Analysis of unequal error protection for LDPCA codes

Y.-C. Sun and W.-J. Tsai

Unequal error protection extensions of low density parity check accumulate (UEP-LDPCA) codes are discussed and several potential applications are proposed. Two UEP-LDPCA schemes are analysed and code parameters are optimised using a density evolution algorithm. Simulation results show that the extensions have the significant unequal error protection property. This activates new directions for existing applications.

Introduction: Many practical applications have been proposed for distributed source coding [1], such as low complexity video encoding [2] and robust image authentication [3]. In these applications, low density parity check accumulate (LDPCA) codes [4] have been widely adopted as the rate-adaptive Slepian-Wolf code.

Although the requirement of the unequal error protection property of the Slepian-Wolf code is seldom addressed, there are many potential applications. Take low complexity video encoding as an example, some studies attempt to remove the feedback channel requirement of the method in [2] based on rate estimation methods [5]. However, if an estimation algorithm fails and the transmitted bits are too few, unrecoverable errors would occur. If the code could provide unequal error protection (UEP), more protections can be given to the important region of an image, such as the facial region in a video conference image, to avoid these errors. The penalty of rate estimation failure thus could be decreased, so the rate estimation methods could be more aggressive. In an image authentication system, for example, when the protection level of a code can be adjusted on every source data, the system could have more flexibility to allocate different error tolerance levels on different areas of an image.

Several UEP designs for LDPC codes have been proposed [6]. In this Letter, we combine the two UEP designs in [6] into a LDPCA coding scheme and analyse the UEP performance. The modifications from ensembles in [6] to the ones in this Letter include: 1. the UEP-LDPCA is a rate-adaptive code, so the check nodes in the Tanner graphs are derived from different rates; 2. the Tanner graphs are modified according to the Slepian-Wolf coding scenario; 3. the modified UEP-LDPCA codes are optimised and evaluated over binary symmetric channels (BSCs).

UEP-LDPCA formulation: The Slepian-Wolf code aims to compress correlated sources by distributed encoding and jointly decoding. The problem can be rephrased as encoding sources with correlated side information, which is available at the decoder side. As a Slepian-Wolf coder, the LDPCA encoder is the concatenation of a LDPC syndrome generator and an accumulator. When encoding, the LDPC syndrome generator encodes the source bits into the syndrome bits according to a full rank encoding matrix. The encoding matrix can be represented by a Tanner graph, and $\rho(x)$ and $\lambda(x)$ represent the degree distribution of check nodes and source nodes [7], respectively. To provide the rate adaptive feature, the syndrome bits are transformed into encoding bits. The compression rate can be computed by the ratio of the amount of encoding bits to the amount of syndrome bits. For generating encoding bits, LDPCA codes use an accumulator for syndrome merging and puncture the accumulated syndrome bits instead of the original syndrome bits. This prevents the loss of edges in the Tanner graphs of the lower rate codes and shows significant improvement on the other codes [4].

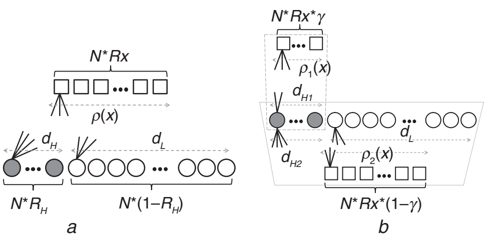

Fig. 1 *Tanner graphs of UEP-LDPCA ensembles*
a Code A
b Code B

For the UEP formulation, the source bits are divided into two groups with different error protection level requirements. The first group has a higher protection level, denoted as HG; while the other group has a lower protection level, denoted as LG. Let R_H denote the ratio of data in HG to all data. For UEP-LDPCA, the objective is to provide different protection levels for different data groups using one code. Fig. 1 shows the ensembles of two UEP-LDPCA codes, code A and code B, modified from [6]. The circular nodes denote the N source nodes and square nodes denote the check nodes.

The key idea of code A in Fig. 1*a* is to construct a partial regular code with two source degrees. For source nodes, the one with the larger connection degree implies it has higher error protection capability because it receives more information from the check nodes. Therefore, the data in HG is connected at a higher degree, d_H. The data in LG is connected at a lower degree, d_L.

The code B in Fig. 1*b* combines two Tanner graphs. The graph in the lower part of Fig. 1*b* is similar to the first ensemble. It protects all the data and has two different degrees corresponding to the different data groups. The upper graph is introduced to protect the data in HG only. The newly introduced graph further eliminates errors in HG that the lower graph fails to correct. The ratio of the upper graph's parity bits to all parity bits is γ.

Note that, in distributed source coding applications, most studies assume the parity bits are transmitted over error-free channels and optimise the code in the Slepian-Wolf coding scenario. In this Letter, we discuss the UEP codes in this scenario. Therefore, the ensembles in Fig. 1 are modified from the original ones proposed in [6].

