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LOCATION CHOICE AND PATENT LICENSING*

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This paper studies the patent licensing decision when firms can endogenously choose their locations. If the insider patentee is the location leader, the royalty is not necessarily the best method of licensing. No licensing can be the best method for an insider patentee with a sufficiently high degree of innovation, but fixed-fee is always the worst. However, if the non-innovating firm is the location leader, the royalty licensing is always the best method; moreover, the fixed-fee licensing (no licensing) is the second best method if the degree of innovation is relatively small (sufficiently large).

1 Introduction

Firms are the major developers of modern technologies. Patent licensing is a strategic behavior for firms to gain competitiveness as well as profits. There has been much existing literature on strategic patent licensing. However, little attention has been paid to the relation between the firm's location and patent licensing. Not only the firm's patent licensing method choice may depend on its location, but also the licensing method may change its optimal location. This paper will discuss the best licensing method when firms can choose their locations.

The literature about the nature of licensing is clarified by the patentee happens to be an outsider or an insider. In the standard models, if the patentee is an outsider, it can be said that fixed-fee licensing is better than royalty licensing, and the reverse happens when the patentee is an outsider. The landmark literature of an outsider patentee with a degree of innovation at least contains: Kamien and Tauman (1984) compare the licensing methods

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of fixed fee and royalties under perfect competition. Under perfect competition the fixed fee is superior to royalties for an outsider patentee. However, the optimal number to license is affected by the degree of cost saving and the number of firms in the product market. Kamien and Tauman (1986) build up a homogenous duopoly model, in order to compare the licensing methods of fixed fee and royalties. They find that the fixed fee dominates royalties for an outsider patentee. By extending the set-ups of Kamien and Tauman (1986) and Katz and Shapiro (1986), Kamien *et al.* (1992) use a general-form demand function to compare three methods of licensing (auction, fixed fee and royalties) in a homogeneous oligopoly model. Muto (1993) further extends Kamien and Tauman (1986) to a heterogeneous duopoly and finds that instead royalties will dominate the fixed fee for the outside patentee in small innovations.

The landmark literature of an insider patentee with a degree of innovation at least contains: Wang (1998) and Wang (2002) establish a homogeneous duopoly and a heterogeneous duopoly, respectively. Wang (1998) finds that royalties dominates the fixed fee for an insider patentee under a homogeneous duopoly. Wang (2002) finds that the fixed fee dominates royalties for an insider patentee if the product differentiation is sufficiently large. Poddar and Sinha (2010) take into account the cost asymmetry among firms and build a bridge to integrate the optimal licensing schemes for insider and outsider patentees.[1] After incorporating trade and transportation costs, Mukherjee (2007) analyzed the optimal licensing scheme in an open economy. Fauli-Oller and Sandonis (2002), Mukherjee (2005) and Sinha (2010) discuss whether or not licensing can promote the social welfare. Chang *et al.* (2009) combine technology licensing and environmental issues, in order to compare the effects of royalty and fixed fee on consumer, producer and social surpluses.

There is also a line of literature on the effects of location or market area on strategic competition. Price discrimination is one example: without spatial considerations, the traditional literature concludes that, in two linear markets, total quantity stays the same under price discrimination and uniform pricing, while price discrimination always reduces social welfare. However, this conclusion does not hold in Greenhut and Ohta (1972) and Holahan (1975), wherein the market area of a firm is endogenously determined, in those two cases; the social welfare is strictly higher under price discrimination. Ohta (1988) finds that, under different set-ups of demand and cost functions, spatial price discrimination is more likely to bring about higher social welfare than uniform pricing. Hwang and Mai (1990) find that with endogenous location choice, price discrimination generates strictly lower total output than uniform pricing; however, price discrimination does not necessarily generate lower social welfare. To sum up, the above literature

[1]Refer to this paper for more literature review on patent licensing.

shows that location is a crucial factor in the strategic behavior of firms and the corresponding levels of social welfare. The distance between two firms is a crucial factor in their price competition. d'Aspremont *et al.* (1979) have proved that if firms are too close to each other, then they will try to reduce the price to expel the opponent from the market and thus enjoy monopoly profit. Therefore, an endogenous location choice set-up is expected to affect the licensing decision of an insider patentee, since it may either accommodate or expel its competitor.

Poddar and Sinha (2004) pioneer in linking the spatial model to patent licensing. However, in their model, the locations of a duopoly are exogenously given, i.e. the two firms, located at the two extreme points of a linear market, engage in price competition. They find that an outside patentee will choose royalty for both drastic and non-drastic innovations, whereas an insider patentee will choose not to license (royalty) when the innovation is drastic (non-drastic), and fixed fee is always behind. In general, however, firms can endogenously choose their locations. Therefore, this paper relaxes the assumption of fixed locations made by Poddar and Sinha (2004), allowing the firms to choose their locations before engaging in price competition. We find that neither royalty is the best nor fixed fee is the worst for an insider patentee. Matsumura *et al.* (2010) also discuss the optimal location under licensing. However, they assume the transportation cost is a quadratic function of the distance and hence do not have the case of price cutting. The Nash equilibrium locations are the two extreme points. Compared with Matsumura *et al.* (2010), this paper will further take into account the price-cutting competition, focus on the optimal location and how the location choice affects the innovating firm's licensing decision.

The major propositions obtained in this paper are as follows: when the locations are exogenously determined, the royalty is the best method of licensing and the equilibrium royalty rate is not affected by the location choice. If the innovating firm can only apply the licensing method of fixed fee, then whether or not to license depends on the location advantage as well as the cost effect. The higher the location advantage of innovating firm or the degree of innovation is, the less likely the licensing will be. When the locations are endogenously determined, the location leader will do his best to expand his market share. If the location leader is the insider patentee, the royalty licensing is not necessarily always the patentee's best method: when the degree of innovation is sufficiently large, no licensing is the best strategy for the patentee. The fixed fee is always the worst licensing method for an insider patentee as the location leader. However, if the non-innovating firm chooses its location first, since the non-innovating firm has the location advantage, the royalty is always the best licensing method; the fixed-fee licensing (no licensing) is the second best method if the degree of innovation is relatively small (sufficiently large).

This paper is organized as follows. Following this introduction, Section 2 is the basic model and no licensing location equilibrium, in which a duopoly is located in a linear market. The patentee is assumed to be an insider in the industry. Section 3 discusses and compares the optimal location decision under no licensing and fixed-fee licensing. Section 4 incorporates the discussion on the optimal locations under the royalty licensing and compares the three licensing methods under endogenously determined locations. Section 5 concludes this paper.

2 The Basic Model without Licensing

Two firms, A and B, produce a homogenous good in a linear market depicted by an interval of [0, 1]. To simplify the analysis, we assume that firm A is on the left of firm B. Consumers are uniformly distributed in this linear market. Consumer x is located at location x in the interval [0, 1]. Each consumer buys exactly one unit of the product from firm A or firm B.

The net utility for consumer x to purchase one unit of product from firm i located at x_i is

$$u_i(x) = v - p_i - t|x - x_i| \tag{1}$$

where v is the reserved utility of consuming this one unit of product, t is the unit transportation cost, p_i is the price charged by firm i, and i = A,B. Therefore, the marginal consumer who feels indifferent about purchasing from either firm A or firm B is located at

$$\hat{x} = \frac{p_B - p_A + t(x_B + x_A)}{2t} \tag{2}$$

Figure 1 depicts this linear market and the location of the marginal consumer.

