IEEE Transactions on Power Systems, Vol. 12, No. 4, November 1997

A New Dual-type Method Used in Solving Optimal Power Flow Problems

Ch'i-Hsin Lin Shin-Yeu Lin, Member, IEEE
Department of Control Engineering
National Chiao Tung University
Hsinchu, TAIWAN

Abstract-In the framework of SQP method for OPF problems, we propose a new dual-type method for solving the QP subproblems induced in the SQP method. Our method achieves some attractive features; it is computationally efficient and numerically stable. The computational formulae of our method are simple, concise and easy to be programmed. We have tested our method for OPF problems on several power systems including a 2500-bus system.

I. INTRODUCTION

Numerous numerical techniques [1]–[10] have been developed for solving optimal power flow (OPF) problems. These methods are based on various mathematical programming techniques such as successive linear programming (SLP) method [1]–[3], successive quadratic programming (SQP) method [4]–[6], Lagrangian Newton method [7]–[9] or the newly developed interior point (IP) method [10]. Each of the above methods has its special features and advantages. Observing the SQP method which possesses a quadratic convergence rate, however, the reduced Hessian is dense. The innovative Lagrangian Newton method [7], [8] successfully exploits the sparsity structure of the system; however, efforts are needed to cope with the difficulties of identifying the binding inequality constraints and the possibility of singular Hessian matrix as pointed out by Monticelli and Liu in [9], and they provided remedied strategies to overcome those pitfalls. Nonetheless, the method in [9] as well as the method in [7] and [8] require sophisticated software programming skill.

PE-549-PWRS-0-01-1997 A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for publication in the IEEE Transactions on Power System. Manuscript submitted August 1, 1995; made available for printing January 16, 1997.

In this paper, we use the framework of SQP method and propose a new dual-type method to solve the QP subproblems. Our method intends to achieve the following features: (i) good convergence rate, (ii) no need to identify the binding constraints, (iii) computational efficiency, (iv) easy programming and (v) numerical stability.

In the framework of SQP method, our method will inherit the advantage of fast convergence as demonstrated in Section V. Features (ii)–(iv) will be achieved by the proposed dual-type method as explained in Section III. To address feature (v), we provide a mathematical proof for the convergence of the proposed dual-type method in the Appendix.

II. STATEMENT OF THE OPTIMAL POWER FLOW PROBLEM

Throughout this paper, if not specifically explained, we assume the following notations:

e, f: state variables represent the real and imaginary part of the complex voltage.

u: control variables including real and reactive power generation, P_G and Q_G, transformer tap ratio, switching capacitor banks,..., etc..

$x : \boxminus (u, e, f)$ denotes the vector of all variables.

$F(x)$: objective function which can be total generation cost, pollution cost, system losses,...,etc.

$g(x)$: real and reactive power mismatch.

$h(e, f)$: functional inequality constraints such as security constraints on line flows for specified lines.

V: vector of voltage magnitude, $V_i \equiv \sqrt{e_i^2 + f_i^2}$.

$\bar{V}$, $\underline{V}$: upper and lower limits of voltage magnitude.

$\bar{u}$, $\underline{u}$: upper and lower limits of control variables u, such as $\bar{P}_G$, $\underline{P}_G$, $\bar{Q}_G$, $\underline{Q}_G$, etc..

$\bar{h}$, $\underline{h}$: upper and lower limits of functional inequality constraints.

k, t : iteration index.

$\alpha(k), \beta(t)$: step-size.

diag$[\Box]$: a diagonal matrix formed by the diagonal terms of the matrix $\Box$.

$\Delta(\cdot)$: the increment of the vector

$P(z)$: penalty function for the violations of constraints.
w : penalty coefficient.
λ : the Lagrange multiplier vector.
$\phi(\lambda)$: the dual function of the QP subproblem.
$\phi^u(\lambda)$: unconstrained dual function.
Ω : the set formed by the inequality constraints of the QP subproblems.
$\gamma, \eta, \sigma_P, \sigma_D$: positive real numbers.
τ_P, τ_D : $\in (0, 1)$.

The OPF problem can be stated as follows:

$$\begin{aligned} &\min_x \quad F(x) \\ &g(x) = 0 \\ &\underline{V} \leq V \leq \bar{V} \\ &\underline{u} \leq u \leq \bar{u} \end{aligned} \tag{1}$$

Remark 1 *For the purpose of explanation, we do not include the functional inequality constraints, $\underline{h} \leq h(e, f) \leq \bar{h}$, in (1), however, this will be treated afterwards.*

III. SOLUTION METHOD

A. The SQP Method

The SQP method uses the following iterations to solve the OPF problem given in (1):

$$x(k+1) = x(k) + \alpha(k)\Delta x(k) \tag{2}$$

where $\alpha(k)$ is a step-size, and $\Delta x(k)$ is the solution of the following QP subproblem:

$$\begin{aligned} &\min_{\Delta x} \frac{\partial F(x(k))}{\partial x}\Delta x + \frac{1}{2}\Delta x^T H \Delta x \\ &g(x(k)) + \frac{\partial g(x(k))}{\partial x}\Delta x = 0, \\ &\underline{V} \leq V(k) + \frac{\partial V(k)}{\partial e}\Delta e + \frac{\partial V(k)}{\partial f}\Delta f \leq \bar{V}, \\ &\underline{u} \leq u(k) + \Delta u \leq \bar{u}, \end{aligned} \tag{3}$$

in which the diagonal matrix H is defined by

$$H \equiv \operatorname{diag}[\frac{\partial^2 F(x(k))}{\partial x^2}] + \frac{1}{2}\eta I \tag{4}$$

where I is an identity matrix, and η is a small positive real number but enough to make H positive definite.

