

Resource Allocation in Cognitive Radio Relay Networks

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Abstract—Cognitive radio has received great attention recently for its ability to improve spectrum efficiency by letting secondary users to access spectrum resource that is unoccupied by primary users. However, cognitive radio also brings new challenges in the design of future wireless networks. In this paper, we investigate the problem of resource allocation in cognitive radio networks. Specifically, we consider the problem of proportional fair scheduling in cognitive radio relay networks. Our problem formulation takes into account the fluctuations of usable spectrum resource, channel quality variations caused by frequency selectivity, and interference caused by different transmit power levels. We prove that the problem is NP-hard and is computationally infeasible to be solved in a timely manner by using brute force algorithms. An easy-to-compute upper bound for the formulated problem is also derived. We then propose two heuristic algorithms that are easy-to-implement, yet achieve performance close to the upper bound. The proposed algorithms can be executed and finished within 1 millisecond. Thus, they can meet the requirement of real-time scheduling. Simulation experiments verify that the proposed algorithms can achieve good proportional fairness among users and enhance system throughput by proper power control.

Index Terms—Cognitive radio, relay networks, resource allocation, proportional fair scheduling, frequency selectivity, power control

I. INTRODUCTION

COGNITIVE radio has redefined the philosophy and framework of wireless networks. Radio spectrum is a limited yet valuable resource. A large portion of operable frequency bands is under legal regulation of governments. According to the report by the Federal Communications Commission (FCC), the usage of spectrum resource is highly unbalanced [1]. Some frequency bands are heavily occupied, while others are underutilized. To better utilize the scarce resource of radio spectrum, researchers have proposed the idea of allowing secondary users to access spectrum holes that are unoccupied by the primary users. To serve this purpose, cognitive radio has been proposed due to its ability to agilely sense, enter, and leave radio spectrum without causing harmful interference to primary users [2]–[4].

Resource allocation, which involves scheduling of spectrum resource that includes frequency bands and time slots, is a

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fundamental problem. It is crucial to the performance of a wireless network in terms of user fairness, system throughput, and bandwidth utilization. A network formed by secondary cognitive stations and users would pose great challenges in the design of efficient resource allocation algorithms due to the inherent nature of wireless channel, the fluctuation of available bandwidth in cognitive radio networks, and the requirement of real-time scheduling.

In this paper, we consider the problem of resource allocation in cognitive radio networks with relay stations. Relay stations are Medium Access Control (MAC) level repeaters that can be used to enhance channel quality of users or to extend cell coverage. Both the IEEE 802.16j [5] and the 3GPP LTE Advanced [6] have incorporated relay stations into their standards. In this paper, we consider a secondary cognitive radio relay network that is co-located with one or more primary networks, as shown in Fig. 1. The network drawn with solid lines in Fig. 1 stands for the Primary Radio (PR) network. The network with dotted lines is the Cognitive Radio (CR) network co-located with the PR network. The nodes in the CR network are equipped with cognitive radios, and are capable of sensing unused channels in its vicinity. Resource allocation is performed by the *Cognitive Radio Base Station (CR BS)* in a centralized manner. We assume that the transmission of the secondary network, that is, the CR network, is time-divided into frames, and synchronized with the PR network. In each frame, the CR BS gathers available channels within the vicinity of each node and the quality of each channel. The gathering can be done through a control channel or from the *Channel State Information (CSI)*. The CR BS then schedules the usage of frequency bands and time-slots for its downstream *Cognitive Radio Relay Stations (CR RSs)* and *Cognitive Radio Mobile Stations (CR MSs)*.

The problems in designing resource allocation algorithms for cognitive radio networks are multi-fold. We identify the main challenges as follows:

- *Fluctuations in available spectrum resource*: The number of usable resource may differ in each area. For example, CR RS₁ in Fig. 1 may have less usable channels than CR RS₂ because CR RS₁ is located near the PR network. It is problematic to ensure Quality of Service (QoS) among nodes in CR networks. The fluctuations in usable channels may cause great variations in available bandwidth and jitters between packets. Thus, in this paper, we adopt *Proportional Fair Scheduling* [7]. Because the goal of cognitive radio is to utilize unoccupied spectrum resource, proportional fair scheduling can serve the purpose well by allocating resource to CR MSs proportionally to

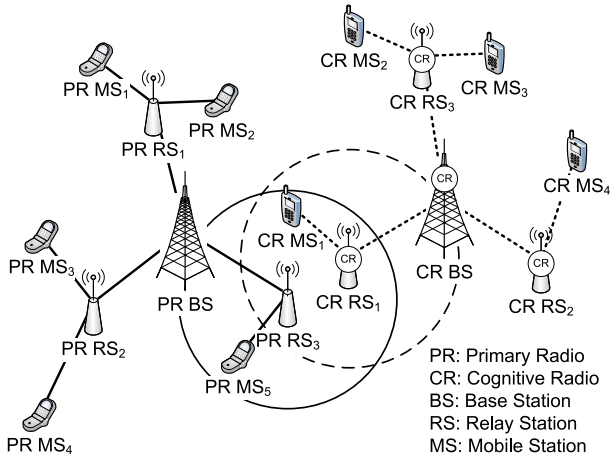


Fig. 1. An illustration of a cognitive radio relay network.

their capabilities such as transmission rates determined by channel quality.

- *Instability of wireless channels:* In a multi-channel wireless system, the frequency selectivity would cause variations of Signal-to-Noise Ratio (SNR), and such phenomenon is location-dependent. As a result, the quality of each channel would differ on each node. The variations in channel quality would pose great challenges on proportional fair scheduling. In addition to achieving long-term proportional fairness among CR MSs, the resource allocation algorithm also needs to take into account the fluctuations in usable channels as well as instability of channel quality.
- *Power control and interference among nodes:* Power control is more important in cognitive radio networks than conventional wireless networks. Because the number of usable channels are determined by the PR networks, it is desired to utilize these channels as efficient as possible. With proper power control, spatial reuse can be achieved and two adjacent CR RSs can transmit over the same channel. For example, assuming that both CR RS₂ and CR RS₃ in Fig. 1 have a common channel available in their vicinity. If they both transmit using full power, only one can access the channel. Otherwise, their transmissions will collide. However, through proper control of transmit power, it is possible for them to transmit over the same channel without causing harmful interference.

In this paper, we take above issues into consideration and design resource allocation algorithms accordingly. The rest of the paper is organized as follows. Section II provides background for cognitive radio and proportional fair scheduling. Section III discusses related work. System model and notations are presented in Section IV. The problems and proposed algorithms with *fixed transmit power* and *adjustable transmit power* are formulated and discussed in Section V and Section VI, respectively. The performance of the proposed algorithms is evaluated in Section VII. Section VIII concludes the paper.

II. BACKGROUND

A. Cognitive Radio

In this section, we provide background on cognitive radio. As the demands on bandwidth increase, researchers have been seeking ways to improve the utilization of scarce spectrum resources. According to several recent surveys and reports by FCC [1] and other organizations [8], [9], the licensed spectrum is underutilized both in time and frequency. Such phenomenon exists even in highly crowded regions such as Washington, D. C. and New York City [8], [9]. To better utilize the limited and scarce spectrum resources, cognitive radio has been considered for its ability to agilely sense and exploit unused licensed spectrum resources, while also keeping legitimate primary users unaffected. The FCC has proposed a *Notice of Proposed Rulemaking* on cognitive radio [10]. The notice may stimulate the redesign of wireless communication architectures so that future wireless devices can coexist with legitimate primary users without causing harmful interference to them. In [11], [12], the FCC has also defined regulations for unlicensed use of TV white space. The IEEE is formulating new wireless air interface based on cognitive radio as well. The IEEE 802.22 working group is now developing physical and MAC layers of wireless regional area networks which aims to utilize unused resources in the spectrum allocated to TV bands [13]–[15].

In general, a cognitive radio performs the following functions [16]:

- *Spectrum sensing:* To sense unused spectrum and the presence of primary users in its vicinity.
- *Spectrum decision:* To determine which spectrum to use and when to use.
- *Spectrum sharing:* To coordinate the use of spectrum with other cognitive radios or primary users.
- *Spectrum mobility:* To leave the spectrum and select other available spectrum when a primary user is sensed.