As a result, the quantities (q) of firms A and B are, respectively,

$$q_A = \hat{x} \qquad \text{and} \qquad q_B = 1 - \hat{x} \tag{3}$$

The profit functions of firms A and B can be expressed as follows:

$$\pi_A = (p_A - c_A)q_A \qquad \text{and} \qquad \pi_B = (p_B - c_B)q_B \tag{4}$$

where $c_i > 0$, i = A, B, are the marginal costs for firms A and B, respectively. The two firms engage in price competition. Assume that the second-order and

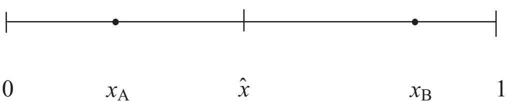

Fig. 1. The Linear City with Firms A and B

stability conditions all hold. The equilibrium prices of these two firms are, respectively,

$$p_A = \frac{2t + tx_A + tx_B + 2c_A + c_B}{3} \quad \text{and} \quad p_B = \frac{4t - tx_A - tx_B + 2c_B + c_A}{3} \tag{5}$$

Substituting the optimal prices in equation (5) into the sales quantities in equation (3), we obtain the optimal quantities of these two firms:

$$q_A = \frac{2t + tx_A + tx_B + c_B - c_A}{6t} \quad \text{and} \quad q_B = \frac{4t - tx_A - tx_B - c_B + c_A}{6t} \tag{6}$$

and the equilibrium profits are

$$\pi_A = \frac{(2t + tx_A + tx_B + c_B - c_A)^2}{18t} \quad \text{and} \quad \pi_B = \frac{(4t - tx_A - tx_B + c_A - c_B)^2}{18t} \tag{7}$$

The model is a three-stage game. In stage 1, the patentee decides whether or not to license to the other firm, as well as how high to set the licensing fee. In stage 2, the two firms sequentially choose their locations. In stage 3, these two firms engage in price competition in order to maximize their own profits. The concept of subgame perfect Nash equilibrium is applied to solve this game, and backward induction will be used.

Suppose that firm A is the innovating firm, and firm B is not. These two firms initially have the same marginal costs before innovation, i.e. $c_A = c_B = c$. The cost reduction after innovation (the degree of innovation) for firm A is $\varepsilon > 0$ with $c > \varepsilon$. As a result, after innovation, the marginal costs of these two firms become $c_A = c - \varepsilon$ and $c_B = c$, respectively. Without losing generality, the unit transportation cost is normalized to be one. Substituting the marginal cost into equation (6), we know that $q_B > 0$ implies $0 < \varepsilon < 4 - x_A - x_B$, in which case the innovation is non-drastic.[2]

2.1 No Licensing

With non-drastic innovation and no licensing, the marginal costs of firms A and B are $c_A = c - \varepsilon$ and $c_B = c$, respectively. Using equations (6) and (7), the outputs and profits of firms A and B can be respectively expressed as (superscript N denotes no licensing):

$$q_A^N = \frac{2 + x_A + x_B + \varepsilon}{6} \qquad q_B^N = \frac{4 - x_A - x_B - \varepsilon}{6} \tag{8}$$

$$\pi_A^N = \frac{(2 + x_A + x_B + \varepsilon)^2}{18} \qquad \pi_B^N = \frac{(4 - x_A - x_B - \varepsilon)^2}{18} \tag{9}$$

[2]In order to focus on the market competition analysis, we will only consider the case of non-drastic innovation.

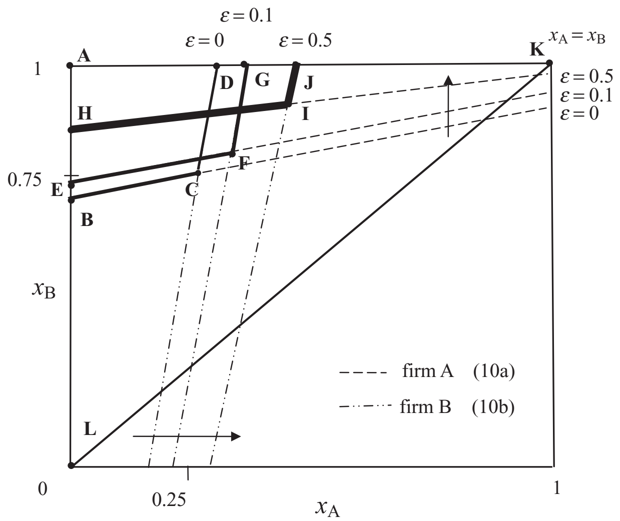

FIG. 2. The Relation between Location and Cost Difference under a Duopoly

A duopoly exists in this linear city only when both firms are not willing or able to monopolize the market via price cutting. This implies that the monopoly profit is no more than a duopolist profit under no licensing for each firm. In this case, both firms have a positive output, requiring the condition $|p_A - p_B| \leq x_B - x_A$ to hold. This condition can be obtained by following d'Aspremont *et al*. (1979). The detailed proof of the conditions for a duopoly to exist without licensing is listed in Appendix A.

The following two conditions guarantee that without licensing, the two firms are not able to monopolize the market via price cutting.

$$x_A^2 + x_B^2 + 2x_A x_B - 8x_A + 28x_B - 20 \geq \varepsilon(8 - 2x_A - 2x_B - \varepsilon) \quad (10a)$$

$$x_A^2 + x_B^2 + 2x_A x_B - 32x_A + 4x_B + 4 \geq -\varepsilon(4 + 2x_A + 2x_B + \varepsilon) \quad (10b)$$

Equations (10a) and (10b) are depicted in the (x_A, x_B) space in Fig. 2, in order to show the relation between the cost difference (ε) and locations.

Since we assume that firm A is on the left of firm B, the condition $x_A < x_B$ will always hold, and ALK is the only feasible area. When $\varepsilon = 0$, the condition preventing A from being a monopoly via price cutting is equation (10a), denoted by the BC line. The area below the BC line is where firm A, without licensing, has an incentive to monopolize the market via price cutting. Similarly, the condition preventing B from being a monopoly via price cutting is equation (10b), denoted by the DC line. The area on the right of the DC line is where firm B has an incentive to monopolize the market via price cutting. When the degree of innovation is zero, these two lines intersect at (0.25, 0.75), making the duopoly locations of these two firms fall in the ABCD area. An increase in the degree of innovation shifts lines (10a) and (10b) upward and rightward, respectively. For example, if ε increases from 0 to 0.1, line BC shifts to EF and line DC shifts to GF, making the duopoly location regime area AEFG. If ε further increases to 0.5, the duopoly location regime then becomes area AHIJ. The feasible duopoly location regime will be affected by the undercutting conditions and the degree of innovation.[3]

Next, the location choice of firm A without licensing in stage 2 will be analyzed. In stage 2, there are two possible sequences of moves where firm A or firm B is the location mover. When firm A is the location leader, firm B then chooses its own optimal location given firm A's location as well as the condition (10a) for firm A not to engage in price cutting. As a result, firm B's profit maximization problem becomes

$$\begin{aligned} &\underset{\{x_B\}}{\text{Max}}\, \pi_B^N(x_A, x_B) = \frac{(4 - x_A - x_B - \varepsilon)^2}{18} \\ &\text{s.t.}\quad x_A^2 + x_B^2 + 2x_A x_B - 8x_A + 28x_B - 20 \geq \varepsilon(8 - 2x_A - 2x_B - \varepsilon) \end{aligned} \tag{11}$$

Equation (11) implies a corner solution for firm B to choose the minimum value of x_B satisfying the constraint, hence making the constraint in equation (11) binding. Therefore, firm B's best response $x_B = x_B(x_A)$ must be on the equality of constraint, with comparative statics of $\partial x_B/\partial x_A > 0$.[4] Substituting firm B's location response into the profit function of firm A's profit function while taking into account the condition (10b) for firm B not to engage in price cutting, we obtain firm A's profit maximization problem as

$$\begin{aligned} &\underset{\{x_A\}}{\text{Max}}\, \pi_A^N(x_A, x_B(x_A)) = \frac{[2 + x_A + x_B(x_A) + \varepsilon]^2}{18} \\ &\text{s.t.}\quad x_A^2 + x_B^2 + 2x_A x_B - 32x_A + 4x_B + 4 \geq -\varepsilon(4 + 2x_A + 2x_B + \varepsilon) \end{aligned} \tag{12}$$

Equation (12) implies that, when firm A is the location leader without licensing, the equilibrium locations are $G^N(x_A, x_B) = \left(15 - \varepsilon - 6\sqrt{6 - \varepsilon}, 1\right)$ in Fig. 3.[5] At this equilibrium location firm A's profit is $\pi_A^{NA} = 2\left(-3 + \sqrt{6 - \varepsilon}\right)^2$, where the superscript NA indicates the case with firm A as the location leader under no licensing.[6]

[3]The case that d'Aspremont *et al.* (1979) discuss corresponds to the area ABCD when the degree of innovation is zero. They also assume that the locations of firms are at symmetric points and restricted in [0, 0.25] for firm A and in [0.75, 1] for firm B. This implies that, in order to prevent monopolization via price cutting, firms will extend the distance in between their locations to reduce the severity of price competition.

