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Predicting Recurrent Financial Distresses with Autocorrelation Structure: An Empirical Analysis from an Emerging Market

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Abstract The dynamic logit model (DLM) with autocorrelation structure (Liang and Zeger Biometrika 73:13–22, 1986) is proposed as a model for predicting recurrent financial distresses. This model has been applied in many examples to analyze repeated binary data due to its simplicity in computation and formulation. We illustrate the proposed model using three different panel datasets of Taiwan industrial firms. These datasets are based on the well-known predictors in Altman (J Financ 23:589–609, 1968), Campbell et al. (J Financ 62:2899–2939, 2008), and Shumway (J Bus 74:101–124, 2001). To account for the correlations among the observations from the same firm, we consider two different autocorrelation structures: exchangeable and first-order autoregressive (AR1). The prediction models including the DLM with independent structure, the DLM with exchangeable structure, and the DLM with AR1 structure are separately applied to each of these datasets. Using an expanding rolling window approach, the empirical results show that for each of the three datasets, the DLM with AR1 structure yields the most accurate firm-by-firm financial-distress probabilities in out-of-sample analysis among the three models. Thus, it is a useful alternative for studying credit losses in portfolios.

Keywords Autocorrelation structure · Dynamic logit model · Expanding rolling window approach · Predictive interval · Predicted number of financial distresses · Recurrent financial distresses

JEL Classification G20 · G33 · C33

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1 Introduction

The prediction of financial distress is an important tool for credit risk management. The well-known models for prediction can be separated into two types: static and dynamic. The static model uses only single-period data of firms. The model includes the following types of methods: the discriminant analysis model (Altman 1968), the Merton model (Merton 1974; Vassalou and Xing 2004), the logit model (Ohlson 1980), the probit model (Zmijewski 1984), and the mixed effect logit model (Alfo et al. 2005). However, the static model might suffer from a loss of predictive power (Shumway 2001) because it ignores the changing characteristics of firms over time. On the other hand, the dynamic model uses multiple-period data of firms so that the prediction involves the effects of time-varying firm characteristics. The dynamic model has proved over time to be more powerful than the static model (Hillegeist et al. 2004; Campbell et al. 2008; Hwang et al. 2011; Glennon and Nigro 2005). Examples of dynamic models are the discrete-time hazard model (Shumway 2001; Chava and Jarrow 2004), the default-intensity model (Duffie et al. 2007), the quantile-regression approach (Li and Miu 2010), and the contingent-claim approach (Tang and Yan 2006). However, these dynamic models solely focus on the point in time when the financial distress first happens to a firm and ignores the possibility that subsequent financial distresses still might happen to that firm. Thus, these models do not generate predictions for firms with financial-distress experiences.

To avoid the above restriction on the dynamic model, we suggest using the dynamic logit model (DLM) to predict recurrent financial distresses. The DLM applies the logit model to the panel data containing not only the first financial distresses but also subsequent financial distresses of firms. Thus, this model has the advantage of using all available information to predict a firm's financial distress at any point in time whether or not that firm has financial-distress experience. Under the independence assumption, the important parameters in DLM can be simply determined by maximizing the log-likelihood function of the panel data under study. However, this independence assumption might not be proper in practice, because repeated observations from the same firm tend to be correlated with one another. If one imposes an improper independence assumption on DLM, then one might suffer from a loss of predictive power.

For more accurately predicting recurrent financial distresses, we consider the DLM with autocorrelation structure. This model is more robust in predicting financial distresses for firms than the DLM with independent structure. Under the autocorrelation structure, the unknown parameters in DLM are estimated using the generalized estimating equations (GEE) approach (Liang and Zeger 1986; Lipsitz et al. 1994). The consistency and asymptotic normality of the resulting estimators are given in Section 2. There are many software packages having the capabilities to implement GEE analyses, for example, SAS, S-Plus, and STATA. Thus, the computation required for the DLM with autocorrelation structure is as simple as that for the DLM with independent structure. There are many examples in the literature in which the GEE approach has proved to be more powerful for analyzing repeated data than the approach using independence estimating equations (IEE). For a detailed introduction to GEE, see for example, the monograph by Hardin and Hilbe (2002).

To implement the proposed model, exchangeable and first-order autoregressive (AR1) autocorrelation structures are used to account for the correlations among the observations from the same firm at different points in time. These two autocorrelation structures assume respectively that the magnitude of the correlation remains unchanged and decreases dramatically as the number of time lags increases. They have the advantage of being formulated using only one nuisance parameter. See Lee and Geisser (1975) and Geisser (1981) for the importance of parsimonious correlation structures in prediction problems based on repeated measurements.

In Section 3, we illustrate the proposed model using three different panel datasets of industrial firms listed on the two major Taiwan stock exchanges: Taiwan Stock Exchange (TWSE) and GreTai Securities Market (GTSM). These panel datasets are based on the well-known predictors suggested by Altman (1968), Campbell et al. (2008), and Shumway (2001). The three prediction models comprise the following: the DLM with independent structure, the DLM with exchangeable structure, and the DLM with AR1 structure. These models are separately applied to each of the three panel datasets. We measure the performance of the proposed models through out-of-sample analysis with two performance metrics. These two performance metrics are the absolute difference (AD) between the actual number of financial distresses (ANFD) and the predicted number of financial distresses (PNFD) as well as the predictive interval (PI) of ANFD. These metrics are based on the actual magnitudes of financial-distress probabilities of firms. Using an expanding rolling window approach (Hillegeist et al. 2004; Chava et al. 2011), the empirical results in Section 3 show that for each of the three panel datasets, the DLM with AR1 structure has the best performance among the three prediction models. Thus, this model has the potential to be a powerful model for studying credit losses in portfolios.

The remainder of this paper is organized as follows. In Section 2, we develop the method for predicting recurrent financial distresses based on DLM. Section 3 presents the empirical results. Section 4 contains the concluding remarks and future research topics. The appendix gives a computational procedure based on the GEE approach for estimating unknown parameters in the DLM with autocorrelation structure.

2 Method

In this section, we first describe the formulation of the DLM with independent structure and estimate its unknown parameters using a maximum likelihood method. Then we give the idea of the DLM with autocorrelation structure and estimate its unknown parameters using the GEE approach. Afterwards, we introduce two out-of-sample performance metrics to measure the performance of the discussed prediction methods.

2.1 The DLM with independent structure

The DLM has the advantage of using all available information to predict each firm's financial distress at any point in time. In the following, we describe the structure of the panel data used in the prediction methods based on the DLM.

Two factors determine the panel data: the sampling criteria and the sampling period. Our sampling criterion is that all industrial firms that are listed on the TWSE or GTSM during the sampling period are included in the sample. All information occurring at the discrete points in time during the sampling period is collected from the Taiwan Economic Journal (TEJ) database. Let the sampling period be $[\xi_1, \xi_2]$. Suppose that there are n selected firms under the particular sampling scheme. We denote the panel data by

$$\left\{\left(Y_{i,j}, x_{i,j}\right) : j = s_i, \cdots, t_i, i = 1, \cdots, n\right\}.$$

Here s_i denotes the first observation time and t_i the last observation time for the i-th firm in the sampling period $[\xi_1, \xi_2]$ The value of $Y_{i,j}=1$ indicates that the financial status of the i-th firm at time j is in distress and $Y_{i,j}=0$ otherwise. Therefore, for the i-th firm, the results of $\sum_{j=s_i}^{t_i} Y_{i,j} = 0$, $\sum_{j=s_i}^{t_i} Y_{i,j} = 1$, and $\sum_{j=s_i}^{t_i} Y_{i,j} > 1$ indicate respectively that the firm either has no, one, or repeated financial-distress experiences during the sampling period. Section 3 provides the definition of financial distress for the sampled firms. Further, we let $x_{i,j}$ be the value of the $d\times 1$ predictor X collected from the i-th firm at time j.