Step-size determination. Concerning the step-size determination rule, a cubic fit or quadratic fit method [14] is popular especially for the unconstrained Lagrangian formulation in the Lagrangian Newton method [7]. However, in the SQP method, while reducing the value of objective function $F(x)$, we should prevent $x(k+1)$ being too far away from the nonlinear constraints in (1). Therefore, we employ Armijo's rule [11], which considers the penalty of violating constraints, for the determination of step-size $\alpha(k)$ as follows:

Let $0 < \tau_P < 1$, $\sigma_P > 0$, then $\alpha(k)$ is set to be $\tau_P^{m(k)}\sigma_P$ where $m(k)$, the power of τ_P, is the smallest nonnegative integer m such that the following inequality holds

$$\begin{aligned} &F(x(k) + \tau_P^m \sigma_P \Delta x(k)) + wP(x(k) + \tau_P^m \sigma_P \Delta x(k)) \\ \leq\ &F(x(k)) + wP(x(k)) - \frac{\gamma}{2}\tau_P^m \sigma_P \Delta x^T(k) H \Delta x(k) \end{aligned} \tag{5}$$

where the penalty function $P(x)$ represents the penalty for the violations on the constraints and is defined by

$$\begin{aligned} P(x) \equiv\ &\max\Big(\max_i\{|g_i(x)|\},\ \max_i\{V_i - \bar{V}_i, \underline{V}_i - V_i\}, \\ &\max_i\{u_i - \bar{u}_i, \underline{u}_i - u_i\},\ 0\Big), \end{aligned} \tag{6}$$

w is a weighting penalty coefficient, and $\gamma \in (0, \frac{1}{2})$. Although Armijo's rule seems inefficient, in most of our test results, the inequality test (5) is passed for $m = 0$ most of the times. Convergence of the SQP method (2) with $\alpha(k)$ determined according to (5) has been shown in [11].

Treatment of discrete control variables. In the QP subproblem (3), we treat all the incremental variables Δx as continuous variables. However, the updated formula (2) may make the updated discrete control variables not having the exact discrete values. To remedy this pitfall, we apply an approximation rule for the update of discrete control variables as follows:

Let u_d be the subvector of u denote the discrete control variables, such as switching capacitor banks, transformer tap ratio,..., etc., then the continuous-value $\Delta u_d(k)$ is the increment of $u_d(k)$ obtained from solving (3). The approximation rule for the update of $u_d(k+1)$ is

$$u_d(k+1) = \lfloor u_d(k) + \alpha(k)\Delta u_d(k) \rfloor, \tag{7}$$

where $\lfloor(\cdot)\rfloor$ denote the closest discrete-value to the value of $(\cdot)$. Then $u_d(k+1)$ obtained from (7) will be the closest discrete-value of u_d to the value of $u_d(k) + \alpha(k)\Delta u_d(k)$.

Comment 1 *When there exist integer variables in a nonlinear programming problem, the computation is very involved. Therefore, heuristic methods are developed to handle integer variables in most of practical application problems such as the approximation rule presented here. Though our heuristic rule works well in our problem as shown in Section V, there is no guarantee that this rule will obtain satisfactory solutions in general nonlinear programming problems consisting of integer variables.*

B. The Proposed Dual-type Method.

Since in (3), all variables Δx are continuous variables, the objective function is strictly convex, and the constraints are linear, we can solve the dual problem of (3) instead of solving (3) directly provided that the solution of (3) exists. This is well-known Duality Theory [14].

The dual problem of the QP subproblem (3) is

$$\max_{\lambda} \phi(\lambda) \tag{8}$$

where the dual function

$$\begin{aligned}\phi(\lambda) &= \min_{\Delta x \in \Omega} \frac{\partial F(x(k))}{\partial x}\Delta x + \frac{1}{2}\Delta x^T H \Delta x \\ &\quad + \lambda^T[g(x(k)) + \frac{\partial g(x(k))}{\partial x}\Delta x],\end{aligned} \tag{9}$$

in which the set Ω denotes the set of inequality constraints in (3) such that $\Omega \equiv \{\Delta x | \underline{V} \le V(k) + \frac{\partial V(k)}{\partial e}\Delta e + \frac{\partial V(k)}{\partial f}\Delta f \le \bar{V},\ \underline{u} \le u(k) + \Delta u \le \bar{u}\}$.

The proposed dual-type method uses the following iterations to solve (8):

$$\lambda(t+1) = \lambda(t) + \beta(t)\Delta\lambda(t), \tag{10}$$

where $\beta(t)$ is a step-size, and $\Delta\lambda(t)$ is obtained from solving the linear equations

$$[\frac{\partial^2 \phi^u(\lambda(t))}{\partial \lambda^2} - \delta I]\Delta\lambda(t) + \frac{\partial \phi(\lambda(t))}{\partial \lambda} = 0, \tag{11}$$

in which $\delta > 0$, I is an identity matrix and the unconstrained dual function ϕ^u is defined by deleting the primal-variable constraints $\Delta x \in \Omega$ in $\phi(\lambda)$ shown in (9) such that

$$\begin{aligned}\phi^u(\lambda) &= \min_{\Delta x} \frac{\partial F(x(k))}{\partial x}\Delta x + \frac{1}{2}\Delta x^T H \Delta x \\ &\quad + \lambda^T[g(x(k)) + \frac{\partial g(x(k))}{\partial x}\Delta x].\end{aligned} \tag{12}$$

The first derivative $\frac{\partial \phi(\lambda(t))}{\partial \lambda}$ and the approximate Hessian matrix $\frac{\partial^2 \phi^u(\lambda(t))}{\partial \lambda^2}$ can be computed based on the formula given in [14] as follows:

$$\frac{\partial \phi(\lambda(t))}{\partial \lambda} = g(x(k)) + \frac{\partial g(x(k))}{\partial x}\Delta\hat{x} \tag{13}$$

$$\frac{\partial^2 \phi^u(\lambda(t))}{\partial \lambda^2} = -\frac{\partial g(x(k))}{\partial x} H^{-1} \frac{\partial g(x(k))^T}{\partial x}, \tag{14}$$

where $\Delta\hat{x}$ in (13) is the solution of $\phi(\lambda(t))$ for a given $\lambda(t)$, that is the constrained minimization problem on the RHS of (9) with $\lambda = \lambda(t)$. We will present the method using Projection Theory to solve $\Delta\hat{x}$ later.

