Design of Low-Overhead Cooperative Beamforming for Information Relaying in Wireless Sensor Networks Under Imperfect Quantized SNR of Source-to-Relay Links

Jwo-Yuh Wu, Chung-Hsuan Hu, Tsang-Yi Wang, and Shang-Ho Tsai

***Abstract*—We consider a wireless sensor network employing cooperative beamforming for information relaying. To reduce the signaling overhead, each relay node quantizes the SNR of the source-to-relay (S-R) link into *one* bit, which is then transmitted through a binary symmetric channel (BSC) with a known crossover probability to the destination. Given the set of the error-corrupted one-bit messages received at the destination, the beamforming design criterion is the maximization of the expected receive SNR, averaged over the conditional bit-flipping distributions of BSCs. We derive an analytic expression for the considered SNR metric, which is a complicated function of the beamforming weights. To facilitate analysis, we further derive a tractable lower bound for the conditional average SNR. By conducting maximization with respect to this lower bound, a closed-form sub-optimal beamformer can be obtained as a solution to a generalized eigenvalue problem. Computer simulations are used to illustrate the performance of the proposed scheme.**

***Index Terms*—Amplify and forward, beamforming, cooperative communications, overhead reduction, quantization, relay.**

I. Introduction

A standard system configuration of wireless sensor networks (WSN) consists of spatially deployed small-size sensing devices connected through wireless links with a fusion center (FC) for performing certain global target tasks such as decision fusion or data aggregation [1], [2]. Sensor nodes are typically powered by battery and, thus, subject to limited signal transmission and communication capabilities. To enable seamless data delivery within WSN, *cooperation* among sensor nodes for signal transmission or information relaying becomes necessary [2]–[4]. Among the various cooperative transmission protocols, cooperative beamforming is a quite popular scheme since it is capable of exploiting the channel state information (CSI) to enhance link reliability [3]–[5]. To facilitate the design of cooperative beamforming for WSN, signaling overheads dedicated to link CSI transmission, exchange, and feedback are thus unavoidable. To meet the high energy-efficiency demand for WSN, such signaling overheads should be properly reduced or even minimized [6]; notably, reduced communication overheads can also enhance system capacity when inband signaling is used. There have been many proposals for low-overhead cooperative beamforming designs [7]–[12], all of which are developed under the idealized assumptions that CSI transmission and feedback are errorless. Such a design paradigm, however, may not be realistic in the WSN scenario. Indeed, since the design of WSN is typically subject to stringent transmit power and decoding complexity constraints, implementation of forward error correction codes for improving the error resilience of quantized CSI (or information bits) may be prohibitive due to unacceptable system complexity and decoding latency [1, Chap. 6]. Hence, transmissions of the local CSI from far-end nodes to the FC via wireless links could be impaired by severe path loss and deep fading [1], resulting in distorted CSI received at the FC. Hence, in addition to the low signaling-overhead requirement, cooperative beamforming designs for WSN should further take account of imperfect CSI transmission/feedback. Transceiver designs in the presence of noisy quantized CSI feedback have been considered in point-to-point multi-antenna wireless systems with limited feedback [13]–[17]. The related study in the context of cooperative communications for WSN, however, remains much to be investigated.

This paper presents an original study of the aforementioned problem in the cooperative beamforming setup. We consider the cooperative beamforming scheme for WSN employing the amplify-and-forward (AF) relaying protocol as in [18]. Also following [18], the information symbols from the source node are assumed to be BPSK modulated so as to reflect the rate and decoding complexity/latency constraints in WSN [11], [12]. The design of the beamforming weights is aimed at achieving SNR maximization, hence BER minimization, at the destination. To reduce the signaling overhead, each relay sensor node quantizes the signal-to-noise ratio (SNR) of the source-to-relay (S-R) link into one bit, which is then sent to the destination for beamforming design. Rather than assuming that the quantized bit is received at the destination without errors, we consider the realistic case that the one-bit message could be flipped by a binary symmetric channel (BSC) with a known nonzero crossover probability. When referring to "BSC" in the sequel, we specifically focus on the transmission link of the one-bit quantized S-R link SNR. Given the one-bit messages received from all relay nodes, the beamforming coefficients are designed at the destination via maximization of the receive SNR averaged with respect to the conditional channel flipping distributions. A closed-form formula for the proposed SNR metric is first derived. The formula is seen to be a highly nonlinear function of the beamforming factors, and direct maximization of this objective function is quite difficult. For analytic tractability, we then derive a lower bound of the conditional average SNR that can be expressed as a generalized Rayleigh quotient [19]. By conducting maximization with respect to this lower bound, a closed-form suboptimal beamformer can be obtained as the solution to a generalized eigenvalue problem. Computer simulations are used to illustrate the performance of the proposed solution. We remark that, in the considered system, quantized S-R-link SNR is forwarded from relays to the destination, and as in [18] we assume perfect feedback of the beamforming coefficients from the destination to relays. It is noted that, in the context of cooperative communications, an alternative relay beamforming scheme based on the "effective channel", i.e., the product of the S-R and relay-to-destination (R-D) link channel gains, estimated at the destination was proposed in [20]. In our system, the destination only needs to estimate the R-D-link CSI using, e.g., conventional LS or LMMSE techniques. To estimate the product channels reliably so as to aid the beamforming design in [20], one must resort to sophistic joint channel estimation schemes which are more computationally demanding (e.g., the convex optimization based technique [21]). Hence, *in terms of algorithmic complexity for CSI acquisition at the destination*, the approach adopted in our paper could be more attractive whenever the destination is subject to stringent complexity constraint. One

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potential application of our scheme is in clustered sensor networks [22], wherein sensor nodes take turn to serve as the "cluster head" (the destination) to perform certain information aggregation/fusion tasks. In this scenario, minimization of the processing complexity/energy at the cluster head, which is active most of the time, is rather crucial for prolonging the network lifetime. The rest of this paper is organized as follows. Section II is the preliminary. Section III presents the main results of this paper. Section IV is the conclusion.