Under the independence assumption, the likelihood function of the panel data is expressed as:

$$L = \prod_{i=1}^{n}\prod_{j=s_i}^{t_i} p_{i,j}^{Y_{i,j}}\left(1 - p_{i,j}\right)^{1-Y_{i,j}}. \tag{1}$$

Here $p_{i,j} = p\left(Y_{i,j} = 1|x_{i,j}\right) \equiv p\left(j, x_{i,j}\right)$ stands for the probability of financial distress happening to the i-th firm at time j. Thus, the probability function $p_{i,j}$ can be of any functional form with values in the interval (0, 1). The DLM considers a linear logistic function for $p_{i,j}$, that is:

$$p_{i,j} = \frac{\exp\left(\alpha + \beta x_{i,j}\right)}{1 + \exp\left(\alpha + \beta x_{i,j}\right)}, \tag{2}$$

where α and β are 1×1 and $1\times d$ vectors of parameters respectively. Plugging Eq. (2) into Eq. (1), the resulting log-likelihood of the panel data becomes:

$$\ell = \sum_{i=1}^{n}\sum_{j=s_i}^{t_i}\left[Y_{i,j}\left(\alpha + \beta x_{i,j}\right) - \log\left\{1 + \exp\left(\alpha + \beta x_{i,j}\right)\right\}\right]. \tag{3}$$

The maximum likelihood estimate $\left(\widehat{\alpha}_I, \widehat{\beta}_I\right)$ of (α, β) in Eq. (3) can be obtained by maximizing ℓ with respect to (α, β), or by solving the normal equations:

$$0 = \frac{\partial \ell}{\partial(\alpha,\beta)^T} = \sum_{i=1}^{n}\sum_{j=s_i}^{t_i}\left\{Y_{i,j} - \frac{\exp\left(\alpha + \beta x_{i,j}\right)}{1 + \exp\left(\alpha + \beta x_{i,j}\right)}\right\}\begin{bmatrix}1\\ x_{i,j}\end{bmatrix}. \tag{4}$$

Using Eq. (2) and replacing the unknown parameters α and β with their maximum likelihood estimates $\widehat{\alpha}_I$ and $\widehat{\beta}_I$, if a firm has the predictor value x_0 at time t_0 then its predicted financial-distress probability based on the DLM with independent structure is:

$$\widehat{p}_I(t_0, x_0) = \frac{\exp\left(\widehat{\alpha}_I + \widehat{\beta}_I x_0\right)}{1 + \exp\left(\widehat{\alpha}_I + \widehat{\beta}_I x_0\right)}. \tag{5}$$

Under the independence assumption, Liang and Zeger (1986) show that the maximum likelihood estimate $\left(\widehat{\alpha}_I, \widehat{\beta}_I\right)$ is consistent for (α, β). Thus, the predicted financial-distress probability $\widehat{p}_I(t_0, x_0)$ converges to the true financial-distress probability $p(t_0, x_0) = \frac{\exp(\alpha+\beta x_0)}{1+\exp(\alpha+\beta x_0)}$. This result shows that the DLM with independent structure is an efficient prediction model if the imposed independence assumption is correct.

Further, through a straightforward calculation, the normal equations in Eq. (4) can also be equivalently expressed as the IEE:

$$0 = \frac{\partial \ell}{\partial(\alpha,\beta)^T} = \sum_{i=1}^{n} D_i^T V_i^{-1}(Y_i - p_i). \tag{6}$$

We use this result in subsection 2.2 to develop the GEE for the DLM with autocorrelation structure. The notations of Y_i, p_i, V_i and D_i in Eq. (6) are defined by

$$Y_i = \begin{bmatrix} Y_{i,s_i} \\ Y_{i,s_i+1} \\ \vdots \\ Y_{i,t_i} \end{bmatrix}, \quad p_i = \begin{bmatrix} p_{i,s_i} \\ p_{i,s_i+1} \\ \vdots \\ p_{i,t_i} \end{bmatrix}, \quad V_i = \begin{bmatrix} V_{i,s_i} & 0 & \cdots & 0 \\ 0 & V_{i,s_i+1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & V_{i,t_i} \end{bmatrix}, \quad D_i = \begin{bmatrix} D_{i,s_i} \\ D_{i,s_i+1} \\ \vdots \\ D_{i,t_i} \end{bmatrix},$$

where

$$V_{i,j} = p_{i,j}(1-p_{i,j}) \equiv Var(Y_{i,j}|x_{i,j}), \quad D_{i,j} = p_{i,j}(1-p_{i,j})\left(1, x_{i,j}^T\right) \equiv \{\partial/\partial(\alpha,\beta)\}p_{i,j}.$$

Under the independence assumption, the quantities p_i, V_i, and D_i stand for $E(Y_i|x_i)$, $Cov(Y_i|x_i)$, and $\{\partial/\partial(\alpha,\beta)\}p_i$, respectively, for each $i = 1, \cdots, n,$ where $x_i = \left(x_{i,s_i}, \cdots, x_{i,t_i}\right)$.

2.2 The DLM with autocorrelation structure

The DLM with autocorrelation structure is developed by assuming that the time series observations $Y_{i,j}$ from the same firm are correlated with one another, but those from different firms are not correlated. Under the autocorrelation assumption, the value of (α, β) in DLM is estimated using the GEE approach. To formulate the GEE, let $\rho_{j,k}$ be the correlation coefficient between $Y_{i,j}$ and $Y_{i,k}$, where $j, k \in \{s_i, \cdots, t_i\}$, for each $i = 1, \cdots, n$. Thus, the covariance matrix of Y_i can be expressed as:

$$Cov(Y_i|x_i) = V_i^{1/2} A_i V_i^{1/2} \equiv G_i, \tag{7}$$

for each $i = 1, \cdots, n$. Subsection 2.1 provides the definitions of Y_i, x_i, and V_i, and

$$A_i = \begin{bmatrix} 1 & \rho_{s_i,s_i+1} & \cdots & \rho_{s_i,t_i} \\ \rho_{s_i+1,s_i} & 1 & \cdots & \rho_{s_i+1,t_i} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{t_i,s_i} & \rho_{t_i,s_i+1} & \cdots & 1 \end{bmatrix}.$$

Note that A_i stands for the correlation matrix of Y_i Using the result from Eq. (7), the associated GEE are similarly formulated as the IEE in Eq. (6) with V_i replaced by G_i Specifically, the GEE are:

$$0 = \sum_{i=1}^{n} D_i^T G_i^{-1}(Y_i - p_i). \tag{8}$$

By way of generation, the GEE alleviate the need to correctly specify the joint distribution of Y_i. But the price paid by the GEE for simple formulation is that the corresponding log-likelihood function of the panel data is not available, and thus the GEE approach is not a maximum likelihood method. A computational procedure for finding the solution to the GEE in Eq. (8) is in the appendix.

In this paper, we consider two types of autocorrelation structures for the time series observations $Y_{i,j}$ from the same firm. One is the exchangeable structure with $\rho_{j,k} = \rho$, and thus the corresponding correlation matrix of Y_i is:

$$A_i = \begin{bmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \cdots & 1 \end{bmatrix},$$

for each $i = 1, \cdots, n$. The other is the AR1 structure with $\rho_{j,k} = \rho^{|j-k|}$, and thus the corresponding correlation matrix of Y_i is:

$$A_i = \begin{bmatrix} 1 & \rho & \cdots & \rho^{t_i - s_i} \\ \rho & 1 & \cdots & \rho^{t_i - s_i - 1} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{t_i - s_i} & \rho^{t_i - s_i - 1} & \cdots & 1 \end{bmatrix},$$

for each $i = 1, \cdots, n$. Given each of these two autocorrelation structures, if ρ=0 then $G_i = V_i$, for each $i = 1, \cdots, n$, and the resulting GEE become the IEE in Eq. (6).

