J Econ (2013) 108:207–229
DOI 10.1007/s00712-012-0265-9

# Bundling strategy and product differentiation

**Hui-Ling Chung · Yan-Shu Lin · Jin-Li Hu**

Received: 31 May 2010 / Accepted: 11 January 2012 / Published online: 14 February 2012


**Abstract** The existing literature shows that a decrease in the degree of substitutability increases a monopoly's incentive to bundle. This paper in addition takes into account competition in the second product market and then re-examines how intra-brand and inter-brand product differentiations affect the incentive to bundle. In order to formally examine the above conjectures, this research builds up a two-firm, two-product model in which product 1 (monopoly product) is produced only by the bundling firm and product 2 (competing product) is produced by both firms. The analysis shows that under both Bertrand and Cournot competitions the incentive to bundle does not necessarily increase with the degree of intra-brand differentiation, while it strictly decreases with the degree of inter-brand differentiation. Moreover, under Bertrand competition bundling always decreases consumer surplus, but may increase the competitor's profit and social surplus. Under Cournot competition bundling always reduces the opponent's profit and social welfare, but may increase consumer surplus.

**Keywords** Bundling strategy · Intra-brand differentiation · Inter-brand differentiation

**JEL Classification** L12 · L13 · L41

---

**Electronic supplementary material** The online version of this article (doi:10.1007/s00712-012-0265-9) contains supplementary material, which is available to authorized users.

---

H.-L. Chung · Y.-S. Lin (✉)
Department of Economics, National Dong Hwa University,
No. 1, Sec. 2, Da Hsueh Rd. Shoufeng, Hualien 97401, Taiwan, ROC
e-mail: ylin@mail.ndhu.edu.tw

J.-L. Hu
Institute of Business and Management, National Chiao Tung University,
Hsinchu, Taiwan

## 1 Introduction

Bundling is a commonly seen marketing strategy. The major purposes of the bundling strategy are to extend the firm's monopoly power in one market to another market, to adopt price discrimination, to deter a potential competitor from entry, and to perform strategic alliances. This is named the so-called leverage theory. In practice, various products that are substitutes or complements can be bundled for sale. An example of bundling substitutes is to bundle products of different flavors; for example, in supermarkets Zero Coca Cola (black can) can be bundled with Classic Coke (red can) to compete against Pepsi Cola. Examples of bundled complements include coffee and coffee-creamer, shampoo and conditioner, computer and printer, etc. As the largest software firm in the world, Microsoft (MS) bundles some of its own products, but not all of them. For example, Microsoft bundles the licensing of Excel and Word or Excel, Word, and Outlook to PC producers (Goto 2009). However, the Exchange Server 2003 product is sold independently of other MS products (Christopher and Brian 2010). The above observations illustrate that product differentiation crucially affects the bundling behavior.

The related literature on bundling and product differentiation can be categorized according to the characteristics of the bundled products: The first strand of literature assumes that the bundled products are independent. Adams and Yellen (1976) find that bundling can lead to oversupply or undersupply with respect to Pareto optimality. Moreover, bundling is a kind of price discrimination that exploits consumer surplus and may reduce social welfare. Carbajo et al. (1990) assume that two firms may engage in Cournot or Bertrand competition, and that demand is inelastic. He shows that imperfect competition creates a strategic incentive to bundle. Bundling under Cournot [Bertrand] competition is harmful [beneficial] to its competitor's profit. In addition, no matter what the competition type is, bundling always reduces consumer surplus, but its effect on social welfare depends on the production cost. Whinston (1990) also assumes inelastic demand and Bertrand competition and find that the bundling firm reduces the competing product's price and takes the market share. In this case, bundling has a strategic foreclosure effect, making the competitor's profit drop. Martin (1999) assumes a linear demand function and homogeneous products under Cournot competition. His findings are consistent with Carbajo et al. (1990). Choi (2004) discusses the effect of bundling on R&D incentives: Bundling makes Bertrand competition more intensive, increases the bundling firm's R&D level, decreases the opponent's R&D level, keeps the industry's R&D level the same, and brings down social welfare. Assuming inelastic demand, Peitz (2008) takes into account a potential entrant that may compete against the incumbent with differentiated products under Bertrand competition. The incumbent firm can use the bundling strategy to deter entry and reduce social welfare.

The second strand of literature assumes the products to bundle are complements. Telser (1979) shows that bundling complementary products increases the net returns of a monopolist. Economides (1993) assumes linear demand and imperfect substitutability between competing products sold by the two firms, where competition is a la Bertrand. He finds that mixed bundling is the dominant strategy for both firms, and such a strategy reduces social welfare. Armstrong and Vickers (2010) establish a model with heterogeneous consumers and elastic demand, allowing consumers to buy from more than one supplier. They find that: (1) When demand elasticity, consumer heterogeneity, purchasing cost, or brand name preference increases, bundling (discount behavior) increases the firm's profit while decreasing consumer surplus. (2) When the shopping cost for buying from more than one firm and the preference for one brand name against another one exist, introducing a discount is more likely to increase social welfare.

The third strand of literature discusses both cases of complementary and substitutive bundled products: Lewbel (1985) extends Adams and Yellen (1976) to discuss the effects of complementarity and substitutability of bundled products on the incentive to bundle. Even if the products to bundle are substitutes, the firm may choose to bundle; on the contrary, when the products to bundle are complements, the firm may choose not to bundle. Venkatesh and Kamakura (2003) use the value added model to study the effects of the degree of substitutability on bundling and optimal pricing. Under low production cost or high reservation price, an increase in the value added or a decrease in the degree of substitutability increases the incentive to bundle.

The above literature review shows that except for Venkatesh and Kamakura (2003), who vary the degree of substitutability between products under a monopoly, most of the existing literature assumes a fixed degree of substitution between products to be bundled. This paper further takes into account product competition in market 2 and allows differentiations both for the bundling products and for the products competing in the same market, in order to examine whether or not a decrease in the substitutability of bundled products will increase the bundling incentive as Venkatesh and Kamakura (2003) show. Moreover, this paper will also inquire whether or not the firm has a higher incentive to bundle with a lower degree of inter-brand differentiation. Particularly, we construct a two-firm, two-product model in which product 1 is produced only by the bundling firm and product 2 is produced by the two firms under Bertrand or Cournot competition. To focus on the effects of production differentiation on bundling, this paper looks at only pure bundling, in which the bundling firm sells only the bundled product. This paper further discusses the effects of bundling behavior on the opponent firm, consumer surplus, and social surplus.

Five major findings are obtained from this theoretical study: (1) The incentive to bundle does not necessarily increase with the degree of intra-brand differentiation. (2) The incentive to bundle decreases with the degree of inter-brand differentiation. (3) Since Bertrand competition is more intensive than Cournot competition, bundling is more likely at a higher degree of inter-brand differentiation under Bertrand competition. (4) Under Bertrand competition, bundling reduces the intensity of market competition and bundling always decreases consumer surplus, but may increase the competitor's profit and social welfare. (5) Under Cournot competition bundling may increase or decrease such intensity, depending on consumers' preferences, and always reduces the competitor's profit and social welfare, but may increase consumer surplus. Our findings are different from those in Venkatesh and Kamakura (2003). Moreover, our results compare to Carbajo et al. (1990) in which under Bertrand competition the competitor's profit and social welfare are always increased by bundling, and under Cournot competition bundling always reduces the consumer surplus, while social welfare may be promoted.

This paper is organized as follows: Sect. 2 is the basic model, which analyzes the demand functions with and without bundling. Sections 3 and 4 discuss the cases under Bertrand and Cournot competitions, respectively.[1] Section 5 concludes this paper.

## 2 The basic model

There are two firms (A and B) and two products (1 and 2) in the market. Product 1 is produced only by firm A (as the monopoly product, A1), and product 2 is produced by both firms A and B (as the competing products, A2 and B2, respectively). The competing products produced by firms A and B can be differentiated. As a result, there are three goods in the market. For simplicity, the marginal production costs of products 1 and 2 are assumed to be constant and both equal to $c$.

This game has two stages. In stage one, firm A decides whether or not to bundle products 1 and 2. In stage 2, firms A and B engage in either Bertrand or Cournot competition. The solution concept of sub-game perfect Nash equilibrium (SPNE) is applied to solve this game, and backward induction is used.

The representative consumer's utility function is assumed to be quadratic and strictly concave, which is well-behaving:[2]

$$u = m + a(q_1^A + q_2^A + q_2^B) - \frac{1}{2}\Big[(q_1^A)^2 + (q_2^A)^2 + (q_2^B)^2 + 2\theta q_1^A q_2^A + 2\alpha q_1^A q_2^B + 2\gamma q_2^A q_2^B\Big], \tag{1}$$

where $\partial^2 u/\partial(q_1^A)^2 = -1 < 0$, $\partial^2 u/\partial(q_1^A)^2 \partial^2 u/\partial(q_2^A)^2 - (\partial^2 u/\partial q_1^A q_2^A)^2 = 1 - \theta^2 > 0$, and $\lambda \equiv (\theta^2 + \gamma^2 + \alpha^2 - 2\alpha\theta\gamma - 1) < 0$. The parameter $m$ is the amount of numeraire goods. The variable $q_j^i$ is the amount of product $j$ produced by firm $i$ with $i = A, B$, $j = 1, 2$. The parameter $\theta$ is the degree of differentiation between products 1 and 2 produced by firm $A$ with $\theta \in (-1, 1)$,[3] hereafter called the degree of intra-brand differentiation. A higher $\theta$ means a lower degree of intra-brand differentiation. When $\theta = 0$ [$\theta < 0, \theta > 0$], the products bundled by firm $A$ are mutually independent [complementary, substitutive]. The parameter $\gamma$ represents the degree of differentiation between competing products produced by firms $A$ and $B$ with $\gamma \in (0, 1]$, hereafter called the degree of inter-brand differentiation. A larger value of $\gamma$ implies a lower degree of inter-brand differentiation. A value $\gamma = 1$ implies the brands are perfect substitutes. The parameter $\alpha$ measures the degree of differentiation between A1 and B2.

