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SPECIAL ISSUE PAPER

Real-time horse gait synthesis[†]

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ABSTRACT

Horse locomotion exhibits rich variations in gaits and styles. Although there have been many approaches proposed for animating quadrupeds, there is not much research on synthesizing horse locomotion. In this paper, we present a horse locomotion synthesis approach. A user can arbitrarily change a horse's moving speed and direction, and our system would automatically adjust the horse's motion to fulfill the user's commands. At preprocessing, we manually capture horse locomotion data from Eadweard Muybridge's famous photographs of animal locomotion and expand the captured motion database to various speeds for each gait. At runtime, our approach automatically changes gaits based on speed, synthesizes the horse's root trajectory, and adjusts its body orientation based on the horse's turning direction. We propose an asynchronous time warping approach to handle gait transition, which is critical for generating realistic and controllable horse locomotion. Our experiments demonstrate that our system can produce smooth, rich, and controllable horse locomotion in real time. Copyright © 2012 John Wiley & Sons, Ltd.

KEYWORDS
character animation; motion synthesis; quadruped gait synthesis

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1. INTRODUCTION

Character animation has been studied in computer graphics for many years. With the rapid development of computer hardware and graphics algorithms, three-dimensional (3D) animation techniques are now wildly used in cartoons, video games, and digital special effects. In computer animation, animals are a very common character. To generate more realistic animal animation, the data-driven approach, which relies on real motion data as synthesis or editing resources, seems to be a good candidate. Nevertheless, it is not convenient and sometimes even difficult to capture quadruped motion, although we are now able to collect a great amount and variety of human motions using commercial motion capture devices.

In this paper, we propose a synthesis approach to animate quadruped motion based on a small motion database. In particular, we focus on generating horse locomotion as it is basic and essential motion while exhibiting large variations. Moreover, this is also a challenging problem as a horse has six different gaits and changes its gaits at different speeds. If we can solve the gait transition problem of a horse, the proposed approach should be applicable to the gait transition problem of other quadrupeds.

We propose a real-time system that allows a user to arbitrarily change a horse's moving speed and direction. Our system automatically changes the horse's gaits according to its speed. We construct a motion database by manually capturing horse postures from Eadweard Muybridge's series pictures of 11 locomotion using MAYA. To let a horse walk or run at arbitrary speed, we expand our horse locomotion data by time warping the captured motion to various speeds on the basis of zoological studies [1]. The studies show that a horse's stride length at stance phase increases with its progressing speed, whereas the contact time of a stride decreases as a power function of its speed.

To be able to generate smooth gait transition between two arbitrary gaits at any phase of a stride, we propose an asynchronous time warping method to handle the gait transition problem. We also develop an on-line root trajectory generation and body orientation adjustment approach to control the global position and orientation of the horse. Instead of using the same generic time to blend the motion of two gaits for all four legs, our asynchronous time warping method performs time warping at each leg separately. We blend each leg's motion of two gaits at its own generic time and gradually adjust each leg's pace to ensure that the four legs would converge to the new gait. Furthermore, our root trajectory generation and body orientation adjustment approach compute the position and orientation based on each foot's ground contact state at each time frame.

[†] Supporting information may be found in the online version of this paper.

The contributions of this paper are proposing the following: (i) a real-time horse gait synthesis approach that can automatically change a horse's gait smoothly according to the speed and direction specified by a user; (ii) an asynchronous time warping method that can generates smooth gait transition between any two gaits; and (iii) a root trajectory generation and body orientation adjustment method that preserves horse gait style while the horse turns and/or changes speed.

2. RELATED WORK

Generating quadrupedal motions receives much research attention in robotics and computer animation. It also benefits from many research studies in zoology and biomechanics. We refer the readers to an excellent survey by Skrba *et al*. [2] for an overview of quadruped animation techniques including quadruped motion capture and simulation.