We also consider other types of autocorrelation structures such as the m-dependence structure with $\rho_{j,k}$=0 for $|j-k| > m$, the banded correlation structure with $\rho_{j,k} = \rho_{|j-k|}$ for $|j-k| \geq 1$, and the unstructured correlation without constraints on $\rho_{j,\ k}$. However, their corresponding estimates of the regression parameters α and β provided by our computational algorithm are not of normal convergence, and thus we do not report the results. The poor computational results might be due to there being too many nuisance parameters $\rho_{j,k}$ involved. The numbers of nuisance parameters in these autocorrelation structures are $\sum_{k=1}^{m} (\xi_2 - \xi_1 + 1 - k)$, ξ_2–ξ_1, and $(\xi_2 - \xi_1)(\xi_2 - \xi_1 + 1)/2$, respectively. Here ξ_1 and ξ_2 are the start and the end points in time for the sampling period of the panel data respectively.

Given each of exchangeable and AR1 structures, set $\left(\widehat{\alpha}_G, \widehat{\beta}_G\right)$ as the solution of the corresponding GEE in Eq. (8). Regardless of whether the imposed covariance matrix G_i is correctly specified for $Cov(Y_i|x_i)$, $\left(\widehat{\alpha}_G, \widehat{\beta}_G\right)$ is consistent for (α, β) and has an asymptotic normal distribution with covariance matrix:

$$V_1 = \lim_{n\to\infty} \left(\sum_{i=1}^{n} D_i^T G_i^{-1} D_i\right)^{-1} \left\{\sum_{i=1}^{n} D_i^T G_i^{-1} Cov(Y_i|x_i) G_i^{-1} D_i\right\} \left(\sum_{i=1}^{n} D_i^T G_i^{-1} D_i\right)^{-1}.$$

If G_i is correctly specified so that $Cov(Y_i|x_i) = G_i$, then V_1 reduces to

$$V_2 = \lim_{n\to\infty} \left(\sum_{i=1}^{n} D_i^T G_i^{-1} D_i\right)^{-1}.$$

Consistent estimators $\widehat{V}_1$ and $\widehat{V}_2$ for V_1 and V_2 can be constructed by replacing the unknown quantities α, β, ρ, and $Cov(Y_i|x_i)$ with their estimates $\widehat{\alpha}_G$, $\widehat{\beta}_G$, $\widehat{\rho}$, and $(Y_i - \widehat{p}_i)(Y_i - \widehat{p}_i)^T$, respectively in each case. Here $\widehat{\rho} = \left\{\sum_{i=1}^{n}(t_i - s_i)(t_i - s_i + 1)/2 - d - 1\right\}^{-1}\sum_{i=1}^{n}\sum_{j=s_i}^{t_i-1}\sum_{k=j+1}^{t_i}\widehat{e}_{i,j}\widehat{e}_{i,k}$ for the exchangeable structure, $\widehat{\rho} = \left\{\sum_{i=1}^{n}(t_i - s_i) - d - 1\right\}^{-1} \times \sum_{i=1}^{n}\sum_{j=s_i}^{t_i-1}\widehat{e}_{i,j}\widehat{e}_{i,j+1}$ for the AR1 structure, $\widehat{e}_{i,j} = \left\{\widehat{p}_{i,j}\left(1 - \widehat{p}_{i,j}\right)\right\}^{-1/2}\left(Y_{i,j} - \widehat{p}_{i,j}\right)$, $\widehat{p}_{i,j} = \frac{\exp\left(\widehat{\alpha}_G + \widehat{\beta}_G x_{i,j}\right)}{1+\exp\left(\widehat{\alpha}_G + \widehat{\beta}_G x_{i,j}\right)}$, $\widehat{p}_i = \left[\widehat{p}_{i,s_i}, \cdots, \widehat{p}_{i,t_i}\right]^T$.

The estimator $\widehat{V}_1$ is called the robust covariance estimator because it is consistent for V_1 regardless of whether or not G_i is correctly specified for $Cov(Y_i|x_i)$. In contrast, $\widehat{V}_2$ is called the naive covariance estimator because it is based on the assumption that G_i is correctly specified for $Cov(Y_i|x_i)$. See Liang and Zeger (1986) and Lipsitz et al. (1994) for a detailed introduction of the asymptotic properties of $\left(\widehat{\alpha}_G, \widehat{\beta}_G\right)$.

Using Eq. (2) and replacing α and β with their estimates $\widehat{\alpha}_G$ and $\widehat{\beta}_G$, if a firm has the predictor value x_0 at time t_0, then its predicted financial-distress probability based on the DLM with autocorrelation structure is:

$$\widehat{p}_G(t_0, x_0) = \frac{\exp\left(\widehat{\alpha}_G + \widehat{\beta}_G x_0\right)}{1 + \exp\left(\widehat{\alpha}_G + \widehat{\beta}_G x_0\right)}. \quad (9)$$

By the consistency of $\widehat{\alpha}_G$ and $\widehat{\beta}_G$, $\widehat{p}_G(t_0, x_0)$ is also consistent for the true financial-distress probability $p(t_0, x_0)$. Thus, the DLM with autocorrelation structure is a reliable prediction model for a firm's financial distress whether or not the autocorrelation structure imposed on the time series observations $Y_{i,j}$ from the same firm is correctly specified.

The same developments for $\left(\widehat{\alpha}_G, \widehat{\beta}_G\right)$, V_1, V_2, $\widehat{\rho}$, $\widehat{V}_1$, $\widehat{V}_2$, and $\widehat{p}_G(t_0, x_0)$ based on the DLM with autocorrelation structure can also be applied to the DLM with independent structure by taking the estimate of the nuisance parameter ρ as zero. Thus, both the consistency and the asymptotic normality of $\left(\widehat{\alpha}_G, \widehat{\beta}_G\right)$ are shared with $\left(\widehat{\alpha}_I, \widehat{\beta}_I\right)$. The principle disadvantage of $\left(\widehat{\alpha}_I, \widehat{\beta}_I\right)$ is that it might not have high efficiency in cases where the autocorrelation is large (Liang and Zeger 1986).

2.3 Measuring prediction performance

For assessing the prediction models introduced in subsections 2.1 and 2.2, there are some standard performance metrics, for example, the out-of-sample type I and type II error rates (Cheng et al. 2010) and the out-of-sample accuracy ratio obtained from the cumulative accuracy profile curve (BCBS 2005). However, these standard performance metrics are only based on the relative ordinal rankings of financial-distress probabilities and not on the actual magnitudes of those probabilities. Thus, they are not suitable to assess whether a prediction model generates financial-distress probabilities that are adequate in absolute terms. Due to the fact that the credit losses of portfolios depend on the actual magnitudes of financial-distress probabilities, a proper prediction model must be able to provide accurate firm-by-firm financial-distress probabilities. Accordingly, we use the out-of-sample AD (AD_{out}) between ANFD and PNFD and the out-of-sample PI (PI_{out}) of ANFD as performance metrics for assessing the prediction models. Both

AD_{out} and PI_{out} are developed using the actual magnitudes of financial-distress probabilities rather than the relative firm-riskiness rankings. Korablev and Dwyer (2007), Duffie et al. (2009), and Chava et al. (2011) also consider similar performance metrics.