It requires collaboration of multiple layers in network stack to achieve above functions. Readers are referred to [17]–[21] for details in cognitive radio technologies.

B. Proportional Fair Scheduling

The goal of *Proportional Fair Scheduling (PFS)* is to allocate bandwidth to each node proportionally to their rates [7]. In wireless networks, channel conditions usually are time-varying. Thus, the available data rate of each Mobile Station (MS) may be different at different time. One way to schedule resources is to serve all MSs equally by using round-robin scheduling to maximize *fairness*. The other way is to always serve the MS with the best channel condition which has the highest data rate to maximize *throughput*. By using PFS, however, the scheduling algorithm aims at maximizing total throughput while also maintaining long-term fairness for all MSs. That is, the scheduling algorithm tries to serve the MS with best channel condition while also maintain acceptable level of performance for other MSs [22]. A simple example can be found in [23].

Next, we discuss how to present PFS mathematically [7]. Considering a network with M nodes, let $r_i(t)$ be the allocated rate of node i at time-slot t , where $1 \leq i \leq M$. Let $R_i(t)$

be the average rate that node i has been serviced until the beginning of time-slot t . The set of average rates $R_i(t)$, $1 \leq i \leq M$ are said to be *proportional fair* if $R_i(t)$'s are feasible, and for all other feasible rates $S_i(t)$'s the following holds [7]:

$$\sum_i \frac{S_i(t) - R_i(t)}{R_i(t)} < 0. \quad (1)$$

It has been shown that if $R_i(t)$'s are proportional fair, the set of long-term rates also maximize the *proportional fair metric*:

$$\sum_i \log(R_i(t)) \quad (2)$$

over all other feasible long-term rates [7]. Eq. (2) is actually another form of Eq. (1). However, Eq. (2) is easier to use to measure the fairness of a scheduling algorithm. Therefore, it is called *proportional fair metric*.

The above definition is based on long-term observation. In each scheduling round, the feasible set of $r_i(t)$'s may change due to channel fluctuations. When performing real-time scheduling, we have only the information about the available rates of each node up to the time when the scheduling decisions are to be made. No future information regarding the rates of each node is available. Hence, we need a way to make the scheduling decisions, which are based on short-term information, to achieve long-term proportional fairness. In [22], conditions under which short-term rates converge to long-term proportional fairness are provided. Let $\rho_i(t)$ be the long-term average throughput of node i up to time t , which is define as:

$$\rho_i(t) = \alpha R_i(t) + (1 - \alpha)\rho_i(t - 1), 0 < \alpha \leq 1. \quad (3)$$

If in each scheduling round we schedule the rates $r_i(t)$'s such that the objective function:

$$O(r) = \sum_i \frac{r_i(t)}{\rho_i(t)} \quad (4)$$

is maximized among other feasible allocation of $r_i(t)$'s, the long-term rates $R_i(t)$'s will converge to proportional fairness.

Based on above argument, one may have a way to achieve proportional fairness. However, the inherent nature of cognitive radio relay networks poses great challenge on the optimization problem. The feasible space of the rates $r_i(t)$'s in each scheduling round can be large due to frequency diversity, node mobility, variations in transmit power of nodes, and interference among nodes, which make the complexity of the optimization problem grows exponentially. As a result, it is computationally infeasible to derive the optimal $r_i(t)$'s during each scheduling round because the frame duration usually is only 5 to 20 ms in wireless network systems. Thus, we need a real-time algorithm that is computationally light, and the results should be as close to the optimal as possible.

III. RELATED WORK

To achieve the desired capability of cognitive radio, multi-layer coordination and collaboration are needed. The design of the physical layer [24]–[26] and MAC layer [27]–[29] has been discussed. Studies about resource allocation in cognitive

radio networks can be found in [30]–[36]. We briefly discuss them in the following paragraphs.

In [30], [31], the authors discuss issues related to design and implementation of cognitive radio MAC in ad-hoc networks. Relay between MSs is also considered in [31]. These studies focus on scenarios of ad-hoc networks, while we consider infrastructure radio networks.

In [32], the authors provide game theoretical analyses in cognitive radio networks for distributed channel allocation. The analyses consider behavior of nodes in distributed environment. By defining the utility of selfish and cooperative users, a deterministic channel allocation can be obtained at the Nash equilibrium point.

The resource allocation algorithm is often designed to maximize certain utility. Two common utilities are *fairness* and *system throughput*. Fair scheduling is discussed in [33], [34]. In [33], the authors consider resource allocation and admission control in distributed cognitive radio networks. The authors propose to jointly combine admission control and power control to minimize the interference to primary users while maintaining the QoS of secondary users. In [34], the authors investigate fair resource allocation strategies that are interference-limited in single-hop OFDM cognitive radio networks. The authors define the received interference as fair metric, and develop an interference limited scheduler that tries to maximize system throughput while limiting the received interference at each user. Both [33], [34] consider only one-hop scenarios, while we incorporate the use of relay stations in this paper.

In [37], the authors study the resource assignment problems with quality constraint and fairness constraint in cognitive radio networks. They show that the problem can be solved in polynomial time when only considering quality constraint. The problem is NP complete when fairness is desired. The authors then propose a tree pruning based algorithm to solve distance constrained resource assignment problem. A major difference between our paper and [37] is that we take relay stations into consideration. In addition, we incorporate power control and interference among MSs into the problem. Also, we adopt proportional fair scheduling.

Scheduling to maximize system throughput is discussed in [35], [36]. In [35], the authors provide algorithms, which consider both total transmit power constraint of secondary users and maximum tolerable interference constraint of primary users, to maximize throughput in OFDM cognitive radio networks. The algorithms also try to be fair in the sense that transmit opportunities are allocated to users that receive less service. In [36], the authors consider the problem of spectrum scheduling in multi-channel cognitive radio networks. The authors present a Markov chain formulation to estimate the expected number of packets that can be transmitted by each secondary user over each sub-channel. Based on the estimation, they propose a scheduler to maximize the aggregated system throughput of the network. These studies emphasize on the throughput performance of cognitive radio networks, while we mainly focus on proportional fairness.

In this paper, we consider a resource allocation problem that differs from the existing papers in the literature. Specifically, we consider proportional fair scheduling in infrastructure

cognitive radio networks with relay stations. The secondary users can either receive service directly from the CR BS or through the help of the CR RSs. We consider power allocation of secondary users to minimize interference. Our scheduling algorithm also exploits sub-channel reuse to boost system throughput. To the best of our knowledge, we are among few of the first authors considering the proportional fair scheduling that takes into account frequency selectivity and transmitting power control in cognitive radio relay networks. We itemize the contributions of this paper as follows:

- We formulate the problem of proportional fair scheduling in cognitive radio relay networks, and prove that the problem is NP-Hard.
- We derive easy-to-compute upper bounds on the problem. The upper bounds can be used to evaluate the performance of resource allocation algorithms.
- We propose an easy-to-implement algorithm for proportional fair scheduling in cognitive radio relay networks.
- We take transmitting power control into the problem formulation, and propose an efficient resource allocation algorithm accordingly.
- We show, through simulation evaluation, that the proposed algorithms have performance comparable to the upper bound.

IV. SYSTEM MODEL AND NOTATIONS

Without loss of generality, we consider a wireless relay network with one CR BS, multiple CR RSs, and multiple CR MSs. An CR MS can attach to the CR BS directly, or it can connect to the CR BS through an CR RS. Which station an CR MS decides to attach to is usually decided by channel quality. We do not concern how the CR MSs are connected. Usually, multiple RSs are connected to one BS, and each RS does not connect to each other. Therefore, we assume the wireless relay network forms a tree-based topology. The transceivers on all stations are cognitive, i.e., they are capable of sensing and utilizing idle channels. The wireless network can be a secondary network that tries to access radio resources which are not used by the primary network, or it can be a member of a group of wireless networks which share common radio resources and use cognitive radio techniques to prevent contentions and collisions. We assume the transmission in the wireless network is frame-by-frame. In each frame, there are multiple sub-channels and time-slots, which are not used by the primary users, can be allocated for transmission. We consider only downlink transmission in this paper. However, our work can be extended to uplink transmission easily with slight changes.