[1] Numerous papers have been published on the effects of competition types on Nash equilibrium. Singh and Vives (1984) and Vives (1985) show that market competition between differentiated duopolists is more severe under Bertrand competition than under Cournot competition, making profits decrease, consumer surplus increase, and social welfare increase. In analyzing bundling behavior, the competition type should be taken into account, along with product differentiation.

[2] This utility function is derived from Dixit (1979) and Singh and Vives (1984), which also ensures the Hessian is negative semi-definite for all values.

[3] When the utility function is quadratic and strictly concave, the intra-brand differentiation must satisfy the condition $|\theta| < 1$.

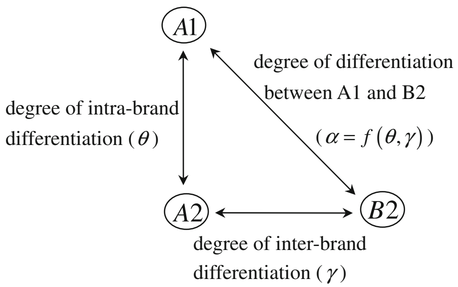



**Fig. 1** The relations in product differentiation

Figure 1 depicts the product differentiation relations among the three products. Without loss of generality, we express $\alpha$ in terms of $\theta$ and $\gamma$ as $\alpha = f(\theta, \gamma)$, with $\alpha \in (-1, 1)$ and $\alpha_\theta = \partial f(\theta, \gamma)/\partial\theta > 0$.[4] Moreover, as long as products A2 and B2 are competing products between, $\theta$ and $\alpha$ should have the same sign.

Following Qiu (1997), the marginal utility of the numeraire is assumed to be one. The inverse demand functions for the three goods are hence derived from the utility function depicted by Eq. (1):

$$p_1^A = a - q_1^A - \theta q_2^A - \alpha q_2^B, \tag{2.1}$$
$$p_2^A = a - \theta q_1^A - q_2^A - \gamma q_2^B, \tag{2.2}$$
$$p_2^B = a - \alpha q_1^A - q_2^B - \gamma q_2^A, \tag{2.3}$$

where $p_j^i$ is the price of product $j$ produced by firm $i$.[5] Assume that firm A combines one unit of product 1 with one unit of product 2 in a bundled package,[6] which is called product A, and $b_A$ is the amount of the bundle. Moreover, firm B produces only product 2. If firm A bundles products 1 and 2, then product 2 (produced by firm B) is denoted as product B, and $b_B$ is the amount of product B.

The amounts of products A and B in the market are respectively:

$$q_1^A = q_2^A = b_A, \tag{3.1}$$
$$q_2^B = b_B. \tag{3.2}$$

[4] Because there are three products in the model, the triangular product differentiation relationships can be expressed in terms of the characteristic functions of two products. Moreover, due to consistency in $\theta$ and $\alpha$, the unreasonable situation $\alpha_\theta < 0$ is ruled out.

[5] Most of the existing literature assumes exactly the same degree of substitution among various kinds of products, for example, Economides (1993), Sutton (1997), and Saggi and Vettas (2002). This article instead assumes different degrees of substitution among the three products.

[6] This assumption does not affect the major findings of this paper. Other bundling ratios of two products require one additional assumption on the fixed degree of product differentiation. Moreover, Belleflamme and Peitz (2010) define bundling as: "The practice of bundling consists in selling two or more products in a single package. The distinguishing feature of bundling is that the bundled goods are always combined in fixed proportions. In contrast, the related practice of tying (or tie-in sales) is less restrictive in that proportions might vary in the mix of goods." Since the tied-in ratio in this paper is 1 to 1, 'bundling' is more appropriate here. Following a referee's advice, the term 'bundling' is used throughout the text.

Substituting Eqs. (3.1) and (3.2) into Eq. (1), we obtain the consumer's utility function that is assumed to be quadratic and strictly concave when firm A bundles its products:

$$U = m + a(2b_A + b_B) - \frac{1}{2}\left[2(1+\theta)b_A^2 + b_B^2 + 2(\alpha+\gamma)b_A b_B\right], \tag{4}$$

where $\partial^2 U/\partial b_A^2 = -2(1+\theta) < 0$, $\delta \equiv (\partial^2 U/\partial b_A^2)(\partial^2 U/\partial b_B^2) - (\partial^2 u/\partial b_A b_B)^2 = 2(1+\theta) - (\gamma+\alpha)^2 > 0$. The inverse demand functions of products A and B can be derived from Eq. (4):

$$p_A = 2a - 2(1+\theta)b_A - (\alpha+\gamma)b_B, \tag{5.1}$$

$$p_B = a - (\alpha+\gamma)b_A - b_B. \tag{5.2}$$

Equations (5.1) and (5.2) show that the competition between firms A and B becomes more intensive as $\gamma$ and $\alpha$ increase, pushing the prices of products A and B down.

In order to further analyze how the competition mode affects bundling behavior, the next two sections study the situations under Bertrand and Cournot competitions.

## 3 Bundling strategy under Bertrand competition

In the second stage, both firms engage in Bertrand competition with and without bundling in the first stage of the game.

### 3.1 Nash equilibrium outcomes without bundling

The demand function for each product can be derived from Eqs. (2.1), (2.2), and (2.3). In Bertrand competition, we exclude the case in which A2 and B2 are perfect substitutes, so that from now on $\gamma \neq 1$. The notations $\Pi$ and $\pi$ present profits with and without bundling, respectively. Therefore, $\pi_j^i$ denotes the profit for firm $i$ to produce product $j$ without firm A's bundling.

Firm A produces products 1 and 2 at the same time. The profit function of firm A without bundling can be expressed as:

$$\underset{\{p_1^A, p_2^A\}}{Max}\ \pi^A = \pi_1^A + \pi_2^A = (p_1^A - c)q_1^A + (p_2^A - c)q_2^A. \tag{6}$$

Denote $\pi_{p_{ij}}^i$ as the first-order derivative of firm $i$'s profit function with respect to $p_{ij}$ when there is no bundling, $i = A, B$, $j = 1, 2$. The first-order conditions of profit maximization for firm A without bundling are:

$$\pi_{p_{A1}}^A = 0 \Leftrightarrow \frac{-1}{\lambda}\left[\beta_1 + 2(\gamma^2 - 1)p_1^A + 2(\theta - \alpha\gamma)p_2^A + (\alpha - \theta\gamma)p_2^B\right] = 0, \tag{7.1}$$

**Table 1** Nash equilibrium outcomes under Bertrand competition without bundling

| | |
|---|---|
| $p_1^A =$ | $\left\{(a+c)\left[\gamma^2-4(1-\theta^2)\right]+(a-c)\alpha\left[\gamma-\alpha\theta+2(1-\theta^2)\right]+2a\alpha^2-\alpha\theta\gamma(3a+c)\right\}/D_1\lambda^2$ |
| $p_2^A =$ | $\left\{(a+c)\left[\alpha^2-4(1-\theta^2)\right]+(a-c)\gamma\left[\alpha-\gamma\theta+2(1-\theta^2)\right]+2a\gamma^2-\alpha\theta\gamma(3a+c)\right\}/D_1\lambda^2$ |
| $p_2^B =$ | $2\left\{(1-\theta)\left[(a-c)(\alpha+\gamma)-2(a+c)(1+\theta)\right]+a(\alpha^2+\gamma^2-2\gamma\theta\alpha)\right\}/D_1\lambda^2$ |
| $q_1^A =$ | $(a-c)\left\{\gamma(\gamma^2+\alpha^2)(\gamma-\alpha)+2\left[\gamma^2\theta(1-\alpha\gamma)-\alpha^2(1-\theta)+(1-\theta^2)(\gamma\theta-\alpha)\right]\right.$ $\left.+3\gamma(\alpha+\gamma\theta^2)+4\left[1-\theta^2(1-\theta)-\theta(1-\alpha\gamma)\right]-5\gamma^2-7\alpha\gamma\theta^2\right\}/D_1\lambda^3$ |
| $q_2^A =$ | $(a-c)\left\{\alpha(\alpha^2+\gamma^2)(\alpha-\gamma)-2\left\{\alpha\theta\left[\left(\theta^2-\alpha-1\right)+\alpha\gamma(\alpha-\gamma)\right]+\gamma\left[(1+\gamma)-\theta(\gamma+\theta)\right]\right\}\right.$ $\left.+3\alpha(\gamma+\alpha\theta)+4\left[(1-\theta^2)(1-\theta)+\alpha\gamma\theta\right]-5\alpha^2-7\alpha\gamma\theta^2\right\}/D_1\lambda^3$ |
| $q_2^B =$ | $2(a-c)(\theta^2-1)\left[\lambda+(1-\theta)(\alpha+\gamma-1)\right]/D_1\lambda^3$ |