People have long been interested in quadruped motions and analyzed them using different motion capture devices. Comparing with human motion, it is hard to capture animal motion. The famous work by Eadweard Muybridge [3] was the first attempt to capture animal motions, in which a series of 24 cameras was used to take photos of animal's locomotion. Kokkevis *et al*. [4] used chronophotography technique to record many phases of a single moment on one image. Different with the aforementioned methods that can only capture two-dimensional information, statistical analysis has also been applied to capture 3D information [5]. In biomechanics and zoology, Alexander *et al*. [6–9] developed dynamic similarity hypothesis, which explains the relationship between animal size, speed, mass, and external forces. Hoyt *et al*. [1] showed that a horse's step length increases with its speed, whereas its feet's time of contact with the ground decreases as a power function of speed. These findings and other research on animal behavior strategies [10,11] help us to develop our gait synthesis approach.

In computer graphics, many methods were proposed to simulate quadruped motion. Inverse kinematics (IK)-based approaches [4,12,13] usually decide animal footprints first, and then combine physically based approaches to control the body movement. Instead of driving quadruped motion by skeletons, mesh-based approaches generate the motion of mesh surface directly [14–17]. Another main stream of generating quadrupeds locomotion is the controller-based approaches, which can produce real-time interaction and deal with unexpected circumstances [18–21]. Although Marsland and Lapper [20] also worked on horse locomotion generation, they focused only on the trot gait in their simulation. In contrast, we address to synthesize all gaits of a horse in real time. Coros *et al*. [21] combined a Proportional-Derivative (PD) controller system and an internal virtual force system to simulate an integrated set of gaits and skills for a physics-based quadruped. Besides, optimization-based approaches were also widely used. Wampler *et al*. proposed a sampling-based derivative-free optimization method under the spacetime optimization formulation to automatically synthesize plausible gaits for different skeletons [22].

There are also many quadruped simulation approaches mixing the aforementioned approaches. Kry *et al*. [23] used the natural vibration modes of the body that are related to morphological parameters such as the shape, size, mass, and joint stiffness to generate efficient locomotion. Tsai *et al*. [24] proposed a physically based method, which is able to adjust the original motion to meet adaptation requirements when animating 3D virtual characters. Sims [25] proposed a novel system creating virtual creatures that move and behave in simulated 3D physical worlds. He used genetic algorithm to evolve the morphologies and the neural systems of those creatures.

In robotics, various quadruped robots have been created. Sony AIBO is a robotic dog, which is able to see and move around while maintaining balance [26]. Boston Dynamics produced "BigDog" and "LittleDog" [27–29]. BigDog has the animal alike legs that can absorb the shocks from the ground. LittleDog, which is capable of walking on rough terrains, is built for studying animal locomotion. One degree of freedom legs were used in SCOUT to produce walking, climbing, and galloping animation [30].

In contrast to the existing work on quadruped animation, we address the gait transition and synthesis problem in this paper. In particular, we focus on horse gaits, which have rarely been studied in computer animation. Our work can be combined with the controller-based approaches to provide the reference trajectory that is required in these approaches. Our asynchronous time warping method can effectively handle the gait transition problem. It allows a horse to change its gait arbitrarily between *walk*, *amble*, *trot*, *rack*, *canter*, and *gallop*. The transition begins when a user requests a larger speed change such that the horse needs to change its gait (according to zoological studies). The speed changing command can be given at any phase of a horse stride. In addition, as a horse can change its gait directly from a slow-speed gait to a fast-speed gait in the reality, our method can also achieve this kind of gait transition and thus results in more natural horse locomotion.

3. OVERVIEW

Our approach combines data-driven and procedural algorithms to generate horse locomotion in real time according to a user's commands. Figure 1 shows an overview of our approach. We first manually capture six different horse locomotion gaits from Muybridge's series pictures. We then warp these motions to obtain motions at various speeds. As the horses in Muybridge's photographs do not translate, we also synthesize their root trajectories so that they move at the correct speed as described in Muybridge's photographs. Note that there is no horse turning motion in Muybridge's photographs. Figure 2 shows the six gaits used in our system.