To compute AD_{out} and PI_{out}, the out-of-sample data are selected. In contrast, the panel data that are used to build the prediction models in subsections 2.1–2.2 are considered as the "in-sample" data. The out-of-sample data are generated in a similar fashion to the in-sample data. In this paper, the out-of-sample period is $(\xi_2, \xi_2 + 1]$, where ξ_2 is the end time of the sampling period of in-sample data. The out-of-sample firms comprise all industrial firms that are listed on the TWSE or GTSM during the out-of-sample period. Suppose that there are n_0 out-of-sample firms. All values of the $d\times 1$ predictor X for the n_0 out-of-sample firms occurring at time ξ_2+1 are collected from the TEJ database. The out-of-sample data are denoted by

$$\left\{\left(\widetilde{Y}_{k,\xi_2+1}, \widetilde{x}_{k,\xi_2+1}\right) : k = 1, \cdots, n_0\right\}.$$

Here $\widetilde{Y}_{k,\xi_2+1} = 1$ indicates that the financial status of the k-th out-of-sample firm is in distress at time ξ_2+1, and $\widetilde{Y}_{k,\xi_2+1} = 0$ otherwise. Further, $\widetilde{x}_{k,\xi_2+1}$ is the value of X collected from the k-th out-of-sample firm at time ξ_2+1, for each $k = 1, \cdots, n_0$.

Using the out-of-sample data $\left\{\left(\widetilde{Y}_{k,\xi_2+1}, \widetilde{x}_{k,\xi_2+1}\right) : k = 1, \cdots, n_0\right\}$ and the result from Eq. (9), we first evaluate the predicted financial-distress probabilities $\widehat{p}_G\left(\xi_2 + 1, \widetilde{x}_{k,\xi_2+1}\right)$, for each $k = 1, \cdots, n_0$. Then we apply the convolution calculation technique (Duan 2010) to these predicted financial-distress probabilities, so that the distribution of the number of financial distresses based on the DLM with autocorrelation structure for the n_0 out-of-sample firms at time ξ_2+1 can be obtained. The statistical characteristics of this distribution such as the mean, variance, and quantile can be evaluated. For the n_0 out-of-sample firms, their PNFD at time ξ_2+1, denoted by $PNFD_{out}(\xi_2+1)$, is taken to be the mean of this distribution. Further, the value of $PNFD_{out}(\xi_2+1)$ can also be equivalently obtained using $PNFD_{out}(\xi_2 + 1) = \sum_{k=1}^{n_0} \widehat{p}_G\left(\xi_2 + 1, \widetilde{x}_{k,\xi_2+1}\right)$. Thus the value of AD_{out} at time ξ_2+1 is defined by

$$AD_{out}(\xi_2 + 1) = |ANFD_{out}(\xi_2 + 1) - PNFD_{out}(\xi_2 + 1)|,$$

where $ANFD_{out}(\xi_2 + 1) = \sum_{k=1}^{n_0} \widetilde{Y}_{k,\xi_2+1}$ and $PNFD_{out}(\xi_2 + 1)$ stands for a prediction of the value of $ANFD_{out}(\xi_2 + 1)$ which has a mean of $\sum_{k=1}^{n_0} p\left(\xi_2 + 1, \widetilde{x}_{k,\xi_2+1}\right)$. On the other hand, for the n_0 out-of-sample firms, the 95 % PI of their $ANFD_{out}$ (ξ_2+1) can be taken as $[p_{0.025}, p_{0.975}]$ Here, $p_{0.025}$ and $p_{0.975}$ denote respectively the 2.5-th and 97.5-th percentiles of the distribution of the number of financial distresses. This 95 % PI of $ANFD_{out}$ (ξ_2+1) is denoted by PI_{out} (ξ_2+1).

The out-of-sample performance metrics AD_{out} (ξ_2+1) and PI_{out} (ξ_2+1) based on the DLM with autocorrelation structure can be similarly defined for the DLM with independent structure by replacing $\widehat{p}_G\left(\xi_2 + 1, \widetilde{x}_{k,\xi_2+1}\right)$ with $\widehat{p}_I\left(\xi_2 + 1, \widetilde{x}_{k,\xi_2+1}\right)$.

3 Empirical studies

In this section, we conduct empirical studies to compare the performance of the three prediction models: the DLM with independent structure, the DLM with exchangeable structure, and the DLM with AR1 structure.

3.1 Data and estimation results

To investigate the performance of the three prediction models, we collect three different panel datasets based on the well-known predictors from Altman (1968), Campbell et al. (2008), and Shumway (2001). For simplicity of presentation, these predictors are called the Altman, Campbell, and Shumway predictors respectively. Table 1 gives their definitions. Further, the panel data used to build the three prediction models are also called the in-sample data in this paper.

The sampling period for each of the three panel datasets is 1986 to 2008. The in-sample firms consist of all industrial firms listed on the TWSE or GTSM during the sampling period. We exclude financial firms from the sample due to the unique capital requirements and regulatory structure of that industry group. The predictor values come from the calendar year-end data collected from the TEJ database. In order to eliminate outliers, the values of each predictor (except the predictor PRICE) are winsorized using a 5/95 percentile interval (Campbell et al. 2008). The resulting predictor values measure the risk of financial distress

Table 1 The definitions of the Altman, Campbell, and Shumway predictors. The results are given respectively in Panels A, B, and C

Variable	Definition
Panel A: Altman predictors	
WCTA	Working capital divided by total assets
RETA	Retained earnings divided by total assets
EBITTA	Earnings before interest and taxes divided by total assets
METL	Market equity divided by total liabilities
STA	Sales divided by total assets
Panel B: Campbell predictors	
NIMTAAVG	$\text{NIMTAAVG} = (1-\omega^3)(1-\omega^{12})^{-1}(NIMTA_4 + \omega^3 NIMTA_3 + \omega^6 NIMTA_2 + \omega^9 NIMTA_1)$, the weighted average of four quarterly *NIMTA* with $\omega = 2^{-1/3}$ and *NIMTA* as net income divided by market-valued total assets[a]
TLMTA	Total liabilities divided by market-valued total assets[a]
EXRETAVG	$\text{EXRETAVG} = (1-\omega)(1-\omega^{12})^{-1}(EXRET_{12} + \cdots + \omega^{11} EXRET_1)$, the weighted average of twelve monthly excess returns (*EXRET*) with $\omega = 2^{-1/3}$ and *EXRET* as the return on the firm minus the TWSE capitalization weighted stock index (TAIEX) return
SIGMA	Annualized square root of the average of the squared deviations in the firm's daily stock returns from zero over the past 3 months
RSIZE	Logarithm of each firm's market equity value divided by the total TWSE market equity value
CASHMTA	Cash and short-term investments divided by market-valued total assets[a]
MB	Market equity value divided by book equity value
PRICE	Logarithm of stock price if the price is below NT$22, and logarithm of NT$22 otherwise
Panel C: Shumway predictors	
SIGMA	Annualized square root of the average of the squared deviations in the firm's daily stock returns from zero over the past 3 months
RSIZE	Logarithm of each firm's market equity value divided by the total TWSE market equity value
NITA	Net income divided by total assets
TLTA	Total liabilities divided by total assets
EXRET	The return on the firm minus the TAIEX return

[a] The market-valued total assets is the sum of the book value of liabilities and the market value of equities

over the 12-month period beginning 4 months after the calendar year end (Hillegeist et al. 2004). Using the TEJ definitions for characteristics such as negative net worth and bankruptcy, we identify the financial-distress filings covering the period from May, 1987 to April, 2010. Table 2 presents the frequency distribution of the in-sample firms in each of the three panel datasets according to the number of financial distresses that a firm experiences during the sampling period. The results in Table 2 show that for each of the three panel datasets, more than 12 % of the in-sample firms have financial-distress experience. Table 3 gives the summary statistics of the predictor values in each of the three panel datasets.