Since $\frac{\partial^2 \phi^u(\lambda(t))}{\partial \lambda^2}$ is at least negative semidefinite, $\frac{\partial^2 \phi^u(\lambda(t))}{\partial \lambda^2} - \delta I$ is negative definite. This ensures that $\Delta\lambda(t) = [\frac{\partial^2 \phi^u(\lambda(t))}{\partial \lambda^2} - \delta I]^{-1}\frac{\partial \phi(\lambda(t))}{\partial \lambda}$ is an ascent direction to maximize $\phi(\lambda)$. However, to guarantee the updated point $\lambda(t+1)$ will increase the value of $\phi(\lambda)$, we develop an Armijo's rule to determine the step-size $\beta(t)$ as follows:

Let $\beta(t) = \tau_D^{m(t)}\sigma_D$, where $0 < \tau_D < 1$, $\sigma_D > 0$, and $m(t)$ is the smallest nonnegative integer m such that

$$\phi(\lambda(t) + \tau_D^m \sigma_D \Delta\lambda(t)) \ge \phi(\lambda(t)) + \frac{\delta \tau_D^m \sigma_D}{2}\|\Delta\lambda(t)\|_2^2 . \tag{15}$$

A sketch of the mathematical proof for the justification of (15) and the convergence of (10) is given in the Appendix.

Remark 2 *Since the objective function $\phi(\lambda)$ in (8) is continuous and quadratic, it is practically suitable to use a cubic fit or quadratic fit method to determine the step-size $\beta(t)$. On account of giving a rigorous mathematical proof, we prefer to use Armijo's rule here.*

Applicability of sparse matrix technique. The nonzero elements of the fixed-dimension, constant matrix $\frac{\partial^2 \phi^u(\lambda(t))}{\partial \lambda^2}$ in (14) as well as $\frac{\partial^2 \phi^u(\lambda(t))}{\partial \lambda^2} - \delta I$ have the same structure as the bus admittance matrix of the power network. Therefore, we may employ a *sparse matrix technique* to solve linear equations (11).

However, to set up $\frac{\partial \phi(\lambda(t))}{\partial \lambda}$ in (11), we need to compute $\Delta\hat{x}$ first as shown in (13).

Applicability of Projection Theory. $\Delta\hat{x}$ is the solution of the constrained minimization problem on the RHS of (9) with $\lambda = \lambda(t)$ which can be solved in two phases using Projection Theory.

Phase 1: Obtain the solution $\Delta\tilde{x}$ of $\phi^u(\lambda(t))$ for a given $\lambda(t)$, that is the unconstrained minimization problem on the RHS of (12) with $\lambda = \lambda(t)$.

Phase 2: Project $\Delta\tilde{x}$ onto the constraint set Ω, and the resulting projection is $\Delta\hat{x}$.

The validity of this two-phase method is justified based on Projection Theory in [12] and is shown in Theorem 1 and Theorem 2 in the Appendix. In the following, we will describe the detailed computational formulae of this two-phase method.

From (12), the solution of the unconstrained minimization problem $\Delta\tilde{x}$ which is $(\Delta\tilde{u}, \Delta\tilde{e}, \Delta\tilde{f})$, can be analytically derived by

$$\Delta\tilde{x} = -H^{-1}[\frac{\partial F(x(k))^T}{\partial x} + \frac{\partial g(x(k))^T}{\partial x}\lambda] \tag{16}$$

Since H is a diagonal positive definite matrix, no extra effort is needed to compute H^{-1} in (16).

The inequality constraints for $(\Delta e, \Delta f)$ and Δu are decoupled, and these inequality constraints are also decoupled for different buses; thus, the projection can be treated separately for each individual bus. The projection of $\Delta\tilde{u}$ onto the set Ω is trivial and can be computed in the following: Let $\Delta\hat{u}_i$ be the projection of $\Delta\tilde{u}_i$ onto the subset $\{\Delta u_i | \underline{u}_i \le u_i(k) + \Delta u_i \le \bar{u}_i\}$, then

$$\Delta\hat{u}_i = \begin{cases} \bar{u}_i - u_i(k), & \text{if } u_i(k) + \Delta\tilde{u}_i > \bar{u}_i, \\ \underline{u}_i - u_i(k), & \text{if } u_i(k) + \Delta\tilde{u}_i < \underline{u}_i, \\ \Delta\tilde{u}_i, & \text{otherwise.} \end{cases} \tag{17}$$

Though the projection of $(\Delta\tilde{e},\ \Delta\tilde{f})$ onto the set Ω is more complicated, by simple geometrical calculation, we can obtain the following: Let $(\Delta\hat{e}_i, \Delta\hat{f}_i)$ be the projection of $(\Delta\tilde{e}_i, \Delta\tilde{f}_i)$ onto the subset $\{(\Delta e_i, \Delta f_i) | \underline{V}_i \le \sqrt{e_i(k)^2 + f_i(k)^2} + \frac{e_i(k)\Delta e_i}{\sqrt{e_i(k)^2+f_i(k)^2}} + \frac{f_i(k)\Delta f_i}{\sqrt{e_i(k)^2+f_i(k)^2}} \le \bar{V}_i\}$ of Ω. Let $\tau_1 = [\bar{V}_i\sqrt{e_i(k)^2 + f_i(k)^2} - (e_i(k)^2 + f_i(k)^2)]$, $\tau_2 = [\underline{V}_i\sqrt{e_i(k)^2 + f_i(k)^2} - (e_i(k)^2 + f_i(k)^2)]$, and $\tau_3 = f_i(k)\Delta\tilde{e}_i - e_i(k)\Delta\tilde{f}_i$ then

$$\Delta\hat{e}_i = \begin{cases} (e_i(k)\tau_1 + f_i(k)\tau_3)/(e_i^2(k) + f_i^2(k)) \\ \quad \text{if } e_i(k)\Delta\tilde{e}_i + f_i(k)\Delta\tilde{f}_i > \tau_1, \\ (e_i(k)\tau_2 + f_i(k)\tau_3)/(e_i^2(k) + f_i^2(k)) \\ \quad \text{if } e_i(k)\Delta\tilde{e}_i + f_i(k)\Delta\tilde{f}_i < \tau_2, \\ \Delta\tilde{e}_i, \quad \text{otherwise,} \end{cases} \tag{18}$$

$$\Delta\hat{f}_i = \begin{cases} (f_i(k)\tau_1 - e_i(k)\tau_3)/(e_i^2(k) + f_i^2(k)), \\ \quad \text{if } e_i(k)\Delta\tilde{e}_i + f_i(k)\Delta\tilde{f}_i > \tau_1, \\ (f_i(k)\tau_2 - e_i(k)\tau_3)/(e_i^2(k) + f_i^2(k)), \\ \quad \text{if } e_i(k)\Delta\tilde{e}_i + f_i(k)\Delta\tilde{f}_i < \tau_2, \\ \Delta\tilde{f}_i, \quad \text{otherwise.} \end{cases} \tag{19}$$