$D_1 \equiv 2[\lambda-3(1-\theta^2)]\lambda^2 < 0, a > c$

$$\pi^A_{p_{A2}} = 0 \Leftrightarrow \frac{-1}{\lambda}\left[\beta_2+2(\theta-\alpha\gamma)p_1^A+2(\alpha^2-1)p_2^A+(\gamma-\alpha\theta)p_2^B\right]=0, \tag{7.2}$$

where $\beta_1 \equiv (a+c)[1-\theta+\gamma(\alpha-\gamma)]+a(\theta\gamma-\alpha)$, and $\beta_2 \equiv (a+c)[1-\theta+\alpha(\gamma-\alpha)]+a(\alpha\theta-\gamma)$. Firm B produces only product 2, and its profit function is:

$$\underset{\{p_2^B\}}{Max}\ \pi_2^B = (p_2^B-c)q_2^B. \tag{8}$$

The first-order condition of profit maximization for firm B is:

$$\pi^B_{p_{B2}} = 0 \Leftrightarrow \frac{-1}{\lambda}\left[\beta_3+(\alpha-\theta\gamma)p_1^A+(\gamma-\alpha\theta)p_2^A+2(\theta^2-1)p_2^B\right]=0, \tag{9}$$

where $\beta_3 \equiv (1-\theta)\left[(a+c)(1+\theta)-a(\theta\gamma-\alpha)\right]$. Simultaneously solving Eqs. (7.1), (7.2), and (9), we obtain the Nash equilibrium prices and outputs (Table 1).[7]

## 3.2 Nash equilibrium outcomes with bundling

The demand functions for all products can be derived from Eqs. (5.1) and (5.2). Let $\Pi_i$ denote firm $i$'s profit when firm A bundles. The profit maximizations for firms A and B with bundling are respectively:

$$\underset{\{p_A\}}{Max}\ \Pi^A = (p_A-2c)b_A, \tag{10}$$

[7] The second-order conditions are respectively $\pi^A_{p_{A1}p_{A1}} = 2(1-\gamma^2)/\lambda, \pi^A_{p_{A2}p_{A2}} = 2(1-\alpha^2)/\lambda$, and $\pi^B_{p_{B2}p_{B2}} = 2(1-\theta^2)/\lambda$. The stability condition is $D_1 \equiv 2[\lambda-3(1-\theta^2)]/\lambda^2$. If the utility function is quardratic and strictly concave (i.e., $\lambda < 0$), then the second-order and stabilty conditions both hold. Accoding to Romano and Yildirim (2005), the Nash equilibrium in the one-period game is unique and interior when the utility function is quardratic and strictly concave both hold.

**Table 2** Nash equilibrium outcomes under Bertrand competition with bundling

| |
|---|
| $p_A = \{-a\delta + (1+\theta)\left[a(\gamma+\alpha-2)\right] - c\,(\gamma+\alpha-4)\}/\varphi$ |
| $p_B = \{-a\delta - 2\left[(a+2c)(1+\theta) - (a-c)(\gamma+\alpha)\right]\}/\varphi$ |
| $b_A = 2(a-c)\left[(1+\theta)(\gamma+\alpha-2)-\delta\right]/\delta\varphi$ |
| $b_B = 2(a-c)(1+\theta)\left[2(\gamma+\alpha-\theta-1)-\delta\right]/\delta\varphi$ |

$\varphi \equiv -\delta - 6(1+\theta) < 0$

$$\underset{\{p_B\}}{Max}\,\Pi^B = (p_B - c)b_B. \tag{11}$$

The first-order conditions of these two firms' profit maximization are respectively:

$$\Pi^A_{p_A} = 0 \Leftrightarrow \frac{1}{\delta}\left[2(a+c-p_A) - (\gamma+\alpha)(a-p_B)\right] = 0, \tag{12.1}$$

$$\Pi^B_{p_B} = 0 \Leftrightarrow \frac{1}{\delta}\{-2\left[a(\gamma+\alpha-\theta-1) - c(1+\theta)\right] + (\gamma+\alpha)p_A - 4(1+\theta)p_B\} = 0, \tag{12.2}$$

where $\Pi^i_{p_i}$ is the first-order derivative of firm $i$'s profit function with respect to $p_i$; $i = A$, B. Simultaneously solving the above two first-order conditions, we obtain the Nash equilibrium prices and outputs for firms A and B as depicted in Table 2.[8]

## 3.3 The degrees of product differentiation and bundling behavior

The bundling decision in stage one can be analyzed by comparing firm A's equilibrium profits with and without bundling. We denote the profit difference of firm A with and without bundling under Bertrand competition by $\Delta\pi^A = \Pi^A - \pi^A$. In the eight combinations,[9] we know that there are only two reasonable combinations of parameter signs: (1) $\theta$, $\gamma$, and $\alpha$ are all positive or (2) $\theta$ and $\alpha$ are negative and $\gamma$ is positive.

[8] The second-order conditions and the stability condition are respectively $\Pi^A_{p_A p_A} = -2/\delta$, $\Pi^B_{p_B p_B} = -4(1+\theta)/\delta$, and $D_2 \equiv -\varphi/\delta^2$. Similarly, if the utility function is quadratic and strictly concave (i.e., $\delta > 0$), the second-order and stability conditions both hold and the Nash equilibrium in the one-period game is unique and interior as Romano and Yildirim (2005) prove.

[9] The possible combinations of the signs $\theta$, $\gamma$, and $\alpha$ are as follows:

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| $\theta$ | + | + | + | + | – | – | – | – |
| $\gamma$ | + | + | – | – | + | + | – | – |
| $\alpha$ | + | – | + | – | + | – | + | – |

From the above table, we know that the reasonable combinations are only (1) and (6). In (1), products 1 and 2 are substitutes and in (6), products 1 and 2 are complements. These two remaining situations are both covered by Cases 1 and 2 in this paper. Therefore, the two cases in this paper cover all reasonable combinations of $\theta$, $\gamma$, and $\alpha$.

Without loss of generality, the two cases $0 < \alpha_\theta < 1$ and $\alpha_\theta > 1$ are discussed, which can cover the two reasonable parameter combinations.[10]

**Case 1** $0 < \alpha_\theta < 1$
In order to obtain comparable and closed-form solutions, we assume that $\alpha = \theta\gamma$, which satisfies the property $0 < \alpha_\theta < 1$. This assumption has two properties: First, when the bundled products are complements [substitutes], the degree of intra-brand differentiation is larger [smaller] than the degree of differentiation between A1 and B2; i.e., $|\theta| > |\alpha|$. Second, as the degree of intra-brand differentiation increases by one unit, the degree of differentiation between A1 and B2 increases by less than one unit (up to $\gamma < 1$).

Real examples of these cases can be seen in daily life. Coca Cola illustrates the case of bundled substitutes. Classic Coke (red can) and Zero Coke (black can) are bundled for promotion in supermarkets. If the consumers differentiate products by the brand name, then the substitution between Zero Coke and Classic Coke is higher than that between Zero Coke and Pepsi Cola. Moreover, the brand name criterion also makes consumers consider the Android platform and a Google cell phone to have higher complementarity than that between the Android platform and an HTC cell phone.[11]

Substituting $\alpha = \theta\gamma$ into Eqs. (7.1), (7.2), (9), (12.1), and (12.2). Using the first-order conditions obtained for profit maximization to solve the optimal pricing and substituting them into Eqs. (6) and (10), we obtain the profit difference of firm A:

$$\Delta\pi^A\Big|_{\alpha=\theta\gamma} = \frac{(a-c)^2\gamma^2(\theta-1)\Omega_1}{4\left[(\gamma-2)^2(\gamma+1)\lambda_1(\lambda_1-6)^2\right]} \gtrless 0, \tag{13}$$

where $\Omega_1 \equiv 64(4\gamma-1) + \gamma^2[\theta(2+\theta)-160] - 8\gamma^3(5+9\theta) + 46\gamma^4(1+\theta) + \gamma^5[\theta^2(4-3\gamma) - 2(3+\theta)] - 3\gamma^6(1+2\theta) + \gamma^7$ and $\lambda_1 \equiv \theta\gamma^2 + \gamma^2 - 2 < 0$. If $\gamma = \bar{\gamma}(\theta)$, then $\Delta\pi^A|_{\alpha=\theta\gamma} = 0$. Figure 2 depicts the relation in Eq. (13) in the domain of $\theta \in (-1, 1) \times \gamma \in (0, 1)$. Based on the above discussion, we obtain the following proposition.

**Proposition 1** *Given Bertrand competition, when $\alpha = \theta\gamma$, bundling is the optimal strategy of firm A if and only if $\gamma > \bar{\gamma}(\theta)$, where $lim_{\theta\to-1}\bar{\gamma}(\theta) \cong 0.304$ and $lim_{\theta\to1}\bar{\gamma}(\theta) \cong 0.329$. Moreover, for a fixed $\gamma \in (0.304, 0.329)$, bundling occurs in equilibrium if and only if $\theta < \bar{\theta}(\gamma)$, with $\bar{\theta} = \bar{\gamma}^{-1}(\cdot)$.*

Figure 2 shows that firm A chooses to bundle its products when the degrees of product differentiation are in the grey area, corresponding to Eq. (13), and depicts three regimes. When the degree of inter-brand differentiation is sufficiently high [low], Eq. (13) is strictly negative [positive]; that is, $\Delta\pi^A|_{\gamma\in(0,0.304)} < 0$

[10] We are grateful to one anonymous referee for this great comment, leading to a more generalized discussion of this section.

[11] Google released the Android platform in November 2007. It was first developed by Google and is now distributed to the Open Handset Alliance for further development. Google later released its own cell phone model, Nexus One, in January 2010.