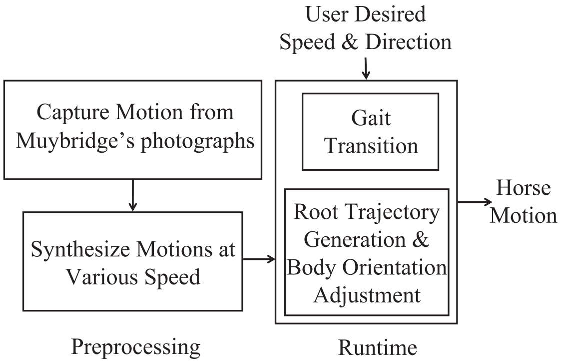

Figure 1. Overview of our horse locomotion generation system.

After preprocessing, we deal with horse gait transition that is needed when a user changes the horse's speed. We propose an asynchronous time warping method to generate horse motion between two different gaits. Our method warps the motion of four legs with different paces so that their motion would gradually transfer to the new gait. Besides, we also propose a root trajectory generation and body orientation adjustment approach to avoid foot sliding and produce turning motion.

The horse skeleton used in this paper has 35 joints and 61 degrees of freedom (DOFs). The root joint has six DOFs for global position and orientation, whereas the other joints have 55 DOFs in total for joint rotation. Figure 3 shows major joints and bones of the horse skeleton. *Hip/Shoulder* is a horse's shoulder or hip joint. *Fetlock* represents the joint between the horse's metatarsal bone and hoof. *Pastern* is a part of the leg of a horse between the *Fetlock* joint and the top of the hoof. *Foot* is at the end of a leg. We set the *Root* joint at the end of the horse's spine and define *Body* a series of bones from *Head* to *Root*.

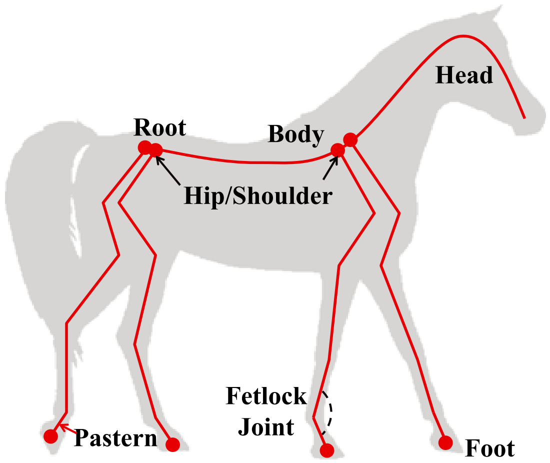

Figure 3. Joints notation.

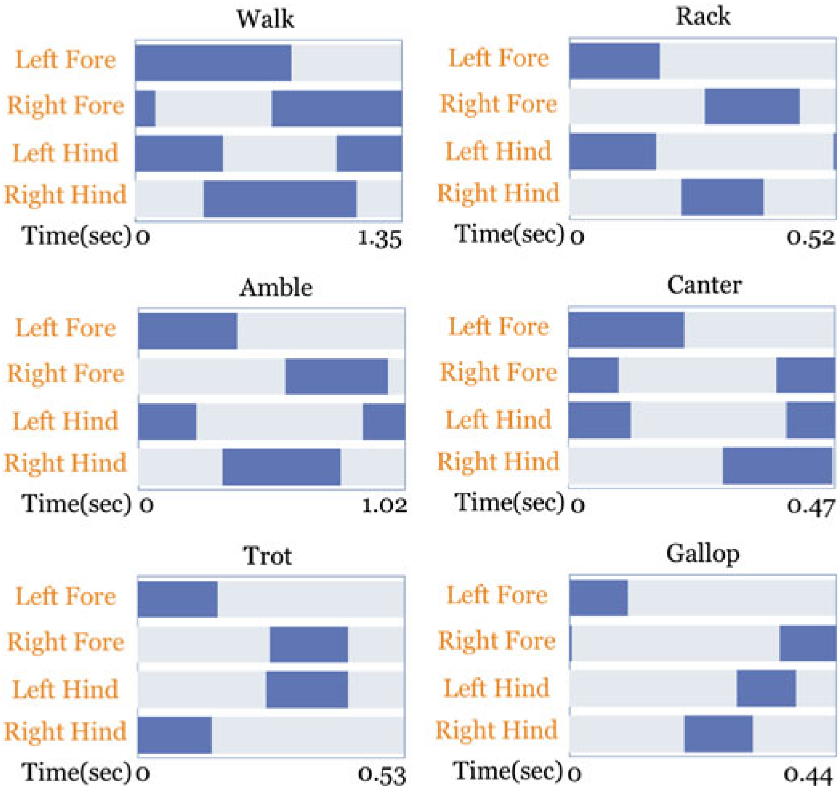

Figure 2. Six gaits of horse locomotion. The dark blue and light blue regions represent the stance and flight phase, respectively.