Remark 3 *The reason that we do not use polar coordinate for bus voltage is the projection of phase angle onto the range $(-2\pi, 2\pi)$ will lose validity.*

C. Summary of the Overall Method.

Our method for solving OPF problem (1) is using the SQP method (2) where $\Delta x(k)$ is the solution of the QP subproblem (3). The proposed iterative dual-type method uses (10) to solve (8), the dual problem of the QP subproblem, instead of solving (3) directly. The $\Delta\lambda(t)$ in (10) is obtained from solving (11) using sparse matrix technique, in which the $\Delta\hat{x}$ needed to set up $\frac{\partial\phi(\lambda(t))}{\partial\lambda}$ can be computed using the simple two-phase method. Consequently, the iterative dual-type method converges to optimal solution λ^*, and the solution $\Delta\hat{x}$ of the constrained minimization problem on the RHS of (9) with $\lambda = \lambda^*$ is $\Delta x(k)$, the solution of (3).

D. The Advantageous Features of the Proposed Dual-type Method.

In the following, we will describe how the proposed dual-type method achieves the four attractive features (ii)–(v) we claimed in Section 1.

In the dual function (9), we put the set of inequality constraints Ω as the domain of primal variables Δx so that we can apply the Projection Theory to circumvent the need of identifying the binding inequality constraints. This address feature (ii).

All the computational requirements of our method for solving OPF problems almost lie in solving the linear equations (11) and the calculations of $\Delta\hat{x}$ in (16)–(19). Equations (16)–(19) are as simple as they show. The approximate Hessian matrix $\frac{\partial^2\phi^u(\lambda(t))}{\partial\lambda^2} - \delta I$ is a sparse constant matrix; then the optimal ordering for the setup of

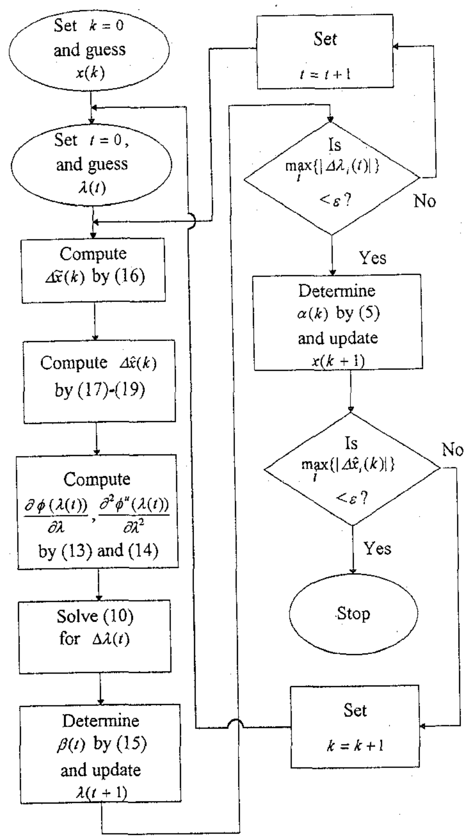

Figure 1: flow chart of our method for solving OPF problems.

memory locations for non-zero elements and fill-ins need only be done once. Therefore, the computational efficiency of our method can be expected. This address feature (iii).

Fig. 1 shows the flow chart of our method. Since all the computational formulae of our method are simple and concise, easy to be programmed is a natural result. This address feature (iv).

Convergence of the SQP method with step-size $\alpha(k)$ determined according to (5) has been shown in [11]. Convergence of the proposed dual-type method for solving the dual problem of QP subproblem is shown in the Appendix. These rigorous mathematical justifications address feature (v).

E. The Inclusion of Functional Inequality Constraints.

For the nonlinear inequality constraints such as security constraints on line flows

$$\underline{h} \le h(e,f) \le \bar{h}, \tag{20}$$

we can convert them into equality constraints by using surplus variable vector z such that

$$\begin{aligned} h(e,f) &= z \\ \underline{h} \le z &\le \bar{h} \end{aligned} \tag{21}$$

Then (21) has the same form of constraints as in (1). Although the inclusion of equality constraints in (21) will increase the dimension of $\frac{\partial^2 \phi^u(\lambda(t))}{\partial \lambda^2}$, in our application, the sparsity is still retained; thus the sparse matrix technique can still apply. Consequently, the five attractive features of our method are still valid.

Comment 2 *If there are too many functional inequality constraints, the increase in the dimension of approximate Hessian matrix and the number of surplus variables will cause extra computational complexity even through the five attractive features still exist. However, the functional inequality constraints in OPF problems are mostly the line flow constraints on specific transmission lines which are generally not too many. Therefore, our approach is suitable for the problem considered in this paper.*

IV. SOME REMARKS

A. Remark on Our Method

There are many dual-type methods in the literature; for example, the dual LP method [2], the Lagrangian relaxation method [14], and the interior point method of primal-dual approach [10],..., etc.. The proposed method for solving the dual problem of QP subproblem is also a dual-type method but differs from all the existing methods. Our method has similarity with the Lagrange relaxation method. However, in the dual function we defined in (9), we put the set of inequality constraints, Ω, as the domain of the primal variables instead of using Lagrange multiplier μ to associate with the inequality constraints in Lagrangian relaxation approaches. This trick enables our method to have a constant sparse approximate Hessian matrix and apply Projection Theory to deal with the difficulties encountered by binding inequality constraints. Consequently, the four attractive features can be achieved as described in Section III.D.