We first state the notations which will be used in later sections. The notations are listed in Table I. We denote \mathcal{M} , \mathcal{R} , \mathcal{R}^+ as the set of CR mobile stations, the set of CR RSs, and the set of CR RSs plus the CR BS, respectively. The set of all links in the networks is denoted by \mathcal{L} , and \mathcal{C} is the set of sub-channels that can be scheduled. For ease of presentation, the number of elements in \mathcal{M} is denoted by M . The number of elements in other sets follows the same notation, e.g., R for \mathcal{R} , L for \mathcal{L} , and so on. N is the number of time-slots in a frame. We refer to one sub-channel by one time-slot as a *tile*,

TABLE I
LIST OF NOTATIONS

Notation	Description
\mathcal{M}	Set of CR Mobile Stations
\mathcal{R}	Set of CR Relay Stations
\mathcal{R}^+	Set of CR Relay Stations and the CR Base Station
\mathcal{L}	Set of all links
\mathcal{C}	Set of sub-channels
\mathcal{P}	Set of available transmit power levels
N	Number of slots in one frame
l_i^c	The set of links connecting to node i 's children.
l_i^p	The link connecting to node i 's parent.
$R_l(c)$	The maximum sustainable rate of sub-channel c on link l in bits/sec, $l \in \mathcal{L}, c \in \mathcal{C}$
$r_l(c, t)$	The actual data transmitted on link l over sub-channel c at time-slot t in bits, $l \in \mathcal{L}, c \in \mathcal{C}$
ρ_m	Long-term average rate of MS m in bits/frame
p_i	The transmit power level of CR RS i
$I_l(c, t)$	Indicator variable as defined in (6)
$e_{(i,j)}$	Indicator variable as defined in (7)
$v_{(i,c)}$	Indicator variable as defined in (8)

which is the minimum unit of resource that can be allocated. For each node i in the network, l_i^c is the set of links connecting to i 's children, and l_i^p is the link connecting to i 's parent. The maximum sustainable rate $R_l(c)$ of link l over sub-channel c is determined by the channel quality, modulation, and coding scheme used.

We assume our system model possesses the following general properties:

- **(P1)** The resource allocation is done by the CR BS in a centralized manner. The CR BS performs resource allocation, including transmissions from the CR BS to CR RSs and from CR RSs to CR MSs, for all links in every frame.
- **(P2)** The transmission is frame-by-frame. CR RSs do not buffer data during the transmission of each frame.
- **(P3)** There is only one transceiver in each node. The nodes operate in half-duplex mode, i.e., they cannot transmit and receive data at the same time which is a common property in most wireless systems.
- **(P4)** If two nodes will interfere with each other, they cannot transmit over the same sub-channel at the same time.
- **(P5)** The transmission rate of a link on a sub-channel cannot exceed the maximum sustainable rate, which is determined by the sub-channel quality and is time varying.
- **(P6)** A node can only transmit on vacant sub-channels in its vicinity to avoid interfering with primary users.

V. FIXED TRANSMIT POWER

The major problem we want to study in this paper is with *adjustable transmit power*. In this section, we first consider

the case of *fixed transmit power* which is the basis of the problem with *adjustable transmit power*. We then extend *fixed transmit power* to *adjustable transmit power* in Section VI. After understanding the case of *fixed transmit power*, it will be easier to comprehend the case of *adjustable transmit power*.

A. Problem Formulation

In this section, we formally state the problem of *proportional fair scheduling for cognitive relay networks with fixed transmit power in relay stations*. The scheduling is done in a *per frame* basis. The goal of the scheduling is to allocate available tiles in each frame so that proportional fairness can be achieved among CR MSs in a long-term scale. In each frame t , we aim to maximize:

$$\sum_{m \in \mathcal{M}} \frac{\lambda_m(t)}{\rho_m(t)}, \quad (5)$$

where $\lambda_m(t)$ is the scheduled rate of CR MS m in frame t , and $\rho_m(t)$ is the long-term average rate of CR MS m until frame t . The scheduling is subject to several constraints which will be discussed later. Since we are concerned with the allocation of resource in each frame, we do not incorporate frame sequence t in the problem formulation.

Before presenting the constraints, we first describe several variables that will be used in the problem formulation. Let $I_l(c, t)$ be the indicator variable such that:

$$I_l(c, t) = \begin{cases} 1, & \text{if sub-channel } c \text{ at time-slot } t \text{ is allocated} \\ & \text{to link } l \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

We take interference among CR RSs into consideration in our problem formulation. Two CR RSs that do not interfere with each other can transmit on the same sub-channel at the same time, which will result in better spatial reuse. On the other hand, two interfering CR RSs cannot transmit on the same sub-channel at the same time, and the scheduling algorithm needs to decide which CR RS the sub-channel should be allocated to. Here, we say an CR RS i interferes an CR RS j if i 's transmission will degrade the channel quality of an CR MS attaching to j when i and j are transmitting on the same sub-channel. Let $e_{(i,j)}$ be the indicator variable such that:

$$e_{(i,j)} = \begin{cases} 1, & \text{if CR RS } i \text{ interferes CR RS } j \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

Note that in the fixed CR RS transmit power scenario, the value of $e_{(i,j)}$ is fixed and can be viewed as input parameters. Let $v_{(i,c)}$ be a 0-1 parameter such that:

$$v_{(i,c)} = \begin{cases} 1, & \text{if sub-channel } c \text{ is vacant around node } i \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

$v_{(i,c)}$ can be obtained by making the cognitive radio on node i detecting vacant channels in its vicinity and report to the CR BS periodically.

Based on the assumptions made in Section IV, we have the following constraints:

- (C1) due to (P2)

$$\sum_{c \in \mathcal{C}} \sum_{t=0}^{\tau} r_{l_i^c}(c, t) \geq \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \sum_{t=0}^{\tau} r_k(c, t), \quad (9)$$

$$\forall i \in \mathcal{R}, \quad 0 \leq \tau < N - 1$$

$$\sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_{l_i^c}(c, t) = \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_k(c, t), \quad \forall i \in \mathcal{R} \quad (10)$$

- (C2) due to (P3)

$$\max_{c \in \mathcal{C}} I_{l_i^c}(c, t) + \max_{k \in \mathcal{I}_i^c, c \in \mathcal{C}} I_k(c, t) \leq 1, \quad \forall i \in \mathcal{R}, \quad 0 \leq t < N \quad (11)$$

- (C3) due to (P4)

$$\sum_{k \in \mathcal{I}_i^c} I_k(c, t) + \sum_{j \in \{\mathcal{R}^+ \setminus i\}} \sum_{m \in \mathcal{I}_j^c} e_{(i,j)} I_m(c, t) \leq 1, \quad (12)$$

$$\forall i \in \mathcal{R}^+, \quad \forall c \in \mathcal{C}, \quad 0 \leq t < N$$

- (C4) due to (P5)

$$r_j(c, t) \leq I_j(c, t) R_j(c), \quad \forall j \in \mathcal{L}, \quad \forall c \in \mathcal{C}, \quad 0 \leq t < N \quad (13)$$

- (C5) due to (P6)

$$\max_{k \in \mathcal{I}_i^c, 0 \leq t < N} I_k(c, t) \leq v_{(i,c)}, \quad \forall i \in \mathcal{R}^+, \quad \forall c \in \mathcal{C} \quad (14)$$

Hence, we formally present the problem of *Proportional Fair Scheduling for Cognitive Relay Networks with Fixed CR RS Transmit Power (PFSCRN-FTP)* as follows:

- **Given:** (i) a cognitive relay network with one CR BS, multiple CR RSs, and multiple CR MSs, (ii) the set of links \mathcal{L} in the tree topology of the network, (iii) the vacant sub-channels $v_{(\dots)}$ in the vicinity of the CR BS and the CR RSs, (iv) The maximum sustainable rate $R_{(.)}$ over each sub-channel on each link, (v) The long-term average rate ρ_m of each CR MS $m \in \mathcal{M}$.
- **To find:** a feasible schedule for the current frame, that is, to determine variables $I_l(c, t)$ and $r_l(c, t)$ subject to constraints (9) to (14) such that the objective function:

$$O(\lambda) = \sum_{m \in \mathcal{M}} \frac{\lambda_m}{\rho_m}, \quad \text{where } \lambda_m = \sum_{c \in \mathcal{C}} \sum_{0 \leq t < N} r_{l_m^c}(c, t) \quad (15)$$

is maximized.

Theorem 1: The problem of PFSCRN-FTP is NP-hard.