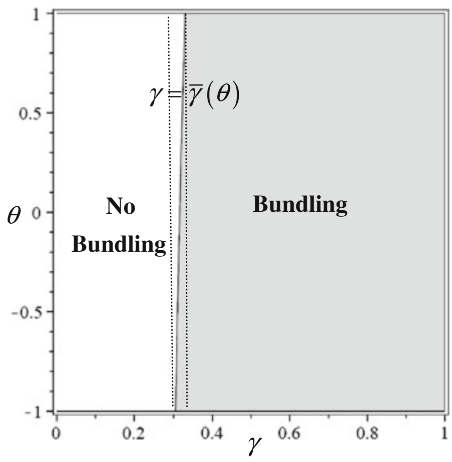



**Fig. 2** Under Case 1 ($\alpha = \theta\gamma$) and Bertrand competition, the relation between degrees of product differentiation and bundling behavior

$[\Delta\pi^A|_{\gamma\in(0.329,1)} > 0]$. However, when $\gamma \in (0.304, 0.329)$, a continuous decrease in $\theta$ eventually makes Eq. (13) strictly positive; at the same time, an increase in $\gamma$ increases the market extension advantage of the firm, hence making firm A tend to bundle.[12] This result is consistent with Telser (1979) and Venkatesh and Kamakura (2003), in which they assume one firm and two products and find that the incentive to bundle increases with a decrease in substitution of the bundled products.

As the lower left-hand corner of Fig. 2 shows, when $\gamma \in (0, 0.304)$, even if the bundled products are highly complementary to each other, firm A still chooses not to bundle. The reason is that in this case the competing products are highly differentiated and the consumers still buy from firm A, even though there is no bundling, and hence firm A still enjoys monopoly profit in product 1. The upper right-hand corner in Fig. 2 shows that when $\gamma \in (0.329, 1)$, firm A still chooses to bundle, even though the bundled products are highly substitutive to each other. The reason is that in this case, even if the competing products are not very differentiated to each other, firm A can still enjoy the market extension advantage by bundling. Summarizing the above analysis, we find that the bundling decision is affected more greatly by the degree of inter-brand differentiation.

**Observation 1** *When $\alpha = \theta\gamma$, the decision to bundle is affected more by the degree of inter-brand differentiation than by the degree of intra-brand differentiation.*

The bundling price is higher than the sum of the two separate prices without bundling: since $p_A = p_1^A + p_2^A + \Delta p_2^A$, where $\Delta p_2^A = (a-c)\gamma^3(\theta^2-1)/2(2-\gamma)(\lambda_1-6) > 0$, $p_A > p_1^A + p_2^A$ must hold. Firm A extends its monopoly power on product 1 to product 2, further raising firm A's profit. In addition, under Bertrand competition firm B's price increases with firm A's price increases; i.e., $\Delta p_2^B = (p_B - p_2^B) =$

[12] By substituting $\alpha = \theta\gamma$ into firm A's equilibrium price and quantity at bundling, we obtain the comparative statics in the bundling regime: $dp_A/d\theta = -8\gamma(a-c)(2+\gamma)/(\lambda_1-6)^2 < 0$ and $db_A/d\theta = (\gamma/4)(dp_A/d\theta) < 0$. In other words, an increase (decrease) in intra-brand differentiation increases (decreases) the effects of extending firm A's monopoly power by bundling, making firm's profit increase (decrease).

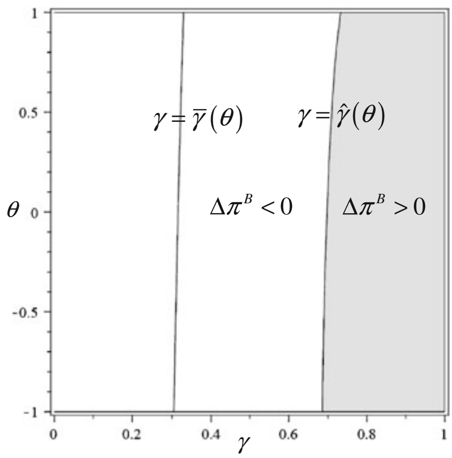



**Fig. 3** Under Case 1 ($\alpha = \theta\gamma$) and Bertrand competition, the bundling effects on the opponent's profit

$[2/\gamma(1+\theta)]\Delta p_2^A > 0$. Therefore, under Bertrand competition firm A's bundling increases both firms' prices, hence reducing the intensity of market competition.[13] This result compares to Choi (2004) in which inelastic demand and a fixed total demand amount are assumed in a Hotelling model, where bundling intensifies market competition. Here, bundling unambiguously increases firm A's quantity in product 2 while reducing firm B's quantity in product 2, but product 1's (monopoly product's) quantity may increase or decrease.[14] Overall, it can be proved that under Bertrand competition bundling reduces the total quantity.

**Corollary 1** *Under Bertrand competition, when $\alpha = \theta\gamma$, bundling increases market prices, hence making market competition less intense.*

To analyze the effects of firm A's bundling on firm B's profit, we substitute the optimal pricing into Eqs. (8) and (11) and obtain the profit difference of firm B:

$$\Delta\pi^B|_{\alpha=\theta\gamma} = \frac{(a-c)^2\gamma^2(\theta-1)\Omega_2}{(\gamma-2)^2(\gamma+1)\lambda_1(\lambda_1-6)^2} \gtrless 0, \tag{15}$$

where $\Omega_2 \equiv 32(2\gamma-1)+2\gamma^2(5\theta-9)-2\gamma^3(9\theta+7)+\gamma^4[\theta(2-\theta)+3]+\gamma^5(\theta+1)^2$. If $\gamma = \hat{\gamma}(\theta)$, then $\Delta\pi^B|_{\alpha=\theta\gamma} = 0$. Figure 3 depicts the relation in Eq. (15) in the domain of $\theta \in (-1, 1) \times \gamma \in (0, 1)$. We now obtain the following proposition.

**Proposition 2** *Under Bertrand competition, when $\alpha = \theta\gamma$, bundling increases the opponent's profit if and only if $\gamma > \hat{\gamma}(\theta)$, where $lim_{\theta\to -1}\,\hat{\gamma}(\theta) \cong 0.685$ and $lim_{\theta\to 1}\,\hat{\gamma}(\theta) \cong 0.732$.*

---

[13] The definition of intensifying competition follows that of Carbajo et al. (1990): Bundling intensifies market competition if prices (quantities) under Bertrand (Cournot) competition decrease (increase).

[14] Comparing the quantities with and without bundling, we obtain $\Delta q_1^A = b_A - q_1^A = -\gamma(a-c)[\gamma^3(1+\theta)-6\gamma+4]/\lambda_1(\lambda_1-6)$, $\Delta q_2^A = b_A - q_2^A = \gamma(a-c)\{(\gamma-1)[\theta\gamma^4(1+\theta)+8] - 2\gamma^2[\gamma(1+4\theta)-5\theta]\}/\ 2\lambda_1(\gamma+1)(\gamma-2)(\lambda_1-6) > 0$, and $\Delta q_2^B = b_B - q_2^B = \gamma^2(a-c)(\theta-1)[\gamma^2(1+\theta)+2(\gamma-3)]/\lambda_1(\gamma+1)(\gamma-2)(\lambda_1-6) < 0$.

Footnote 14 and Corollary 1 tell us: First, firm A can extend its market advantage to competing products and hence compel firm B's quantity to drop. Second, firm A's bundling helps firm B increase its own price. When $\gamma > \hat{\gamma}(\theta)$ [$\gamma \in (\bar{\gamma}(\theta), \hat{\gamma}(\theta))$], and firm A's market advantage extension effect is weak [strong], the negative impact on firm B's profit is smaller [larger] than the positive impact.

The welfare and consumer surplus effects of bundling behavior can be further analyzed from the consumer's and government's viewpoints. According to the previous modeling set-up, social welfare is defined as the sum of profits (producer surplus) and consumer surplus. Previous sections have discussed the effects of bundling on profits. Consumer surpluses with and without bundling are respectively $CS = U - (p_A b_A + p_B b_B)$ and $cs = u - (p_1^A q_1^A + p_2^A q_2^A + p_2^B q_2^B)$. Substituting the equilibrium values in all stages of the game with and without bundling into consumer surplus functions for comparison, we find that bundling reduces consumer surplus, that is $\Delta cs = CS - cs = (a-c)^2\gamma^2(\theta-1)\Omega_3/8(\gamma-2)^2 8(\gamma-2)^2(\gamma+1)\lambda_1(\lambda_1-6)^2 < 0$, where $\Omega_3 \equiv 64(2\gamma-3) + 72\gamma^2(\theta-1) + 48\gamma^3 + 2\gamma^4(17+15\theta-2\theta^2) -18\gamma^5(1+\theta) - \gamma^6(1+\theta)^2(3-\gamma)$. Similarly, social welfare difference with and without bundling is $\Delta w = W - w = (a-c)^2\gamma^2(\theta-1)\Omega_4/8(\gamma-2)^2(\gamma+1)\lambda_1 (\lambda_1-6)^2 \gtreqless 0$ with $\Delta w|_{\gamma=\tilde{\gamma}(\theta)} = 0$, where $\Omega_4 \equiv 576(2\gamma-1) + 8\gamma^2(19\theta-67) - 144\gamma^3(2\theta+1) + 6\gamma^4(25+23\theta-2\theta^2) - 2\gamma^5(11+3\theta-8\theta^2) - 3\gamma^6(1+\theta)^2(3-\gamma)$, $\lim_{\theta\to-1}\tilde{\gamma}(\theta) \cong 0.869$, and $\lim_{\theta\to 1}\tilde{\gamma}(\theta) \cong 1$. The above discussion generates Proposition 3.

