the total energy of the particle, consisting of its potential kinetic energies, is conserved. The motion of the particle is simulated and by monitoring its kinetic energy an interfering strategy is adopted which ensures that potential energy is systematically reduced. In this way the particle is forced to follow a trajectory towards a local minimum in potential energy, $\hat{\mathbf{y}}$. It is noted that if the trajectory leaves the domain of interest at the point θ_p where one or more of the components θ_{pi} take on values such that either $\theta_{pi}>\pi$ or $\theta_{pi}<0$, then the constraints are imposed by continuing the trajectory at the point θ'_{pi} with components identical to θ_p except for the components corresponding to the violated constraints. These components are replaced as follows

$$\theta'_{pi}=\theta_{pi}-m\pi \quad \text{if} \quad \theta_{pi}>\pi$$

and

$$\theta'_{pi}=\theta_{pi}+m\pi \quad \text{if} \quad \theta_{pi}<0; \quad m=1,2,3,\ldots \tag{14}$$

Here the value of m is chosen in such a way that θ'_{pi} satisfies the constraints. By uninterrupting the motion of the particle with conserved total energy, other lower local minima, including in particular the global minimum, are obtained and recorded when the particle is traveling along its path. The motion of the particle is stopped once a termination criterion is satisfied. The same procedure is applied to the other starting point. As the process of searching for the global minimum continues, a Bayesian argument is used to establish the probability of the current overall minimum value of F being the global minimum, given the number of starts and number of times this value has been achieved. The multi-start procedure is terminated once a target probability, typically 0·998, has been exceeded. The main advantage of this multi-start global optimization algorithm is that it can determine the global optimal solution in a very efficient and effective way.

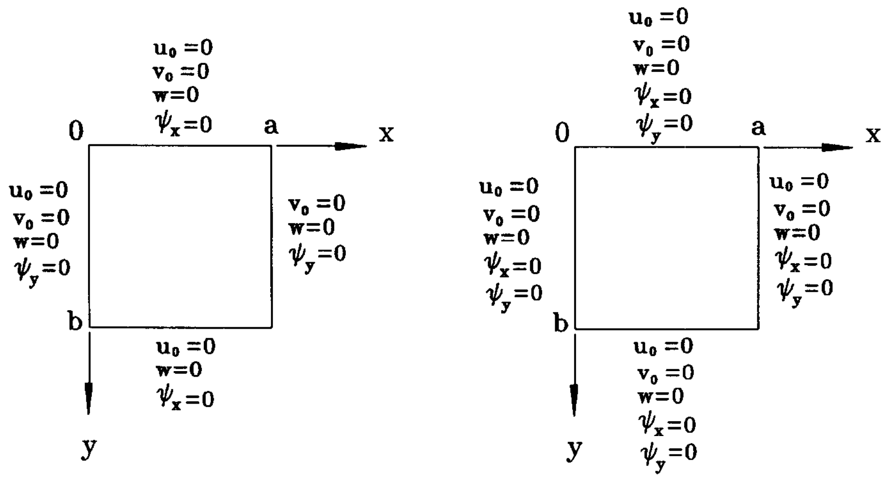
NUMERICAL EXAMPLES

The aforementioned constrained global optimization technique will be applied to the design of symmetrically laminated composite plates with simply supported or fixed edges subjected to the center point load P. The boundary conditions for the two types of support are shown in Fig. 2. The material properties used in the following design are given as

$$\frac{E_1}{E_0}=181{\cdot}0, \quad \frac{E_2}{E_0}=10{\cdot}3, \quad \frac{G_{12}}{E_0}=7{\cdot}17$$

$$\frac{G_{23}}{E_0}=3{\cdot}0, \quad \nu_{12}=0{\cdot}25, \quad G_{12}=G_{13}, \quad E_0=1{\cdot}0 \text{ GPa}$$

The advantage of using the present method in designing laminated composite plates is first illustrated by means of an example on the design of simply supported symmetric four-layered and centrally loaded plates with various aspect ratios. The results obtained by the present approach are listed in Table 1 in comparison with those obtained by using other method. The minimization routine BCONF of the IMSL mathematical package[20] has been used to solve the above optimal design problems for determining the fiber angles and



(a) simple support (b) Fixed support

Fig. 2. Boundary conditions of laminated plates.