Proof: Please refer to Appendix A. ■

B. Upper Bound for PFSCRN-FTP

The problem of PFSCRN-FTP formulated in Section V-A is an integer programming problem, which is NP-hard and computationally infeasible to solve. If we drop the integrality constraint on the problem, the solution to the linear programming problem will be an upper bound for the PFSCRN-FTP problem. However, the linear programming problem still has $RN + C^2RN + RCN + LCN + RLCN$ constraints. Therefore, the computational time to solve the problem is still too high. For performance evaluation purpose, we need an upper bound on the PFSCRN-FTP problem. In this section, we derive an computationally light upper bound for the PFSCRN-FTP

problem. The upper bound has $R + RC + LC$ constraints and can be solved easily with off-the-shelf linear programming tools.

Define $\omega_l(c) \equiv \frac{1}{N} \sum_{t=0}^{N-1} I_l(c, t)$ as the sub-channel utilization rate of link l over sub-channel c . Summing up and rewriting constraints C1 to C5, we get:

$$\sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_{1_i^p}(c, t) \geq \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_k(c, t), \quad \forall i \in \mathcal{R} \quad (16)$$

$$\max_{c \in \mathcal{C}} \omega_{1_i^p}(c) + \max_{k \in \mathcal{I}_i^c, c \in \mathcal{C}} \omega_k(c) \leq 1, \quad \forall i \in \mathcal{R}^+ \quad (17)$$

$$\sum_{k \in \mathcal{I}_i^c} \omega_k(c) + \sum_{j \in \{\mathcal{R}^+ \setminus i\}} \sum_{m \in \mathcal{I}_j^c} e_{(i,j)} \omega_m(c) \leq 1, \quad (18)$$

$$\forall i \in \mathcal{R}^+, \forall c \in \mathcal{C}$$

$$\sum_{t=0}^{N-1} r_l(c, t) \leq N \omega_l(c) R_l(c), \quad \forall l \in \mathcal{L} \quad (19)$$

$$\max_{k \in \mathcal{I}_i^c} \omega_k(c) \leq v_{(i,c)}, \quad \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C} \quad (20)$$

The constraints (16)–(20) are necessary conditions for any feasible solution to the original PFSCRN-FTP problem. To maximize the objective function $O(\lambda)$, the solution subjected to constraints (16)–(20) is an upper bound of the PFSCRN-FTP problem. However, the above constraints still have high complexity. To further reduce the complexity of the problem, we drop the integrality constraint. Therefore, (16) and (19) can be rewrite as:

$$\sum_{c \in \mathcal{C}} \omega_{1_i^p}(c) R_{1_i^p}(c) \geq \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \omega_k(c) R_k(c), \quad \forall i \in \mathcal{R} \quad (21)$$

$$\sum_{t=0}^{N-1} r_l(c, t) = N \omega_l(c) R_l(c), \quad \forall l \in \mathcal{L} \quad (22)$$

Also, we replace (17) and (18) with

$$\sum_{k \in \mathcal{I}_i^c} \omega_k(c) + \sum_{j \in \{\mathcal{R}^+ \setminus i\}} \sum_{m \in \mathcal{I}_j^c} e_{(i,j)} \omega_m(c) \leq \frac{1}{R+1}, \quad (23)$$

$$\forall i \in \mathcal{R}^+, \forall c \in \mathcal{C}.$$

Theorem 2 shows the solution to the new problem is within $\frac{1}{R+1}$ of the solution to the original PFSCRN-FTP problem. It provides a worst case gap between the upper bound and the optimal solution. In Section VII, the simulation results will show that the upper bound derived from Theorem 2 is close to the optimal solution of the original problem.

Theorem 2: Let σ_{UB} be the solution to the following linear programming problem:

Maximize:

$$\sum_{m \in \mathcal{M}} \frac{\lambda_m^*}{\rho_m} \quad (24)$$

Subject to:

$$\lambda_m^* = N \sum_{c \in \mathcal{C}} \omega_{1_m^p}(c) R_{1_m^p}(c) \quad (25)$$

$$\sum_{c \in \mathcal{C}} \omega_{1_i^p}(c) R_{1_i^p}(c) \geq \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \omega_k(c) R_k(c), \quad \forall i \in \mathcal{R} \quad (26)$$

Algorithm 1 Greedy Algorithm for PFSCRN-FTP

- 1: Let T be the set of CR RSs that have been scheduled.
 - 2: Let U be the set of CR RSs that interfere with each other.
 - 3: Let M be the candidate CR MSs to be scheduled.
 - 4: $M \leftarrow \phi$
 - 5: **for all** $c \in \mathcal{C}$ **do**
 - 6: $T \leftarrow \phi$
 - 7: **for all** $r \in \mathcal{R}$ such that $v_{(r,c)} = 1$ and $r \notin T$ **do**
 - 8: $U \leftarrow \phi$
 - 9: **for all** $i \in \mathcal{R}$ **do**
 - 10: **if** $e_{(r,i)} = 1$ and $v_{(i,c)} = 1$ **then**
 - 11: $T \leftarrow i, \quad U \leftarrow i$
 - 12: **end if**
 - 13: **end for**
 - 14: **for all** $u \in U$ **do**
 - 15: $m_{(u,c)} = \arg \max_{m \in u's \text{ children}} \frac{R_{1_m^p}(c)}{\rho_m}$
 - 16: $M \leftarrow M \cup \{m_{(u,c)}\}$
 - 17: **end for**
 - 18: **end for**
 - 19: **end for**
 - 20: Sort elements in M in descending order of contribution to the objective function
 - 21: Denote by $n_{(l,c)}$ the available time-slots on link l over sub-channel c
 - 22: **for all** $m_{(u,c)} \in M$ **do**
 - 23: $D_m = R_{m_{(u,c)}}(c) n_{(1_{m_{(u,c)}^p}, c)}$
 - 24: $SchedData \leftarrow 0$
 - 25: **while** $SchedData < D_m$ and CR BS has available sub-channels **do**
 - 26: $c' \leftarrow \arg \max_{c \in \mathcal{C} \text{ and } v_{(BS,c)}=1} R_{1_u^p}(c)$
 - 27: Allocate available time-slots of sub-channel c' on link 1_u^p to CR RS u
 - 28: $SchedData \leftarrow SchedData + R_{1_u^p}(c')$
 - 29: **end while**
 - 30: Allocate time-slots of sub-channel c on link $1_{m_{(u,c)}^p}$ to $m_{(u,c)}$
 - 31: **end for**
-

$$\sum_{k \in \mathcal{I}_i^c} \omega_k(c) + \sum_{j \in \{\mathcal{R}^+ \setminus i\}} \sum_{m \in \mathcal{I}_j^c} e_{(i,j)} \omega_m(c) \leq \frac{2}{R+1}, \quad (27)$$

$$\forall i \in \mathcal{R}^+, \forall c \in \mathcal{C}$$

$$\max_{k \in \mathcal{I}_i^c} \omega_k(c) \leq v_{(i,c)}, \quad \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C} \quad (28)$$

Let σ_{LP} be the solution to the reduced linear programming problem with constraints (16)–(20). Let σ be the solution to the original PFSCRN-FTP problem. The following holds:

$$\sigma \leq \sigma_{LP} \leq (R+1)\sigma_{UB} \quad (29)$$

Proof: Please refer to Appendix B. ■

C. Proposed Greedy Algorithm for PFSCRN-FTP

If we solve the PFSCRN-FTP problem by brute force algorithm, the complexity is $O((MR)^{NC})$. In real systems, however, the frame duration is less than 20 *ms*. Thus, brute force algorithm cannot meet the requirement of real-time scheduling. Therefore, we propose a heuristic greedy

algorithm for the PFSCRN-FTP problem. As we will show later, the proposed algorithm is easy to implement and has performance comparable to the upper bound. The algorithm is shown in Algorithm 1.