**Proposition 3** *Under Bertrand competition, when* $\alpha = \theta\gamma$, *bundling decreases social welfare if and only if* $\gamma \in (\bar{\gamma}(\theta), \tilde{\gamma}(\theta))$, *where* $lim_{\theta\to-1}\tilde{\gamma}(\theta) \cong 0.869$, $lim_{\theta\to 1}\tilde{\gamma}(\theta) \cong 1$. *Otherwise, when*$\gamma > \tilde{\gamma}(\theta)$, *bundling promotes social welfare.*

Figure 4 depicts the $\Delta cs$ and $\Delta w$ functions in the domain of $\theta \in (-1, 1) \times \gamma \in (0, 1)$. They show that in the parameters' region in which A has an incentive to bundle, bundling reduces consumer surplus, because under Bertrand competition bundling increases both firms' prices and decreases the total quantity. With respect to social welfare, if $\gamma > \tilde{\gamma}(\theta)$ [$\gamma \in (\overline{\gamma}(\theta), \tilde{\gamma}(\theta))$], then under Bertrand competition the bundling behavior increases profits more [less] than it decreases consumer surplus, hence making social welfare increase [decrease]. Therefore, under Bertrand competition whether or not bundling should be prohibited depends mainly on the parameter $\gamma$. Under Bertrand competition when the degree of inter-brand differentiation is very low, bundling should not be prohibited; otherwise, bundling should be prohibited.

Provided that bundled products are independent, bundling is not always beneficial to firm A under Bertrand competition. The previous literature finds that it is always profitable for firm A to bundle under Bertrand competition; for example, Carbajo et al. (1990), Whinston (1990), and Peitz (2008). However, we find that the bundling decision depends crucially on the degree of inter-brand differentiation. Moreover, when there are degrees of intra-brand differentiation and inter-brand differentiation, the bundling effects on the opponent's profit and social welfare are ambiguous. These results differ significantly from those of previous studies.

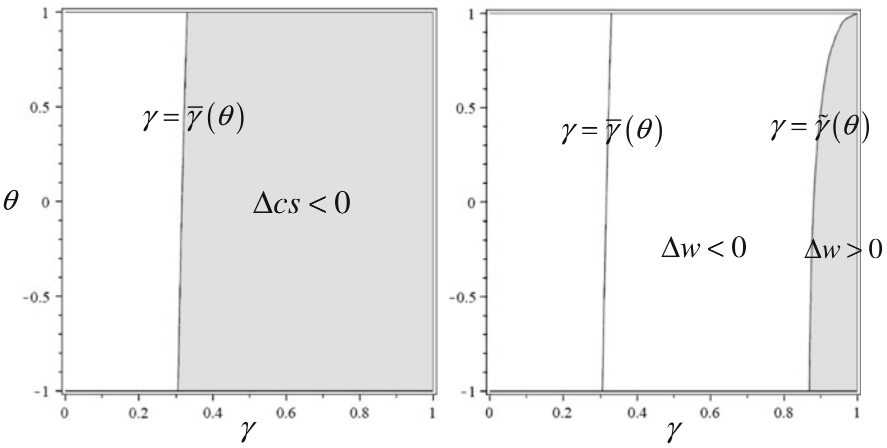



**Fig. 4** Under Case 1 ($\alpha = \theta\gamma$) and Bertrand competition, the bundling effects on consumer surplus and social welfare

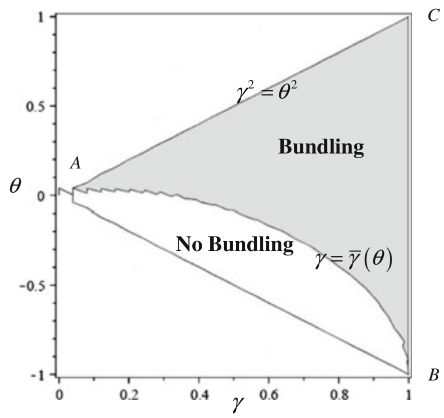
**Case 2** $\alpha_\theta > 1$
In order to obtain comparable and closed-form solutions and to make the degree of differentiation between A1 and B2 satisfy the condition $\alpha_\theta > 1$, we assume that $\alpha = \theta/\theta\gamma$.[15] This set-up implies the following: (1) When the bundled products are complements (substitutes), the degree of differentiation between A1 and B2 is larger (smaller) than the degree of intra-brand differentiation; that is, $|\alpha| > |\theta|$. (2) When the degree of intra-brand differentiation increases by one unit, the degree of differentiation between A1 and B2 increases by more than one unit (up to $\gamma^{-1}$ units). Take the example of Coca Cola and Pepsi Cola as bundled substitutes again. If the consumers differentiate products by taste, then the substitution between Zero Coke (black can) and Classic Coke (red can) is smaller than that between Zero Coke and Pepsi Cola. When the bundled products are complements, by product synergy the consumers consider the Android platform and a Google cell phone to have lower complementarity than that between the Android platform and an HTC cell phone.

Here we substitute $\alpha = \theta/\gamma$ into Eqs. (7.1), (7.2), (9), (12.1), and (12.2). Moreover, in order to make the utility function quadratic and strictly concave (i.e., $\lambda < 0$), the conditions $\gamma^2 > \theta^2$ is needed. Substituting the optimal pricing obtained from the first-order conditions with and without bundling into Eqs. (6) and (10), we get the profit difference function as $\Delta\pi^A|_{\alpha=\theta/\gamma} = \Delta\pi^A(\theta, \gamma)$, and when $\gamma = \bar{\gamma}(\theta)$, we obtain that $\Delta\pi^A|_{\alpha=\theta/\gamma} = 0$. The function of $\Delta\pi^A|_{\alpha=\theta/\gamma} = 0$ and the condition $\gamma^2 > \theta^2$ are depicted in Fig. 5 in the domain of $\theta \in (-1, 1) \times \gamma \in (0, 1)$.

**Proposition 4** *Given Bertrand competition, when* $\alpha = \theta/\gamma$, *bundling is the optimal strategy of firm A if and only if* $\gamma > \bar{\gamma}(\theta)$, *where* $lim_{\theta\to-1}\,\bar{\gamma}(\theta) \cong 1$ *and*

[15] Cases 1 and 2 are consistent with those in the existing literature. For example, Martin (1999) is a special case in this paper, with $\theta = 0$ and $\gamma = 1$. This paper hence covers a broader scope of product differentiation to study the bundling decision.

**Fig. 5** Under Case 2 ($\alpha = \theta/\gamma$) and Bertrand competition, the relation between degrees of product differentiation and bundling behavior




*$lim_{\theta \to 0}\, \bar{\gamma}(\theta) \cong 0$. Moreover, for a fixed $\gamma \in (0, 1)$, bundling occurs in equilibrium if and only if $\theta > \bar{\theta}(\gamma)$, with $\bar{\theta} = \bar{\gamma}^{-1}(\cdot)$.*

In Fig. 5, lines $AB$ and $AC$ are the functions $\gamma = \bar{\gamma}(\theta)$ and $\gamma = \theta$, respectively. Therefore, the $ABC$ area is the regime where firm A has an incentive to bundle. When $\alpha_\theta > 1$ (i.e., $\alpha = \theta/\gamma$), the degree of differentiation between A1 and B2 increases with and is faster than the degree of intra-brand differentiation. Consequently, a higher degree of intra-brand differentiation is adverse to firm A; at the same time, this adverse effect enlarges as the degree of intra-brand differentiation increases ($\theta \downarrow$) or the degree of inter-brand differentiation gets higher ($\gamma \downarrow$). To sum up, when $\alpha = \theta/\gamma$, both the degrees of intra-brand and inter-brand differentiation are the main factors in firm A's decision to bundle. Our result is different from that in Venkatesh and Kamakura (2003), who point out that an increase in value added or a decrease in the degree of substitutability will increase the incentive to bundle.

Recall that in Case 1 where $0 < \alpha_\theta < 1$ (i.e., $\alpha = \theta\gamma$), an increase in the degree of intra-brand differentiation increases the degree of differentiation between A1 and B2 at a lower velocity. In Case 1, an increase in the degree of intra-brand differentiation is beneficial to firm A with the magnitude increasing with the degree of inter-brand differentiation. Therefore, the effects of increasing the degree of inter-brand differentiation on firm A's profit difference under $\alpha = \theta\gamma$ and $\alpha = \theta/\gamma$ are quite different, which then impacts its incentive to bundle.

Comparing the prices with and without bundling, we get $p_A = p_1^A + p_2^A + \Delta p_2^A$ where $\Delta p_2^A = (a - c)(1 + \theta)(\gamma^2 + \theta)(\gamma^2 - \theta)^2(\gamma^2 + 2\gamma + 2\theta\gamma + \theta)/2D_1D_2 > 0$ and $\Delta p_2^B = p_B - p_2^B = [2\gamma/(\gamma^2 + \theta)]\Delta p_2^A > 0$. These results imply that under Bertrand competition and when $\alpha = \theta\gamma$, firm A can still extend its monopoly power in product 1 to product 2, hence increasing the prices for firm A by bundling. Firm B's price also increases with firm A's bundling. Therefore, in this case firm A's bundling increases prices and makes market competition less intense.