II. Preliminary

A. Cooperative Beamforming System

We consider the dual-hop cooperative beamforming WSN as in [18] and is depicted in Fig. 1, in which L sensor nodes employ the AF protocol to collaboratively relay the common source signal $x[n] \in \{-1, 1\}$ from the source node to the destination. During the signal broadcasting phase, the received signal at the ith relay node is

$$y_{s_i}[n] = \sqrt{P_s} h_{s,i} x[n] + v_i[n], \tag{2.1}$$

where P_s is the source transmit power, $h_{s,i} \sim \mathcal{CN}(0, \sigma_s^2)$ is the channel gain of the ith S-R link[1], and $v_i[n] \sim \mathcal{CN}(0, \sigma_v^2)$ is the receive noise at the ith relay. Based on (2.1) the instantaneous SNR of the ith S-R link is thus

$$\gamma_{s_i} \triangleq \frac{P_s |h_{s,i}|^2}{\sigma_v^2}. \tag{2.2}$$

At the information relaying phase, the received signal at the destination then reads

$$y_d[n] = \sum_{i=1}^{L} h_{r,i} G_i g_i y_{s_i}[n] + w[n], \tag{2.3}$$

where $h_{r,i} \sim \mathcal{CN}(0, \sigma_r^2)$ denotes the ith R-D channel gain, $G_i = \frac{1}{h_{s,i}\sqrt{P_s(1+\gamma_{s_i}^{-1})}}$ is the power normalization factor, g_i is the ith beamforming weight, and $w[n] \sim \mathcal{CN}(0, \sigma_w^2)$ represents the receive noise at the destination. With (2.1), $y_d[n]$ in (2.3) can be expressed as [18]

$$y_d[n] = \sum_{i=1}^{L} \frac{h_{r,i} g_i}{\sqrt{1+\xi_{s_i}}} x[n] + \sum_{i=1}^{L} \frac{h_{r,i} g_i \sqrt{\xi_{s_i}}}{\sqrt{1+\xi_{s_i}}} \bar{v}_i[n] + w[n], \tag{2.4}$$

where $\xi_{s_i} \triangleq 1/\gamma_{s_i}$ is the reciprocal of the SNR of the ith S-R link, and $\bar{v}_i[n] \sim \mathcal{CN}(0, 1)$. To design the beamforming weights g_i's, one commonly used approach is to conduct SNR maximization based on the knowledge of the CSI of the S-R and R-D communication links (e.g., [23]–[25]). This paper focuses on the low-overhead cooperative beamforming scheme, wherein the ith relay node quantizes the SNR of the ith S-R link (see (2.2)) into one bit $q_i \in \{0, 1\}$. Assuming that (i) $\{q_1, \ldots, q_L\}$ are received at the destination without errors, and (ii) the CSI of all the R-D links is perfectly known at the destination, the SNR conditioned on either $x[n] = 1$ or $x[n] = -1$, is shown to be [18]

$$\gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \mathbf{q}) = \frac{\left| \sum_{i=1}^{L} g_i h_{r,i} \phi(q_i) \right|^2}{\sum_{i=1}^{L} |g_i|^2 |h_{r,i}|^2 (1 - \phi^2(q_i)) + \sigma_w^2}, \tag{2.5}$$

[1]The notation $\mathcal{CN}(0, \sigma^2)$ denotes the complex Gaussian random variable with zero mean and variance σ^2.

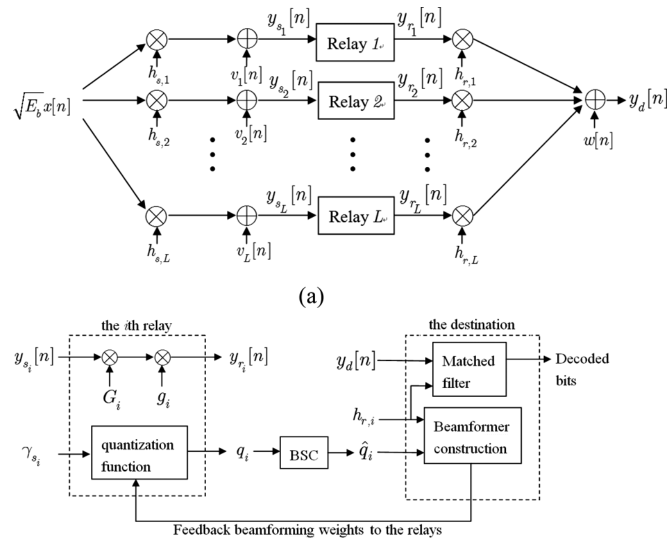

Fig. 1. (a) Low-overhead cooperative beamforming system diagram; (b) Depiction of AF relay and S-R SNR quantizer.