4. SYNTHESIZE MOTIONS AT VARIOUS SPEEDS

The speed range of each gait is shown in Figure 4, where orange squares denote 11 motions captured from Eadweard Muybridge's photographs [3]. Our system applies the rules of Hoyt *et al.* [1] to synthesize motions at different speeds based on these 11 motions. Hoyt *et al.* found that a horse's step length increases whereas the time of contact decreases when its speed increases. Mathematically,

$$\begin{aligned} T_c &= a v^b \\ D &= v T_c \end{aligned} \tag{1}$$

where T_c is the time of contact, v is the speed, and a and b are two constants. b is a negative number and significantly different from -1. D is the step length of horse. a and b are different for different gaits. We obtain a and b by fitting Eadweard Muybridge's measurement data to the rules of Hoyt *et al.* as shown in Table 1.

Given a new speed v, we first decide the horse's gait based on Figure 4 and then apply Equation 1 to compute the corresponding step length. Once the step length is obtained, we adjust the foot position to match the step length. The adjustment is performed on the sagittal plane of the horse. Define XY the sagittal plane with X-axis along the horse heading direction and Y-axis along the upward direction. The new foot position P' is computed as follows:

$$\begin{aligned} P'_x &= (P_x - S_x)\frac{D'}{D} + S_x \\ P'_y &= (P_y - S_y)\frac{D'}{D} + S_y + G \end{aligned} \tag{2}$$

where P is the original foot position, D and D' are the original and new step length, respectively, S is the position of the *Hip/Shoulder*, and G is an offset ensuring that the stance leg would not penetrate or leave the ground. Note that Equation 2 is used to adjust the position of each foot. We apply IK to alter the horse's *Foot* positions from P to P'. After we adjust the foot position at all frames of the new motion, we warp the motion to match the desired speed. As there is no translation data in Eadweard Muybridge's photographs [3], namely, the horse was moving at the same position in his photographs, we compute the translation of the root joint based on the desired speed. In some gaits, there exists a flight phase, where the flying distance is taken into account when computing the root trajectory of these gaits.

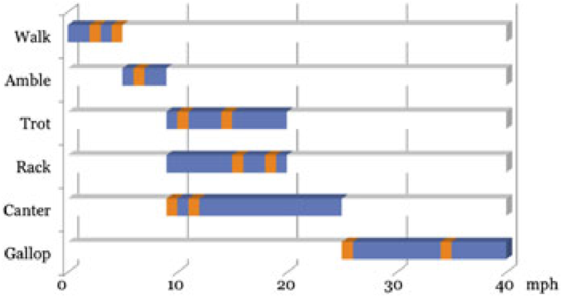

Figure 4. Speed range of different gaits. Our motion database contains 11 motions shown in orange portions.

Table 1. Values of *a* and *b*.

Gait	*a*	*b*
Walk	1.688057	−0.71476
Amble	1.826353	−0.51021
Trot	1.407662	−0.74782
Rack	2.386013	−0.90153
Canter	2.005463	−0.96045
Gallop	1.14589	−0.42978

5. GAIT TRANSITION

When the horse changes its speed, its gait may change. It is important to preserve the smoothness during gait transition. There are some interesting properties of a horse's leg motions under gait transition. First, the loading of each leg is not equal, for example, in gait gallop, only the left fore leg is used for acceleration, whereas the other legs are ancillary. Second, in different gaits, horse has different step order. Therefore, changing the speed of each leg synchronously would not work for gait transition.