The design of the *Greedy PFSCRN-FTP Algorithm* consists of two parts: (1) To find and schedule available sub-channels for each CR RS. (2) To allocate sub-channels and time-slots of all hops in a greedy-based approach. The algorithm works as follows:

- *Resolve conflicts among interfering CR RSs and schedule their available sub-channels:* It is possible that two CR RSs that interfere with each other have access to the same sub-channels. Thus, conflicts need to be solved and the algorithm will decide for each CR RS the sub-channels that it can utilize. The algorithm scans through all the sub-channels (line 5-19). For each sub-channel c , we scan through each CR RS r (line 7-18), and record all CR RSs that interfere with r in the set U (line 9-13). All of the CR RSs in the set U have access to sub-channel c , but are interfered with each other. Thus, only one of the CR RSs in U can access the sub-channel c at a given time. Others must avoid transmitting over c at that time to avoid collisions. We select the CR RS that has CR MS m with maximum value of $\frac{R_{l_p}(c)}{\rho_m}$ to have access to sub-channel c (line 14-17). The procedure continues until all sub-channels are scanned and all available sub-channels of CR RSs are scheduled.
- *Allocate the resource between the CR BS and the CR RSs:* The resource between the CR BS and the CR RSs may not be enough to transmit all the data of the CR MSs selected. We allocate the resource between the CR BS and the CR RSs in a greedy approach. First, the CR MSs are sorted in descending order of their contributions to the objective function (line 20). For each m in the sorted order and the CR RS u that m attaches to, the amount of service that m receives is initialized to the number of bits that can be transmitted using all the available time-slots between m and u over the scheduled sub-channel c (line 23). We allocate the sub-channels between the CR BS and u in the order of transmission rate. That is, the sub-channel c' with highest rate $R_{l_u}(c')$ is allocated first, and then the second highest sub-channel is allocated, and so on (line 26). The allocation continues until all m 's requirement is fulfilled, or there is no available sub-channels left between the CR BS and u . In the later case, the time-slots allocated between m and u over sub-channel c is shrank to match the data transmitted from the CR BS to u .

Proposition 1: The computational complexity of the proposed Greedy PFSCRN-FTP is $O(CR^2)$. The algorithm of the proposed Greedy PFSCRN-FTP is shown in Algorithm 1. Based on the algorithm, it is easy to see that the time complexity is $O(C \times R \times R)$ which equals $O(CR^2)$.

VI. ADJUSTABLE TRANSMIT POWER

The PFSCRN-FTP problem we discussed in Section V-A assumes fixed transmit power of CR RSs. With the transmit power of CR RSs fixed, the interference pattern among the CR

RSs and the maximum sustainable rate from CR RSs to the CR MSs are also fixed. They can be viewed as input parameters to the algorithm during the resource allocation procedure. However, power control and interference are crucial issues in wireless networks. In the case of cognitive radio networks, they are especially important because the main purpose of cognitive radio is to utilize unused spectrum resources as efficient as possible while keeping the original primary users unaffected. Better spectrum utilization and system throughput can be achieved with proper control of CR RS transmit power. In this section, we consider the problem of resource allocation with adjustable CR RS transmit power.

A. Problem Formulation

The transmit power of an CR RS is assumed to be selected from a set of discrete power levels. The scheduler can decide which power level an CR RS should use. The decision normally involves factors such as interference among CR RSs and transmission rates between CR RSs and CR MSs. With the transmit power of CR RSs being adjustable, the interference indicator $e_{(i,j)}$ between two CR RSs (i,j) and the maximum sustainable rate $R_l(c)$ on an CR RS-CR MS link l over a sub-channel c become functions of the transmit power p . Let p_i denote the transmit power level of CR RS i . The problem of *Proportional Fair Scheduling of Cognitive Radio Networks - Adjustable CR RS Transmission Power (PFSCRN-ATP)* has following constraints:

- **C1'**
$$p_i \in \mathcal{P}, \forall i \in \mathcal{R} \quad (30)$$

- **C2'** due to **P2**

$$\sum_{c \in \mathcal{C}} \sum_{t=0}^{\tau} r_{l_i^p}(c, t) \geq \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \sum_{t=0}^{\tau} r_k(c, t), \quad (31)$$

$$\forall i \in \mathcal{R}, 0 \leq \tau < N - 1$$

$$\sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_{l_i^p}(c, t) = \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \sum_{t=0}^{N-1} r_k(c, t), \forall i \in \mathcal{R} \quad (32)$$

- **C3'** due to **P3**

$$\max_{c \in \mathcal{C}} I_{l_i^p}(c, t) + \max_{k \in \mathcal{I}_i^c, c \in \mathcal{C}} I_k(c, t) \leq 1, \forall i \in \mathcal{R}, 0 \leq t < N \quad (33)$$

- **C4'** due to **P4**

$$\sum_{k \in \mathcal{I}_i^c} I_k(c, t) + \sum_{j \in \{\mathcal{R}^+ \setminus i\}} \sum_{m \in \mathcal{I}_j^c} e_{(i,j)}(p_i) I_m(c, t) \leq 1, \quad (34)$$

$$\forall i \in \mathcal{R}^+, \forall c \in \mathcal{C}, 0 \leq t < N$$

- **C5'** due to **P5**

$$r_j(c, t) \leq I_j(c, t) R_j(c, p_i), \forall i \in \mathcal{R}^+, \forall j \in \mathcal{I}_i^c, \quad (35)$$

$$\forall c \in \mathcal{C}, 0 \leq t < N$$

- **C6'** due to **P6**

$$\max_{k \in \mathcal{I}_i^c, 0 \leq t < N} I_k(c, t) \leq v_{(i,c)}, \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C} \quad (36)$$

The problem of PFSCRN-ATP can be formulated as follows:

- **Given:** (i) a cognitive relay network with one CR BS, multiple CR RSs, and multiple CR MSs, (ii) the set of links \mathcal{L} in the tree topology of the network, (iii) the set of available transmit power levels for CR RSs, (iv) the vacant sub-channels $v_{(\cdot,\cdot)}$ in the vicinity of the CR BS and the CR RSs, (v) the maximum sustainable rate $R_{\cdot}(\cdot,\cdot)$ over each sub-channel on each link using each power level, (vi) the long-term average rate ρ_m of each CR MS $m \in \mathcal{M}$.
- **To find:** a feasible schedule for the current frame, that is, to determine variables $I_l(c,t)$, $r_l(c,t)$, and transmit power levels of CR RSs subject to constraints (30) to (36) such that the objective function:

$$O(\lambda) = \sum_{m \in \mathcal{M}} \frac{\lambda_m}{\rho_m}, \text{ where } \lambda_m = \sum_{c \in \mathcal{C}} \sum_{0 \leq t < N} r_{l_m}^p(c,t) \quad (37)$$

is maximized.

The problem formulation of PFSCRN-ATP is similar to that of PFSCRN-FTP. The interference indicator $e_{(i,j)}(p)$ between two CR RSs (i,j) and the maximum sustainable rate $R_l(c,p)$ on an CR RS and CR MS link l over sub-channel c are functions of the CR RS transmit power p . Note that it is possible for each CR RS to transmit with different power levels. An CR RS with its attaching CR MSs being near to it can be scheduled to transmit using low power level to minimize interference, while an CR RS that is experiencing bad channel quality can be scheduled to transmit using high power level to boost transmission speed. Similar to that in Theorem 1, we have the following theorem:

Theorem 3: The problem of PFSCRN-ATP is NP-Hard.

Theorem 3 can be proved by reducing the problem of PFSCRN-FTP in Section V-A to an instance of PFSCRN-ATP. Because the power levels of CR RSs in PFSCRN-FTP can be viewed as one of the many possible combinations of the CR RS transmit power levels in PFSCRN-ATP, the proof is easy to follow. The PFSCRN-FTP problem is already computationally infeasible to solve. With one more variable being added, the problem of PFSCRN-ATP is even harder to solve.

B. Upper Bound for PFSCRN-ATP

The upper bound of PFSCRN-ATP can be derived by using Theorem 2 if the transmit power levels of CR RSs are given. An algorithm for determining the transmit power levels of CR RSs is provided in next subsection.