where $\mathbf{q} \triangleq [q_1, \ldots, q_L]^T$, $\mathbf{g} \triangleq [g_1, \ldots, g_L]^T$, $\mathbf{h_r} \triangleq [h_{r,1}, \ldots, h_{r,L}]^T$,

$$\phi(q_i) \triangleq \begin{cases} \frac{1}{1-e^{-\tau_i/(\bar{\gamma}_s)}} \int_0^{\tau_i} \frac{1}{\sqrt{1+(1/\mu)}} \exp(-\frac{\mu}{\bar{\gamma}_s}) d\mu, & \text{when } q_i = 0; \\ e^{\tau_i/(\bar{\gamma}_s)} \int_{\tau_i}^{\infty} \frac{1}{\sqrt{1+(1/\mu)}} \exp(-\frac{\mu}{\bar{\gamma}_s}) d\mu, & \text{when } q_i = 1 \end{cases} \tag{2.6}$$

is the mean of $1/\sqrt{1+\xi_{s_i}}$ given that $\gamma_{s_i} = 1/\xi_{s_i}$ belongs to the quantization interval associated with q_i, $\bar{\gamma}_s \triangleq P_s(\sigma_s^2/\sigma_v^2)$ is the average S-R link SNR, and $\tau_i > 0$ is the quantization threshold determined according to (66) in [18, p-4780]. We note the following: (i) for a finite L, an analytic expression of the SNR is difficult to find [18]; the expression (2.5) is obtained in [18] based on asymptotic analyses in the regime $L \to \infty$; (ii) the result (2.6) is directly obtained based on (33) and (34) in [18] together with the one-bit quantization assumption. The optimal g_i's, which maximize $\gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \mathbf{q})$ in (2.5) subject to the total power constraint

$$\sum_{i=1}^{L} |g_i|^2 = P_d, \tag{2.7}$$

are shown to be [18]

$$g_i \propto \frac{h_{r,i}^{-1} \phi(q_i)}{1 + \xi_{r_i} - \phi^2(q_i)}, \text{ where } \xi_{r_i} \triangleq 1/\gamma_{r_i} = \frac{\sigma_w^2}{P_d |h_{r,i}|^2}. \tag{2.8}$$

B. Problem Statement

The main purpose of this paper is to study the problem of cooperative beamforming design for WSN, under the assumption that the transmitted one-bit message q_i from each relay node is subject to communication channel impairments. More specifically, it is assumed that q_i is sent over a BSC with a crossover probability p_i, $1 \leq i \leq L$. From the perspective of SNR maximization, we propose a new beamforming design method which takes account of the imperfect reception of $\{q_1, \ldots, q_L\}$.

III. Main Results

This section introduces the main results of this paper. Section III-A derives the conditional average SNR, which is the proposed design metric for the beamforming factors. Section III-B then derives a lower bound of the considered SNR metric. An analytic suboptimal beamforming scheme is also obtained via the maximization of the lower bound. Computer simulations are given in Section III-C to illustrate the performance of the proposed solution.

A. Conditional Average SNR

Let $\hat{q}_i \in \{0,1\}$ be the received quantized message associated with $q_i, 1 \leq i \leq L$. Conditioned on the $\hat{q} = [\hat{q}_1 \cdots \hat{q}_L]^T$, the main purpose is to derive the conditional SNR averaged with respect to all possible transmitted $\tilde{q} = [\tilde{q}_1 \cdots \tilde{q}_L]^T$'s that are flipped to $\hat{q}$ by the BSC. Recall that, the SNR conditioned on $\hat{q} = q = [q_1 \cdots q_L]^T$ is given by $\gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \mathbf{q})$ in (2.5). Hence, the expected $\gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \tilde{\mathbf{q}})$ given $\hat{\mathbf{q}}$ is thus

$$\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}}) \triangleq E_{\tilde{\mathbf{q}}|\hat{\mathbf{q}}}\left[\gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \tilde{\mathbf{q}})|\hat{\mathbf{q}}\right] = \sum_{\tilde{\mathbf{q}}} \gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \tilde{\mathbf{q}}) \times \Pr(\tilde{\mathbf{q}}|\hat{\mathbf{q}}), \tag{3.1}$$

where $\Pr(\tilde{\mathbf{q}}\,|\hat{\mathbf{q}}\,)$ denotes the probability that $\tilde{\mathbf{q}}$ is flipped into $\hat{\mathbf{q}}$. The conditional average SNR (3.1) is obtained by averaging over all possible transmitted $\tilde{\mathbf{q}}$'s given the received $\hat{\mathbf{q}}$. To fix the idea, let us define [2]

$$S_l(\hat{\mathbf{q}}) \triangleq \left\{ \tilde{\mathbf{q}} \Big| \sum_{i=1}^{L} \tilde{q}_i \oplus \hat{q}_i = l \right\},\ 0 \leq l \leq L, \tag{3.2}$$

which denotes the set consisting of all possible $\tilde{\mathbf{q}}$'s that differ from $\hat{\mathbf{q}}$ in exactly l bits; there are thus $C_l^L = \frac{L!}{l!(L-l)!}$ possible $\tilde{\mathbf{q}}$'s in $S_l(\hat{\mathbf{q}})$. Associated with each $\tilde{\mathbf{q}} \in S_l(\hat{\mathbf{q}})$, we further collect all indices at which $\tilde{q}_i$ differs from $\hat{q}_i$ to obtain

$$I_l(\tilde{\mathbf{q}}, \hat{\mathbf{q}}) \triangleq \{i\,|\tilde{q}_i \neq \hat{q}_i\}. \tag{3.3}$$

With (3.2) and (3.3), the conditional average SNR is given as

$$\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}}) = \sum_{l=0}^{L} \sum_{\tilde{\mathbf{q}} \in S_l(\hat{\mathbf{q}})} \Pr(\tilde{\mathbf{q}}\,|\hat{\mathbf{q}})\gamma_{dq}(\mathbf{g}, \mathbf{h_r}, \tilde{\mathbf{q}}), \tag{3.4}$$

where

$$\Pr(\tilde{\mathbf{q}}\,|\hat{\mathbf{q}}) = \left(\prod_{k \in I_l(\tilde{\mathbf{q}}, \hat{\mathbf{q}})} p_k \right) \left(\prod_{m \in I_l^c(\tilde{\mathbf{q}}, \hat{\mathbf{q}})} (1 - p_m) \right), \tag{3.5}$$

and $I_l^c(\tilde{\mathbf{q}}, \hat{\mathbf{q}})$ denotes the complement of $I_l(\tilde{\mathbf{q}}, \hat{\mathbf{q}})$. Through further manipulations an explicit formula of $\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}})$ is shown in the following theorem.