B. Remark on the Objective Function of OPF

Observing from the objective function of (3), if the considered OPF problem is an economic dispatch control problem, the SQP method (2) is a Newton-type method. However, if the criterion is to minimize the system losses, the SQP method (2) is a Jacobi-type method. The Jacobi-type method associated with our dual-type method for solving the OPF problems is still very computationally efficient as we will demonstrate by numerical examples in next section.

C. Remark on No Feasible Solution

It is possible that the QP subproblem (3) does not have any feasible solution. If so, the objective value of the dual problem (8) will be unbounded. This is owing to the magnitude of some components of $\Delta\lambda(t)$ increase as iteration t increases; in other words, the magnitude of some components of $\frac{\partial \phi(\lambda(t))}{\partial \lambda}$ do not decrease as t increases as can be observed from (11). Investigating further, we found from (13) and (16) that components of $|\Delta\tilde{x} - \Delta\hat{x}|$ with irreducible magnitude should be the major factor causing the above problem. This implies that if we push all the primal variables x to satisfy the inequality constraints in Ω, the objective value of (8) will be unbounded. To remedy this infeasible situation, we may release the constraints with larger magnitude of $|\Delta\tilde{x} - \Delta\hat{x}|$ or $|h(x(k)+\Delta\tilde{x}) - h(x(k)+\Delta\hat{x})|$ when $\max_i |\Delta\lambda_i(t)|$ does not decrease. In fact, the above reasoning is similar to the way of handling infeasible solution in [8].

Table I: The final objective value and CPU time consumption of the tested OPF problems with economic criterion in Case (i).

IEEE system	no. of lines	no. of G-buses	final obj. value (100 MVA base)	CPU time (seconds)
6-bus	11	3	432.711 (433.444)	0.08
9-bus	9	3	321.093 (321.490)	0.09
11-bus	17	3	620.384 (620.000)	0.15
30-bus	41	6	154.694 (154.000)	0.41
57-bus	86	29	4845.123 (4845.000)	0.57
118-bus	179	54	2638.720	0.72
244-bus	445	46	36819.537	6.11
2500-bus	3152	124	50577.994	578.75

V. TEST RESULTS

We tested our method for three cases of OPF problems on several power systems using a Spark-10 workstation.

Case (i): We consider the OPF with economic criterion with fixed transformer tap ratio, without switching capacitor banks, and no security constraints on line flows. We use total generation cost $\sum_i a_i P_{G_i}^2 + b_i P_{G_i} + c_i$ as the objective function of the OPF problem. The coefficients a_i, b_i, and c_i of the generation cost curve are various for different generation buses. The parameters we select are as follows: $\varepsilon = 10^{-3}$, $w = 100$, $\sigma_D = \sigma_P = 1$, $\tau_D = \tau_P = 0.9$, $\delta = \eta = 1.0$, and $\gamma = 0.1$. We have tested the OPF problems in this case on eight systems. All computer runs begin from a flat start with initial voltages being $e_i = 1.0$ and $f_i = 0.0$ for all buses i's. Table I shows the final objective value and the CPU times consumption of each OPF problem in Case (i).

We use IMSL subroutines to verify our solution by running the same problems with same initial guesses. IMSL subroutines is a nonlinear programming tool implemented by the well-known Han-Powell algorithm [14]. The

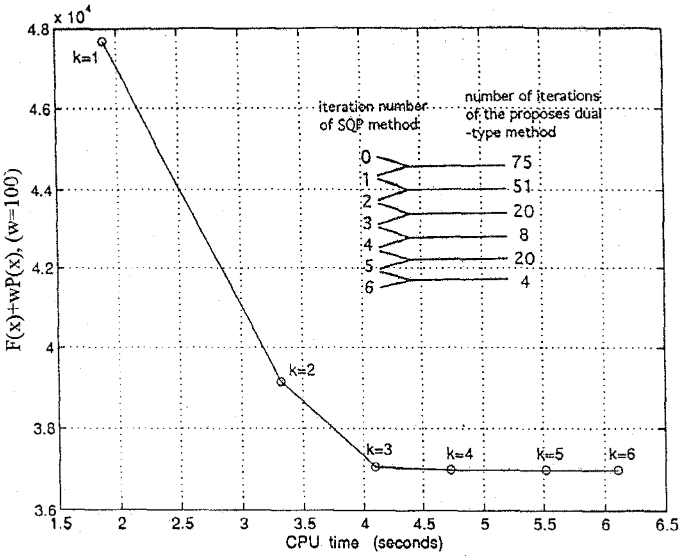

Figure 2(a): The detailed progression of our method for solving the OPF problem on IEEE 224-bus system.

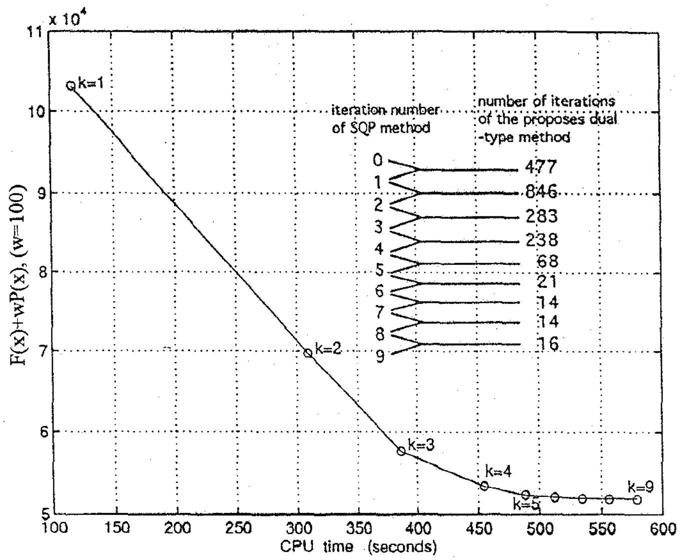

Figure 2(b): The detailed progression of our method for solving the OPF problem on IEEE 2500-bus system.

final objective values inside the parenthesis listed in Table I are obtained by IMSL subroutines. The verification cannot continue for systems with more than 60 buses, because IMSL subroutines can not execute due to the large memory requirement. We observed that our method is 30 times faster than IMSL subroutine in the case of modified 57-bus system and experienced an exponential growth of speed-up ratio as system size increases.