C. Proposed Greedy Algorithm for PFSCRN-ATP

In general, the decision on which power level an CR RS should use involves two major issues. First, the power level should be chosen such that the interference to other CR RSs is minimized. Second, the power level should not be too low such that the spectrum cannot be fully utilized. In a cell with R CR RSs and P possible power levels, a brute force algorithm needs to explore all of the P^R combinations of power levels. Together with the complexity of the PFSCRN-FTP problem, a brute force algorithm that solves the problem of PFSCRN-ATP has complexity of $O(P^R(\text{MR})^{\text{NC}})$. Thus,

Algorithm 2 Arg-Max Algorithm for Determining Power for CR RSs

```

1: for all  $r \in \mathcal{R}$  do
2:   for all available power  $p$  do
3:      $u_{(r,p)} \leftarrow 0$ 
4:     for all  $c \in \mathcal{C}$  and  $v_{(r,c)} = 1$  do
5:        $g_{(r,c,p)} \leftarrow \max_{m \in r's \text{ children}} \frac{R_{l_m}^p(c,p)}{\rho_m}$ 
6:        $u_{(r,p)} \leftarrow u_{(r,p)} + g_{(r,c,p)}$ 
7:     end for
8:   end for
9: end for
10: Let  $p(r)$  be the scheduled power level of CR RS  $r$ 
11:  $p(r) \leftarrow \arg \max_{\text{all power level } p} u_{(r,p)}$ 
12: If there are multiple maximum  $u_{(r,p)}$ , set  $p(r)$  as the lowest  $p$  among them

```

brute force algorithm cannot meet the requirement of real-time scheduling.

In this section, we present an algorithm to decide transmit power levels of CR RSs. The algorithm is shown in Algorithm 2. We first calculate a score for each combination of (CR RS, power level) (r,p) (line 1-9). The score is determined by selecting the CR MS m with most contribution $\frac{R_{l_m}^p(c,p)}{\rho_m}$ to the objective function on each available sub-channel c of r and summing up the contribution of these CR MSs (line 5-6). The power levels of CR RSs are determined as follows.

Priority is given to the (r,p) combination with highest score. That is, for each CR RS r , its transmit power level p is set to the one that has highest score among all other (r,p') combinations (line 11). If there are multiple (r,p) combinations that have the same scores, ties are broken by giving priority to the lowest power level (line 12). Generally, the power level of an CR RS is chosen to be the lowest one such that maximum contribution to the objective function can be achieved.

With the transmit power of CR RSs determined using Algorithm 2, the scheduling problem is solved by using a modified version of Algorithm 1. The modified heuristic greedy algorithm for PFSCRN-ATP is shown in Algorithm 3. The flow basically follows Algorithm 1, with the interference indicator $e_{(i,j)}(p)$ between two CR RS (i,j) and maximum sustainable rate $R_l(c,p)$ on link l over sub-channel c being changed to functions of transmit power level p .

Proposition 2: The computational complexity of the proposed Greedy PFSCRN-ATP is $O(CPR + CR^2)$. The computational complexity of the proposed Greedy PFSCRN-ATP depends on Algorithm 2 and Algorithm 3, which have time complexity $O(C \times P \times R)$ and $O(C \times R \times R)$, respectively. Therefore, the total complexity is $O(CPR + CR^2)$.

VII. PERFORMANCE EVALUATION

The problem formulation and the proposed algorithms in previous sections are not specific to any particular system. However, for performance evaluation purpose, we construct a simulation environment that mimics the system of an IEEE 802.16j relay network. We have conducted extensive simulations to evaluate the performance of the proposed algorithms.

Algorithm 3 Greedy Algorithm for PFSCRN-ATP

```

1: Let  $p(r)$  be the scheduled power determined by Algorithm 2
2: Let  $T$  be the set of CR RSs that have been scheduled.
3: Let  $U$  be the candidate CR RSs to be scheduled.
4: Let  $M$  be the candidate CR MSs to be scheduled.
5:  $M \leftarrow \phi$ 
6: for all  $c \in \mathcal{C}$  do
7:    $T \leftarrow \phi$ 
8:   for all  $r \in \mathcal{R}$  such that  $v_{(r,c)} = 1$  and  $r \notin T$  do
9:      $U \leftarrow \phi$ 
10:    for all  $i \in \mathcal{R}$  do
11:      if  $e_{(r,i)}(p(r)) = 1$  and  $v_{(i,c)} = 1$  then
12:         $T \leftarrow i, U \leftarrow i$ 
13:      end if
14:    end for
15:    for all  $u \in U$  do
16:       $m_{(u,c)} = \arg \max_{m \in u's \text{ children}} \frac{R_{1m}^p(c, p(u))}{\rho_m}$ 
17:       $M \leftarrow M \cup \{m_{(u,c)}\}$ 
18:    end for
19:  end for
20: end for
21: Denote by  $n_{(l,c)}$  the available time-slots on link  $l$  over sub-channel  $c$ 
22: for all  $m_{(u,c)} \in M$  do
23:    $D_m = R_{m_{(u,c)}}(c, p(u))n_{(1m_{(u,c)}, c)}$ 
24:    $SchedData \leftarrow 0$ 
25:   while  $SchedData < D_m$  and CR BS has available sub-channels do
26:      $c' \leftarrow \arg \max_{c \in \mathcal{C} \text{ and } v_{(BS,c)}=1} R_{1u}^p(c)$ 
27:     Allocate available time-slots of sub-channel  $c'$  on link  $l_u^p$  to CR RS  $u$ 
28:      $SchedData \leftarrow SchedData + R_{1u}^p(c')$ 
29:   end while
30:   Allocate time-slots of sub-channel  $c$  on link  $l_{m_{(u,c)}}^p$  to  $m_{(u,c)}$ 
31: end for

```

As discussed earlier, proportional fair scheduling aims at maximizing total throughput while also maintaining long-term fairness for all MSs. Therefore, the goals of our simulations are: (i) to examine how good the proportional fairness is achieved by our proposed algorithms, and (ii) to compare the system throughput of each algorithm.

A. Simulation Setup

In this section, setups which are common in all simulations are described. We assume the queues of each CR MS always have backlogged traffic. Table II summarizes the simulation parameters.

We adopt the path loss model and parameters presented in [38]. Frequency gains are assumed to be independent of the sub-channels. We use Jake's model [39] to generate waveforms for each sub-channel and each CR BS – CR MS pair or CR RS – CR MS pair. The code of Jake's model is a modified version of the mphy module in ns-miracle [40].

TABLE II
PARAMETERS IN SIMULATION

Parameter	Value
CR BS transmit power	43dBm (20 watts)
Default CR RS transmit power	34dBm (2.5 watts)
Available transmit power levels of CR RS	22/26/30/34 dBm (0.15/0.4/1/2.5 watts)
CR BS-CR RS, CR RS-CR RS shadowing standard deviation	3.5dB
CR BS-CR MS, CR RS-CR MS shadowing standard deviation	8dB
CR BS, CR RS antenna gain	15dB
Noise power	-147dBm/Hz
CR BS height	30m
CR RS height	15m
Frame duration	10ms
Slots per frame	48
Number of sub-channels	64
Sub-channel bandwidth	10MHz
Carrier frequency	2.5GHz

TABLE III
AMC MODE IN IEEE 802.16

AMC mode	bits/s/Hz	SINR threshold (dB)
QPSK- $\frac{1}{2}$	1.0	7.6
QPSK- $\frac{3}{4}$	1.5	10.3
16QAM- $\frac{1}{2}$	2.0	14.3
16QAM- $\frac{3}{4}$	3.0	17.4
64QAM- $\frac{2}{3}$	4.0	21.0
64QAM- $\frac{3}{4}$	4.5	22.0

CR MSs are assumed to move with pedestrian speed of 3 km per hour. Therefore, a sub-channel has identical gain during the span of the whole frame. Adaptive Modulation and Coding (AMC), which allows modulation and coding scheme to be adjusted according to channel quality, is used and the thresholds presented in Table III are adopted.

We construct two scenarios to evaluate the performance of the proposed algorithms. In each scenario, the CR BSs are placed at the center of the simulated cell. In scenario 1, the CR RSs are located closer to the CR BS, and CR MSs are uniformly distributed in the cell. With CR BS being in the center, we distribute 4 CR RSs in a radius of 1200 m and all CR MSs in a radius of 1800 m. In scenario 2, the CR RSs are located farther to the CR BS, and CR MSs are uniformly distributed around the CR RSs. We distribute 4 CR RSs in a radius of 1500 m and each CR MS in a radius of 300 m with a random CR RS being in the center.