Theorem 3.1: The conditional average SNR (3.4) admits the following form

$$\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}}) = \sum_{l=0}^{L} \left\{ \sum_{k_1=1}^{L} \sum_{k_2=k_1+1}^{L} \cdots \sum_{k_{l-1}=k_{l-2}+1}^{L} \times \sum_{k_l=k_{l-1}+1}^{L} \frac{\left| \sum_{i=1}^{L} c_i(l, k_1, \ldots, k_l) g_i \right|^2}{\sum_{i=1}^{L} |g_i|^2 d_i(l, k_1, \ldots, k_l)} \right\} \tag{3.6}$$

where

$$c_i(l, k_1, \ldots, k_l) \triangleq \sqrt{\eta \prod_{j=1}^{l} p_{k_j} \left(\prod_{j=1}^{l} (1 - p_{k_j}) \right)^{-1}}\ h_{r,i}\phi(\rho_i), \tag{3.7}$$

in which $\eta \triangleq \prod_{l=1}^{L}(1 - p_l)$ and $\phi(\cdot)$ is defined in (2.6),

$$d_i(l, k_1, \ldots, k_l) \triangleq |h_{r,i}|^2 \left[1 - \phi^2(\rho_i)\right] + \frac{\sigma_w^2}{P_d}, \tag{3.8}$$

and

$$\rho_i \triangleq \begin{cases} \hat{q}_i^t = \hat{q}_i \oplus 1, & i = k_1, k_2, \ldots, k_l \\ \hat{q}_i, & \text{otherwise.} \end{cases} \tag{3.9}$$

Proof: See Appendix. □

To maximize the conditional average SNR $\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}})$ given in (3.6) with respect to the beamforming weights g_i's, we shall first rewrite $\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}})$ in a more tractable form. Through further rearranging the indices in the multiple summations in (3.6), $\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}})$ can be expressed as a single sum of Rayleigh quotients. This is established in the next theorem.

Theorem 3.2*:* Let $\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}})$ be defined in (3.6). Then we have

$$\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}}) = \sum_{m=1}^{M} \frac{\left|\mathbf{c}_m^H \mathbf{g}\right|^2}{\mathbf{g}^H \mathbf{D}_m \mathbf{g}}, \tag{3.10}$$

in which $M = \sum_{l=0}^{L} C_l^L$, and, for each [3], $1 \leq m \leq M$,

$$\mathbf{c}_m^H \triangleq [c_1(l, k_1, \ldots, k_l), \ldots, c_L(l, k_1, \ldots, k_l)], \tag{3.11}$$

$$\mathbf{D}_m \triangleq diag\left\{d_1(l, k_1, \ldots, k_l), \ldots, d_L(l, k_1, \ldots, k_l)\right\}, \tag{3.12}$$

for certain $l, k_1, \ldots, k_l$. Given a particular set of indices $l, k_1, \ldots, k_l$ in the multiple summations in (3.6), the corresponding index m in (3.10) is determined according to

$$m = \delta(l) + \sum_{s_0=0}^{l-1} C_{s_0}^L + \sum_{\lambda=1}^{l-1} \sum_{s_\lambda=k_{\lambda-1}+1}^{k_\lambda-1} C_{l-\lambda}^{L-s_\lambda} + (k_l - k_{l-1}), \tag{3.13}$$

where $k_0 = 0$ and $\delta(\cdot)$ denotes the Kronecker delta function.

Proof: See Appendix. □

Based on (3.10), the beamforming weights can be obtained by solving the following optimization problem

$$\text{Maximize} \quad \sum_{m=1}^{M} \frac{\left|\mathbf{c}_m^H \mathbf{g}\right|^2}{\mathbf{g}^H \mathbf{D}_m \mathbf{g}} \text{ s.t. } \|\mathbf{g}\|_2^2 = P_d \tag{3.14}$$

where P_d denotes the total transmit power. However, since the cost function in (3.14) is a highly nonlinear function of g, a closed-form solution to (3.14) is hard to find. In the next subsection we propose an alternate approach to finding suboptimal beamforming weights.

B. Closed-Form Suboptimal Solution

To facilitate analysis, we go on to derive in the following theorem a tractable lower bound for $\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}})$. By conducting maximization with respect to this lower bound, we can then obtain a closed-form suboptimal solution.

Theorem 3.3*:* Let $\mathbf{c}_m$ and $\mathbf{D}_m$ be defined in (3.11) and (3.12). The following inequality holds:

$$\sum_{m=1}^{M} \frac{\left|\mathbf{c}_m^H \mathbf{g}\right|^2}{\mathbf{g}^H \mathbf{D}_m \mathbf{g}} \geq \frac{\mathbf{g}^H \mathbf{c}\mathbf{c}^H \mathbf{g}}{\mathbf{g}^H \mathbf{D} \mathbf{g}}, \tag{3.15}$$

[2]The notation $\oplus$ denotes the binary addition operation.

[3]The dependence of the index m in (3.10) on $l, k_1, \ldots, k_l$ is omitted to simplify notation.

where $\mathbf{c} \triangleq \sum_{m=1}^{M} \mathbf{c}_m$ and $\mathbf{D} \triangleq \sum_{m=1}^{M} \mathbf{D}_m$.