Given the in-sample data, Table 4 reports the estimation results of the three prediction models using each set of the Altman, Campbell, and Shumway predictors. Table 4 shows that the values of the estimated coefficients in each of the three models all agree with their expected signs. Also, the table indicates that the robust standard errors for most of coefficient estimates are of larger magnitude than the associated naive standard errors.

3.2 Prediction results

To compare the prediction performance of the models based on each set of the Altman, Campbell, and Shumway predictors, we collect the corresponding out-of-sample data with the predictor values in 2009 and financial statuses in 2010 (covering the period from May, 2010 to April, 2011) from the TEJ database. The out-of-sample data based on the Altman predictors consist of 24 financial distresses and 1,387 firm-year observations, the Campbell predictors consist of 22 financial distresses and 1,242 firm-year observations, and the Shumway predictors consist of 23 financial distresses and 1,368 firm-year observations.

Using the out-of-sample data and the estimation results in Table 4, Fig. 1 and Table 5 present the out-of-sample prediction performance of the three models. Figure 1 gives the distributions of the number of financial distresses in 2010. Panel (a) of Fig. 1 presents the results for the three models based on the Altman predictors. The panel shows that the distributions generated from the three models are quite different. From Panel (a) of Fig. 1, we see that the mean of the distribution generated by the DLM with AR1 structure is the closest to the ANFD among the models. Also, the distribution generated by the DLM with AR1 structure shifts to the left relative to the distribution produced by each of the other two

Table 2 The frequency and percent frequency (in parentheses) distributions of the in-sample firms in each of the three panel datasets according to the number *N* of financial distresses that a firm has experienced during the sampling period. These panel datasets are collected from the TEJ database for the period of 1986 to 2008

N	Altman predictors	Campbell predictors	Shumway predictors
0	1,495 (87.12 %)	1,209 (84.84 %)	1,435 (86.65 %)
1	66 (3.85 %)	79 (5.54 %)	68 (4.11 %)
2	73 (4.25 %)	71 (4.98 %)	73 (4.41 %)
3	29 (1.69 %)	29 (2.04 %)	32 (1.93 %)
4	28 (1.63 %)	19 (1.33 %)	25 (1.51 %)
5	9 (0.52 %)	9 (0.63 %)	14 (0.85 %)
6	7 (0.41 %)	3 (0.21 %)	3 (0.18 %)
7	4 (0.23 %)	5 (0.35 %)	3 (0.18 %)
8	1 (0.06 %)	0 (0.00 %)	2 (0.12 %)
9	4 (0.23 %)	1 (0.07 %)	1 (0.06 %)
Total firms	1,716 (100 %)	1,425 (100 %)	1,656 (100 %)

Table 3 Summary statistics of predictors in each of the three panel datasets. These panel datasets are collected from the TEJ database for the period of 1986 to 2008. Panels A, B, and C give the results for the Altman, Campbell, and Shumway predictors respectively

Variable	Mean	Median	Standard deviation	Minimum	Maximum
Panel A: Altman predictors					
570 financial distresses, 14,269 firm-year observations, and 1,716 firms					
WCTA	0.167	0.160	0.170	−0.141	0.490
RETA	0.060	0.082	0.143	−0.325	0.272
EBITTA	0.046	0.053	0.077	−0.138	0.175
METL	3.575	2.112	3.830	0.301	14.890
STA	0.758	0.664	0.440	0.164	1.827
Panel B: Campbell predictors					
491 financial distresses, 12,858 firm-year observations, and 1,425 firms					
NIMTAAVG	0.003	0.008	0.018	−0.049	0.025
TLMTA	0.346	0.312	0.202	0.063	0.749
EXRETAVG	−0.006	−0.007	0.045	−0.090	0.087
SIGMA	0.447	0.432	0.155	0.205	0.755
RSIZE	−8.149	−8.231	1.480	−10.567	−5.348
CASHMTA	0.084	0.060	0.074	0.007	0.270
MB	1.514	1.340	0.799	0.473	3.315
PRICE	2.642	2.996	0.648	−2.408	3.091
Panel C: Shumway predictors					
544 financial distresses, 13,655 firm-year observations, and 1,656 firms					
SIGMA	0.459	0.439	0.166	0.207	0.809
RSIZE	−8.231	−8.310	1.523	−10.769	−5.389
NITA	0.030	0.038	0.075	−0.155	0.149
TLTA	0.394	0.388	0.166	0.122	0.716
EXRET	−0.054	−0.059	0.410	−0.824	0.751

models, and the distribution has a smaller mean. The results in Panel (a) of Fig. 1 are the same as those in Panels (b) and (c) of Fig. 1 for the distributions generated by the models based on the Campbell and Shumway predictors respectively.

Table 5 gives the values of PNFD, AD, and 95 % PI produced from the distributions in Fig. 1. Among the three models, Table 5 shows that the value of AD from the DLM with AR1 structure is the smallest one for each set of the Altman, Campbell, and Shumway predictors. Also, the 95 % PI from the DLM with AR1 structure is the only one containing the ANFD for the Altman and Shumway predictors. By the results presented in Fig. 1 and Table 5, we conclude that the DLM with AR1 structure has the best out-of-sample prediction performance in 2010 among the three models.

3.3 Robustness performance

In this section, we use an expanding rolling window approach to assess the robustness of the advantage of the DLM with AR1 structure, that is, it has the best out-of-sample prediction performance among the three models. For simplicity, we use the same predictors to generate the in-sample and out-of-sample data. Also, we use the same computational procedures as in

Table 4 Estimation results of the three models. Panels A, B, and C present the results based on the Altman, Campbell, and Shumway predictors respectively. These panel data are collected for the period of 1986 to 2008 to study recurrent financial distresses. The three models are the DLM with independent structure, the DLM with exchangeable structure, and the DLM with AR1 structure

Variable	Independent			Exchangeable			AR1		
	Coefficient estimate	Naive standard error	Robust standard error	Coefficient estimate	Naive standard error	Robust standard error	Coefficient estimate	Naive standard error	Robust standard error
Panel A: Altman predictors									
570 financial distresses, 14,269 firm-year observations, and 1,716 firms									
Intercept	−3.507	0.126***	0.177***	−3.493	0.142***	0.176***	−3.749	0.170***	0.236***
WCTA	−0.878	0.333***	0.469*	−0.981	0.357***	0.453**	−0.672	0.431	0.561
RETA	−10.455	0.436***	0.575***	−9.640	0.464***	0.549***	−8.917	0.556***	0.712***
EBITTA	−1.762	0.800**	1.092	−1.879	0.823**	1.016*	−0.782	0.953	1.249
METL	−0.109	0.026***	0.039***	−0.090	0.026***	0.036**	−0.118	0.036***	0.056**
STA	−0.304	0.123**	0.181*	−0.267	0.133**	0.178	−0.295	0.163*	0.218
Panel B: Campbell predictors									
491 financial distresses, 12,858 firm-year observations, and 1,425 firms									
Intercept	−8.841	0.701***	0.982***	−8.172	0.768***	0.950***	−8.197	0.771***	0.889***
NIMTAAVG	−29.127	2.922***	3.877***	−27.021	2.899***	3.661***	−21.638	2.961***	3.509***
TLMTA	2.807	0.432***	0.659***	2.497	0.443***	0.602***	2.489	0.461***	0.603***
EXRETAVG	−4.492	1.264***	1.254***	−3.669	1.218***	1.126***	−3.174	1.194***	1.031***
SIGMA	2.069	0.425***	0.459***	1.564	0.414***	0.400***	1.457	0.423***	0.388***
RSIZE	−0.346	0.055***	0.073***	−0.339	0.062***	0.076***	−0.361	0.064***	0.071***
CASHMTA	−1.660	0.955*	1.189	−1.462	0.953	1.075	−2.172	1.088**	1.061**
MB	1.341	0.099***	0.155***	1.140	0.097***	0.127***	1.061	0.099***	0.124***
PRICE	−0.862	0.099***	0.143***	−0.783	0.100***	0.133***	−0.749	0.103***	0.128***
Panel C: Shumway predictors									
544 financial distresses, 13,655 firm-year observations, and 1,656 firms									
Intercept	−9.250	0.413***	0.596***	−9.275	0.545***	0.815***	−8.869	0.576***	0.657***