Table 1. Optimal solutions obtained via different design methods for a simply supported symmetric four-layered and centrally-loaded plate ($a/h=10$, $\bar{W}_c=W_c[E_o bh^3/(pa^3)]\times 10^3$)

	Design method									
	(i) Present method			(ii) Previous method[13]				(iii) BCONF[20]		
Aspect ratio b/a	Fiber angles (degrees)	Normalized layer group thickness ($\bar{h}_i=h_i/h$)	$\bar{W}_c$	Fiber angles (degrees)	Normalized layer group thickness ($\bar{h}_i=h_i/h$)	$\bar{W}_c$	Difference $\frac{\text{(ii)}-\text{(i)}}{\text{(i)}}$ %	Fiber angles (degrees)	Normalized layer group thickness ($\bar{h}_i=h_i/h$)	$\bar{W}_c$
0·5	$[58{\cdot}9°/-59{\cdot}7°]_s$	$[0{\cdot}09575/0{\cdot}40425]_s$	0·84	$[72{\cdot}0°/-56{\cdot}0°]_s$	$[0{\cdot}25/0{\cdot}25]_s$	0·89	5·95	$[59{\cdot}5°/-56{\cdot}2°]_s$	$[0{\cdot}1107/0{\cdot}3893]_s$	0·86
1·0	$[45°/-45°]_s$	$[0{\cdot}09660/0{\cdot}40340]_s$	2·76	$[45°/-46{\cdot}0°]_s$	$[0{\cdot}25/0{\cdot}25]_s$	3·21	16·30	$[45°/-46{\cdot}0°]_s$	$[0{\cdot}1121/0{\cdot}3879]_s$	2·78
1·2	$[41{\cdot}2°/-41{\cdot}5°]_s$	$[0{\cdot}09649/0{\cdot}40351]_s$	3·64	$[40{\cdot}0°/-41{\cdot}0°]_s$	$[0{\cdot}25/0{\cdot}25]_s$	4·23	16·21	$[40{\cdot}9°/-41{\cdot}0°]_s$	$[0{\cdot}1126/0{\cdot}3874]_s$	3·69

thicknesses of layer groups. The BCONF routine can minimize a function of n variables subject to side constraints using a quasi-Newton method and finite-difference gradient. In solving the optimal design problems, the present method has used ten starting points to find the global optimal solution with probability 0·998 while the BCONF routine has used 28 starting points in obtaining the results listed in Table 1. Furthermore, the present method has no convergence problem while the successful use of BCONF routine greatly depends on the choice of the starting point. For the BCONF routine, divergence of solution may occur if the starting point is close to the bounds of the constraints. Therefore, this proves that the present approach is comparatively efficient and can yield plates with greater stiffness. The present method is then used to study the optimal design parameters of simply supported or clamped laminated composite plates. The optimal layer group parameters (fiber angles and normalized thicknesses) of the simply supported plates with various aspect ratios ($b/a=0{\cdot}5$, 1·0, 1·2), numbers of layer groups ($NL=4$, 6, 8) and length-to-thickness ratios ($a/h=5$, 100) are tabulated in Tables 2 and 3 while those for the plates with fixed edges are in Tables 4 and 5. It is noted that as shown in Tables 2 and 3 for the simply supported plates the plate thickness is controlled by one layer group whose fiber angle normally falls in the range from approximately $-61°$ to $-40°$. For example, the layer group of fiber angle $-56{\cdot}0°$ contributes 82% of plate thickness for the four-layered plate with $b/a=0{\cdot}5$ and $a/h=5$ in Table 2. On the other