[4]A total differentiation of equation (10a) generates $\partial x_B/\partial x_A = (4 - x_A - x_B - \varepsilon)/(x_A + x_B + 14 + \varepsilon) > 0$.

[5]Substituting $x_B = 1$ into equation (10b), we obtain $x_A = 15 - \varepsilon - 6\sqrt{6 - \varepsilon}$ and $\partial x_A/\partial \varepsilon > 0$.

[6]The superscript *ij*, hereafter, indicates the case with firm *j* as the location leader under licensing strategy *i*, where *i* = no licensing (N), fixed-fee licensing (F), royalty licensing (R) and *j* = A, B.

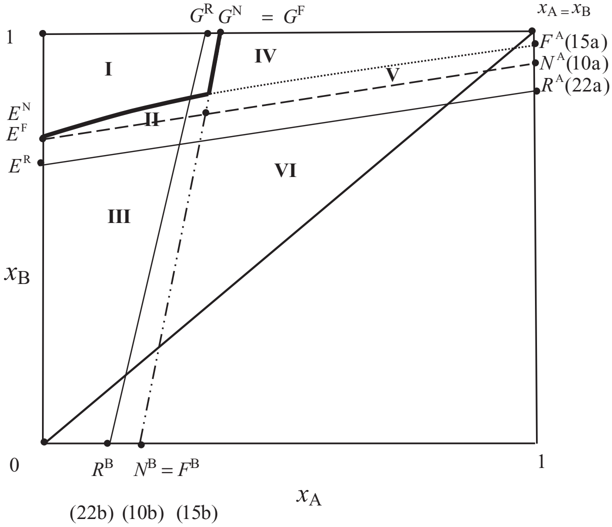

FIG. 3. The Conditions of No Monopolization via Price Cutting under Licensing and No Licensing

When firm B is the location leader, firm A chooses its own optimal location given firm B's location as well as the condition (10b) for firm B not to engage in price cutting. Therefore, firm A's optimal location is $x_A = x_A(x_B)$, with $\partial x_A/\partial x_B > 0$.[7] Firm B maximizes its own profit subject to the condition that firm A does not engage in price cutting. As a result, when firm B is the location leader and firm A does not license, the equilibrium locations are $E^N(x_A, x_B) = (0, -14 - \varepsilon + 6\sqrt{6+\varepsilon})$ in Fig. 3.[8] Meanwhile, firm A's equilibrium profit is $\pi_A^{NB} = 2(-2 + \sqrt{6+\varepsilon})^2$. The above results are summarized in Proposition 1.

Proposition 1: Assume that there is no licensing. When firm A is the location leader, the equilibrium locations are $G^N(x_A, x_B) = (15 - \varepsilon - 6\sqrt{6-\varepsilon}, 1)$ in Fig. 3. When firm B is the location leader, the equilibrium locations are $E^N(x_A, x_B) = (0, -14 - \varepsilon + 6\sqrt{6+\varepsilon})$ in Fig. 3. No matter which one is the location leader, the optimal locations move rightward (close to market 1 in Fig. 1) as the degree of innovation increases.

3 Licensing with the Fixed Fee

After firm A licenses to firm B with fixed-fee strategy, the marginal costs become $c_A = c - \varepsilon$ and $c_B = c - \varepsilon$, respectively. Consequently, their profits are

[7]A total differentiation of equation (10b) generates $\partial x_A/\partial x_B = (16 - x_A - x_B - \varepsilon)/(x_A + x_B + 2 + \varepsilon) > 0$.

[8]Substituting $x_A = 0$ into equation (10a), we obtain $x_B = -14 - \varepsilon + 6\sqrt{6+\varepsilon}$ and $\partial x_B/\partial \varepsilon > 0$.

$$\pi_A = (2 + x_A + x_B)^2/18 \quad \text{and} \quad \pi_B = (4 - x_A - x_B)^2/18$$

The fixed licensing fee is denoted by *F*. In a take-or-leave-it bargaining process, the licensing fee, *F*, will be the difference in firm B's profits before and after licensing (Wang, 1998; Aoki and Hu, 1999, 2003). As a result, the profits of firms A and B after licensing will be (superscript F denotes the regime of fixed-fee licensing)

$$\pi_A^F = \frac{(2 + x_A + x_B)^2}{18} + \frac{(4 - x_A - x_B)^2}{18} - \frac{(4 - x_A - x_B - \varepsilon)^2}{18} \tag{13}$$

$$\pi_B^F = \frac{(4 - x_A - x_B - \varepsilon)^2}{18} \tag{14}$$

In any licensing agreement, these two firms should have no incentive to monopolize the market via price-cutting competition. According to Appendix B, the following two inequalities must hold to guarantee that the market is duopolistic:

$$x_A^2 + x_B^2 + 2x_A x_B - 8x_A + 28x_B - 20 \geq \varepsilon(4 + 2x_A + 2x_B + \varepsilon) \tag{15a}$$

$$x_A^2 + x_B^2 + 2x_A x_B - 32x_A + 4x_B + 4 \geq -\varepsilon(4 + 2x_A + 2x_B + \varepsilon) \tag{15b}$$

As shown in Appendix B, an increase in the degree of innovation shifts line (15a) up, line (15b) rightward, and the duopolistic location regime to the upper right.

Figure 3 compares the conditions of not licensing (equations (10a) and (10b)) and fixed-fee licensing (equations (15a) and (15b)). It is shown that the conditions of licensing are above the conditions of not licensing, indicating that it is easier for firm A to engage in price-cutting competition under fixed-fee licensing than under not licensing. However, firm B faces the same conditions under both regimes, implying that firm B also enjoys the same profits under both. Therefore, when these two firms are located at regime I, the conditions for duopoly are satisfied under both fixed-fee licensing and not licensing conditions, preventing monopolization from occurring.[9]

Before the analysis of the optimal location choices under the fixed fee, let us first discuss how location choices affect firm A's decision to license. Moreover, here we will focus on the licensing decision of firm A under a duopoly,

[9]In regime II, the monopolist profit via price cutting is higher than the duopolist profit under fixed-fee licensing for firm A; however, the duopolist profit without licensing is higher than the monopoly profit via price cutting for firm A. Therefore, in regime II, firm A will choose not to license and not to monopolize via price cutting. Similarly, whether or not firm A licenses, firm B's profit is higher under a duopoly than under monopolization via price cutting. Consequently, in regime II, both firms coexist in the market without licensing. This paper focuses on the situations where both firms will not engage in price cutting under all three licensing methods, which is in regime I.

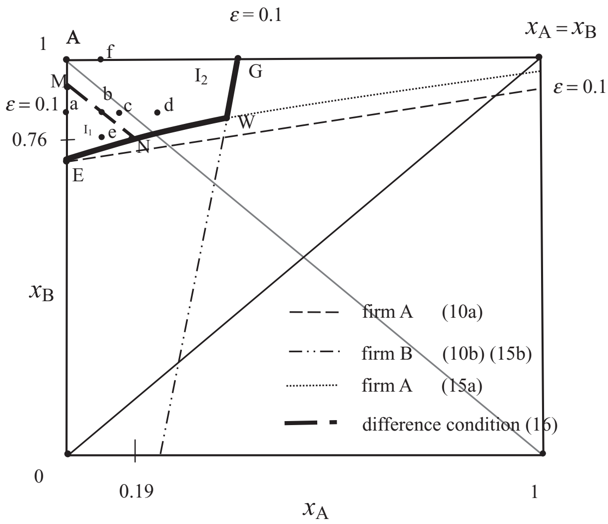

FIG. 4. The Licensing Decision under a Duopoly ($\varepsilon = 0.1$)

represented by regime I of Fig. 3. According to equations (9) and (13), the profit difference for firm A without licensing and with fixed-fee licensing under a duopoly is

$$\pi_A^N - \pi_A^F = 2\varepsilon[\varepsilon - 2 + 2(x_A + x_B)] \tag{16}$$

When $\varepsilon \geq (<)2 - 2(x_A + x_B)$, equation (16) is positive (negative), i.e. $\pi_A^N \geq (<)\pi_A^F$, and the firm will choose not to license (to license). As a result, the licensing decision crucially depends on the degree of innovation and locations of these two firms.