Proof: See Appendix. □

With the aid of (3.15), a suboptimal beamformer can be obtained based on maximization of the lower bound derived in (3.15):

$$\text{Maximize} \quad \frac{\mathbf{g}^H \mathbf{c}\mathbf{c}^H \mathbf{g}}{\mathbf{g}^H \mathbf{D} \mathbf{g}} \ \text{s.t.} \ \|\mathbf{g}\|_2^2 = P_d. \tag{3.16}$$

The solution to (3.16), denoted by $\tilde{\mathbf{g}}$, is precisely the dominant eigenvector of $\mathbf{D}^{-1}\mathbf{c}\mathbf{c}^H$. Since the matrix $\mathbf{D}^{-1}\mathbf{c}\mathbf{c}^H$ is of rank-one, we have

$$\tilde{\mathbf{g}} = c_1 \mathbf{D}^{-1}\mathbf{c}, \tag{3.17}$$

where c_1 is chosen so that $\|\tilde{\mathbf{g}}\|_2^2 = P_d$.

C. Simulation Results

In this section computer simulations are used to illustrate the performance of the proposed method. We consider a cooperative beamforming system with four relays $(L = 4)$ in a quasi-static fading environment, wherein the channels remain constant within a coherent interval and can vary independently across different intervals. During each coherent interval, the channel gains of both the S-R and R-D links are drawn independently according to $\mathcal{CN}(0,1)$. At each relay node, the SNR of the S-R link is computed, and is then quantized into one bit; the quantization threshold is designed in accordance with the rule in [18, p-4779]. The quantized bit is transmitted from each relay with on-off signaling over a BSC, with crossover probability p_i drawn independently from the uniform distribution over the interval $[0.05, 0.1]$. At the destination, the received one-bit SNR message is decoded for beamforming design. In each coherent interval, a total number of $T = 5000$ BPSK source symbols are generated; the symbol decision rule at the destination follows the scheme in [18, p-4776]. The total power of transmit beamforming is set to be $P_d = 1$. The BER curves in our simulations are obtained by averaging over the detection results of 100000 coherent intervals (channel realizations). For fixed average S-R SNR $\bar{\gamma}_s = 20$ dB, Fig. 2 compares the BER curves of the proposed beamformer (3.17) with the solution in [18] at various average R-D SNR, defined to be $\bar{\gamma}_d \triangleq P_d\sigma_r^2/\sigma_w^2 = \sigma_w^{-2}$[2]. As can be seen from the figure, the proposed scheme outperforms the method in [18], especially when SNR is high; this is not unexpected since the solution in [18] is designed under the idealized assumption that the one-bit message is received at the destination without errors. Also, it is seen from the figure that the performance improvement is slight when SNR is below 10 dB. This phenomenon is caused by the fact that, for the considered cooperative beamforming scheme, S-R-link CSI mismatch is not a dominant factor for the BER performance in the low-to-medium SNR regime; this fact has been confirmed by the simulation results provided in [18, p-4780]. With fixed $\bar{\gamma}_s = 20$ dB and assuming that the cross-over probabilities p_i's of the BSC are identical for all i (thus $p_i = p,\ 1 \le i \le 4$), Fig. 3 further shows the BER of the two methods for $0 \le p \le 0.2$ with respect to three different average R-D link SNR $\bar{\gamma}_d = 5$,10, and 15 dB. It can be seen that, when $p = 0$ (i.e., the one-bit message is perfectly received), the proposed scheme and the method in [18] yields an identical performance. The result is not unexpected since, with $p = 0$, the considered objective function (3.10) reduces to the single term (2.5)($M = 1$), and, hence, equality holds in (3.15): this then implies that the proposed solution (3.17) is exactly the beamformer given by (2.8). For $p > 0$, our solution is seen to be quite robust against the increase of p, thereby confirming the advantage of the proposed design.

D. Extension to Multiple-Bit Case

It is noted that multiple-bit quantization of the S-R link SNR is considered in the problem formulation in [18]. Simulation study therein shows that the beamformer designed based on even one-bit quantization can perform quite close to that designed in accordance with the full S-R link CSI. Hence, to minimize the signaling overheads for CSI transmission, the problem formulation of our approach focuses on the one-bit assumption. We would like to remark that the proposed approach can be directly extended to the multiple-bit case. Specifically, assume that (i) B bits $B \ge 1$ are used at each relay node for S-R link SNR quantization, and (ii) the transmission of the B bits from the ith relay to the destination is over B parallel BSC's each with crossover probability p_i, $1 \le i \le L$. By following essentially the same procedures as in Section III-A, we can derive an analogous formula for the corresponding average receive SNR as in Proposition 3.1, based on which the proposed analytic beamforming design as in Section III-B can be directly used to obtain a suboptimal solution. Due to space limitation, we refer the details to the supplementary results [26].

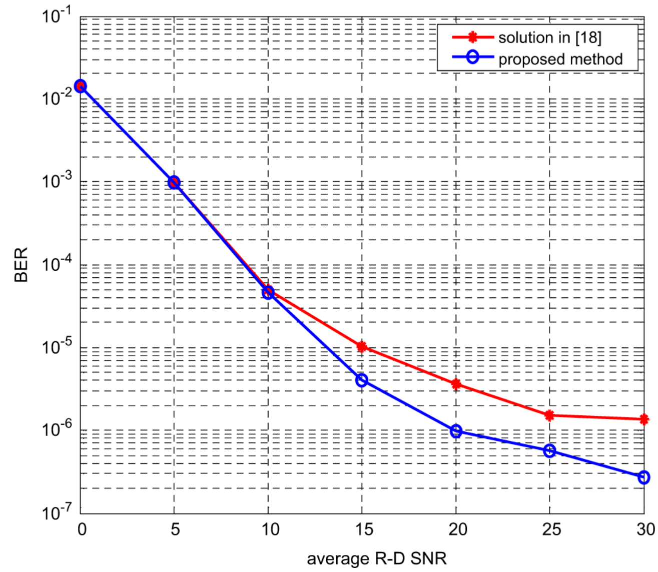

Fig. 2. Simulated BER of the proposed beamformer (3.17) and the solution in [18].