Table 4 (continued)

Variable	Independent			Exchangeable			AR1		
	Coefficient estimate	Naive standard error	Robust standard error	Coefficient estimate	Naive standard error	Robust standard error	Coefficient estimate	Naive standard error	Robust standard error
SIGMA	1.692	0.331***	0.381***	1.041	0.350***	0.390***	0.156	0.342	0.345
RSIZE	−0.226	0.042***	0.057***	−0.292	0.055***	0.080***	−0.304	0.060***	0.067***
NITA	−12.278	0.778***	1.053***	−9.794	0.860***	1.268***	−5.862	0.835***	1.128***
TLTA	5.585	0.378***	0.669***	4.964	0.459***	0.775***	5.160	0.510***	0.783***
EXRET	−0.134	0.124	0.122	−0.134	0.126	0.135	−0.045	0.114	0.125

The notations ***, **, and * indicate the significance of the parameter based on the Wald chi-squared test at the 1 %, 5 %, and 10 % levels respectively

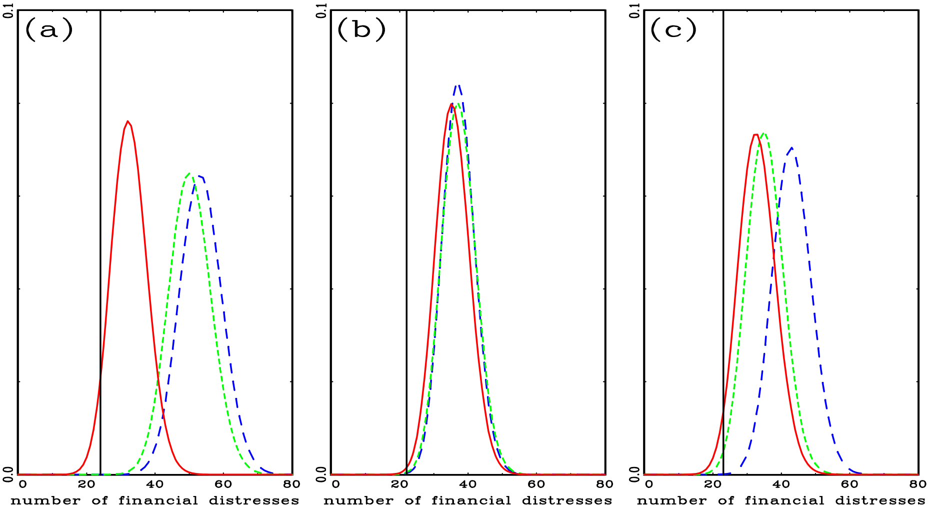

Fig. 1 Plot of the distributions of the number of financial distresses in 2010. The distributions are generated by applying Duan's (2010) convolution calculation technique to the predicted financial-distress probabilities of the out-of-sample firms based on the three models. These predicted financial-distress probabilities are computed using the predictor values of the out-of-sample firms in 2009. In each panel, the blue dashed, the green short dashed, and the red solid curves denote the distributions produced using the DLM with independent structure, the DLM with exchangeable structure, and the DLM with AR1 structure respectively. Also, the location of the black vertical solid curve stands for the value of ANFD. Panels (**a**), (**b**), and (**c**) show the results based on the Altman, Campbell, and Shumway predictors respectively

Table 5 The numerical values of PNFD, AD, and 95 % PI for the out-of-sample firms in 2010. The results are produced from the distributions of the number of financial distresses in Fig. 1. These distributions are obtained by applying Duan's (2010) convolution calculation technique to the predicted financial-distress probabilities of the out-of-sample firms in 2010 based on the three models. The three models are the DLM with independent structure, the DLM with exchangeable structure, and the DLM with AR1 structure. Panels A, B, and C present the numerical results based on the Altman, Campbell, and Shumway predictors respectively

	Independent	Exchangeable	AR1
Panel A: Altman predictors			
1,387 out-of-sample firms, ANFD=24			
PNFD	53.404	50.234	32.484
AD	29.404	26.234	8.484
PI	[41.057, 65.262]	[37.980, 62.035]	[22.074, 42.604][a]
Panel B: Campbell predictors			
1,242 out-of-sample firms, ANFD=22			
PNFD	37.220	37.247	35.507
AD	15.220	15.247	13.507
PI	[27.742, 46.259]	[27.257, 46.823]	[25.497, 45.107]
Panel C: Shumway predictors			
1,368 out-of-sample firms, ANFD=23			
PNFD	43.082	35.221	32.922
AD	20.082	12.221	9.922
PI	[31.770 53.951]	[24.450, 45.662]	[22.169, 43.458][a]

The notation [a] indicates that the 95 % PI contains the ANFD given in the same panel of the table subsections 3.1 and 3.2 to obtain the distributions of the number of financial distresses based on the three models in each year during the period of 2001 to 2010.

For the first window, we estimate the updated coefficients for each of the three models using the in-sample data comprising the predictor values from 1986 to 1999 and financial statuses from 1987 to 2000. The updated coefficients are combined with the predictor values of out-of-sample firms in 2000 to predict the financial-distress probabilities and to generate the distribution of the number of financial distresses for out-of-sample firms in 2001. The out-of-sample performance based on AD and 95 % PI during 2001 is measured for each of the three models. For the second window, we estimate the updated coefficients for each of the three models using the in-sample data comprising the predictor values from 1986 to 2000 and financial statuses from 1987 to 2001. The updated coefficients are combined with the predictor values of out-of-sample firms in 2001 to predict the financial-distress probabilities and to generate the distribution of the number of financial distresses for out-of-sample firms in 2002. The out-of-sample performance based on AD and 95 % PI during 2002 is measured for each of the three models. The process is continued so that the updated coefficients for each of the three models in the last window are based on the in-sample data comprising the predictor values from 1986 to 2008 and financial statuses from 1987 to 2009. The last set of updated coefficients is combined with the predictor values of out-of-sample firms in 2009 to predict the financial-distress probabilities and to generate the distribution of the number of financial distresses for out-of-sample firms in 2010. Thus, the out-of-sample performance based on AD and 95 % PI during 2010 is measured for each of the three models. The process carried out in the last window is the same as that performed in subsections 3.1 and 3.2. Table 6 gives the numbers of firm-year observations in the in-sample and out-of-sample data for each of the ten windows.

Figures 2 and 3 and Table 7 present the out-of-sample prediction performance of the three models based on each set of the Altman, Campbell, and Shumway predictors. Figures 2 and 3 give the out-of-sample prediction comparisons in terms of AD and 95 % PI respectively. From Fig. 2, we see that the DLM with AR1 structure has the best performance among the three models for most of the windows, in the sense of yielding the smallest values of AD. From Fig. 3, we see that the 95 % PI from the DLM with AR1 structure is of the best performance among the three models in terms of the number of times that the 95 % PI contains ANFD over the ten windows.