Based on the regime I duopoly in Fig. 3 and incorporating equation (16), we can determine the domain for firm A to license. Figure 4 depicts the condition for licensing under a duopoly (line MN) at $\varepsilon = 0.1$.

In Fig. 4, the feasible location for a duopoly to exist is still in regime I. Equation (16) as the licensing condition is labeled as the MN line, which severs regime I into subregimes I_1 and I_2. Equation (16) ($\varepsilon = 2–2(x_A + x_B)$) intersects with the duopoly condition (equation (15a)) at (0.19, 0.76). In subregime I_1, licensing dominates not licensing to firm A. In subregime I_2, not licensing dominates licensing to firm A.

The result substantially differs from the findings of the previous literature. If these two firms are initially located at $(x_A, x_B) = (0, 1)$, which is equivalent to the set-up of Poddar and Sinha (2004), with the exogenous location assumption, then under the fixed fee, firm A's best response is not to license. However, this paper allows the two firms to change their own locations, and hence firms can enhance their own location advantage via their location choice while affecting the licensing decision. On the other hand, the degree of innovation will also affect the licensing decision. In this paper, if firm B moves from 1 to another location (such as 0.76) and firm A still stays at location 0, then firm A will choose to license to firm B.

The economic meanings of this result can be explained by decomposing the right-hand side of equation (16) into two effects:

$$\pi_A^N - \pi_A^F = \underbrace{2\varepsilon^2}_{\text{Cost difference effect}} + \underbrace{4\varepsilon[-1+(x_A+x_B)]}_{\text{Location advantage effect}} \tag{16a}$$

The first item on the right side is $2\varepsilon^2$, representing the effect from the cost differences before and after the licensing. Moreover, this cost difference effect strictly increases with the degree of innovation, and the insider patentee tends not to license as the degree of innovation increases. The second item on the right side is $4\varepsilon[-1 + (x_A + x_B)]$, representing the location advantage effect. The location advantage effect is positive when $x_A + x_B > 1$. That is, if the insider patentee initially has a higher market share (hinterland) than its opponent, it must not to license. In contrast, when $x_A + x_B < 1$, the insider patentee will be possible to license.

Whether or not the insider patentee chooses to license depends on the relative strength of the cost and location advantages. Figure 4 can help explain this. Let us first fix the location of firm B and gradually move the innovating firm A from 0 towards firm B in a sequence of a, b, c and d. When the innovation firm A is located at point a, the location advantage is negative, since it is below the negative 45° line, moreover, the location advantage effect is larger than the cost difference effect, making the innovating firm A license. At point b, the location advantage effect is still negative, and its magnitude equals that of the positive cost advantage effect, making the innovating firm A indifferent between licensing or not. However, as the innovating firm continually moves rightward, the location advantage becomes trivial. When the innovating firm A is at point c, although firm B still enjoys a relatively higher location advantage, its location advantage is dominated by the cost difference advantage, making the innovating firm not license. Finally, the innovating firm enjoys a location advantage at point d, making it not license. Similar illustrations can be made with points e, b and f.

Moreover, an increase in the degree of innovation will shift the MN line in Fig. 4 leftward, shrinking the area left of the MN line. As a result, a higher degree of innovation will make the area left of the MN line disappear, making not licensing the best response of the innovating firm A. When the degree of innovation increases up to 0.43, the intersection of equation (16a) of licensing and equation (13a) of not licensing under a duopoly is at (0, 0.78) in Fig. 4. That is, any degree of innovation greater than 0.43 will make not licensing better response for the innovating firm A. The above discussion leads to the following lemma.

Lemma 1: (i) If the insider patentee initially has a higher market share (hinterland) than its opponent, it must not to license. In contrast, when $x_A + x_B < 1$, the insider patentee will be possible to license. (ii) As the degree of innovation increases, the insider patentee tends not to license to the opponent firm.

Summarizing the two points in Lemma 1, we know that the cost advantage effect is always positive and increases with the degree of innovation, discouraging the innovating firm from licensing. The location advantage depends on the relative locations of these firms. The innovating firm tends to license when the location advantage is negative. The factor of location does indeed affect the licensing decision, leading to outcomes different from those in the previous literature with fixed locations.

We then apply the same analysis in Section 2.1 to discuss the optimal location of firm A under the fixed fee and compare the corresponding profit of firm A to that without licensing. Similarly, the cases for firm A or B to the location leader will be studied. To order to avoid duplication of similar analysis in Section 2.1, the detailed mathematical process will be omitted. When firm A is the location leader, given firm A's location, firm B takes into account the condition (15a) for firm A not to engage in price cutting and its own profit maximization in equation (14) in order to choose its own optimal location. The objective function in equation (14) implies that firm B will choose the minimum x_B satisfying the constraints, hence making its best response $x_B = x_B(x_A)$ take place on the constraint with comparative statics $\partial x_B/\partial x_A > 0$.[10] Substituting firm B's location best response function and no price-cutting condition (equation (15b)) into firm A's profit function (equation (13)), we can obtain the first-order condition for firm A's optimal location as

$$\frac{\partial \pi_A^F(x_A, x_B(x_A))}{\partial x_A} = \frac{2(2 + x_A + x_B - \varepsilon)}{14 + x_A + x_B - \varepsilon} \tag{17}$$

where the denominator is always positive and the condition $p_A - c > 0$ implies that the numerator is positive.

The first-order condition for firm A's optimal location is strictly positive, making the optimal locations be on the constraints, i.e. $G^F(x_A, x_B) = \left(15 - \varepsilon - 6\sqrt{6 - \varepsilon}, 1\right)$ in Fig. 3. At this equilibrium location firm A's profit is π_A^{FA}.[11]

Because equations (10b) and (15b) imply the same conditions, the optimal locations for firm A as the location leader will be the same under no

[10]Total differentiation of equation (15a) generates $\partial x_B/\partial x_A = (4 - x_A - x_B + \varepsilon)/(x_A + x_B + 14 - \varepsilon) > 0$.

[11]Equilibrium profits can be obtained by substituting the equilibrium locations into equation (11): $\pi_A^{FA} = 30 - 16\varepsilon/3 - 12\sqrt{6 - \varepsilon} + \varepsilon^2/9 + (4\varepsilon/3)\sqrt{6 - \varepsilon}$

licensing (with profit π_A^{NA}) and under the fixed-fee licensing (with profit π_A^{FA}). Lemma 1 implies that $\pi_A^{NA} > \pi_A^{FA}$ since point G is more advantageous to firm A. As a result, no licensing is the best strategy for firm A.

When firm B is the location leader, given firm B's location, firm A takes into account the condition (15b) for firm B not to engage in price cutting to determine its own optimal location, making its best response in location as $x_A = x_A(x_B)$; and we have $\partial x_A/\partial x_B > 0$. By maximizing firm B's profit subject to the condition (15a) for firm A not to engage in price cutting, we obtain the equilibrium locations with firm B as the location leader: $E^F(x_A, x_B) = \left(0, -14+\varepsilon+\sqrt{2(108-12\varepsilon+\varepsilon^2)}\right)$ in Fig. 3.[12] Firm A's equilibrium profit is $\pi_A^{FB} = 20 - \left[2\varepsilon + 4\sqrt{2(108-12\varepsilon+\varepsilon^2)}\right]/3$. It is then straightforward that if $\varepsilon \lesseqgtr 42-24\sqrt{3} \approx 0.43$, then $\pi_A^{FB} - \pi_A^{NB} \gtreqless 0$. The above discussion can be summarized in Propositions 2 and 3.

Proposition 2: Under the fixed-fee licensing, when firm A is the location leader, the equilibrium locations are $G^F(x_A, x_B) = \left(15-\varepsilon-6\sqrt{6-\varepsilon}, 1\right)$ in Fig. 3; when firm B is the location leader, the equilibrium locations are $E^F(x_A, x_B) = \left(0, -14+\varepsilon+\sqrt{2(108-12\varepsilon+\varepsilon^2)}\right)$ in Fig. 3.