Table 2. Optimal solutions of simply supported symmetrically laminated composite plates subjected to center point load ($a/h=5$, $\bar{W}_c=W_c[E_o bh^3/(pa^3)]\times 10^3$)

Aspect ratio (b/a)	Optimal solution	Number of layer groups (NL)		
		4	6	8
0·5	Fiber angle	$[54{\cdot}3°/-56{\cdot}0°]_s$	$[86{\cdot}1°/3{\cdot}3°/-54{\cdot}8°]_s$	$[86{\cdot}1°/3{\cdot}3°/68{\cdot}8°/-54{\cdot}3°]_s$
	Layer group thickness h_i/h	$[0{\cdot}08806/0{\cdot}41194]_s$	$[0{\cdot}01540/0{\cdot}07210/0{\cdot}41250]_s$	$[0{\cdot}00556/0{\cdot}01598/0{\cdot}06748/0{\cdot}41098]_s$
	Deflection $\bar{W}_c$	2·9758	2·5426	2·5424
1·0	Fiber angle	$[45°/-45{\cdot}0°]_s$	$[79{\cdot}6°/38{\cdot}9°/-44{\cdot}1°]_s$	$[81{\cdot}3°/27{\cdot}8°/45{\cdot}3°/-44{\cdot}1]_s$
	Layer group thickness h_i/h	$[0{\cdot}09143/0{\cdot}40857]_s$	$[0{\cdot}01465/0{\cdot}07977/0{\cdot}40558]_s$	$[0{\cdot}01309/0{\cdot}02601/0{\cdot}05833/0{\cdot}40257]_s$
	Deflection $\bar{W}_c$	6·35447	6·3159	6·2953
1·2	Fiber angle	$[41{\cdot}6°/-42{\cdot}0°]_s$	$[-41{\cdot}3°/-42{\cdot}9°/-41{\cdot}9°]_s$	$[84{\cdot}5°/28{\cdot}9°/40{\cdot}40°/-41{\cdot}9°]_s$
	Layer group thickness h_i/h	$[0{\cdot}09156/0{\cdot}40844]_s$	$[0{\cdot}09211/0{\cdot}00007/0{\cdot}40782]_s$	$[0{\cdot}01184/0{\cdot}03601/0{\cdot}05052/0{\cdot}40146]_s$
	Deflection $\bar{W}_c$	7·9575	7·9440	7·8605

Table 3. Optimal solutions of simply supported symmetrically laminated composite plates subjected to center point load ($a/h=100$, $\bar{W}_c=W_c[E_o bh^3/(pa^3)]\times 10^3$)

Aspect ratio (b/a)	Optimal solution	Number of layer groups (NL)		
		4	6	8
0·5	Fiber angle	$[59{\cdot}9°/-60{\cdot}3°]_s$	$[60{\cdot}0°/-60{\cdot}2°/66{\cdot}1°]_s$	$[-57{\cdot}0°/60{\cdot}0°/65{\cdot}7°/-60{\cdot}4°]_s$
	Layer group thickness h_i/h	$[0{\cdot}10249/0{\cdot}39751]_s$	$[0{\cdot}10204/0{\cdot}39776/0{\cdot}00020]_s$	$[0{\cdot}00060/0{\cdot}10280/0{\cdot}00040/0{\cdot}39620]_s$
	Deflection $\bar{W}_c$	0·2710	0·2709	0·2709
1·0	Fiber angle	$[45°{\cdot}0/-45{\cdot}0°]_s$	$[-45{\cdot}0°/45{\cdot}0°/-45{\cdot}0°]_s$	$[-45{\cdot}2°/-45{\cdot}2°/-45{\cdot}2°/45{\cdot}0°]_s$
	Layer group thickness h_i/h	$[0{\cdot}10368/0{\cdot}39632]_s$	$[0{\cdot}00050/0{\cdot}10397/0{\cdot}39553]_s$	$[0{\cdot}01309/0{\cdot}02601/0{\cdot}05833/0{\cdot}40257]_s$
	Deflection $\bar{W}_c$	1·4954	1·4954	1·4952
1·2	Fiber angle	$[41{\cdot}0°/-40{\cdot}9°]_s$	$[41{\cdot}0°/-40{\cdot}9°/-40{\cdot}9°]_s$	$[-41{\cdot}0°/41{\cdot}0°/-37{\cdot}6°/-41{\cdot}4°]_s$
	Layer group thickness h_i/h	$[0{\cdot}10392/0{\cdot}39608]_s$	$[0{\cdot}04850/0{\cdot}05530/0{\cdot}39620]_s$	$[0{\cdot}00003/0{\cdot}10388/0{\cdot}00001/0{\cdot}39608]_s$
	Deflection $\bar{W}_c$	2·0973	2·0971	2·0971