To appreciate more about our method, we show in Figs. 2(a) and 2(b) the detailed progression of our method when solving the OPF problems on IEEE 244-bus and IEEE 2500-bus systems. Each circle in the figures represents one iteration of the SQP method. Thus, from Figs. 2(a) and 2(b), we see that our method inherit a good convergence rate of the SQP method. The CPU time consumed in between circles represents the CPU time consumed by the proposed dual-type method for solving (3) completely. We also indicate in Figs. 2(a) and 2(b) the number of iterations of the dual-type method used in each iteration of SQP method. Because of the flat start, the proposed dual-type method takes more iterations to solve (3) in the first few iterations of SQP method. We also observe that the number of iterations used in the dual-type method for solving (3) is linearly proportional with system size, and a reasonably good solution is already obtained in about one half or two-thirds of the total CPU time listed in Table I. As indicated in Table I, we can obtain the solution of the systems with size in the order of hundred buses, within 10 seconds. In fact, for the 2500-bus system, we are actually obtain a solution of a nonlinear programming problem with 5248 variables, 5000 equality constraints, and 2748 inequality constraints in 580 seconds. This shows the computational efficiency of our method, and the numerical stability is manifested by the successful test results on the large complex 2500-bus system running from a flat start.

Table II: The final objective value and CPU time consumption of the tested OPF problems with system losses criterion in Case (ii).

IEEE system	no. of lines	no. of G-buses	final object value (100MVA base)	CPU time (seconds)
6-bus	11	3	9.99	0.07
9-bus	9	3	0.37	0.20
11-bus	17	3	6.50	0.25
30-bus	41	6	5.47	0.49
57-bus	86	29	8.82	0.56
118-bus	179	54	13.35	0.67
244-bus	445	46	37.82	5.57
2500-bus	3152	124	461.50	441.23

Case (ii): The setup of this case is the same as Case (i) except for using the system losses criterion. We let the total active system losses $\sum_l P_l$ be the objective function, where P_l denote the active loss of transmission line l. From a flat start, the final objective value and CPU time consumption of the OPF problems on eight systems are listed in Table II. Comparing with the CPU time in Table I, we see that the computational efficiency are about the same. These results show that the Jacobi-type method associated with the proposed dual-type method are still very efficient in solving OPF problems with system losses criterion.

Case (iii): The purpose of this case is to investigate the

Table III: The final objective value and CPU time consumption of the tested OPF problems with economic criterion, consisting of switch capacitor banks and security constraints on line flows in Case (iii).

IEEE system	no. of sw. cap. banks installed	no. of secur. constr. on line flows	final objective value (100 MVA base)	CPU time (seconds)
6-bus	2	2	432.87 (432.90)	0.05 (0.11)
9-bus	2	2	321.02 (321.58)	0.11 (0.10)
11-bus	2	2	620.59 (620.21)	0.14 (0.16)
30-bus	4	4	155.39 (152.54)	0.57 (0.49)
57-bus	5	10	4850.73 (4849.17)	0.53 (0.54)
118-bus	10	20	2636.94 (2638.62)	1.26 (1.10)
244-bus	15	20	36819.41 (36815.08)	9.53 (11.81)

performances of the approximation rule (7) for the update of discrete control variables and the way we handle functional inequality constraints described in Section III.E. We consider the OPF problem with economic criterion as in Case (i) but installing several switching capacitor banks and assuming security constraints of line flows on several specified lines in each tested system as indicated in Table III. With the approximation rule (7) and security constraints, the corresponding final objective value and CPU time consumption for the tested OPF problems on eight systems are shown in Table III. We also test all the OPF problems by assuming the installed switching capacitor banks are continuous variables. The corresponding final objective values shown inside the parenthesis are also listed in Table III; they are almost the same as the objective values obtained with approximation rule. This implies that the approximation rule for the update of discrete control variables is qualified for application. Furthermore, at the presence of the security constraints on line flows, the CPU time are only slightly larger than those listed in Table I. This indicates that our way of handling functional inequality constraints are suitable.

Remark 4 *Our machine is small and out of memory when tested the 2500-bus system with 30 switching capacitor banks and 50 security constraints.*

VI. CONCLUSION

The proposed dual-type method for solving the QP subproblems in the framework of SQP method is a new method in OPF literature and also a new dual-type method in nonlinear programming methodologies. This method is general, theoretically sound, and computationally efficient. The exploitation of the sparsity structure of power system network and capability of coping with difficulties encountered by inequality constraints make this method attractive for applications on other power system optimization problems.

VII. APPENDIX

The Special Structure of H.

According to [13], almost all the cost criteria can be formulated as a functions of real power generation. Thus, the diagonal terms of the diagonal positive definite matrix H in (4) corresponding to e and f have the same values as $\frac{1}{2}\eta$.

Theorem 1 *The solution $\Delta\hat{x}$ of the constrained minimization problem on the RHS of (9) can be solved in two phases:*

Phase 1: Compute $\Delta\tilde{x} = -H^{-1}[\frac{\partial F(x(k))^T}{\partial x} + \frac{\partial g(x(k))^T}{\partial x}\lambda]$ which is (16).

Phase 2: Project $\Delta\tilde{x}$ onto Ω, the resulting projection is $\Delta\hat{x}$.

Proof: Since the square terms of Δx contains a scaling matrix H, the basic idea of this proof is using a coordinate transformation to transform the minimization problem into a projection problem as follows.