Proposition 3: (i) When A is location leader, no licensing dominates the fixed-fee licensing for firm A. (ii) When firm B is the location leader, the fixed-fee licensing dominates no licensing for firm A if $\varepsilon < 0.43$; otherwise, no licensing is firm A's optimal licensing method if $\varepsilon > 0.43$.

Assuming that the two firms' locations are fixed at the two extreme points in Fig. 1, Poddar and Sinha (2004) obtain the conclusion that no licensing is always the best licensing method for the insider patentee. However, Lemma 1 in this paper points out that, when locations are given and firm A lacks the location advantage, the fixed fee is still likely to dominate no licensing. Moreover, in the case of location endogenously determined, when firm A is the location leader who enjoys the location advantage as well as the cost advantage, no licensing dominates the fixed-fee licensing for an insider patentee. If firm B is the location leader and hence firm A has no location advantage, when the degree of innovation is sufficiently small, the fixed-fee licensing which gives firm B a larger market share will dominate no licensing and firm A can compensate its own profit loss by the fixed-fee revenues.

[12]Substituting $x_A = 0$ into equation (15a) obtains $x_B = -14+\varepsilon+\sqrt{2(108-12\varepsilon+\varepsilon^2)}$.

4 Licensing with the Royalty

The superscript R denotes the cases under royalty. The royalty rate is r. After licensing the marginal cost of firm A is $c_A = c - \varepsilon$ and that of firm B is $c_B = c - \varepsilon + r$, with $0 < r \le \varepsilon$. The after licensing equilibrium output of firm B is $q_B = (4 - x_A - x_B - r)/6$, with $r \le (4 - x_A - x_B)$ to guarantee a positive output level for firm B. Under a non-drastic innovation and licensing with the royalty, the profits of the two firms are as follows:

$$\pi_A^R = \frac{(2 + x_A + x_B + r)^2}{18} + r\left(\frac{4 - x_A - x_B - r}{6}\right) \tag{18}$$

$$\pi_B^R = \frac{(4 - x_A - x_B - r)^2}{18} \tag{19}$$

Before solving the optimal locations for both firms, in a backward induction way we first discuss the three licensing methods (royalties, fixed fees and no licensing) given any location combinations. The innovating firm chooses the royalty rate, in order to maximize its own profit. The first-order condition ensues that the royalty rate is $r = (16 - x_A - x_B)/4$. However, here we only discuss the case of a non-drastic innovation in which the innovating firm cannot monopolize the whole market, and hence this royalty rate does not satisfy the conditions of $0 < r \le \varepsilon$ and $r \le (4 - x_A - x_B)$. As a result, the optimal royalty rate satisfying the above constraints is $r = \varepsilon$. Substituting $r = \varepsilon$ into equations (18) and (19), we obtain the profits for firms A and B, respectively,

$$\pi_A^R = \frac{(2 + x_A + x_B + \varepsilon)^2}{18} + \varepsilon\left(\frac{4 - x_A - x_B - \varepsilon}{6}\right) \tag{20}$$

$$\pi_B^R = \frac{(4 - x_A - x_B - \varepsilon)^2}{18} \tag{21}$$

Similar to the cases under no licensing and fixed fees, under royalties there are also constraints for price cutting. The constraint for firm A and B is, respectively,

$$x_A^2 + x_B^2 + 2x_A x_B - 8x_A + 28x_B - 20 \ge \varepsilon(-16 + x_A + x_B + 2\varepsilon) \tag{22a}$$

$$x_A^2 + x_B^2 + 2x_A x_B - 32x_A + 4x_B + 4 \ge -\varepsilon(-8 + 2x_A + 2x_B + \varepsilon) \tag{22b}$$

Equations (22a) and (22b) are shown on the (x_A, x_B) space in Fig. 3.[13]

The judgment of whether or not the innovating firm will license with royalty can be obtained by comparing the innovating firm's profits under no licensing and licensing with royalty or fixed fee. Equations (9) and (20) show

[13]The process to obtain equations (22a) and (22b) is similar to equations (10a) and (10b) or equations (15a) and (15b), as shown in Appendixes A and B, and therefore we will not repeat here.

that licensing with royalty is always superior to no licensing for the innovating firm, i.e. $\pi_A^R > \pi_A^N$. Equations (13) and (20) can be used for comparing the innovating firm's profits under licensing with royalty and fixed fee. When $\varepsilon < 8 + x_A + x_B$, $\pi_A^R > \pi_A^F$ and the reverse holds. After incorporating the non-drastic innovation constraint ($0 < \varepsilon < (4 - x_A - x_B)$), we obtain an unambiguous result, $\pi_A^R > \pi_A^F$; that is, the royalty is superior to the fixed cost for the innovating firm. Summarizing the above outcomes, we have lemma 2 here.

Lemma 2: When the locations are exogenous and the degree of innovation is non-drastic, the royalty licensing is the best method of licensing for the insider patentee.

When the locations are exogenous and the innovation is non-drastic such that the innovating firm cannot monopolize the whole market, the profit of licensing by royalty is higher than that by fixed fee. This is because in addition to licensing revenues, royalty enable the innovating firm to enjoy an advantage in marginal cost. That is, the licensing method of royalty has a cost effect found by Wang (1998, 2002). This finding is consistent with Poddar and Sinha (2004) that the royalty is a better licensing method, no matter whether the locations can be considered or not. However, after locations being endogenized, whether or not royalties stay as the best licensing method needs further formal analysis.

Similar to the analysis of cases under fixed fees, the two situations in which firm A or firm B plays as the location leader will be analyzed. Using the same approach, when firm A is the location leader, we can obtain the optimal location $G^R(x_A, x_B) = \left(15 - r - 2\sqrt{6(9-r)}, 1\right)$ in Fig. 3.[14] The corresponding equilibrium for firm A can be obtained by substituting equilibrium locations into equation (18): $\pi_A^{RA} = 30 - 10r/3 - (4 - r/3)\sqrt{6(9-r)}$. In stage one, firm A has to determine the royalty rate r. The first-order condition $\partial\pi_A^{RA}/\partial r > 0$ implies that firm A's profit increases with the royalty rate, making $r = \varepsilon$, which satisfies the constraints.

Since constraints (22b) and (10b) are different, firm A's optimal locations are different under royalties licensing and no licensing. Lemma 2 hence cannot apply to compare firm A's profits under royalties licensing and no licensing. The relative magnitude of firm A's profits under no licensing (π_A^{NA}) and royalties licensing (π_A^{RA}), $\pi_A^{NA} - \pi_A^{RA} = -4\left[3\sqrt{6-\varepsilon} - (\varepsilon/3)\right] + [4 - (\varepsilon/3)]\sqrt{6(9-\varepsilon)}$, depends on ε.

[14]Firm B's maximization problem is: given firm A's location to maximize equation (19), subject to equation (22a), to obtain firm B's best response $x_B = x_B(x_A)$ that take place on the constraint with comparative statics $\partial x_B/\partial x_A > 0$. Then, firm A's maximization problem is to maximize equation (18), subject to equation (22b), and find the first-order condition for firm A's optimal location is strictly positive, implying that firm A's optimal location is a corner solution (on the constraint).

TABLE 1
FIRM A IS THE LOCATION LEADER

Optimal location and profits for firm A	No licensing		Fixed-fee licensing		Royalty licensing	
	G^{N}	π_A^{NA}	G^{F}	π_A^{FA}	G^{R}	π_A^{RA}
$\varepsilon = 0.1$	(0.3261, 1)	0.6521	(0.3261, 1)	0.6437	(0.2849, 1)	0.6801
$\varepsilon = 0.43$	(0.4095, 1)	0.8190	(0.4095, 1)	0.7593	(0.2285, 1)	0.9114
$\varepsilon = 1$	(0.5836, 1)	1.1672	(0.5836, 1)	0.9264	(0.1436, 1)	1.2633
$\varepsilon = 1.435$	(0.7455, 1)	1.4920	(0.7455, 1)	1.0123	(0.0927, 1)	1.4920
$\varepsilon = 1.5$	(0.7721, 1)	1.5442	(0.7721, 1)	1.0368	(0.0836, 1)	1.5213
$\varepsilon = 1.99$	(0.9950, 1)	1.99	(0.9950, 1)	1.1100	(0.0393, 1)	1.7272
$\varepsilon = 2$	(1, 1)	2	(1, 1)	1.1111	(0.0385, 1)	1.7309

Table 1 contains a simple numerical simulation to illustrate how the change in ε affects the optimal locations and profits under three licensing methods. It further shows that $\pi_A^{RA} = \pi_A^{NA}$ at $\varepsilon = 1.435$. Combining the results under the fixed-fee licensing, we obtain that $\pi_A^{RA} > \pi_A^{NA} > \pi_A^{FA}$ if $\varepsilon < 1.435$ while $\pi_A^{NA} > \pi_A^{RA} > \pi_A^{FA}$ if $\varepsilon > 1.435$.