**Corollary 2** *Under Bertrand competition, when $\alpha = \theta/\gamma$, bundling make prices increase and market competition intensity decrease.*

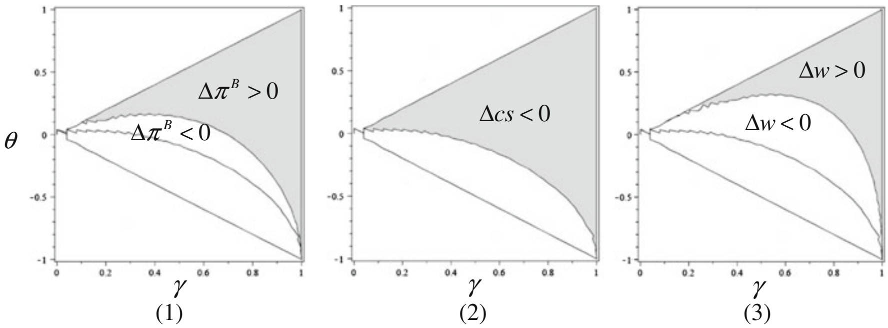



**Fig. 6** Under Case 2 ($\alpha = \theta/\gamma$) and Bertrand competition, the bundling effects on the opponent's profit, consumer surplus, and social welfare

We are able to analyze the effects of bundling by substituting the optimal pricing with and without bundling into firm B's profit, consumer surplus, and social welfare, as Fig. 6 shows. Proposition 5 summarizes the comparison of Figs. 5 and 6.

**Proposition 5** *Under Bertrand competition, when $\alpha = \theta/\gamma$, firm A's bundling may increase firm B's profit, reduce consumer surplus, and increase social welfare.*

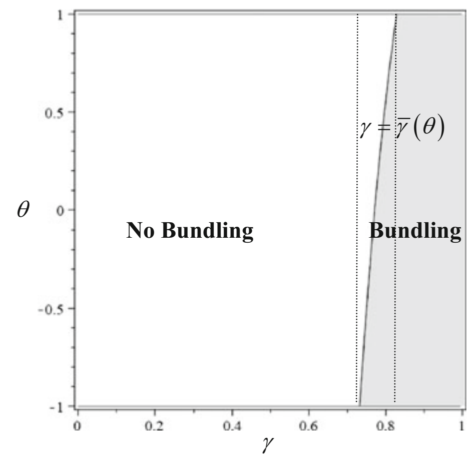
Figure 6(1) shows that under Bertrand competition firm A's bundling may increase firm B's profit. Corollary 2 says that firm A's bundling increases firm B's price. Substituting the optimal price into the demand also incurs $\Delta q_2^B = b_B - q_2^B < 0$.[16]

Similar to the case of $\alpha = \theta\gamma$, there are one positive effect and one negative effect from firm A's bundling on firm B's profit. When the product characteristics lie in the gray area, the effect from reducing market competition intensity by firm A's bundling dominates that of quantity reduction for firm B, hence increasing firm B's profit. Because bundling increases both firms' prices and reduces the total quantity, consumer surplus drops after firm A bundles, which is depicted by Fig. 6(2). To sum up, when the effect from reducing market competition intensity (increasing profits) dominates that of total quantity (consumer surplus) reduction, bundling increases social welfare. Fig. 6(3) shows that under Bertrand competition and when $\alpha = \theta/\gamma$, whether or not bundling should be prohibited depends on the degree of product differentiation.

We are finally able to answer the two questions raised before for the Bertrand competition case. Will firm A have a higher incentive to bundle when the degree of intra-brand differentiation is higher or the degree of inter-brand differentiation is lower? Figures 2 and 5 show that no matter whether $\alpha = \theta\gamma$ or $\alpha = \theta/\gamma$, a decrease in the degree of inter-brand differentiation increases firm A's incentive to bundle, which confirms the conventional conjecture. Moreover, when $\alpha = \theta\gamma$ and $\gamma \in (0.304, 0.329)$, firm A's incentive to bundle increases with the degree of intra-brand differentiation. However, when $\alpha = \theta/\gamma$, firm A's incentive to bundle decreases with the degree of intra-brand differentiation. Therefore, the effect of the degree of inter-brand differentiation on the incentive to bundle depends on consumers' preferences.

[16] In the regime where firm A has an incentive to bundle, it can be shown that $\Delta q_2^B = -(a-c)\gamma^2(1+\theta)^2 [\gamma^5(4+\gamma) + 4\gamma^3(\theta^2-3) + \gamma^2(\theta-2)(\gamma^2+\theta) + \theta^2(4\gamma+\theta)]/D_1 D_2 \lambda_2 (1+\gamma)(\gamma+\theta) < 0$.

**Fig. 7** Under Case 1 ($\alpha = \theta\gamma$) and Cournot competition, the relation between degrees of product differentiation and bundling behavior




## 4 Bundling strategy under Cournot competition

This section discusses the Nash equilibrium outcomes with and without bundling under Cournot competition. The solution steps are similar to those under Bertrand competition. This section presents the two cases $0 < \alpha_\theta < 1$ and $\alpha_\theta > 1$, respectively. Under Cournot competition, the effects of firm A's bundling on profits, consumer surplus, and social welfare are $\Delta\Pi$, $\Delta CS$, and $\Delta W$, respectively.

### 4.1 The degrees of product differentiation and bundling behavior

**Case 1.** $0 < \alpha_\theta < 1$
In order to consistently compare the results under Cournot and Bertrand competitions, we still adopt the functional form $\alpha = \theta\gamma$. Using the first-order conditions for profit maximization with and without bundling, we obtain the equilibrium quantities as $q_1^A, q_2^A, q_2^B, b_A$, and $b_B$ (see Appendix for details). Substituting the above equilibrium quantities into firm A's profit functions with and without bundling, we find that the profit difference of firm A is:

$$\Delta\Pi^A|_{\alpha=\theta\gamma} = \frac{\varphi_1}{4}[\gamma^3(1+\theta)(4+\gamma) - 16(\gamma^2 + 2\gamma - 2)], \tag{17}$$

where $\varphi_1 \equiv (a-c)^2\gamma^2(\theta-1)/(2+\gamma)^2(\theta\gamma^2+\gamma^2-8)^2 < 0$. If$\gamma = \bar{\gamma}(\theta)$, then $\Delta\Pi^A|_{\alpha=\theta\gamma} = 0$. Figure 7 corresponds to the situation of Eq. (17) in the domain of $\theta \in (-1, 1) \times \gamma \in (0, 1]$.

Figure 7 shows that under Cournot competition, when $\alpha = \theta\gamma$, bundling is the optimal strategy of firm A if and only if $\gamma > \bar{\gamma}(\theta)$, where $lim_{\theta\to-1}\,\bar{\gamma}(\theta) \cong 0.73$ and $lim_{\theta\to 1}\,\bar{\gamma}(\theta) \cong 0.83$. Moreover, for a fixed $\gamma \in (0.73, 0.83)$, bundling occurs in equilibrium if and only if $\theta < \bar{\theta}(\gamma)$, with $\bar{\theta} = \bar{\gamma}^{-1}(\cdot)$. Comparing Figs. 2 and 7, we find that when $\alpha = \theta\gamma$, even when the degree of inter-brand differentiation is high, firm A is more likely to bundle products under Bertrand competition than under Cournot competition since Bertrand competition is more severe. These findings are still consistent with those under the Bertrand competition case, but with different magnitudes.

Using the optimal quantities, we know that firm A's bundling increases its own quantity in product 2 while decreasing firm B's quantity and increasing the total quantity of product 2. Therefore, under Cournot competition bundling intensifies market competition for product 2.[17]

**Proposition 6** *Under Cournot competition and when $\alpha = \theta\gamma$, bundling increases the total quantity of product 2 and hence intensifies market competition for product 2.*

We would like to further discuss the effects of firm A's bundling on firm B's profit, consumer surplus, and social welfare under Cournot competition and when $\alpha = \theta\gamma$. The calculation process is similar to that under Bertrand competition and hence we only use Fig. 8 to explain the results. Figure 8 shows that Firm A's bundling reduces firm B's profit and social welfare, but may increase consumer surplus.[18] The economic intuition behind it goes like this: Firm A's bundling makes firm B's quantity and price both drop, resulting in firm B's profit dropping. Proposition 6 shows that bundling intensifies the quantity competition, which is beneficial to consumers. However, firm A's price increases and firm B's price decreases with bundling—that is, $p_A = p_1^A + p_2^A + \Delta p_2^A$ and $p_B = p_2^B + \Delta p_2^B$ where $\Delta p_2^A = (a-c)\gamma^3(\theta^2-1)/2(2+\gamma)(\theta\gamma^2+\gamma^2-8) > 0$, $\Delta p_2^B = [-2/\gamma(1+\theta)]\Delta p_2^A < 0$ must hold.

Based on the above analysis, under Cournot competition the effects of firm A's bundling on consumer surplus are summarized as: (1) Bundling intensifies quantity competition and reduces firm B's price, making consumer surplus increase. (2) Firm A's price increases with bundling, which is adverse to consumer surplus. As the degree of intra-brand differentiation increases and the degree of inter-brand differentiation decreases, firm A's price increases more. Consequently, in region I of Fig. 8(2), the negative effect of bundling dominates the positive effect, making consumer surplus drop; on the contrary, in region II the positive effect of bundling dominates the negative effect, making consumer surplus increase. Since firm B's profit reduction dominates the increase in firm A's profit and consumer surplus, bundling reduces social welfare under Cournot competition and when $\alpha = \theta\gamma$.

**Case 2.** $\alpha_\theta > 1$

Under this set-up, the optimal quantities are: $q_1^A, q_2^A, q_2^B, b_A$, and $b_B$ (see Appendix for details). Assuming that the utility function is quadratic and strictly concave

[17] In order to analyze the effects of firm A's bundling on product 2, we define $\Delta q_2^A$ and $\Delta q_2^B$ as the impacts of firm A's bundling on its own product 2's quantity and firm B's quantity, respectively. Moreover, $\Delta q_2^A = b_A - q_2^A = (a-c)\gamma(\theta\gamma^2-4)/2(2+\gamma)(\theta\gamma^2+\gamma^2-8) > 0$ and $\Delta q_2^B = b_B - q_2^B = \omega\Delta q_2^A < 0$, with $\omega = 2\gamma(1-\theta)/(\theta\gamma^2-4) < 0$ and $|\omega| < 1$. In other words, firm A's bundling increases the total quantity of product 2.