We propose an *asynchronous time warping* method to solve the gait transition problem. The basic idea of our method is to blend the motion of four legs in the current and new gaits separately while adjusting each leg's speed so that their motion gradually converges to a posture in the new gait at the end of transition. Given the actual time T of the posture of the original gait M_1 at the beginning of the transition, we map T to four generic time for different legs. During the transition, the incremental of the generic time for each leg is different so that some feet falling behind in the transition process would speed up to catch up with the other feet. The accompanying video illustrates our idea.

We first determine the length of gait transition. As horses usually change their gaits within two to three stride cycles, we set the duration of transition at 2.5 cycles, that is, 2.5 in generic time. Furthermore, we assume that the incremental of generic time during gait transition linearly varies from the incremental of current gait Δt_1 to that of the new gait Δt_2. Hence, the length of gait transition in generic time is

$$\sum_{i=1}^{B} w_1(i)\,\Delta t_1 + w_2(i)\,\Delta t_2 \approx 2.5 \tag{3}$$

where B is the number of actual time frames of the blending process, $w_1(i) = 1 - \frac{i}{B}$, and $w_2(i) = \frac{i}{B}$. L_1 and L_2 are the number of frames of a stride cycle of the current and new gait, respectively. We obtain B by solving Equation 3, $B = (5L_1L_2 + L_2 - L_1)/(L_1 + L_2)$. B represents the total number of frames during gait transition.

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We then compute the incremental of generic time of each leg. To have a common reference for the generic time of the four legs, we arbitrarily assign one leg as a reference leg. The incremental of the generic time of the reference leg Δt^* gradually varies from the incremental of the current gait to that of the new gait, that is,

$$\Delta t^* (i) = w_1 (i)\, \Delta t_1 + w_2 (i)\, \Delta t_2 \tag{4}$$

where i denotes the ith frame in the transition. To compute the incremental of generic time of the other legs, we need to know the length of the gait transition of these legs in generic time. This can be performed by firstly determining the ending posture of the transition in M_2, and then finding the generic time of each leg in that ending posture. Specifically, the ending posture is obtained by $M_2(t_2^*(0) + \sigma^*)$, where $t_2^*(0)$ is the generic time of the reference leg when the transition begins and σ^* is the length of transition in generic time by summing up the incremental in each frame,

$$\sigma^* = \sum_{i=1}^{B} \Delta t^* (i) \tag{5}$$

Given the ending posture, we can determine the generic time of each leg in the ending posture and obtain the length of transition of each leg σ^j. The incremental of the generic time of the other legs then can be computed as follows:

$$\Delta t^j (i) = \Delta t^* (i) \frac{\sigma^j}{\sigma^*} \tag{6}$$

where $j \neq *$ denotes the jth leg and σ^j is the length of transition of the jth leg in generic time. Note that σ^j is usually different for different legs.

Having the incremental of the generic time of each leg, we can then compute the motion in the gait transition. We divide the posture of a horse into six parts: $Q = \{p^r, q^1, q^2, q^3, q^4, q^b\}$, where p^r is the global position of the root joint, q^1, q^2, q^3, and q^4 are the joint angles of the right-fore leg, left-fore leg, right-back leg, and left-back leg, respectively, and q^b are joint angles of the *Body*. We further define M_1, M_2, and M_B as the motion of the current gait, new gait, and gait transition, respectively. To ease the definition of notation, $q(var)$ may refer to part of the posture at a generic time or a time frame depending on the meaning of var. We first compute the blended posture of legs at the ith frame of the transition,

$$q^j (i) = w_1 (i) \cdot q_1^j \left(t_1^j (i)\right) + w_2 (i) \cdot q_2^j \left(t_2^j (i)\right) \tag{7}$$

where q_1^j and q_2^j are the postures of the jth leg in the current gait and new gait. The generic time $t_1^j(i)$ and $t_2^j(i)$ are computed as follows:

$$t_1^j (i) = t_1^j (0) + \sum_{f=1}^{i} \Delta t^j (f) \tag{8}$$

$$t_2^j (i) = t_2^j (0) + \sum_{f=1}^{i} \Delta t^j (f) \tag{9}$$

where $t_1^j(0)$ and $t_2^j(0)$ are the generic time of the jth leg in the current and new gaits when the transition begins.