In order to investigate the effect of endogenous x on the profit, we decompose the optimal profit discrepancy into two parts:

$$
\begin{aligned}
&\left[\pi_A^{RA}\big|_{x_A=G^{RA}} - \pi_A^{NA}\big|_{x_A=G^{NA}}\right] \\
&= \left[\pi_A^{RA}\big|_{x_A=G^{RA}} - \pi_A^{NA}\big|_{x_A=G^{RA}}\right] + \left[\pi_A^{NA}\big|_{x_A=G^{RA}} - \pi_A^{NA}\big|_{x_A=G^{NA}}\right] \\
&= \frac{1}{3}\varepsilon\left[-6+\sqrt{6(9-\varepsilon)}\right] + \left[\frac{2}{3}\varepsilon - 4\sqrt{6(9-\varepsilon)} + 12\sqrt{6-\varepsilon}\right]
\end{aligned}
\tag{23}
$$

The first part on the right-hand side of equation (23) denotes the *licensing effect* for firm A. Given the same locations under royalty and no licensing, Lemma 2 implies that $\pi_A^{RA}\big|_{x_A=G^{RA}} > \pi_A^{NA}\big|_{x_A=G^{RA}}$, making royalties licensing dominates no licensing for firm A. The second part is the *location effect* for firm A, which is negative since $\pi_A^{NA}\big|_{x_A=G^{RA}} < \pi_A^{NA}\big|_{x_A=G^{NA}}$.[15] The sign of equation (23) depends on the relative magnitudes of these two effects. Moreover, it can be further shown that the location effect is more likely to be greater than the licensing effect as the degree of innovation increases.

When firm B is the location leader, by the same way, we obtain the equilibrium location $E^{R}(x_A, x_B) = \left(0, -14 + \frac{1}{2}\left[r + \sqrt{9r^2 - 120r + 864}\right]\right)$

[15]Substituting the optimal locations under royalties licensing into the constraint of non-drastic innovation $\varepsilon < 4 - x_A^{RA} - x_B^{RA}$, we obtain that $\varepsilon < 3$ must hold. Under the constraint $\varepsilon < 3$, the first part on the right-hand side of equation (23) must be positive. Similarly, substituting the optimal locations under no licensing into the constraint of non-drastic innovation $\varepsilon < 4 - x_A^{NA} - x_B^{NA}$, we have $\varepsilon < 2$ must hold, which implies that the second part on the right-hand side of equation (23) must be negative.

TABLE 2
FIRM B IS THE LOCATION LEADER

Optimal location and profits for firm A	*No licensing*		*Fixed-fee licensing*		*Royalty licensing*	
	E^{N}	π_A^{NB}	E^{F}	π_A^{FB}	E^{R}	π_A^{RB}
$\varepsilon = 0.1$	(0, 0.7189)	0.4415	(0, 0.7157)	0.4457	(0, 0.6453)	0.4729
$\varepsilon = 0.43$	(0, 0.7845)	0.5740	(0, 0.7844)	0.5741	(0, 0.4809)	0.6921
$\varepsilon = 1$	(0, 0.8745)	0.8340	(0, 0.9284)	0.7621	(0, 0.2204)	1.0394
$\varepsilon = 1.2$	(0, 0.8997)	0.9337	(0, 0.9871)	0.8174	(0, 0.1366)	1.1512
$\varepsilon = 1.5$	(0, 0.9317)	1.0911	(0, 1.0831)	0.8893	(0, 0.0189)	1.3082

in Fig. 3.[16] Meanwhile, firm A's profit at E^{R} is $\pi_A^{RB} = 20 - 2\left(r + \sqrt{9r^2 - 120r + 864}\right)/3$. In stage one, firm A has to determine the royalty rate r. The first-order condition $\partial\pi_A^{RB}/\partial r = 0$ ensues that the royalty rate is $r = (20 - \sqrt{58})/3 \approx 4.128$. However, here we only discuss the case of a non-drastic innovation in which the innovating firm cannot monopolize the whole market, and hence this royalty rate does not satisfy the conditions of $0 < r \le \varepsilon$ and $r \le (4 - x_A - x_B)$. As a result, the optimal royalty rate satisfying the above constraints is $r = \varepsilon$, implying that firm A's profit increases with the royalty rate, making $r = \varepsilon$, which satisfies the constraints. Firm A's profit difference under royalties licensing and no licensing is $\pi_A^{RB} - \pi_A^{NB} \underset{<}{\overset{>}{=}} 0$ if $\varepsilon \underset{>}{\overset{<}{=}} 80(42 - 24\sqrt{3})/49 \approx 2.67$. Similarly, Table 2 lists the numerical examples when firm B is the location leader. The value ε must satisfy the conditions for locations to be positive, and therefore $\pi_A^{RB} > \pi_A^{NB}$ must hold. Moreover, we can prove that $\pi_A^{RB} > \pi_A^{FB}$ also holds.[17]

Summarizing the analysis of equilibrium locations without licensing (in Section 2), under fixed-fee licensing (in Section 3) and under royalties licensing (in this Section), we come up with Propositions 4 and 5.

Proposition 4: Under the royalties licensing, the equilibrium locations are $G^{R}(x_A, x_B) = \left(15 - \varepsilon - 2\sqrt{6(9-\varepsilon)}, 1\right)$ in Fig. 3 when firm A is the location leader and $E^{R}(x_A, x_B) = \left(0, -14 + \frac{1}{2}\left(\varepsilon + \sqrt{9\varepsilon^2 - 120\varepsilon + 864}\right)\right)$ in Fig. 3 when firm B is the location leader.

[16]Firm A's maximization problem is: given firm B's location to maximize equation (18), subject to equation (22b), to obtain firm B's best response $x_A = x_A(x_B)$ that take place on the constraint with comparative statics $\partial x_A/\partial x_B > 0$. Then, Firm B's maximization problem is maximize equation (19), subject to equation (22a), to find the first-order condition for firm B's optimal location is strictly positive, implying that firm B's optimal location is a corner solution (on the constraint).

[17] $\pi_A^{RB} - \pi_A^{FB} = \frac{2}{3}\left[2\sqrt{2(108 - 12 + \varepsilon^2)} - \sqrt{9\varepsilon^2 - 120\varepsilon + 864}\right] \underset{<}{\overset{>}{=}} 0$, if $\varepsilon \underset{>}{\overset{<}{=}} 24$. It must be $\pi_A^{RB} > \pi_A^{FB}$, because $\varepsilon < 24$.

Proposition 5: When firms sequentially choose their locations, if the insider patentee is the location leader, then no licensing is the best method for a sufficiently large degree of innovation while the fixed-fee licensing is always the worst method for the insider patentee. If the non-innovating firm is the location leader, the royalty licensing is always the best method for the insider patentee; moreover, no licensing (the fixed fee) is the second best method when the degree of innovation is sufficiently large (small).

Proposition 5 comes up with a new result compared with the previous literature: when the locations are exogenous, the existing literature shows that, under a non-drastic innovation, the royalty licensing is always the best method for an insider patentee, as Lemma 2 shows. This paper instead finds that given a sufficiently large degree of innovation, when the innovating firm is the location leader, no licensing is the best method for an insider patentee, which is different from Proposition 6 in Poddar and Sinha (2004). Consequently, the royalty licensing becomes only the second-best method and the fixed fee is the worst method. When the non-innovating firm is the location leader, the royalty licensing is still the best method for the insider patentee; moreover, no licensing (the fixed fee) is the second best method when the degree of innovation is sufficiently large (small), which is different from Proposition 5 in Poddar and Sinha (2004).