Table 7 summarizes the out-of-sample prediction performance in Figs. 2 and 3 for the three models. Table 7 gives the sample average and standard deviation of the values of AD as well as the number of times that the 95 % PI contains ANFD over the ten windows. For each set of the Altman, Campbell, and Shumway predictors, Table 7 shows that the values of AD generated by the DLM with AR1 structure over the ten windows not only have the smallest sample average but also have the lowest volatility, and the number of times that the 95 % PI contains ANFD over the ten windows is the largest. These results confirm the robustness of the advantage of the DLM with AR1 structure having the best out-of-sample prediction performance among the three models.

4 Concluding remarks and future research topics

In this paper, we propose to use the DLM with autocorrelation structure to predict recurrent financial distresses. We construct the model by applying the logit model to the panel data containing not only the first financial distresses but also subsequent financial distresses of firms. Thus the proposed model has the advantage of using all of the available information to

Table 6 The numbers of firm-year observations in the in-sample and out-of-sample data. The data are collected in each of the ten windows for investigating the robustness performance of the three models based on each set of the Altman, Campbell, and Shumway predictors. Panels A and B present the results for the in-sample and out-of-sample data respectively

Window (predictor sampling period)	Altman predictors		Campbell predictors		Shumway predictors	
	Financial distresses	Total firm-years	Financial distresses	Total firm-years	Financial distresses	Total firm-years
Panel A: In-sample data						
First window (1986–1999)	139	4,071	87	3,641	121	3,771
Second window (1986–2000)	173	4,812	119	4,359	154	4,507
Third window (1986–2001)	212	5,641	156	5,161	192	5,330
Fourth window (1986–2002)	252	6,632	196	6,098	232	6,289
Fifth window (1986–2003)	304	7,752	247	7,120	284	7,354
Sixth window (1986–2004)	366	9,012	304	8,209	345	8,539
Seventh window (1986–2005)	435	10,290	369	9,330	414	9,752
Eighth window (1986–2006)	486	11,579	414	10,477	464	11,018
Ninth window (1986–2007)	537	12,924	462	11,664	514	12,341
Tenth window (1986–2008)	570	14,269	491	12,858	544	13,655
Panel B: Out-of-sample data						
First window (2000)	34	741	32	718	33	736
Second window (2001)	39	829	37	802	38	823
Third window (2002)	40	991	40	937	40	959
Fourth window (2003)	52	1,120	51	1,022	52	1,065
Fifth window (2004)	62	1,260	57	1,089	61	1,185
Sixth window (2005)	69	1,278	65	1,121	69	1,213
Seventh window(2006)	51	1,289	45	1,147	50	1,266
Eighth window (2007)	51	1,345	48	1,187	50	1,323
Ninth window (2008)	33	1,345	29	1,194	30	1,314
Tenth window (2009)	24	1,387	22	1,242	23	1,368

predict a firm's financial distress at any point in time whether or not the firm has financial-distress experience. Further, the model assumes that the financial statuses collected for the same firm at different time points are correlated with one another, but those obtained from different firms are not correlated. Such an autocorrelation assumption is more appropriate than the usual independence assumption, because repeated observations from the same firm tend to be correlated with one another. We estimate the unknown parameters in the proposed model using the GEE approach. From the theoretical results in Liang and Zeger (1986) and Lipsitz et al. (1994), the estimated financial-distress probability based on the proposed

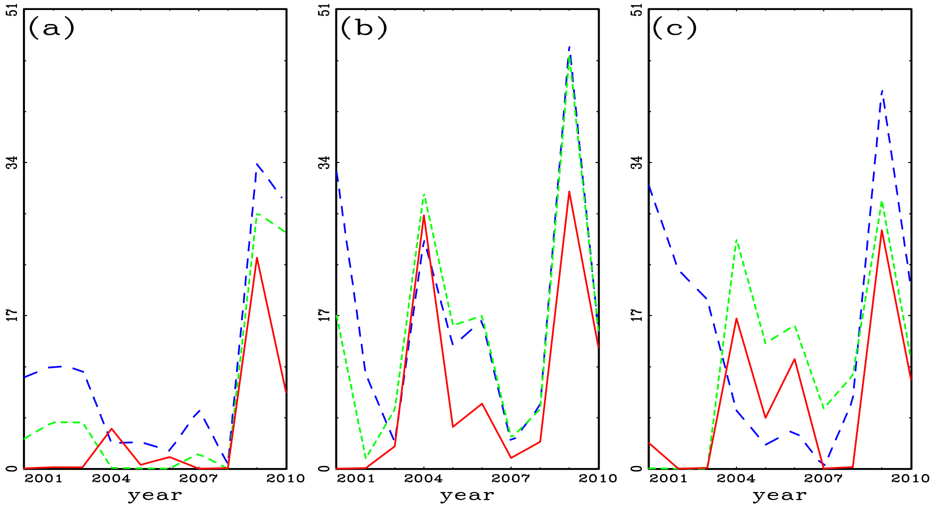

Fig. 2 The out-of-sample AD between ANFD and PNFD produced by the three models using an expanding rolling window approach for the period of 2001 to 2010. In each panel, the blue dashed, the green short dashed, and the red solid curves denote the values of AD produced using the DLM with independent structure, the DLM with exchangeable structure, and the DLM with AR1 structure respectively. Panels (**a**), (**b**), and (**c**) show the results based on the Altman, Campbell, and Shumway predictors respectively

model is consistent for the true financial-distress probability whether or not the imposed autocorrelation structure is correct. Thus, the DLM with autocorrelation structure is a reliable prediction model.

The performance of the proposed model in predicting recurrent financial distresses is illustrated using three panel datasets based on the well-known predictors from Altman (1968), Campbell et al. (2008), and Shumway (2001). Its robustness assessment is also investigated using different numbers of years of data. To implement the proposed method, we use two different autocorrelation structures: exchangeable and AR1, to account for the correlations among the observations from the same firm at different points in time. The data collected from the TEJ database are based on all industrial firms listed on the two major Taiwan stock exchanges: TWSE and GTSM. Using an expanding rolling window approach, our empirical results show that for each of the three panel datasets, the DLM with AR1 structure has better performance than the other two discussed models: the DLM with independent structure and the DLM with exchangeable structure. The DLM with AR1 structure yields more accurate PNFD and PI in out-of-sample analysis. Thus, from the empirical results, the DLM with AR1 structure has the potential to be a powerful model for studying credit losses in portfolios.