The *Body* postures q^b of M_B can be obtained by simply blending the body postures corresponding to the generic time of four legs. We use slerp to blend the *Body* postures by

$$q^b(i) = \frac{1}{4}\left[w_1(i)\sum_{j=1}^{4} q_1^b\left(t_1^j(i)\right) + w_2(i)\sum_{j=1}^{4} q_2^b\left(t_2^j(i)\right)\right], \tag{10}$$

where q_1^b and q_2^b are the body posture of M_1 and M_2, respectively. The translation of the root p^r will be explained in the following section.

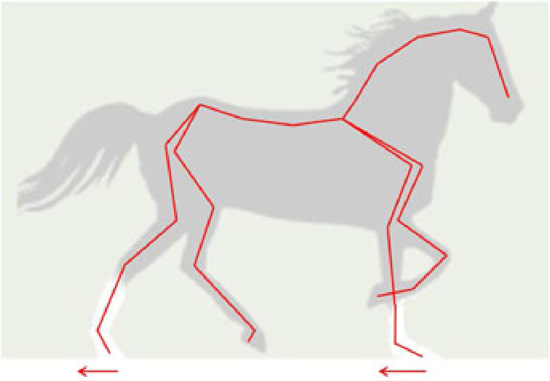
6. ROOT TRAJECTORY GENERATION AND BODY ORIENTATION ADJUSTMENT

In this section, we describe our approach for planning the horse's root trajectory and adjusting body orientation in real time. The basic idea is to utilize the movement of landing feet to calculate the amount of root translation to avoid foot sliding. We first determine the phase of foot at each time frame. We then compute the root translation and the orientation of the horse body when the horse moves straightly or turns.

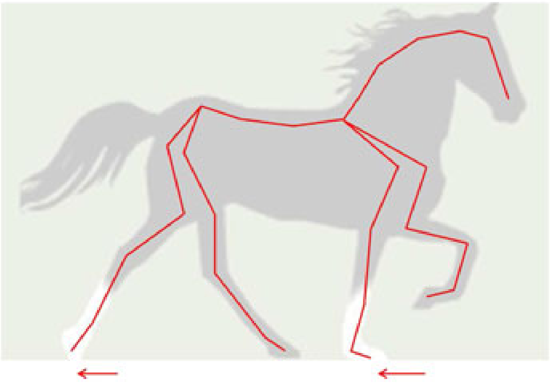
6.1. Determine Foot Phase

We divide the phase of a horse's feet into stance, striking, and flight phases, which are determined based on three parts of a horse: pastern, hip/shoulder, and foot. Pastern plays a role of being a spring to lessen the concussion from the ground. When a horse's foot strikes on the ground, pastern commences to bend. As the leg becomes vertical, depending on the concussion, pastern could become horizontal. Although the leg is gradually away from the ground, pastern bends from horizontal to vertical then bends backward. After the leg leaves the ground, pastern keeps bending in the air, and then progressively stretches to the front. Because of the function and characteristics of pastern, we determine if a foot is on the ground based on the bending degree of pastern.

It is easier to determine whether a leg is on the ground or not. When a leg's foot position is in the front of its hip/shoulder position, this leg is about to land on the ground or is just landing on the ground. In this situation, we check this leg's fetlock joint angle to determine the bending degree of pastern. If pastern bends beyond a threshold, this leg is marked as on the ground. We analyzed Muybridge's photographs to set the threshold at $\{158, 152, 165, 169\}$ degrees for the pastern in the left forelimb, right forelimb, left hindlimb and right hindlimb. On the other hand, when a leg's foot position is in the back of its hip/shoulder position, the leg is preparing to leave the ground or just leaving the ground. In this case, if pastern bend over a threshold, we label this leg in the air. The threshold we obtained from Muybridge's photographs is $\{181, 181, 180, 180\}$ degrees. In addition to the aforementioned criteria, a leg is also considered to be in the air if the height difference between the hip/shoulder and foot is too small.



(a) Current Posture



(b) Next Posture

Figure 5. This figure illustrates the foot sliding problem if we do not adjust the root position. To avoid foot sliding, our system moves the root position forward when a landing foot moves backward. The red arrows denote the offset d^j of each foot. (a) Current posture and (b) the next posture.