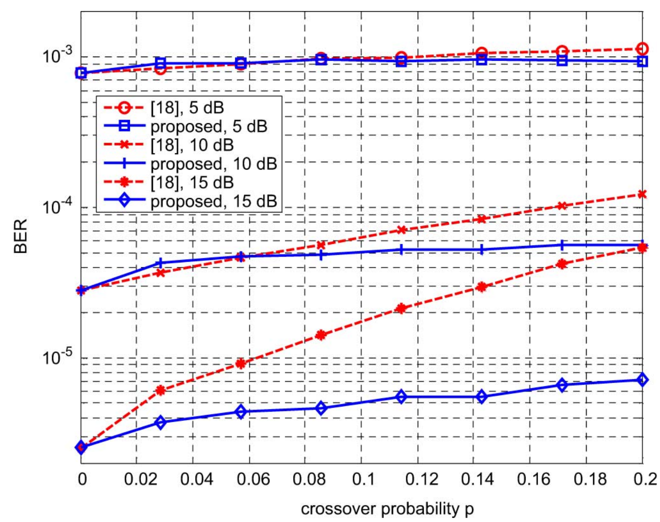

Fig. 3. BER results of two methods with respect to different cross-over probabilities.

IV. Conclusions

We study the problem of low-overhead cooperative beamforming design for WSN, under the assumption that the SNR of each S-R link is quantized into one bit, which is transmitted through a BSC to the destination. Given the received one-bit message, the performance measure is the conditional receive SNR averaged over the conditional bit-flipping distributions. A closed-form expression for the considered objective function is derived, and is seen to be a complicated function of the beamforming coefficients. A tractable lower bound of the conditional average SNR is derived. By conducting maximization of this lower bound, a suboptimal beamformer can be obtained by solving a generalized eigenvalue problem. Simulation results confirmed the performance advantage of the proposed design as compared with an existing solution designed based on the perfect knowledge of the one-bit S-R SNR messages. Our report has presented an original study of the cooperative beamforming design in the presence of CSI transmission/feedback errors. We will extend the current results by taking into account the effect of the estimation errors of the R-D link CSI.

APPENDIX

Proof of Theorem 3.1: To derive (3.6), we shall find an explicit expression for $\beta_l \triangleq \sum_{\tilde{\mathbf{q}}\in S_l(\hat{\mathbf{q}})} \Pr(\tilde{\mathbf{q}}\,|\hat{\mathbf{q}})\gamma_{dq}(\mathbf{g},\mathbf{h_r},\tilde{\mathbf{q}}), l=0,\ldots,L$. The term β_0 represents the case with $\mathbf{q}=\hat{\mathbf{q}}$. It then follows immediately that

$$\beta_0 = \Pr(\tilde{\mathbf{q}}=\hat{\mathbf{q}}\,|\hat{\mathbf{q}})\gamma_{dq}(\mathbf{g},\mathbf{h_r},\hat{\mathbf{q}}) \overset{(a)}{=} \underbrace{\prod_{k=1}^{L}(1-p_k)}_{=\Pr(\tilde{\mathbf{q}}=\hat{\mathbf{q}}|\hat{\mathbf{q}})} \frac{\left|\sum_{i=1}^{L} h_{r,i}g_i\phi(\hat{q}_i)\right|^2}{\sum_{i=1}^{L}|h_{r,i}|^2|g_i|^2[1-\phi^2(\hat{q}_i)]+\sigma_w^2}, \quad \text{(A1)}$$

where (a) follows from (2.5) and (3.5). The term β_1 represents the event that the true q differs from $\hat{\mathbf{q}}$ in one bit. Given $\hat{\mathbf{q}}$, there are totally $|S_1(\hat{\mathbf{q}})|=C_1^L=L$ possible candidate $\tilde{\mathbf{q}}$'s. Therefore, β_1 can be accordingly expressed as

$$\begin{aligned}\beta_1 &= \sum_{\tilde{\mathbf{q}}\in S_1(\hat{\mathbf{q}})} \Pr(\tilde{\mathbf{q}}\,|\hat{\mathbf{q}})\gamma_{dq}(\mathbf{g},\mathbf{h_r},\tilde{\mathbf{q}}) \\ &= \sum_{k_1=1}^{L}\left[p_{k_1}\prod_{j\neq k_1}(1-p_j)\right]\gamma_{dq}(\mathbf{g},\mathbf{h_r},\tilde{\mathbf{q}}) \\ &= \sum_{k_1=1}^{L}\left[p_{k_1}\prod_{j\neq k_1}(1-p_j)\right] \\ &\quad\times\frac{\left|h_{r,k_1}g_{k_1}\phi(\hat{q}_{k_1}^t)+\sum_{i\neq k_1}h_{r,i}g_i\phi(\hat{q}_i)\right|^2}{\left|h_{r,k_1}\right|^2\left|g_{k_1}\right|^2\left[1-\phi^2(\hat{q}_{k_1}^t)\right]+\sum_{i\neq k_1}\left|h_{r,i}\right|^2|g_i|^2\left[1-\phi^2(\hat{q}_i)\right]+\sigma_w^2} \\ &\overset{(b)}{=} \sum_{k_1=1}^{L}\frac{\left|\sum_{i=1}^{L}c_i(1,k_1)g_i\right|^2}{\sum_{i=1}^{L}|g_i|^2 d_i(1,k_1)},\end{aligned} \quad \text{(A2)}$$