Scenario 1 simulates conditions where CR RSs are being used for *signal enhancement purpose*. The CR RSs are placed in areas where the signals from CR BS is under severe fading. Scenario 2 simulates conditions where CR RSs are being used for *range extension purpose*. The CR RSs are placed away from the CR BS to achieve larger cell coverage. For each scenario, we test the performance of the proposed algorithms with two system loads, namely *low system load* with 20 CR MSs and *high system loads* with 40 CR MSs. In each

scenario, we also evaluate the proposed algorithms for fixed transmit power and adjustable transmit power.

Although we discuss some related studies in Section III, to the best of our knowledge, there is no existing work that can be perfectly applied to the environment we consider. For comparison purpose, we use a *random scheduling* algorithm. In fixed transmit power, the random scheduling selects a random MS in each frame for each sub-channel. In adjustable transmit power, each RS first randomly selects an available transmit power level. It then randomly selects an MS for each sub-channel. In addition to random scheduling, we also compare the results with the upper bound.

B. Proportional Fair Metric

In this section, we present the performance of the proposed algorithms in terms of proportional fairness. The fairness of the proposed algorithms is evaluated through the proportional fair metric, which is defined in Eq. (2) in Section II-B. For each scenario, 40 random topologies are generated according to the configuration described in Section VII-A. The rates of each CR MS are recorded after 2000 frames and the proportional fair metrics are calculated. The results are the average of the 40 randomly generated topologies. The 95% confidence intervals show that the performance are consistent throughout all the randomly generated topologies.

Fig. 2 and Fig. 3 show the proportional fair metrics of scenario 1 and scenario 2, respectively. Both cases of *Fixed Transmit Power (FTP)* and *Adjustable Transmit Power (ATP)* for CR RSs, along with their upper bounds, are presented. The upper bounds of FTP case in both figures are derived as that presented in Section V-B. The upper bounds of ATP in both figures are derived by using the power levels determined by Algorithm 2 in Section VI-C along with the linear constraints in Section V-B. We also include the random scheduling for comparison. As the figures show, our proposed algorithms represented by the bars of FTP and ATP in Figs. 2–3 can achieve near-to-optimal proportional fairness in both low and high system loads of all scenarios. In Fig. 2, the performance of random scheduling is lower than our proposed algorithms. In Fig. 3, our proposed algorithms significantly outperform the random scheduling. This is because in scenario 1, the CR RSs are placed closer to the CR BS. The channel condition of each CR MS would be less variant. On the contrary, in scenario 2, the CR RSs are placed far apart from each other as well as from the CR BS, and the channel condition of each CR MS varies a lot. Our proposed algorithms are especially effective in achieving proportional fairness in such diverse environment. The reason behind this has been discussed in earlier sections. From the results in Fig. 2 and Fig. 3, we can see that the proposed algorithms are insensitive to the changes of scenario and system load. The random scheduling, however, varies a lot in different scenarios. Besides, we can see that the proportional fair metrics of our proposed algorithms are less than the upper bounds derived with Theorem 2 by 1.3% to 5.3%.

C. System Throughput

The performance of the proposed algorithms in terms of overall system throughput is also evaluated. Overall system

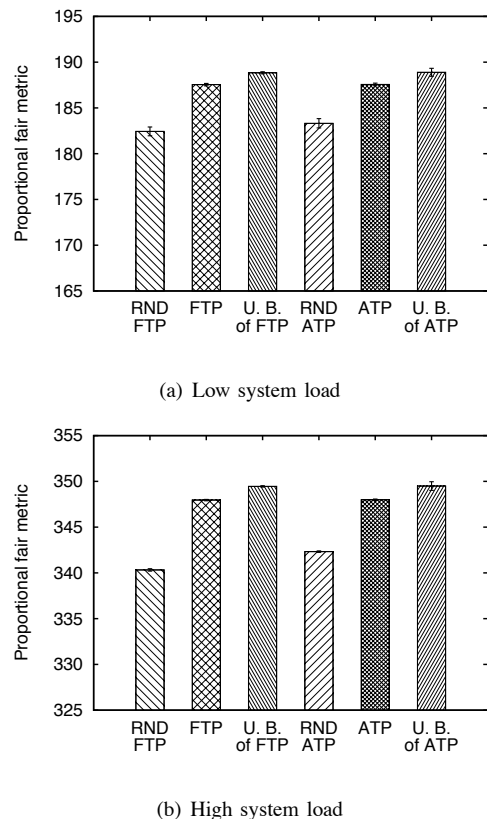
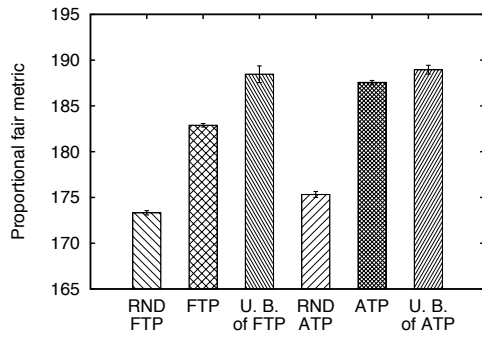


Fig. 2. Proportional fair metrics of scenario 1 with 95% of confidence interval. (RND: Random Scheduling; FTP: Fixed Transmit Power; U.B.: Upper Bound; ATP: Adjustable Transmit Power.)

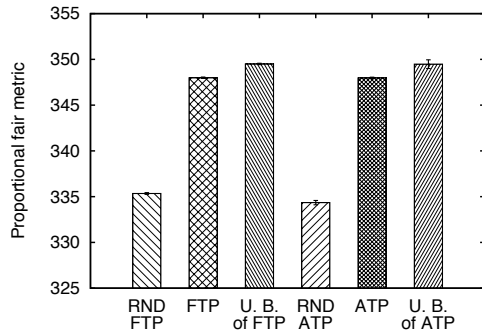
throughput is the sum of the services that each CR MS receives during the whole simulation period. Each result shown here is the average of 40 random topologies. For each topology, the simulation is run for 2000 frames.

Fig. 4 depicts the overall system throughput of the proposed algorithms as well as the random scheduling with both low and high system loads in scenario 1. As shown in the figure, we observe the following facts: (1) The overall system throughput of 20 and 40 CR-MSs does not differ much. (2) The overall system throughput of FTP and ATP does not differ much, either. In scenario 1, CR RSs are placed closer to the CR BS, which forms a smaller cell coverage and leads to smaller distance between CR RSs. Also, CR MSs are uniformly spread in the cell. That is, no hot-spot is formed. As a result, CR RSs will easily interfere with each other even with the lowest transmit power level. In such circumstance where cell coverage is small and CR MSs are evenly distributed, transmit power control will not help much. Also, in scenario 1, the channel condition of each CR MS does not differ much. Hence, the throughput does not differ much in all settings.

Fig. 5 depicts the overall system throughput of the proposed algorithms as well as the random scheduling in scenario 2. It is worth to mention that the gain of throughput from transmit power control of CR RSs is more significant in scenario 2. In scenario 2, the CR RSs are placed farther from the CR BS. Also, CR MSs tend to form hot-spot near the CR RSs. In systems where CR RSs are apart far enough and CR MSs are clustered in a certain extent, the interference among CR RSs



(a) Low system load



(b) High system load

Fig. 3. Proportional fair metrics of scenario 2 with 95% of confidence interval. (RND: Random Scheduling; FTP: Fixed Transmit Power; U. B.: Upper Bound; ATP: Adjustable Transmit Power.)

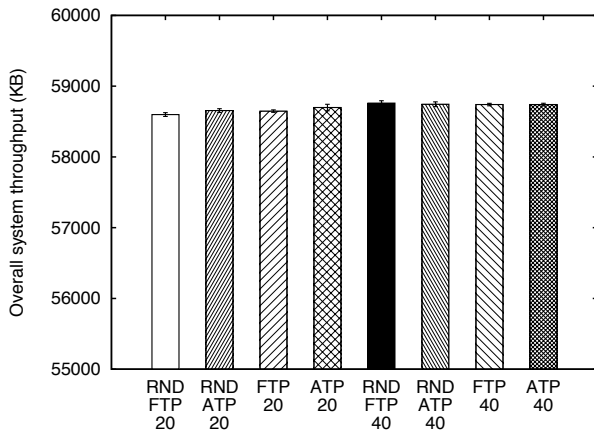


Fig. 4. Overall system throughput of scenario 1 with 95% of confidence interval. (RND: Random Scheduling; FTP: Fixed Transmit Power; ATP: Adjustable Transmit Power. 20 stands for 20 CR MSs. 40 stands for 40 CR MSs.)

will be highly controllable through adjustment of transmit power levels. Thus, as shown in Fig. 5, the performance of ATP is better than that of FTP. In addition, please note that the difference in throughput between ATP/20 and ATP/40 is less than that between FTP/20 and FTP/40. With the help of transmit power control, interference among CR RSs can be minimized and more CR MSs can be served in each frame. As a result, the throughput degradation caused by bad-channel-quality CR MSs can be compensated.