There are some possible extensions for the prediction models considered in this paper. First, a measure of aggregate systemic risk can be generated using the predicted financial-distress probabilities based on each of the three models. For example, using the expanding rolling window approach in subsection 3.3, systemic risk can be the equal-weighted (or value-weighted) average of predicted financial-distress probabilities for out-of-sample firms collected in each window. It is of interest to examine whether this type of measure of aggregate systemic risk can predict the future market return, economic downturn, TED spread, or loss-given-default rate. Caselli et al. (2008), Acharya et al. (2010), Allen et al. (2010), Adrian and Brunnermeier (2011), and Kelly (2011) all consider similar studies on systemic risk. Second, each of the three models can be extended to predict financial

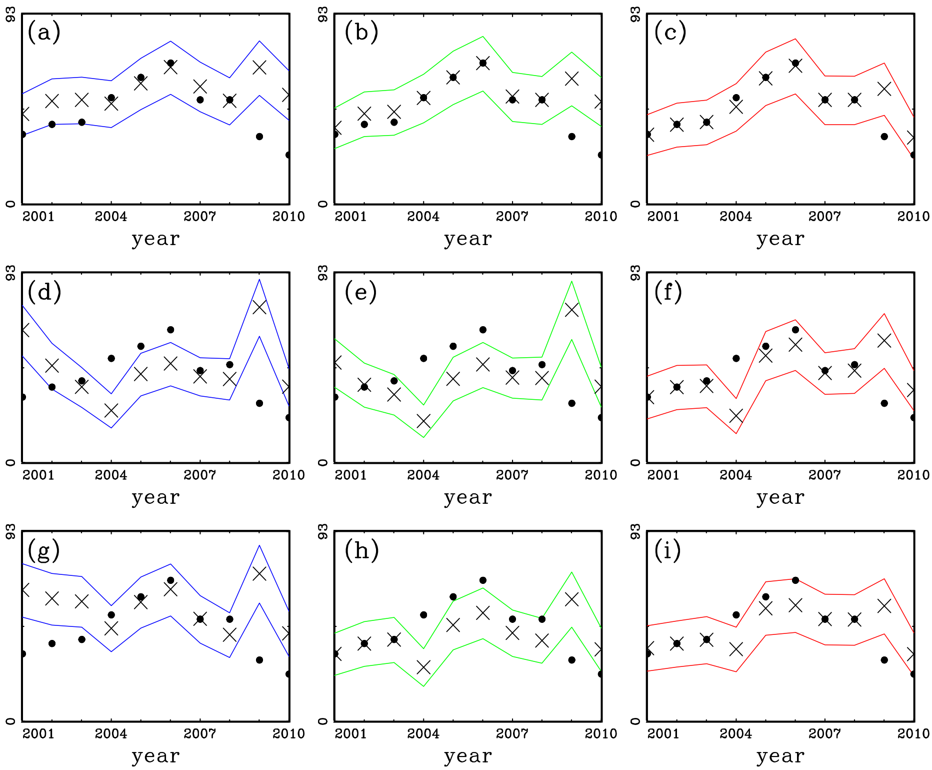

Fig. 3 The out-of-sample 95 % PI of ANFD produced by the three models using an expanding rolling window approach for the period of 2001 to 2010. The blue, the green, and the red solid curves denote the out-of-sample 95 % PI of ANFD produced using the DLM with independent structure, the DLM with exchangeable structure, and the DLM with AR1 structure respectively. In each panel, the black stars and the solid circles stand for PNFD and ANFD respectively. Panels **(a)**–**(c)** show the results based on the Altman predictors, **(d)**–**(f)** based on the Campbell predictors, and **(g)**–**(i)** based on the Shumway predictors

distresses over different future periods (Campbell et al. 2008; Duan et al. 2011). It is of interest to study the term structure of forward financial-distress probabilities based on each of the three models. Third, we study the performance of the three models in this paper only using firm-specific variables. The macroeconomic variables such as the real GDP growth rate and interest rate have been considered in Salas and Saurina (2002) for assessing credit risk. Researchers might study the effects of macroeconomic variables on the discussed models for predicting recurrent financial distresses. Finally, each of the three models assumes that the firm-specific effects on financial distress prediction are constant. However, in practice, these firm-specific effects should depend on business cycles (Pesaran et al. 2006), especially in cases of severe economic downturns. Thus, it would be more sensible to allow the parameters of logistic function to evolve with the effect of changes in macroeconomic dynamics. To do so, the idea of the varying coefficient model (Fan and Zhang 2008) can be considered. Specifically, the logistic function in Eq. (2) is replaced by

$$p_{i,j} = \frac{\exp\{\alpha(z_j) + \beta(z_j)x_{i,j}\}}{1 + \exp\{\alpha(z_j) + \beta(z_j)x_{i,j}\}},$$

Table 7 The sample average and standard deviation of the values of AD and the number N of times that the 95 % PI contains ANFD over the ten windows for the three models based on each set of the Altman, Campbell, and Shumway predictors. The three models are the DLM with independent structure, the DLM with exchangeable structure, and the DLM with AR1 structure

	Independent	Exchangeable	AR1
Panel A: Altman predictors			
Average	11.071	7.010	3.847
Standard deviation	11.575	10.922	7.411
N	8	8	9
Panel B: Campbell predictors			
Average	17.413	15.934	9.096
Standard deviation	14.084	13.595	11.468
N	4	4	7
Panel C: Shumway predictors			
Average	15.568	11.442	7.399
Standard deviation	13.786	10.414	8.928
N	5	5	8

where $\beta(z_j) = \{\beta_1(z_j), \cdots, \beta_d(z_j)\}$, and each of $\alpha(z_j), \beta_1(z_j), \cdots, \beta_d(z_j)$ is an unknown but smooth function of the value z_j collected at time j from the $k\times 1$ macroeconomic variable Z. Thus, the resulting prediction models would allow the effects of firm-specific predictors on credit risk to change with observable macroeconomic dynamics factors.

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Appendix: A computational procedure for finding the solution $\left(\widehat{\alpha}_G, \widehat{\beta}_G\right)$ of GEE

The value of $\left(\widehat{\alpha}_G, \widehat{\beta}_G\right)$ can be computed using the Fisher-scoring algorithm. Set $\theta = (\alpha, \beta)^T$. Given a starting value $\widehat{\theta}_0$ for θ, iterate

$$\widehat{\theta}_{m+1} = \widehat{\theta}_m + \left\{\sum_{i=1}^{n} D_i\left(\widehat{\theta}_m\right)^T G_i\left(\widehat{\theta}_m, \widehat{\rho}_m\right)^{-1} D_i\left(\widehat{\theta}_m\right)\right\}^{-1} \left[\sum_{i=1}^{n} D_i\left(\widehat{\theta}_m\right)^T G_i\left(\widehat{\theta}_m, \widehat{\rho}_m\right)^{-1} \left\{Y_i - p_i\left(\widehat{\theta}_m\right)\right\}\right],$$

until $\widehat{\theta}_{m+1} = \widehat{\theta}_m \equiv \left(\widehat{\alpha}_G, \widehat{\beta}_G\right)^T$ and $\widehat{\rho}_{m+1} = \widehat{\rho}_m \equiv \widehat{\rho}$. Here the nuisance parameter ρ in the m-th iteration is estimated by $\widehat{\rho}_m = \left\{\sum_{i=1}^{n} (t_i - s_i)(t_i - s_i + 1)/2 - d - 1\right\}^{-1} \sum_{i=1}^{n} \sum_{j=s_i}^{t_i-1} \sum_{k=j+1}^{t_i} \widehat{e}_{i,j}^{(m)} \widehat{e}_{i,k}^{(m)}$ for the exchangeable structure, $\widehat{\rho}_m = \left\{\sum_{i=1}^{n} (t_i - s_i) - d - 1\right\}^{-1} \sum_{i=1}^{n} \sum_{j=s_i}^{t_i-1} \widehat{e}_{i,j}^{(m)} \widehat{e}_{i,j+1}^{(m)}$ for the AR1 structure, $\widehat{e}_{i,j}^{(m)} = \left[\widehat{p}_{i,j}^{(m)}\left\{1 - \widehat{p}_{i,j}^{(m)}\right\}\right]^{-1/2} \left(Y_{i,j} - \widehat{p}_{i,j}^{(m)}\right)$, and $\widehat{p}_{i,j}^{(m)} = p_{i,j}\left(\widehat{\theta}_m\right)$. Liang and Zeger

(1986) suggest taking the starting value $\widehat{\theta}_0$ as the maximum likelihood estimate of θ produced in subsection 2.1 under the independence assumption.

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