where (b) follows after some straightforward manipulations. The term β_2 stands for the event that $\hat{\mathbf{q}}$ differs from the true q in two bits. Given $\hat{\mathbf{q}}$, there are totally $|S_2(\hat{\mathbf{q}})|=C_2^L=L(L-1)/2$ possible candidate $\tilde{\mathbf{q}}$'s in this case. By repeating the above arguments, it can be directly verified that [see (A3) at the bottom of the page]. Based on the same idea and procedures, it can be readily shown that, for $l=1,\ldots,L$, see (A4) at the bottom of the page. (3.6) follows since $\bar{\gamma}_{dq}(\mathbf{g},\mathbf{h_r},\hat{\mathbf{q}})=\sum_{l=0}^{L}\beta_l$. □

$$\begin{aligned}\beta_2 &= \sum_{k_1=1}^{L}\sum_{k_2=k_1+1}^{L}\left\{\begin{array}{c}\eta\frac{p_{k_1}p_{k_2}}{(1-p_{k_1})(1-p_{k_2})}\times \\ \frac{\left|\sum_{i=k_1,k_2}h_{r,i}g_i\phi(\hat{q}_i^t)+\sum_{i\neq k_1,k_2}h_{r,i}g_i\phi(\hat{q}_i)\right|^2}{\sum_{i=k_1,k_2}\left|h_{r,i}\right|^2|g_i|^2\left(1-\phi^2(\hat{q}_i^t)\right)+\sum_{i\neq k_1,k_2}\left|h_{r,i}\right|^2|g_i|^2\left(1-\phi^2(\hat{q}_i)\right)+\sigma_w^2}\end{array}\right\} \\ &= \sum_{k_1=1}^{L}\sum_{k_2=k_1+1}^{L}\frac{\left|\sum_{i=1}^{L}c_i(2,k_1,k_2)g_i\right|^2}{\sum_{i=1}^{L}|g_i|^2 d_i(2,k_1,k_2)}.\end{aligned} \quad \text{(A3)}$$

$$\begin{aligned}\beta_l &= \sum_{k_1=1}^{L}\sum_{k_2=k_1+1}^{L}\cdots\sum_{k_l=k_{l-1}+1}^{L}\left\{\begin{array}{c}\frac{\eta\prod_{j=1}^{l}p_{k_j}}{\prod_{n=1}^{l}(1-p_{k_n})}\times \\ \frac{\left|\sum_{i=k_1,\ldots,k_l}h_{r,i}g_i\phi(\hat{q}_i^t)+\sum_{i\neq k_1,\ldots,k_l}^{L}h_{r,i}g_i\phi(\hat{q}_i)\right|^2}{\sum_{i=k_1,\ldots,k_l}\left|h_{r,i}\right|^2|g_i|^2\left(1-\phi^2(\hat{q}_i^t)\right)+\sum_{i\neq k_1,\ldots,k_l}^{L}\left|h_{r,i}\right|^2|g_i|^2\left(1-\phi^2(\hat{q}_i)\right)+\sigma_w^2}\end{array}\right\} \\ &= \sum_{k_1=1}^{L}\sum_{k_2=k_1+1}^{L}\cdots\sum_{k_l=k_{l-1}+1}^{L}\frac{\left|\sum_{i=1}^{L}c_i(l,k_1,\ldots,k_l)g_i\right|^2}{\sum_{i=1}^{L}|g_i|^2 d_i(l,k_1,\ldots,k_l)}\end{aligned} \quad \text{(A4)}$$

Proof of Theorem 3.2: All we have to do is to rewrite the multiple summations in (3.6) as a single summation in the form of (3.10), and then to provide an explicit relation between m in (3.10) and the multiple indices $l, k_1, \ldots, k_l$ involved in (3.6). For ease of discussion, recall that $\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}}) = \sum_{l=0}^{L} \beta_l$, where β_0 is defined in (A1) and β_l for $l \neq 0$ is given by (A4). Note that, for each $0 \leq l \leq L$, there are totally C_l^L terms in β_l. The main procedures for deriving (3.10) can be summarized as follows: (1) exhaustively list all the $C_0^L + C_1^L + \cdots + C_L^L$ terms in (3.6) in the increasing order of l; (2) particularly, in $\beta_l (l \geq 1)$, the C_l^L terms in the l-fold multiple summations are listed in the following way: starting from $k_1 = 1$, exhaustively list all involved terms indexed by this k_1 in the remaining summations, and then proceed to $k_1 = 2$, and so forth. Based on such procedures, for given $l, k_1, \ldots, k_l$, the corresponding m given in (3.13) can then be obtained by induction and some straightforward manipulations.

The first term in (3.10) (indexed by $m = 1$) is simply β_0, thus $\frac{|\mathbf{c}_1^H \mathbf{g}|^2}{\mathbf{g}^H \mathbf{D}_1 \mathbf{g}} = \frac{\left|\sum_{i=1}^{L} c_i(l=0) g_i\right|^2}{\sum_{i=1}^{L} |g_i|^2 d_i(l=0)}$. Now consider $l \neq 0$, and our purpose is to determine for the particular indices $(l, k_1, \ldots, k_l)$ the corresponding m. For this we first note that the total number of terms contained in $\beta_0, \ldots, \beta_{l-1}$ is $\sum_{i=0}^{l-1} C_i^L$. For the particular l, let us likewise exhaustively list all the terms in β_l and collect them into a set $\mathcal{L}_l$. Assume that the considered term indexed by $(l, k_1, \ldots, k_l)$ is exactly the K-th element in $\mathcal{L}_l$. Then it follows immediately that $m = \sum_{i=0}^{l-1} C_i^L + K$. Hence it suffices to determine K.