5 Concluding Remarks

This paper uses a linear market model in which two firms engage in location choice, patent licensing and price competition. In this spatial model, the royalty is a better method of licensing for the insider patentee when locations are exogenously determined. Under the licensing method of fixed fees, when the insider patentee has the location advantage, it tends not to license to its opponent. However, if the opponent has the location advantage, the insider patentee tends to license. The licensing decision also depends on the degree of innovation. When the degree of innovation is small (sufficiently large), the insider patentee will choose (choose not) to license. Our results compare to those of Poddar and Sinha (2004), in which for the insider patentee, no licensing always dominates fixed-fee licensing.

When locations are endogenously chosen, both firms will try to locate themselves as close as possible to the center of the linear market, with or without licensing. If the insider patentee is the location leader, it will occupy the best position to enjoy a dominant location advantage and not license to its opponent when the degree of innovation is sufficiently large. This result differs from the generally accepted previous conclusion that the royalty licensing is the best method for an insider patentee. When the non-innovating firm is the location leader, the royalty licensing is still the best method for the insider patentee.

After endogenizing the location choices, this paper discusses only the case of the insider patentee. Further considerations and extensions will be interesting topics for future research.

Appendix A

Firm A chooses not to license to firm B in order to prevent price cutting and become a monopoly. The profit for firm A is

$$\pi_A = \begin{cases} (p_A - c_A)q_A & |p_A - p_B| \le x_B - x_A \\ p_A - c_A & p_A < p_B - (x_B - x_A) \\ 0 & p_A > p_B + (x_B - x_A) \end{cases}$$

The profit for firm B is

$$\pi_B = \begin{cases} (p_B - c_B)q_B & |p_A - p_B| \le x_B - x_A \\ p_B - c_B & p_B < p_A - (x_B - x_A) \\ 0 & p_B > p_A + (x_B - x_A) \end{cases}$$

Under a duopoly, the first-order conditions for profit maximization are, respectively,

$$\partial\pi_A/\partial p_A = (p_B - 2p_A + x_B + x_A + c_A)/2$$

$$\partial\pi_B/\partial p_B = [(2 - x_B - x_A) + p_A - 2p_B + c_B]/2$$

Simultaneously solving for the above two first-order conditions, we obtain the Nash equilibrium prices as equation (5) and the profits as equation (7) in the context

$$\pi_A = \frac{(2 + x_A + x_B + c_B - c_A)^2}{18} \qquad \pi_B = \frac{(4 - x_A - x_B + c_A - c_B)^2}{18}$$

The profit for firm A to cut its price in order to become a monopoly can be obtained by substituting the monopoly price constraint into firm A's profit function, with an ensuing $\pi_A = P_B - (x_B - x_A) - c_A$, and then the best response function of firm B $p_B = (4 - x_A - x_B + 2c_B + c_A)/3$, making the equilibrium profit of firm A as a monopoly be $\pi_A^M = (4 + 2x_A - 4x_B + 2c_B - 2c_A)/3$. Similarly, if firm B cuts its price in order to become a monopoly, then its profit will be $\pi_B^M = (2 + 4x_A - 2x_B + 2c_A - 2c_B)/3$.

In order to guarantee that the price-cutting competition will not take place, the following two inequalities must hold:

$$\pi_A = \frac{(2 + x_A + x_B + c_B - c_A)^2}{18} \ge \pi_A^M = \frac{4 + 2x_A - 4x_B + 2c_B - 2c_A}{3}$$

$$\pi_B = \frac{(4 - x_A - x_B + c_A - c_B)^2}{18} \ge \pi_B^M = \frac{2 + 4x_A - 2x_B + 2c_A - 2c_B}{3}$$

The marginal costs when firm A is the innovator not to license are $c_A = c - \varepsilon$ and $c_B = c$, respectively. The conditions of a duopoly without licensing are

$$x_A^2 + x_B^2 + 2x_Ax_B - 8x_A + 28x_B - 20 \ge \varepsilon(8 - 2x_A - 2x_B - \varepsilon) \tag{10a}$$

$$x_A^2 + x_B^2 + 2x_Ax_B - 32x_A + 4x_B + 4 \ge -\varepsilon(4 + 2x_A + 2x_B + \varepsilon) \tag{10b}$$

Totally differentiating equations (10a), defined as f^{N}, and (10b), defined as g^{N}, we obtain

$$\frac{\mathrm{d}x_{\mathrm{B}}}{\mathrm{d}\varepsilon}=-\frac{f_{\varepsilon}^{\mathrm{N}}}{g^{\mathrm{N}}x_{\mathrm{B}}}=-\frac{-8+2x_{\mathrm{A}}+2x_{\mathrm{B}}+2\varepsilon}{28+2x_{\mathrm{A}}+2x_{\mathrm{B}}+2\varepsilon}>0$$

$$\frac{\mathrm{d}x_{\mathrm{A}}}{\mathrm{d}\varepsilon}=-\frac{f_{\varepsilon}^{\mathrm{N}}}{g^{\mathrm{N}}x_{\mathrm{A}}}=-\frac{4+2x_{\mathrm{A}}+2x_{\mathrm{B}}+2\varepsilon}{-32+2x_{\mathrm{A}}+2x_{\mathrm{B}}+2\varepsilon}>0$$

When the degree of innovation increases, the line corresponding to equation (10a) shifts up, and the line corresponding to equation (10b) shifts to rightward.

Partial differentiations of profits of firms A and B under a monopoly or a duopoly without licensing yield

$$\frac{\partial\pi_{\mathrm{A}}^{\mathrm{N}}}{\partial\varepsilon}=\frac{2+x_{\mathrm{A}}+x_{\mathrm{B}}+\varepsilon}{9}<\frac{\partial\pi_{\mathrm{A}}^{\mathrm{M}}}{\partial\varepsilon}=\frac{2}{3}$$

$$\frac{\partial\pi_{\mathrm{B}}^{\mathrm{N}}}{\partial\varepsilon}=\frac{4-x_{\mathrm{A}}-x_{\mathrm{B}}-\varepsilon}{9}>\frac{\partial\pi_{\mathrm{B}}^{\mathrm{M}}}{\partial\varepsilon}=-\frac{2}{3}$$

When the degree of innovation increases, the marginal profit of firm A as the innovator will increase under a monopoly as well as under a duopoly, while its marginal profit under a monopoly will be strictly higher than that under a duopoly. The marginal profit of firm B due to an increase in the degree of innovation is higher under a duopoly than under a monopoly.

Appendix B

Firm A licenses to firm B to prevent price cutting. The profit of firm A is

$$\pi_{\mathrm{A}}=\begin{cases}(p_{\mathrm{A}}-c_{\mathrm{A}})q_{\mathrm{A}} & |p_{\mathrm{A}}-p_{\mathrm{B}}|\le x_{\mathrm{B}}-x_{\mathrm{A}}\\ p_{\mathrm{A}}-c_{\mathrm{A}} & p_{\mathrm{A}}<p_{\mathrm{B}}-(x_{\mathrm{B}}-x_{\mathrm{A}})\\ 0 & p_{\mathrm{A}}>p_{\mathrm{B}}+(x_{\mathrm{B}}-x_{\mathrm{A}})\end{cases}$$

The profit of firm B is

$$\pi_{\mathrm{B}}=\begin{cases}(p_{\mathrm{B}}-c_{\mathrm{B}})q_{\mathrm{B}} & |p_{\mathrm{A}}-p_{\mathrm{B}}|\le x_{\mathrm{B}}-x_{\mathrm{A}}\\ p_{\mathrm{B}}-c_{\mathrm{B}} & p_{\mathrm{B}}<p_{\mathrm{A}}-(x_{\mathrm{B}}-x_{\mathrm{A}})\\ 0 & p_{\mathrm{B}}>p_{\mathrm{A}}+(x_{\mathrm{B}}-x_{\mathrm{A}})\end{cases}$$