Neglecting constant terms $\lambda^T g(x(k))$and letting $\Delta y = H^{\frac{1}{2}}\Delta x$, where the diagonal positive definite matrix $H^{\frac{1}{2}}$ is defined by $H^{\frac{1}{2}} \cdot H^{\frac{1}{2}} = H$, we can rewrite the constrained minimization problem on the RHS of (9) as

$$\min_{H^{-\frac{1}{2}}\Delta y \in \Omega} \frac{1}{2}\|\Delta y - [H^{-\frac{1}{2}}\frac{\partial F(x(k))^T}{\partial x} + H^{-\frac{1}{2}}\frac{\partial g(x(k))^T}{\partial x}\lambda]\|_2^2 \tag{22}$$

Since the constraints $H^{-\frac{1}{2}}\Delta y \in \Omega$ is equivalent to $\Delta y \in H^{\frac{1}{2}}\Omega$, where $H^{\frac{1}{2}}\Omega$ is defined as $\{H^{\frac{1}{2}}\Delta x | \Delta x \in \Omega\}$. Thus, we can rewrite (22) as

$$\min_{\Delta y \in H^{\frac{1}{2}}\Omega} \frac{1}{2}\|\Delta y - [H^{-\frac{1}{2}}\frac{\partial F(x(k))^T}{\partial x} + H^{-\frac{1}{2}}\frac{\partial g(x(k))^T}{\partial x}\lambda]\|_2^2 \tag{23}$$

The minimization problem in (23) is simply a projection problem of projecting $\Delta\tilde{y} = H^{-\frac{1}{2}}[\frac{\partial F(x(k))^T}{\partial x} + \frac{\partial g(x(k))^T}{\partial x}\lambda]$ onto the set $H^{\frac{1}{2}}\Omega$. Let $\Delta\hat{y}$ be the projection of $\Delta\tilde{y}$, then $\Delta\hat{x} = H^{-\frac{1}{2}}\Delta\hat{y}$. In fact, the above projection process is equivalent to project $\Delta\tilde{x}(= H^{-\frac{1}{2}}\Delta\tilde{y} = H^{-1}[\frac{\partial F(x(k))^T}{\partial x} + \frac{\partial g(x(k))^T}{\partial x}\lambda])$ onto the set Ω and the projection is $\Delta\hat{x}$ as we stated in the two-phase procedures of this Theorem. In the following, we will prove this claim. The set Ω are decoupled for each individual bus i, and the simple bounded inequality constraints (such as the constraints for real power

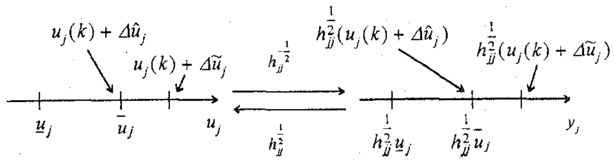

Figure 3: ($h_{jj}^{\frac{1}{2}}$ is the jth diagonal element of $H^{\frac{1}{2}}$).

and reactive power generations) $\underline{u}_i \leq u_i(k) + \Delta\tilde{u}_i \leq \bar{u}_i$ are decoupled from the constraints on voltage magnitude $\underline{V}_i - V_i(k) \leq \frac{\partial V(k)}{\partial e}\Delta e_i + \frac{\partial V(k)}{\partial f}\Delta f_i \leq \bar{V}_i - V_i(k)$. Since $H^{\frac{1}{2}}$ is diagonal, and the diagonal terms of $H^{\frac{1}{2}}$ corresponding to e_i and f_i are the same as $\frac{1}{\sqrt{2}}\eta^{\frac{1}{2}}$, Fig. 3 and Fig. 4 geometrically show the equivalence of projecting $\Delta\tilde{x}$ onto the set Ω and $\Delta\tilde{y}$ onto the set $H^{\frac{1}{2}}\Omega$ using coordinate transformation. This proves our claim.□

Remark 5 *In general, if H does not posses special structure, more complicated formula are needed to obtain $\Delta\tilde{x}$ and $\Delta\hat{x}$, however, the simplicity of the two-phase procedures still hold.*

Theorem 2 *The $\Delta\hat{x}$ obtained from (17)-(19) is the projection of $\Delta\tilde{x}$ onto the set Ω.*

Proof: The result is trivial by inspection from Fig. 3 and Fig. 4.□

Theorem 3 *The dual-type method (10) with $\beta(t)$ determined according to (15) is an ascent method.*

Proof: First, we can rewrite (8) as $\min[-\phi(\lambda)]$. From (14), $-[\nabla^2\phi^u(\lambda(t)) - \delta I]$ is positive definite. Using Decent Lemma in [12] and by simple calculations, we can set $\beta(t) = \tau_D^{m(t)}\sigma_D$, where $m(t)$ is the smallest nonnegative integer m that the following inequality holds

$$-\phi(\lambda(t) + \tau_D^m\sigma\Delta\lambda(t)) \leq -\phi(\lambda(t)) - \frac{\delta\tau_D^m\sigma_D}{2}||\Delta\lambda(t)||_2^2,$$

which is (15). We then have

$$\phi(\lambda(t) + \beta(t)\Delta\lambda(t)) \geq \phi(\lambda(t)) + \frac{\delta\beta(t)}{2}||\Delta\lambda(t)||_2^2. \quad (24)$$

This shows that (10) is an ascent method as long as $||\Delta\lambda(t)|| \neq 0$. In fact, the condition $||\Delta\lambda(t)|| = 0$ implies $\frac{\partial\phi(\lambda(t))}{\partial\lambda} = 0$ which is the necessary condition when $\phi(\lambda)$ achieves its maximum. Thus, (10) is an ascent method to maximize $\phi(\lambda)$.

Combining Theorem 3 with the two-phase method shown in Theorems 1 and 2, and also by Duality Theory [14], we have the following theorem which is the main theoretical result of the proposed dual-type method.□