hand, aspect ratio b/a has some effects on the optimal fiber angles of the simply supported plates. For instance, the optimal fiber angles change from [54·3/−56·0]s for $b/a=0{\cdot}5$ to [41·6/−42·0]s for $b/a=1{\cdot}2$ as shown in Table 2. As for the clamped plates, the results in Tables 4 and 5 show that for thin plates ($a/h=100$) the optimal fiber angles tend to be a combination of 0° and 90°. However, for some thick plates ($a/h=5$) or for thin plates with $b/a=1$ and $NL=6$ and 8, the optimal fiber angles may deviate from 0° or 90°. For example, it is interesting to point out that for the clamped plate with $b/a=1{\cdot}0$, the optimal fiber angles change from [0/90]s for $a/h=100$ to [45/−45]s for $a/h=5$ as shown in Table 4. On the other hand, aspect ratio b/a may have some effects on the optimal fiber angles of the clamped plates. For instance, for $a/h=5$ the optimal fiber angles change from [50·4/−52·6]s for $b/a=0{\cdot}5$ to [40·7/−38·1]s for $b/a=1{\cdot}2$ as shown in Table 4. similar to the simply supported plates, plate thickness is generally controlled by one layer group. For example, the layer group of 90° possesses 93% of the thickness of the four-layered plate with $a/h=100$ and $b/a=0{\cdot}5$ in Table 5. It is also worth noting that the number of layer groups has insignificant effects on the total plate thickness for both the simply supported and clamped laminated composite plates which have been optimally designed. In general, the use of only six layer groups can yield the approximately global maximum stiffness for the plates. Hence, in view of the fact that the use of fewer number of layer groups in design can greatly reduce the time for manufacturing composite laminates,

Table 4. Optimal solutions of clamped symmetrically laminated composite plates subjected to center point load ($a/h=5$, $\bar{W}_c=W_c[E_o bh^3/(pa^3)]\times 10^3$)