The first-order conditions of profit maximization for the two firms are, respectively,

$$\partial\pi_{\mathrm{A}}/\partial p_{\mathrm{A}}=[p_{\mathrm{B}}-2p_{\mathrm{A}}+(x_{\mathrm{B}}+x_{\mathrm{A}})+c_{\mathrm{A}}]/2$$

$$\partial\pi_{\mathrm{B}}/\partial p_{\mathrm{B}}=[2-x_{\mathrm{B}}-x_{\mathrm{A}}+p_{\mathrm{A}}-2p_{\mathrm{B}}+c_{\mathrm{B}}]/2$$

The Nash equilibrium prices are

$$p_{\mathrm{A}}=\frac{2+x_{\mathrm{A}}+x_{\mathrm{B}}+2c_{\mathrm{A}}+c_{\mathrm{B}}}{3}\qquad\text{and}\qquad p_{\mathrm{B}}=\frac{4-x_{\mathrm{A}}-x_{\mathrm{B}}+2c_{\mathrm{B}}+c_{\mathrm{A}}}{3}$$

The Nash equilibrium profits are

$$\pi_A = \frac{(2 + x_A + x_B + c_B - c_A)^2}{18} \quad \text{and} \quad \pi_B = \frac{(4 - x_A - x_B + c_A - c_B)^2}{18}$$

Because in this regime, firm A licenses to firm B, the two firms have the same marginal costs as $c_A = c_B = c - \varepsilon$. Since firm A can exploit part of firm B's profit as the fixed licensing fee, the profits after licensing become (11) and (12) in the context.

The profits for firms A and B to become a monopoly via price cutting can be obtained by the approach in Appendix A, yielding $\pi_A^M = (4 + 2x_A - 4x_B + 2c_B - 2c_A)/3$ and $\pi_B^M = (2 + 4x_A - 2x_B + 2c_A - 2c_B)/3$.

As a result, to avoid a monopoly via price cutting, the following two constraints must be satisfied:

$$\pi_A^F = \frac{(2 + x_A + x_B)^2}{18} + \frac{(4 - x_A - x_B)^2}{18} - \frac{(4 - x_A - x_B - \varepsilon)^2}{18} \geq \frac{4 + 2x_A - 4x_B + 2\varepsilon}{3}$$

$$\pi_B^F = \frac{(4 - x_A - x_B - \varepsilon)^2}{18} \geq \frac{2 + 4x_A - 2x_B - 2\varepsilon}{3}$$

After re-arrangements, these two constraints become

$$x_A^2 + x_B^2 + 2x_A x_B - 8x_A + 28x_B - 20 \geq \varepsilon(4 + 2x_A + 2x_B + \varepsilon) \quad (15a)$$

$$x_A^2 + x_B^2 + 2x_A x_B - 32x_A + 4x_B + 4 \geq -\varepsilon(4 + 2x_A + 2x_B + \varepsilon) \quad (15b)$$

Total differentiations of equations (15a), defined as f^F, and (15b), defined as g^F, yield

$$\frac{dx_B}{d\varepsilon} = -\frac{f_\varepsilon^F}{g^F x_B} = -\frac{-(4 + 2x_A + 2x_B + 2\varepsilon)}{28 + 2x_A + 2x_B - 2\varepsilon} > 0$$

$$\frac{dx_A}{d\varepsilon} = -\frac{f_\varepsilon^F}{g^F x_A} = -\frac{4 + 2x_A + 2x_B + 2\varepsilon}{-32 + 2x_A + 2x_B + 2\varepsilon} > 0$$

When the degree of innovation increases, the line corresponding to equation (13a) shifts up, and the line corresponding to equation (13b) shifts to rightward. Moreover,

$$\frac{\partial \pi_A^F}{\partial \varepsilon} = -\frac{4 - x_A - x_B - \varepsilon}{9} < \frac{\partial \pi_A^M}{\partial \varepsilon} = \frac{2}{3}$$

$$\frac{\partial \pi_B^F}{\partial \varepsilon} = \frac{4 - x_A - x_B - \varepsilon}{9} > \frac{\partial \pi_B^M}{\partial \varepsilon} = -\frac{2}{3}$$

That is, when the degree of innovation increases, firm A's marginal profit to choose price cutting will be higher than that under a duopoly; however, the marginal profit of firm B under a duopoly is higher than that under a monopoly.

References

Aoki, R. and Hu, J. L. (1999). 'Licensing vs. Litigation: the Effect of the Legal System on Incentives to Innovate', *Journal of Economics and Management Strategy*, Vol. 8, pp. 133–160.

Aoki, R. and Hu, J. L. (2003). 'Time Factors of Patent Litigation and Licensing', *Journal of Institutional and Theoretical Economics*, Vol. 159, pp. 280–301.

Chang, M. C., Hu, J. L. and Tzeng, G. H. (2009). 'Decision Making on Strategic Environmental Technology Licensing: Fixed-fee versus Royalty Licensing Methods', *International Journal of Information Technology and Decision Making*, Vol. 8, pp. 609–624.

d'Aspremont, C., Jaskold, J. and Thisse, J. (1979). 'On Hotelling's Stability in Competition', *Econometrica: Journal of the Econometric Society*, Vol. 47, pp. 1145–1150.

Fauli-Oller, R. and Sandonis, J. (2002). 'Welfare Reducing Licensing', *Games and Economic Behavior*, Vol. 41, pp. 192–205.

Greenhut, M. L. and Ohta, H. (1972). 'Output under Alternative Spatial Pricing Techniques', *American Economic Review*, Vol. 62, pp. 705–713.

Holahan, W. L. (1975). 'The Welfare Effects of Spatial Price Discrimination', *American Economic Review*, Vol. 65, pp. 498–503.

Hwang, H. and Mai, C. C. (1990). 'Effects of Spatial Price Discrimination on Output, Welfare, and Location', *American Economic Review*, Vol. 80, pp. 567–575.

Kamien, M. and Tauman, Y. (1984). 'The Private Value of a Patent: a Game Theoretic Analysis', *Journal of Economics*, Vol. 4, pp. 93–118.

Kamien, M. and Tauman, Y. (1986). 'Fees versus Royalty and the Private Value of a Patent', *Quarterly Journal of Economics*, Vol. 101, pp. 471–491.

Kamien, M., Oren, S. and Tauman, Y. (1992). 'Optimal Licensing of Cost Reducing Innovation', *Journal of Mathematical Economics*, Vol. 21, pp. 483–508.

Katz, M. and Shapiro, C. (1986). 'How to License Intangible Property', *Quarterly Journal of Economics*, Vol. 101, pp. 567–590.

Matsumura, T., Matsushima, N. and Stamatopoulos, G. (2010). 'Location Equilibrium with Asymmetric Firms: the Role of Licensing', *Journal of Economics*, Vol. 99, pp. 267–276.

Mukherjee, A. (2005). 'Innovation, Licensing and Welfare', *Manchester School*, Vol. 73, pp. 29–39.

Mukherjee, A. (2007). 'Optimal Licensing Contract in an Open Economy', *Economics Bulletin*, Vol. 12, pp. 1–6.

Muto, S. (1993). 'On Licensing Policies in Bertrand Competition', *Games and Economic Behavior*, Vol. 5, pp. 257–267.

Ohta, H. (1988). *Spatial Price Theory of Imperfect Competition*, College Station, Texas A&M University Press.

Poddar, S. and Sinha, U. B. (2004). 'On Patent Licensing and Spatial Competition', *Economic Record*, Vol. 80, pp. 208–218.

Poddar, S. and Sinha, U. B. (2010). 'Patent Licensing from a High-cost Firm to a Low-cost Firm', *Economic Record*, Vol. 86, pp. 384–395.

Sinha, U. B. (2010). 'Strategic Licensing, Exports, FDI, and Host Country Welfare', *Oxford Economic Papers*, Vol. 62, pp. 114–131.

Wang, X. H. (1998). 'Fee vs. Royalty Licensing in a Cournot Duopoly Model', *Economics Letters*, Vol. 60, pp. 55–62.

Wang, X. H. (2002). 'Fee versus Royalty Licensing in a Differentiated Cournot Duopoly', *Journal of Economics and Business*, Vol. 54, pp. 253–266.