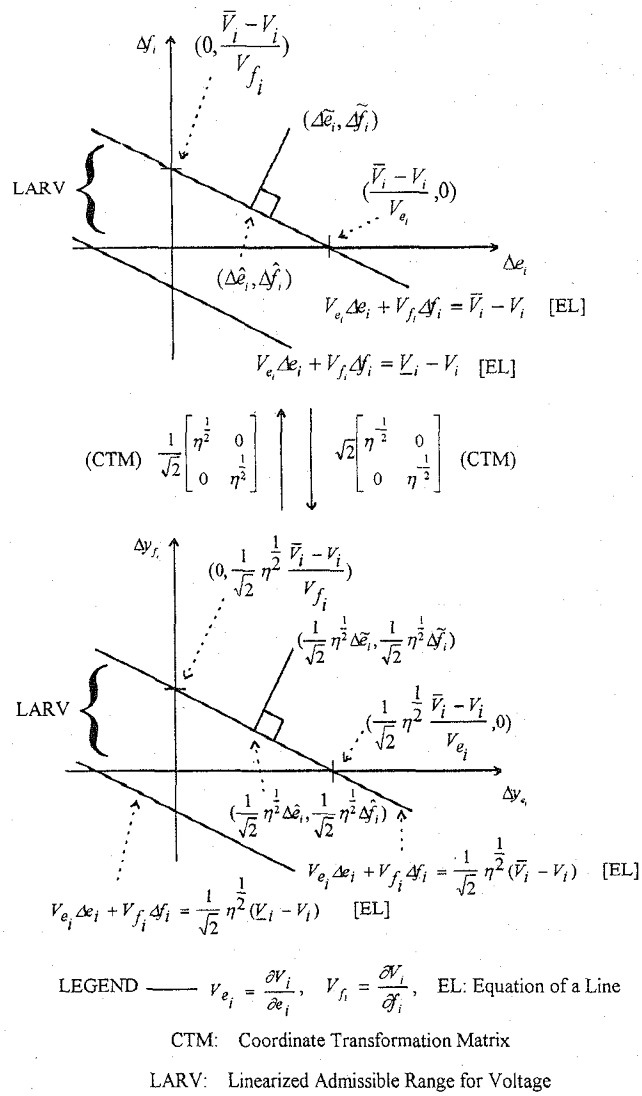

Figure 4: (($\Delta y_{e_i}, \Delta y_{f_i}$) is the transformed coordinate of ($\Delta e_i, \Delta f_i$)).

Theorem 4 *The dual-type method (10) converges to a point λ^* such that $\frac{\partial\phi(\lambda^*)}{\partial\lambda} = 0$ and maximize $\phi(\lambda)$. Furthermore, $\Delta\hat{x}$, the solution of the constrained minimization problem on the RHS of (9) with $\lambda = \lambda^*$, equals $\Delta x(k)$, the optimal solution of (3).*

Proof: the proof can be similarly developed from the proof of Proposition 2.1 of Section 3.2.2 in [12].□

VIII. ACKNOWLEDGMENT

The authors wish to thank Professor Yu-Chi Wu for several helpful discussions on the setup of numerical tests. Professor Wu and his colleagues have done excellent research work in [10].

This research work is supported in part by National Science Council in Taiwan under grant #NSC83-0404-E009-115.

References

[1] B. Stott, J. L. Marinho, and O. Alsac, "Review of linear programming applied to power system rescheduling," pp.142154, *PICA* 1979.

[2] B. Stott and J. L. Marinho, "Linear Programming for Power-System Network Security Applications," *IEEE Trans. on Power Apparatus and Systems,* vol.PAS-98, no.3, pp.837-848, May/June 1979.

[3] B. Scott, O. Alsac, and A. Monticelli, "Security analysis and optimization," *Proc. IEEE.* vol.75, no.12, pp.1623-1664, Dec. 1987.

[4] T. C. Giras and S. N. Talukdar, "Quasi-Newton method for optimal power flows," *International Journal of Electrical Power & Energy Systems,* vol.3, no.2, pp.59-64, Apr. 1981.

[5] S. N. Talukdar and T. C. Giras, "A fast and robust variable metric method for optimum power flows," *IEEE Trans. on PAS,* vol.101, no.2,pp.415-420, 1982

[6] R. C. Burchett, H. H. Happ, and D. R. Vierath, "Quadratically convergent optimal power flow," *IEEE Trans. on Power Apparatus and Systems,* vol.PAS-103, no.11, pp.3267-3275, Nov. 1985.

[7] D. I. Sun, B. Ashley, B. Brewer, A. Hughes, and W. F. Tinney, "Optimal power flow by Newton approach," *IEEE Trans. on Power Apparatus and Systems,* vol.PAS-103, no.10, pp.2864-2880, Oct. 1984.

[8] D. I. Sun, T. I. Hu, G. S. Lin, C. J. Lin, and C. M. Chen, "Experiences with implementing optimal power flow for reactive scheduling in the Taiwan power system," *IEEE Trans. on Power Systems,* vol.3, no.3, Aug. 1988.

[9] A. Monticelli and W. E. Liu, "Adaptive movement penalty method for the Newton optimal power flow," *IEEE Trans. on Power System,* pp.334-340, 1992.

[10] Y. C. Wu, A. S. Debs and R. E. Marsten, "A direct nonlinear predictor-corrector primal-dual interior point algorithm for optimal power flows," *IEEE Trans. on Power System,* pp.876-883, May. 1994.

[11] D. P. Bertsekas, "Constrained optimization and Lagrange multiplier methods," *Academic Press,* 1982.

[12] D. P. Bertsekas and J. N. Tsitsiklis, "Parallel and distributed computation: numerical methods," *Prentice-Hall Englewood Cliffs,* **NJ**, 1989.

[13] A. S., Debs, "Modern power systems control and operation," *Kluwer Academic Publishers,* 1988.

[14] D. Luenberger, "Linear and nonlinear programming, 2nd ed," *Addison-Wesley Reading,* **MA**, 1984.

BIOGRAPHY

CH'I-HSIN LIN was born in Taiwan, ROC, on Aug. 29, 1965. He received the B.S. degree in electrical engineering from Feng Chia University, Taiwan, the M.S. degree in electrical engineering from National Tsing Hua University, Taiwan, and Ph.D degree in control engineering from Chiao Tung University, Taiwan, in 1989, 1991, and 1996, respectively. He is currently serving as an officier in the army.

SHIN-YEU LIN was born in Taiwan, ROC. He received the B.S. degree in electronics engineering from National Chiao Tung University, the M.S. degree in electrical engineering from University of Texas at El Paso and the D. Sc. degree in systems science and mathematics from Washington University in St. Louis, Missouri, in 1975, 1979, and 1983, respectively.

From 1984 to 1985, he was with Washington University working first as a Research Associate and then a Visiting Assistant Professor. From 1985 to 1986, he was with GTE Laboratory working as a Senior MTS. He joined the Department of Control Engineering at National Chiao Tung University in 1987 and has been a Professor since 1992. His major research interests are Large-Scale Power Systems, Optimization Theory and Applications, and Distributed and Parallel Computations.