[18] Under Cournot competition and when $\alpha = \theta\gamma$, the effects of firm A's bundling on firm B's profit, consumer surplus, and social welfare are respectively: $\Delta\Pi^B|_{\alpha=\theta\gamma} = -\varphi_1[\gamma^2(3+\theta)-16] < 0$, $\Delta CS = -\varphi_1[\gamma^3(1+\theta)(\gamma-4)+4\gamma^2(1-\theta)-32(1-\gamma)]/8$, and $\Delta W = \varphi_1[\gamma^3(1+\theta)(3\gamma+4) - 4\gamma^2(3\theta+13)+32(5-\gamma)]/8 < 0$.

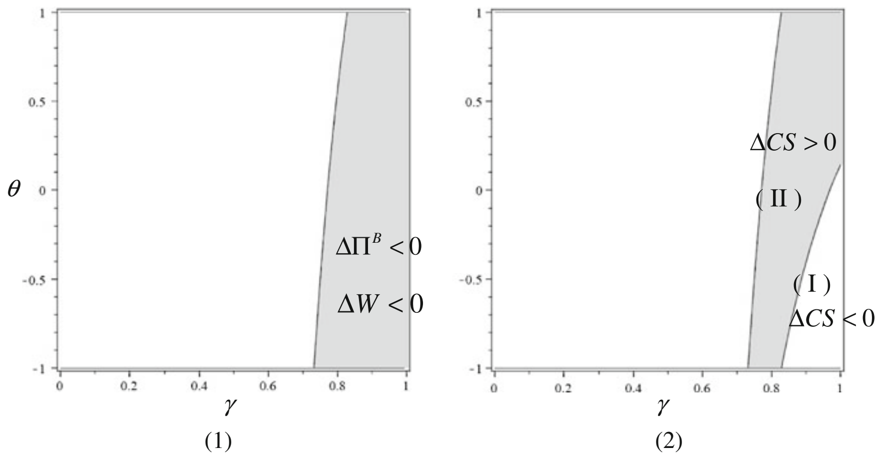



**Fig. 8** Under Case 1 ($\alpha = \theta\gamma$) and Cournot competition, the bundling effects on the opponent's profit, consumer surplus, and social welfare

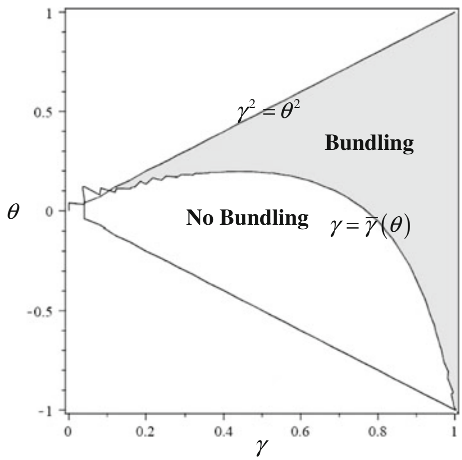



**Fig. 9** Under Case 2 ($\alpha = \theta/\gamma$) and Cournot competition, the relation between degrees of product differentiation and bundling behavior

(i.e., $\gamma^2 > \theta^2$) and depicting firm A's profit difference function $\Delta\Pi^A|_{\alpha=\theta/\gamma} = \Delta\Pi^A(\theta, \gamma)$ in the $\theta \in (-1, 1) \times \gamma \in (0, 1]$ regime in Fig. 9, we find that when $\theta$ and $\gamma$ lie in the gray regime, firm A chooses to bundle. If $\gamma = \bar{\gamma}(\theta)$, then $\Delta\Pi^A|_{\alpha=\theta/\gamma} = 0$.

A comparison of Figs. 5 and 9 shows that since Bertrand competition is more intensive than Cournot competition, when $\alpha = \theta/\gamma$, firm A is more likely to bundle under Bertrand competition than under Cournot competition. At the same time, the incentive to bundle under Cournot competition is affected by factors similar to those under Bertrand competition.

Using the optimal quantities, we know that the effects of firm A's bundling on its own product 2's quantity depend on the degrees of intra-brand and inter-brand

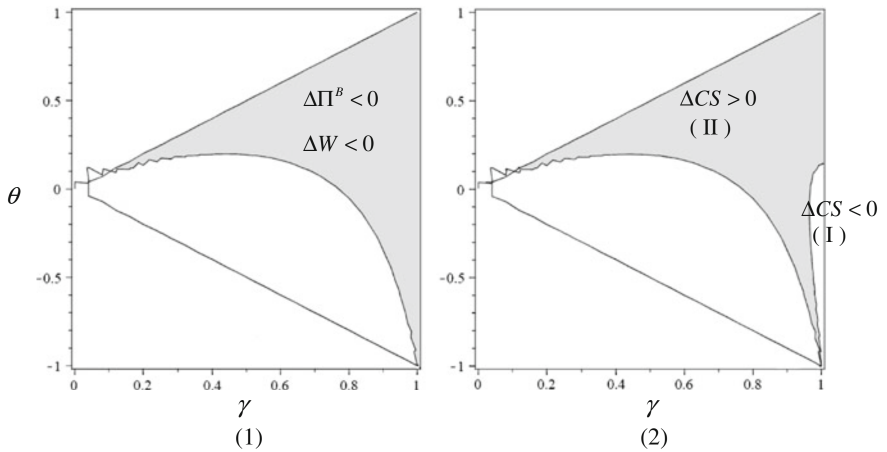



**Fig. 10** Under Case 2 ($\alpha = \theta/\gamma$) and Cournot competition, the bundling effects on the opponent's profit, consumer surplus, and social welfare

differentiation—that is, $\Delta q_2^A = b_A - q_2^A \gtreqless 0$, if $\theta \lesseqgtr \gamma^2$; moreover, firm A's bundling always reduces firm B's quantity. However, the change in total quantity of product 2 still depends upon the relative magnitudes of $\theta$ and $\gamma^2$. Under Cournot competition bundling also increases the sum of the prices of bundled products and decreases firm B's price.

**Proposition 7** *Under Cournot competition and when $\alpha = \theta/\gamma$, if the degree of intra-brand differentiation is sufficiently high (i.e., $\theta < \gamma^2$), then bundling increases the total quantity of product 2 and intensifies market competition. On the contrary, if the degree of intra-brand differentiation is sufficiently low (i.e., $\theta > \gamma^2$), bundling decreases the total quantity of product 2 and makes market competition less intense.*

Under Cournot competition and when $\alpha = \theta/\gamma$, firm A's bundling decreases the competitor's profit and social welfare, but may increase consumer surplus. Figure 10(1) shows that firm A's bundling reduces firm B's quantity and price, making firm B's profit unambiguously drop. In region I of Fig. 10(2), the positive effect of bundling on reducing firm B's price is lower than its negative effects on increasing the sum of the bundled products' prices, and hence bundling decreases consumer surplus. On the contrary, in region II the positive effect of bundling dominates its negative effects, and hence bundling increases consumer surplus. Summarizing and comparing the above results under Cournot and Bertrand competitions, we come up with the following proposition.

**Proposition 8** *The comparison of equilibrium outcomes under Cournot and Bertrand competitions shows that: (1) No matter what the consumer preference is (i.e., whether $0 < \alpha_\theta < 1$ or $\alpha_\theta > 1$), given a degree of intra-brand differentiation, the firm is more likely to bundle under Bertrand competition than for Cournot competition due* *to a higher degree of competition. (2) Under Cournot competition, bundling always reduces the competitor's profit and social welfare, but may increase consumer surplus.*

Under the assumption that the bundled products are independent, Carbajo et al. (1990) find under Cournot competition that bundling always increases the profit of the bundling firm, hence making bundling a dominant strategy. This paper instead assumes elastic demand and simultaneously takes into account the degrees of intra-brand differentiation and inter-brand differentiation, finding that bundling may not always be a dominant strategy and bundling may increase consumer surplus if the bundled products are independent. This result is inconsistent with the previous literature. This paper further notes that under Cournot competition bundling should be prohibited (the per se rule). However, whether or not bundling should be prohibited under Bertrand competition depends on the product differentiations (the rule of reason).

## 5 Concluding remarks

The existing literature shows that a decrease in the degree of substitutability increases a monopoly's incentive to bundle. This paper further takes into account product competition in market 2 and allows differentiations both for the bundling products and for the products competing in the same market, in order to examine whether or not a decrease in the substitutability of bundled products will increase the bundling incentive as Venkatesh and Kamakura (2003) show, who vary the degree of substitutability between products under a monopoly. Moreover, this paper will also inquire whether or not the firm has a higher incentive to bundle with a lower degree of inter-brand differentiation.

Particularly, we construct a two-firm, two-product model in which product 1 is produced only by the bundling firm and product 2 is produced by the two firms under Bertrand or Cournot competition. To focus on the effects of production differentiation on bundling, this paper looks at only pure bundling, in which the bundling firm sells only the bundled product. This paper further discusses the effects of bundling behavior on the opponent firm, consumer surplus, and social surplus.