Towards this end, we recall that, for a fixed l, thus totally l flipped bits, the indices $k_1 < k_2 < \cdots < k_l$, where $k_1, \ldots, k_l \in \{1, \ldots, L\}$, denote the locations at which bit errors occur. Consequently, it is noted that the first C_{l-1}^{L-1} terms [4] in $\mathcal{L}_l$ are those indexed by $k_1 = 1$, the next C_{l-1}^{L-2} terms in $\mathcal{L}_l$ are those indexed by $k_1 = 2$, and so on. Hence, in $\mathcal{L}_l$ there are $\sum_{s_1=1}^{k_1-1} C_{l-1}^{L-s_1}$ terms listed before the terms indexed by a given k_1.

Now let us further list the terms indexed by such a k_1 to obtain a set $\mathcal{S}_{k_1}$. By following the same argument, the first $C_{l-2}^{L-(k_1+1)}$ terms in $\mathcal{S}_{k_1}$ are indexed by $k_2 = k_1 + 1$, the next $C_{l-2}^{L-(k_1+2)}$ terms are indexed by $k_2 = k_1 + 2$, and so forth. Hence, it can be readily deduced that there are totally $\sum_{s_2=k_1+1}^{k_2-1} C_{l-2}^{L-s_2}$ terms listed before the terms indexed by a given k_2 in $\mathcal{S}_{k_1}$. As a result, for a given pair of k_1 and k_2, a total number of $\sum_{s_1=1}^{k_1-1} C_{l-1}^{L-s_1} + \sum_{s_2=k_1+1}^{k_2-1} C_{l-2}^{L-s_2}$ terms are listed before the terms indexed by such (k_1, k_2) in $\mathcal{L}_l$. By induction, it can be concluded that there are totally $\sum_{\lambda=1}^{l-1} \sum_{s_\lambda=k_{\lambda-1}+1}^{k_\lambda-1} C_{l-\lambda}^{L-s_\lambda}$ terms listed before the terms indexed by a given set of $k_0 = 0, k_1, \ldots, k_{l-1}$ in $\mathcal{L}_l$. Hence, for the considered $l, k_1, \ldots, k_l$, the associated K can be computed as

$$
\begin{aligned}
K &= \sum_{\lambda=1}^{l-1} \sum_{s_\lambda=k_{\lambda-1}+1}^{k_\lambda-1} C_{l-\lambda}^{L-s_\lambda} + (k_l - (k_{l-1}+1) + 1) \\
&= \sum_{\lambda=1}^{l-1} \sum_{s_\lambda=k_{\lambda-1}+1}^{k_\lambda-1} C_{l-\lambda}^{L-s_\lambda} + (k_l - k_{l-1}). \qquad \text{(A5)}
\end{aligned}
$$

With (A5), the desired index m is thus

$$
m = \begin{cases} \sum_{i=0}^{l-1} C_i^L + \sum_{\lambda=1}^{l-1} \sum_{s_\lambda=k_{\lambda-1}+1}^{k_\lambda-1} C_{l-\lambda}^{L-s_\lambda} + (k_l - k_{l-1}), & l > 0; \\ 1, & l = 0. \end{cases} \qquad \text{(A6)}
$$

The assertion follows from (A6). □

[4]Since $k_1 = 1$ and the relation $k_1 < k_2 < \cdots < k_l$ must hold, the plausible values of k_i, $2 \leq i \leq l$, take only $L-1$ levels, namely, $\{2, 3, \ldots, L\}$. Hence, there are totally C_{l-1}^{L-1} possible error patterns (equal to the total number of combinations of $l-1$ out of $L-1$ levels).

Proof of Theorem 3.3: By the Cauchy-Schwartz inequality, we have

$$
\underbrace{\left(\sum_{m=1}^{M} \frac{\left|\mathbf{c}_m^H \mathbf{g}\right|^2}{\mathbf{g}^H \mathbf{D}_m \mathbf{g}}\right)}_{\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}})} \left(\sum_{m=1}^{M} \mathbf{g}^H \mathbf{D}_m \mathbf{g}\right) \geq \left(\sum_{m=1}^{M} \left|\mathbf{c}_m^H \mathbf{g}\right|\right)^2. \qquad \text{(A7)}
$$

Then,

$$
\begin{aligned}
\bar{\gamma}_{dq}(\mathbf{g}, \mathbf{h_r}, \hat{\mathbf{q}}) &\geq \frac{\left(\sum_{m=1}^{M} \left|\mathbf{c}_m^H \mathbf{g}\right|\right)^2}{\left(\sum_{m=1}^{M} \mathbf{g}^H \mathbf{D}_m \mathbf{g}\right)} \geq \frac{\left|\sum_{m=1}^{M} \mathbf{c}_m^H \mathbf{g}\right|^2}{\left(\sum_{m=1}^{M} \mathbf{g}^H \mathbf{D}_m \mathbf{g}\right)} \\
&= \frac{\left|\mathbf{c}^H \mathbf{g}\right|^2}{\mathbf{g}^H \mathbf{D} \mathbf{g}} = \frac{\mathbf{g}^H \mathbf{c} \mathbf{c}^H \mathbf{g}}{\mathbf{g}^H \mathbf{D} \mathbf{g}}, \qquad \text{(A8)}
\end{aligned}
$$

which proves (3.15). □

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