To determine if a foot is striking the ground, we label the frame at which a leg strikes the ground in our motion database. These labels are then used to determine if a foot strikes the ground at the runtime stage.

6.2. Adjust Root Movement and Body Orientation

We divide the problem of adjusting root movement and body orientation into two cases: advancing and turning. In advancing, we only need to deal with the foot sliding problem and do not need to alter the horse posture. In turning, in addition to handling the foot sliding problem, we also need to rotate the horse body to follow the new direction given by the user.

Advancing. For advancing postures, we avoid food sliding by moving the root position. In other words, we divert the sliding offset d from the landing foot to the root translation at each frame. Figure 5 illustrates our strategy. If there are more than one landing foot, we take the average offset of the landing feet to adjust the root position,

$$p^r(i) = p^r(i-1) + \frac{1}{N}\sum_{j=1}^{N} d^j \tag{11}$$

where $p^r(i)$ is the root position at frame i and $p^r(0)$ is the initial root position when the landing foot first strikes the ground. N is the number of landing feet.

Turning. For turning postures, which are not in our captured motion database, our system generates them by adjusting the direction of the horse's body. When the user inputs a turning direction to the horse, the global orientation of the bones on the *Body* begins to turn toward the desired orientation. To ensure the motion smoothness, the joint angles along the body is gradually adjusted from the *head* joint to the *root* at every frame. For example, the *head* joint rotates θ at the first frame, and then *head* and *fore-neck* joints rotate 2θ and θ at the second frame. The turning angle θ is determined by the horse's speed. We set θ as 0.01° at slow speed. When the horse moves at faster speeds, θ increases to 0.1 and 0.2. A joint would stop turning once the joint's global orientation satisfies the desired orientation.

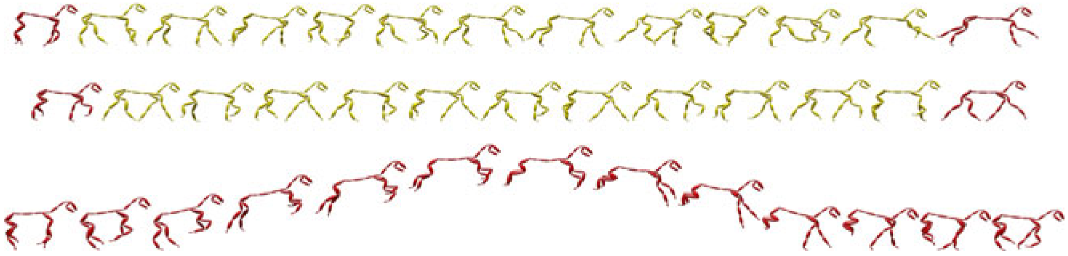


Figure 6. Top: snapshots of gait transition from *canter* to *gallop*. Middle: snapshots of gait transition from *trot* to *rack*. Bottom: snapshots of a leap motion. Yellow color denotes that the horse is under gait transition.

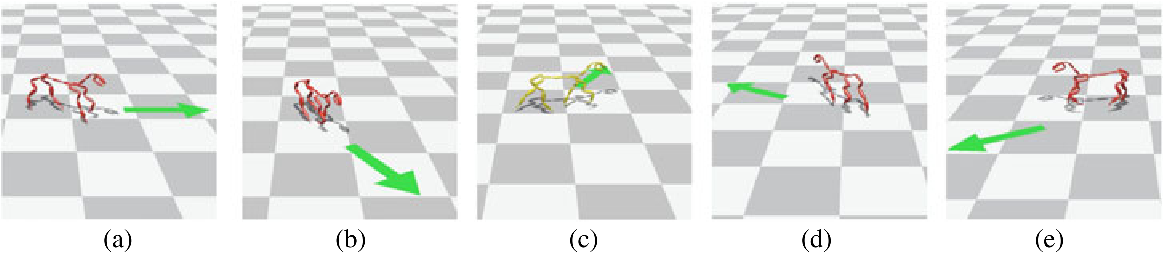


Figure 7. Snapshots of a horse's turning motions. The yellow horse denotes that the horse is under both gait transition and turning.