Five major findings are obtained from this theoretical study: (1) The incentive to bundle does not necessarily increase with the degree of intra-brand differentiation. (2) The incentive to bundle decreases with the degree of inter-brand differentiation. (3) Since Bertrand competition is more intensive than Cournot competition, bundling is more likely at a higher degree of inter-brand differentiation under Bertrand competition. (4) Under Bertrand competition, bundling reduces the intensity of market competition and bundling always decreases consumer surplus, but may increase the competitor's profit and social welfare. (5) Under Cournot competition bundling may increase or decrease such intensity, depending on consumers' preferences, and always reduces the competitor's profit and social welfare, but may increase consumer surplus. From theoretical point of view, our findings are different from those in Venkatesh and Kamakura (2003). From empirical point of view, we suggest that under Cournot competition bundling should be prohibited; however, whether or not bundling should be prohibited under Bertrand competition depends on the product differentiations.

## Appendix

The mathematical computations under Cournot competition are listed here.

**Case 1.** $0 < \alpha_\theta < 1$
In the second stage, both firms engage in Cournot competition. The profit function of firm A without bundling can be expressed as:

$$\underset{\{q_1^A, q_2^A\}}{Max}\ \pi^A = \pi_1^A + \pi_2^A = (p_1^A - c)q_1^A + (p_2^A - c)q_2^A. \tag{A.1}$$

The first-order conditions of profit maximization for firm A without bundling are:

$$\pi_{q_{A1}}^A = 0 \Leftrightarrow a - 2q_1^A - 2\theta q_2^A - \theta\gamma q_2^B - c = 0, \tag{A.2}$$

$$\pi_{q_{A2}}^A = 0 \Leftrightarrow a - 2\theta q_1^A - 2q_2^A - \gamma q_2^B - c = 0. \tag{A.3}$$

Firm B produces only product 2, and its profit function is:

$$\underset{\{q_2^B\}}{Max}\ \pi_2^B = (p_2^B - c)q_2^B. \tag{A.4}$$

The first-order condition of profit maximization for firm B is:

$$\pi_{q_{B2}}^B = 0 \Leftrightarrow a - \theta\gamma q_1^A - \gamma q_2^A - 2q_2^B - c = 0. \tag{A.5}$$

Simultaneously solving Eqs. (A.2), (A.3), and (A.5), we obtain the Nash equilibrium outputs and prices.

| Nash equilibrium outcomes under Cournot competition without bundling ($\alpha = \theta\gamma$) |
|---|
| $q_1^A = (a-c)/[2(1+\theta)]$ |
| $q_2^A = (a-c)(2-\theta\gamma)/[2(1+\theta)(2+\gamma)]$ |
| $q_2^B = (a-c)/(2+\gamma)$ |
| $p_1^A = [(a+c)(2+\gamma) - (a-c)\theta\gamma]/[2(2+\gamma)]$ |
| $p_2^A = p_2^B = [a + c(1+\gamma)]/(2+\gamma)$ |

Let $\Pi^i$ denote firm $i$'s profit when firm A bundles, $i = A, B$:

$$\underset{\{b_A\}}{Max}\ \Pi^A = (p_A - 2c)b_A, \tag{A.6}$$

$$\underset{\{b_B\}}{Max}\ \Pi^B = (p_B - c)b_B. \tag{A.7}$$

The first-order conditions of these two firms' profit maximization are respectively:

$$\Pi^A_{b_A} = 0 \Leftrightarrow 2a - 4(1+\theta)b_A - \gamma(1+\theta)b_B - 2c = 0, \tag{A.8}$$

$$\Pi^B_{b_B} = 0 \Leftrightarrow a - \gamma(1+\theta)b_A - 2b_B - c = 0. \tag{A.9}$$

The Nash equilibrium outputs and prices can also be computed.

| Nash equilibrium outcomes under Cournot competition with bundling ($\alpha = \theta\gamma$) |
|---|
| $b_A = (a-c)(\theta\gamma + \gamma - 4)/\left[(1+\theta)(\theta\gamma^2 + \gamma^2 - 8)\right]$ |
| $b_B = 2(a-c)(\gamma - 2)/(\theta\gamma^2 + \gamma^2 - 8)$ |
| $p_A = 2\left\{\gamma(1+\theta)\left[a + (\gamma-1)c\right] - 4(a+c)\right\}/(\theta\gamma^2 + \gamma^2 - 8)$ |
| $p_B = \left\{\gamma\left[\gamma c(1+\theta) + 2(a-c)\right] - 4(a+c)\right\}/(\theta\gamma^2 + \gamma^2 - 8)$ |

**Case 2.** $\alpha_\theta > 1$
Using similar calculations as in Case 1, the Nash equilibrium outputs and prices with and without bundling are listed here.

| Nash equilibrium outcomes under Cournot competition without bundling($\alpha = \theta/\gamma$) |
|---|
| $q^A_1 = (a-c)(\gamma-2)(\gamma^2 - 2\theta\gamma + 2\gamma - \theta)/\gamma D_5$ |
| $q^A_2 = (a-c)(2\gamma-\theta)(\gamma^2 + 2\theta\gamma - 2\gamma - \theta)/\gamma^2 D_5$ |
| $q^B_2 = 2(a-c)(1-\theta)(\gamma^2 - 2\theta\gamma - 2\gamma + \theta)/\gamma D_5$ |
| $p^A_1 = \left\{(a-c)\theta\left[\theta^2(1-2\gamma) + \gamma(2-\gamma)\right] + \gamma^2(a+c)(\gamma^2-4) + \theta^2\left[\gamma^2(3a+c) + 2c\right]\right\}/\gamma^2 D_5$ |
| $p^A_2 = \left\{(a-c)\left[2\gamma^3(1-\theta^2) + \theta\gamma^2(\gamma^2-1)\right] + (a+c)(\theta^2 - 4\gamma^2) + \gamma^2\left[\theta^2(3a+c) + 2\gamma^2 c\right]\right\}/\gamma^2 D_5$ |
| $p^B_2 = 2\left\{(a-c)\gamma\left[\gamma^2(1-\theta) + \theta(1+\theta)\right] - 2\gamma^2(a+c) + \theta^2(2a\gamma^2 + c) + \gamma^4 c\right\}/\gamma^2 D_5$ |
| $D_5 \equiv 2\left[\gamma^4 - 2\gamma^2(2-\theta^2) + \theta^2\right]/\gamma^2 < 0.$ |

| Nash equilibrium outcomes under Cournot competition with bundling ($\alpha = \theta/\gamma$) |
|---|
| $b_A = (a-c)(4\gamma - \theta - \gamma^2)/\gamma D_6$ |
| $b_B = 2(a-c)\left[2\gamma(1+\theta) - \theta - \gamma^2\right]/\gamma D_6$ |
| $p_A = 2\left\{-(a-c)\gamma(1+\theta)(\theta+\gamma^2) - 2\gamma^2\left[2(a+c) - \theta(2a+c)\right] - c(\theta^2+\gamma^4)\right\}/\gamma^2 D_6$ |
| $p_B = \left\{-2\gamma(a-c)(\theta+\gamma^2) + 2\gamma^2\left[2(a+c) - \theta(2a+c)\right] - c(\theta^2+\gamma^4)\right\}/\gamma^2 D_6$ |
| $D_6 \equiv -\left[\gamma^4 - 2\gamma^2(4+3\theta) + \theta^2\right]/\gamma^2 > 0.$ |

## References


Adams WJ, Yellen JL (1976) Commodity bundling and the burden of monopoly. Q J Econ 90(3):475–498
Armstrong M, Vickers J (2010) Competitive non-linear pricing and bundling. Rev Econ Stud 77(1):30–60
Belleflamme P, Peitz M (2010) Industrial organization: markets and strategies. Cambridge University Press, Cambridge
Carbajo J, Meza D, Seidmann DJ (1990) A strategic motivation for commodity bundling. J Ind Econ 38(3):283–298
Choi JP (2004) Tying and innovation: a dynamic analysis of tying arrangements. Econ J 114(492):83–101
Christopher V, Brian WH (2010) E-mail management: an exchange 2010 upgrade is worth considering, KM World, vol 19, No 4, pp 12–14. http://www.kmworld.com/Articles/Editorial/Feature/E-mail-managementAn-Exchange-2010-upgrade-is-worth-considering-66059.aspx
Dixit A (1979) A model of duopoly suggesting a theory of entry barriers. Bell J Econ 10(1):20–32
Economides N (1993) Mixed bundling in duopoly. Discussion Paper EC-93-29, Stern School of Business, New York University
Goto A (2009) Innovation and competition policy. Jpn Econ Rev 60(1):55–62
Lewbel A (1985) Bundling of substitutes or complements. Int J Ind Organ 3(1):101–107
Martin S (1999) Strategic and welfare implications of bundling. Econ Lett 62(3):371–376
Peitz M (2008) Bundling may blockade entry. Int J Ind Organ 26(1):41–58
Qiu LD (1997) On the dynamic efficiency of Bertrand and Cournot equilibria. J Econ Theory 75(1):213–229
Romano R, Yildirim H (2005) On the endogeneity of Cournot–Nash and Stackelberg equilibria: games of accumulation. J Econ Theory 120(1):73–107
Saggi K, Vettas N (2002) On intrabrand and interbrand competition: the strategic role of fees and royalties. Eur Econ Rev 46(1):189–200
Singh N, Vives X (1984) Price and quantity competition in a differentiated duopoly. RAND J Econ 15(4):546–554
Sutton J (1997) One smart agent. RAND J Econ 28(4):605–628
Telser LG (1979) A theory of monopoly of complementary goods. J Business 52(2):211–230
Venkatesh R, Kamakura W (2003) Optimal bundling and pricing under a monopoly: contrasting complements and substitutes from independently valued products. J Business 76(2):211–231
Vives X (1985) On the efficiency of Bertrand and Cournot equilibria with product differentiation. J Econ Theory 36(1):166–175
Whinston MD (1990) Tying, foreclosure, and exclusion. Am Econ Rev 80(4):837–859