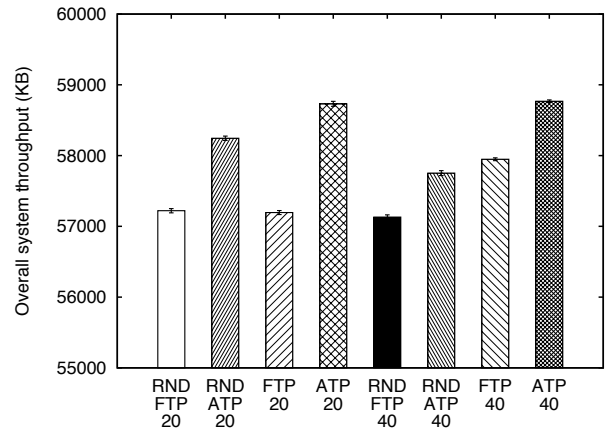


Fig. 5. Overall system throughput of scenario 2 with 95% of confidence interval. (RND: Random Scheduling; FTP: Fixed Transmit Power; ATP: Adjustable Transmit Power. 20 stands for 20 CR MSs. 40 stands for 40 CR MSs.)

D. Algorithm Running Time

We have also implemented the proposed algorithms with C++ and compiled them by using GNU g++ v4.1.2 with optimization flag O3. The implemented algorithms are executed with 64 sub-channels, 48 time-slots, 4 CR RSs, 40 CR MSs, and 2000 frames. As discussed earlier, a frame duration usually is only 5 to 20 ms. Table IV shows the average running time retrieved using GNU profiler. It indicates that the proposed Greedy PFSCRN-FTP and Greedy PFSCRN-ATP require 0.38 ms and 0.75 ms, respectively. Because it is computationally infeasible to run the brute force algorithms, we can only estimate the running time. For brute force FTP, we use the same setting of 40 CR MSs, 4 CR RSs, 64 sub-channels, 48 time-slots as parameters. Because we have already known the running time of the proposed PFSCRN-FTP is 0.38 ms, by using the time complexity shown in the 2nd column of Table IV, we can estimate that it takes more than 100 years for the brute force FTP. By using the same way, the estimated running time of the brute force ATP algorithm is also more than 100 years. As shown in the table, both of the proposed greedy algorithms have running times much less than 5 ms, which can meet the requirement of scheduling frames in real-time. We have also implemented and tested the random scheduling algorithms with the same settings. The random FTP and random ATP require 0.14 ms and 0.23 ms, respectively. Although they are slightly less than our proposed algorithms, our proposed algorithms outperform random scheduling algorithms as shown in Figs. 2–5.

VIII. SUMMARY

In this paper, we investigate the problem of resource allocation in cognitive radio relay networks. Specifically, we consider the problem of proportional fair scheduling in cognitive radio relay networks with the effect of frequency selectivity, transmit power control, and the volatility of usable frequency bands. The problems are formally formulated and proved to be NP-hard. They are computationally infeasible to derive optimal solutions in a timely manner by using brute-force algorithms. Upper bounds of the formulated problems are

TABLE IV
ALGORITHM RUNNING TIME

Algorithm	Time complexity	Average running time (per frame)
Brute force algorithm (FTP)	$O((MR)^{NC})$	more than 100 years
Greedy PFSCRN-FTP	$O(CR^2)$	0.38 ms
Random algorithm (FTP)	$O(CR^2)$	0.14 ms
Brute force algorithm (ATP)	$O(P^R(MR)^{NC})$	more than 100 years
Greedy PFSCRN-ATP	$O(CPR + CR^2)$	0.75 ms
Random algorithm (ATP)	$O(CR^2)$	0.23 ms

derived. We then propose two greedy algorithms which are easy-to-implement, yet achieve performance close to the upper bounds. Simulation experiments verify that our proposed algorithms can provide good proportional fairness among users. They can also achieve high system throughput. Besides, the proposed algorithms are insensitive to the changes of scenario and system loads. Because the goal of cognitive radio is to utilize unoccupied spectrum resource such as TV white space, the proposed algorithms which adopt PFS can serve the purpose well by allocating resource to CR MSs proportionally to their capabilities.

APPENDIX A
PROOF OF THEOREM 1

Proof: To proof Theorem 1, we reduce the problem of Integer Knapsack into an instance of the problem of PFSCRN-FTP.

The Integer Knapsack problem:

- **Given:** $(s_1, s_2, \dots, s_n, p_1, p_2, \dots, p_n, T) \in \mathbb{N}^{2n+1}$
- **To find:** $(x_1, x_2, \dots, x_n) \in \mathbb{N}^n$, such that $\sum_i s_i x_i \leq T$ and $\sum_i p_i x_i$ is maximized.

Considering an CR relay network with n CR MSs, one CR BS, one CR RS, and n sub-channels available for scheduling. The rates between the CR BS and the CR RS are T bits/sec on all sub-channels, and the rates between CR MS i and the CR RS are ns_i bits/sec on all sub-channels. Let there be two time-slots in each frame. Let x_i be the number of sub-channels allocated to CR MS i . A feasible schedule of the PFSCRN-FTP problem requires that:

$$\sum_i \lambda_i = \sum_i ns_i x_i \leq nT, \quad (38)$$

which leads to:

$$\sum_i s_i x_i \leq T. \quad (39)$$

Let ρ_i equal $\frac{ns_i}{p_i}$. The solution to the problem of PFSCRN-FTP seeks to maximize:

$$\sum_i \frac{\lambda_i}{\rho_i} = \sum_i ns_i x_i \frac{p_i}{ns_i} = \sum_i p_i x_i. \quad (40)$$

Hence the integer knapsack problem can be reduced to an instance of the PFSCRN-FTP problem. This completes the proof. ■

APPENDIX B
PROOF OF THEOREM 2

Proof: Let $I_l^*(c, t)$ be the optimum solution to the original PFSCRN-FTP problem subject to constraints (9)-(14). Define $\omega_l^*(c) \equiv \frac{1}{N} \sum_{t=0}^{N-1} I_l^*(c, t)$ as the corresponding sub-channel utilization rate of the optimum solution. Because $I_l^*(c, t)$ satisfies constraints (9)-(14), we have the following:

$$\begin{aligned} \sum_{c \in \mathcal{C}} \omega_{1_i}^*(c) R_{1_i}(c) &\geq \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \omega_k^*(c) R_k(c), \quad \forall i \in \mathcal{R} \\ \Rightarrow \sum_{c \in \mathcal{C}} \frac{\omega_{1_i}^*(c)}{(R+1)} R_{1_i}(c) &\geq \sum_{k \in \mathcal{I}_i^c} \sum_{c \in \mathcal{C}} \frac{\omega_k^*(c)}{(R+1)} R_k(c), \quad \forall i \in \mathcal{R}. \end{aligned} \quad (41)$$

Similarly,

$$\sum_{k \in \mathcal{I}_i^c} \frac{\omega_k^*(c)}{(R+1)} + \sum_{j \in \{\mathcal{R} \setminus i\}} e_{(i,j)} \sum_{m \in \mathcal{I}_j^c} \frac{\omega_m^*(c)}{(R+1)} \leq \frac{1}{R+1}, \quad (42)$$

$\forall i \in \mathcal{R}^+, \forall c \in \mathcal{C}$

and

$$\max_{k \in \mathcal{I}_i^c} \frac{\omega_k^*(c)}{(R+1)} \leq v_{(i,c)}, \quad \forall i \in \mathcal{R}^+, \forall c \in \mathcal{C}. \quad (43)$$

Because $\frac{\omega_l^*(c)}{(R+1)}$ satisfies constraints (26)-(28), it is considered as a feasible solution. This leads to the fact that the optimum solution of Eq. (24)-(28) is within $\frac{1}{R+1}$ of the optimum solution of the PFSCRN-FTP problem. This completes the proof. ■

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