We also deal with the foot sliding problem when the horse is turning. If a foot is striking on the ground, our systems first rotates the horse's *Body* to the desired orientation and records the striking position as the goal position R, which is the position the foot should step on. Once the striking foot becomes a stance foot, our system adjusts the orientation and position of root so that the foot position is close to the goal position. IK is then applied to fine tune the leg posture ensuring that the stance foot is kept at the goal position.

7. EXPERIMENTAL RESULTS

We demonstrate the results of our horse locomotion system. In our real-time demo video, we show the gait transition process and the turning process. Motions of horse leaping, climbing up, and walking down a slope are also demonstrated in the accompanying video.

Gait transition. We demonstrate six types of horse locomotion gaits at different speeds. In the accompanying video, we illustrate a gait transition process from the slowest speed to the fastest speed. In this case, the horse changes gaits from *walk*, *amble*, *trot*, *rack*, *canter* to *gallop*. Our system allows the user to arbitrarily transit between any two gaits, which may not be the two gaits whose speed ranges are continuous. Figure 6 shows some snapshots of two examples of gait transition: *canter* to *gallop* and *trot* to *rack*. In our video, we show a gait transition from *gallop* to *amble*.

Turning. Figure 7 shows the snapshots of five turning motions, where the yellow horse denotes that the horse is under both gait transition and turning. According to zoological studies, a horse uses slower gaits and proceeds a smaller distance per time frame when the horse turns. Our horse turns 0.01° every time frame at the speed under 5 mph, 0.1° at speed between 6 to 25 mph, and 0.2° at speed above 25 mph. It turns 0.05° and 1.15° if there is also a gait transition between different speeds. Our system allows the horse to turn its heading direction during the gait transition and jumping.

Leaping. We also captured a leap motion from Eadweard Muybridge's work [3]. Our system searches our motion database to find a motion that is closest to the beginning of the leap motion. We mark the gait of the closest motion as the leaping-start gait. When the user enter a jump command, our system first change the horse's gait to the leaping-start gait and then connects to the leap motion by linearly blending. Similarly, our system finds the leaping-end gait in our database and changes the horse gait to the gait before leaping or a desired gait the user specified. Figure 6 shows a leaping motion generated by our system.

Climbing up and walking down a slope. We can also generate the motion of the horse going up and down a slope by modifying our turning algorithm. The modification is very simple. Instead of turning the global orientation of the horse body in the horizontal plane (XZ-plane), we can simply adjust the body's orientation along the vertical direction (Y-axis). Please see the accompanying video for the result.

Limitations and discussion. Currently, we manually capture animal locomotion using MAYA. Although the manual capture process is reliable, it is time-consuming. In the future, we would like to apply visual tracking techniques to improve motion capture of animal locomotion. Besides, our gait synthesis approach does not take into account the physical properties of horse skeleton, such as mass or moment of inertia. We can improve this problem by applying zoological studies or physics-based approaches to our gait synthesis and transition approach. Finally, although we only demonstrate horse gate synthesis in this paper, our approach can be applied to synthesize other quadruped gaits.

8. CONCLUSION

In this paper, we present a horse gait synthesis approach that allows user to arbitrarily change the speed and direction of a horse locomotion in real time. The core technique of our approach is the asynchronous time warping method, which would automatically adjust the horse's gait based on its speed and generate smooth gait transition. The proposed asynchronous time warping method is flexible and effective. It can generate smooth transition between two arbitrary gaits, even between a slow-speed gait and a fast-speed gait. This allows our system to generate more natural locomotion as horses also change their gaits in a similar way. Our results demonstrate that our approach can generate flexible and controllable horse locomotion in real time. In the future, we would like to apply our approach to other types of quadruped gaits. As our approach is simple and effective, we believe it can also benefit quadruped or multiped motion simulation that requires a reference trajectory.

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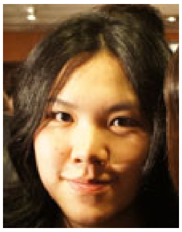

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