Codes optimisation: For these two UEP-LDPCA codes, we further optimise the degree distribution for minimising the error rates of the data in HG. Let Λ denote the parameter vectors of UEP-LDPCA codes. In code A, Λ has two parameters, (d_H, d_L), to be optimised. For the LDPCA code construction method in [4], the degree distribution of check nodes would be approximately concentrated on at most two consecutive values. In other words, the degree distribution of check nodes could be written as $\rho(x) = \rho_d{*}x^j + (1-\rho_d){*}x^{j+1}$ at any compression rates, R_X. For a given $\lambda(x)$, the parameters in the concentrated degree distribution could be derived as

$$j = \left\lfloor \frac{1}{R_x((\lambda_{d_H}/d_H) + (\lambda_{d_L}/d_L))} \right\rfloor \tag{1}$$

$$\rho_d = (j^2 + j)R_x((\lambda_{d_H}/d_H) + (\lambda_{d_L}/d_L)) - j \tag{2}$$

For the second UEP-LDPCA code, the parameter vector has four parameters, $(d_{H1}, d_{H2}, d_L, \gamma)$. For check nodes, there are two degree distributions of check nodes in the second code. These degree distributions could be derived in the same manner as the first code. For fixed R_X, the decoding error probability of HG data would decrease when γ increases. However, the $R_X{*}(1-\gamma)$ should be large enough to guarantee the error probability of the low-protected group acceptable. In [6], the authors add a constraint on $R_X{*}\gamma$. Because the R_X is not fixed in the rate-adaptive coding scenario, we add a constraint on γ directly.

We adopt the density evolution method [7] to estimate the decoding error probability during the optimisation process. The correlation between sources and side information is modelled as BSC with crossover probability p. For a fixed decoding iteration, the estimated decoding error probability of two data groups could be written as $\mathrm{P_H}(R_X, p, \Lambda)$ and $\mathrm{P_L}(R_X, p, \Lambda)$. It depends on R_X, p, and Λ. For a single R_X, [6] optimises degree parameters to minimise the decoding error probability of HG data. However, for the rate adaptive LDPCA code, it is hard to calculate decoding error probability at all rates within a specified range of compression rate, $[R_{X,\min}, R_{X,\max}]$. Therefore, we minimise the decoding error probability at $R_{X,\min}$ and $R_{X,\max}$ simultaneously [8]. To guarantee the low decoding error probability within the rate range, the cost function is defined as

$$P_{H,max} = \max(P_H(R_{X,min}, p_{\min} - \delta_{\min}, \boldsymbol{\Lambda}), P_H(R_{X,max}, p_{\max} - \delta_{\max}, \boldsymbol{\Lambda})) \tag{3}$$

The channel parameters of BSCs, $p_{\min}$ and $p_{\max}$, are corresponding to the Slepian-Wolf limits. The small positive values, δmin and δmax, indicate the performance gaps between the proposed codes and the Slepian-Wolf limits. To guarantee consistent performance within the rate range, δmin and δmax should satisfy (4) below:

$$H(p_{\min}) - H(p_{\min} - \delta_{\min}) = H(p_{\max}) - H(p_{\max} - \delta_{\max}) \quad (4)$$

Consider the problem of designing an UEP-LDPCA code for $R_X \in$ [0.175 0.7] with $R_H = 0.1$. The search space is $d_H \leq 30$, $d_{H1} \leq 30$, $d_{H2} \leq 30$, $d_L \leq 30$ and $\gamma \leq 0.03$. The optimised parameters of the two UEP-LPDCA ensembles are $(d_H, d_L) = (19,3)$ and $(d_{H1}, d_{H2}, d_L, \gamma) = (1, 19, 3, 0.025)$.

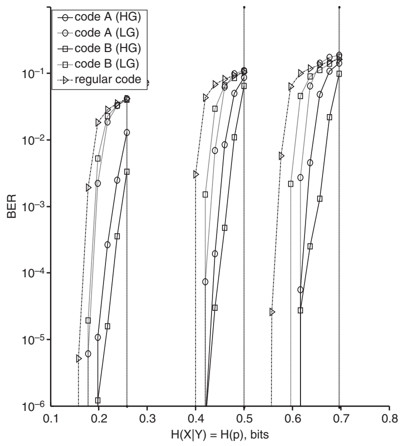

Fig. 2 *Comparison of decoding error rate for compression rate RX = 17/66, 33/66, 46/66*

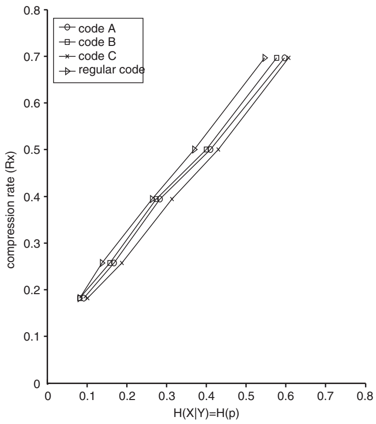

Fig. 3 *Comparison of overall compression performance*

Experimental results: We conducted finite length simulation to evaluate the proposed UPE-LDPCA codes. The source length is set as 16434 bits and 2000 source blocks were simulated with the maximum number of decoding iterations as 100. Fig. 2 shows the bit error rates (BERs) of two data groups, HG and LG. A regular code with source degree 3 is also evaluated for comparison. Define UEP gain as the ratio of the BER of LG to the BER of HG. It could be found that the proposed codes provide significant UEP gain. Especially, the UEP gain of code B is larger than 10^2. It is interesting to note that the UEP gain also increases when operating rate decreases. This property is good because the operating rates of real applications are usually very low. The overall compression performance is shown in Fig. 3. The performance of the proposed codes is in between the regular code and the optimised irregular code [8] (code C). One important reason for the inferiority to the optimised irregular code is that the proposed codes are restricted on the partial regular degree distribution.

Conclusion: In this Letter, two UEP-LDPCA codes are proposed and evaluated. The codes are for the applications with UEP requirements. The experimental result shows significant UEP gain compared with previous studies. However, considering overall coding performance, there is a performance gap between the proposed UEP-LPDCA codes and the optimised irregular code. This might motivate the investigation of advanced UEP-LDPCA to improve overall performance while maintaining the UEP property.

12 July 2012
doi: 10.1049/el.2012.2470

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