Aspect ratio (b/a)	Optimal solution	Number of layer groups (NL)		
		4	6	8
0·5	Fiber angle	$[50{\cdot}4°/-52{\cdot}6°]_s$	$[-4{\cdot}1°/-71{\cdot}3°/52{\cdot}5°]_s$	$[5{\cdot}4°/74{\cdot}5°/-46{\cdot}5°/-55{\cdot}4°]_s$
	Layer group thickness h_i/h	$[0{\cdot}08676/0{\cdot}41324]_s$	$[0{\cdot}01865/0{\cdot}06280/0{\cdot}41855]_s$	$[0{\cdot}021490/0{\cdot}05745$ $0{\cdot}00002/0{\cdot}42104]_s$
	Deflection $\bar{W}_c$	2·5162	2·4438	2·4327
1·0	Fiber angle	$[45{\cdot}0°/-45{\cdot}0°]_s$	$[84{\cdot}1°/22{\cdot}2°/-44{\cdot}4°]_s$	$[86{\cdot}2°/7{\cdot}6°/47{\cdot}9°-42{\cdot}9°]_s$
	Layer group thickness h_i/h	$[0{\cdot}08596/0{\cdot}41404]_s$	$[0{\cdot}02265/0{\cdot}06050/0{\cdot}41585]_s$	$[0{\cdot}01309/0{\cdot}02601/$ $0{\cdot}05833/0{\cdot}40257]_s$
	Deflection $\bar{W}_c$	5·8337	5·6573	5·6154
1·2	Fiber angle	$[40{\cdot}7°/-38{\cdot}1°]_s$	$[96{\cdot}3°/-20{\cdot}4°/38{\cdot}3°]_s$	$[94{\cdot}4°/-7{\cdot}85°/-44{\cdot}3°/37{\cdot}1°]_s$
	Layer group thickness h_i/h	$[0{\cdot}08486/0{\cdot}41514]_s$	$[0{\cdot}02255/0{\cdot}06593/0{\cdot}41152]_s$	$[0{\cdot}02089/0{\cdot}04075/$ $0{\cdot}03364/0{\cdot}40472]_s$
	Deflection $\bar{W}_c$	7·1491	6·9241	6·8802

Table 5. Optimal solutions of clamped symmetrically laminated composite plates subjected to center point load ($a/h=100$, $\bar{W}_c=W_c[E_o bh^3/(pa^3)]\times 10^3$)

Aspect ratio (a/b)	Optimal solution	Number of layer groups (NL)		
		4	6	8
0·5	Fiber angle	$[0°/90°]_s$	$[90°/0°/90°]_s$	$[0°/90°/0°/90°]_s$
	Layer group thickness h_i/h	$[0{\cdot}03519/0{\cdot}46481]_s$	$[0{\cdot}00450/0{\cdot}03590/0{\cdot}45960]_s$	$[0{\cdot}03515/0{\cdot}04072/$ $0{\cdot}00001/0{\cdot}42412]_s$
	Deflection $\bar{W}_c$	0·1170	0·1170	0·1168
1·0	Fiber angle	$[0°/90°]_s$	$[4{\cdot}9°/85{\cdot}7°/-7{\cdot}3°]_s$	$[3{\cdot}0°/81{\cdot}3°/8{\cdot}0°/-79{\cdot}1°]_s$
	Layer group thickness h_i/h	$[0{\cdot}08521/0{\cdot}41479]_s$	$[0{\cdot}02650/0{\cdot}01043/0{\cdot}45960]_s$	$[0{\cdot}03998/0{\cdot}03607/$ $0{\cdot}05806/0{\cdot}36589]_s$
	Deflection $\bar{W}_c$	0·7747	0·7724	0·7720
1·2	Fiber angle	$[0°/90°]_s$	$[0°/90°/0°]_s$	$[0°/90°/90°/0°]_s$
	Layer group thickness h_i/h	$[0{\cdot}08521/0{\cdot}41479]_s$	$[0{\cdot}00170/0{\cdot}04510/0{\cdot}43820]_s$	$[0{\cdot}00172/0{\cdot}04055/$ $0{\cdot}05050/0{\cdot}43721]_s$
	Deflection $\bar{W}_c$	0·9771	0·9764	0·9758

the optimal number of layer groups obtained herein demonstrates one of the merits of the present design method.

CONCLUSION

Optimal lamination arrangements of laminated composite plates designed for maximum stiffness subject to side constraints were investigated via a constrained multi-start global minimization approach. Results for symmetrically laminated multi-layer plates of various aspect ratios, different numbers of layer groups and boundary conditions subject to center point load were obtained. The effects of aspect ratio, length-to-thickness ratio and number of layer groups on the optimal lamination arrangements were studied. It has been shown that the present constrained optimization technique can yield the global optimal design of laminated composite plates without considering many design variables. The constrained multi-start global minimization algorithm is of promise for further applications to the optimal design of more complex laminated composite structures.

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