Performance Analysis of Energy Detection Based Spectrum Sensing with Unknown Primary Signal Arrival Time

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***Abstract*—Spectrum sensing in next-generation wireless cognitive systems, such as overlay femtocell networks, is typically subject to timing misalignment between the primary transmitter and the secondary receiver. In this paper, we investigate the performance of the energy detector (ED) when the arrival time of the primary signal is modeled as a uniform random variable over the observation interval. The exact formula for the detection probability is derived and corroborated via numerical simulation. To further improve the detection performance, we propose a robust ED based on the Bayesian principle. Computer simulation confirms the effectiveness of the Bayesian based solution when compared with the conventional ED.**

***Index Terms*—Cognitive radio, spectrum sensing, energy detection.**

I. Introduction

COGNITIVE radio (CR) is a widely known opportunistic spectrum access technique for enhancing the cell-wide spectrum utilization efficiency [1-2]. In order to detect the idle frequency band so as to gain the channel access, spectrum sensing performed at the CR users is indispensible. In the literature, the detection of idle spectrum is typically considered as a binary hypothesis test, and a commonly used signal model under both hypotheses is [1-2]

$$\begin{array}{lll} \mathcal{H}_0 : x[n] = v[n], & 0 \le n \le N-1 & \text{(idle)} \\ \mathcal{H}_1 : x[n] = s[n] + v[n], & 0 \le n \le N-1 & \text{(occupied)} \end{array} \tag{1.1}$$

where N is the length of the data record, $s[n]$, $x[n]$, $v[n]$ are, respectively, the signal of the primary user, the received signal at the CR terminal, and the measurement noise. The hypothesis model (1.1) implicitly assumes perfect synchronization between the primary transmitter and the CR receiver. Such an assumption, however, is not valid in many practical situations. For example, in an overlay femto cell network [3], the signal of the macro mobile subscriber, synchronized with the macro base station (BS), will arrive at a femto BS asynchronously. The spectrum detection at the femto BS is typically subject to timing misalignment of the primary signal [4], [5]. Also, in heavy-traffic networks in which primary users may dynamically enter the network, time delays observed in the sensing period is unavoidable, especially when a long sensing duration is adopted for obtaining good sensing performance. Thus, in the aforementioned cases, a more reasonable signal model for the binary hypothesis test is thus

$$\begin{array}{lll} \mathcal{H}_0 : x[n] = v[n], & 0 \le n \le n_0 - 1 & \text{(idle)} \\ \mathcal{H}_1 : \left\{ \begin{array}{ll} x[n] = v[n], & 0 \le n \le n_0 - 1 \\ x[n] = s[n] + v[n], & n_0 \le n \le N-1 \end{array} \right. & & \text{(occupied)} \end{array} \tag{1.2}$$

where n_0 accounts for the primary signal arrival time. Therefore, in contrast to the spectrum sensing schemes in the literature focusing on the synchronized signal model (1.1) [1-2], this paper considers the spectrum detection aimed for tackling signal timing uncertainty under the hypothesis (1.2).

Among the existing spectrum sensing schemes, the energy detector (ED) [6] is quite popular mainly because it involves only the partial knowledge (the second moment) of the primary signal and is thus cost-effective to implement [1-2]. Even though various performance characteristics of the ED have recently been investigated, e.g., [7-9], the discussions in all these works were based on the idealized model (1.1). In this paper, we study the detection performance of ED under the hypothesis (1.2). As the detection of arrival is the main focus, as in [10], we consider the scenario that the primary user is present only after spectrum sensing is started. Motivated by the fact that, in high-traffic random access networks, the traffic patterns of primary users are typically unknown to the secondary users, the signal arrival time n_0 is assumed to be uniformly distributed over the observation window $0 \le n \le N-1$. Specific technical contributions of this paper can be summarized as follows. Firstly, conditioned on a fixed n_0, the exact formula for the conditional detection probability under the hypothesis model (1.2) is derived. The average detection probability can then be accordingly obtained by taking the expectation with respect to n_0. To the best of our knowledge, the performance study shown in this paper is the original contribution in the literature that is tailored for the ED scheme in the realistic sensing environment characterized by the model (1.2). Secondly, to further exploit the prior knowledge about n_0 for improving the detection performance, we also propose a robust ED based on the Bayesian formulation [6]. Simulation study shows that the Bayesian based solution improves the receiver operation characteristics (ROC). Moreover, under a prescribed detection probability threshold, the Bayesian scheme does lead to a smaller false-alarm

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probability, thereby enhancing spectrum utilization efficiency of the CR networks. Finally, we would like to remark that the problem of ED based spectrum sensing in the presence of signal arrival timing misalignment was recently addressed in [4] within the OFDMA system framework. Rather than developing robust sensing schemes, the conventional ED was considered in [4] for spectrum detection. In addition, analyses of the associated ROC characteristics therein (conditioned on a fixed set of delays) were not exact, but instead resorted to the Chi-square approximation of the true data distribution [4, p-5305]. Given these facts, the distinctive features of current paper in contrast with [4] are: (i) derivations of the exact conditional and average detection probabilities for the conventional ED under the timing-misaligned signal model (1.2); (ii) development of a Bayesian ED robust against timing uncertainty.

II. Performance Analysis

A. Exact Detection Probability of ED Test Under (1.2)

The test statistic of the conventional ED is by definition given by

$$T=\sum_{n=0}^{N-1}|x[n]|^2. \tag{2.1}$$

Under the alternative hypothesis $\mathcal{H}_1$ in (1.2) and conditioned on a fixed n_0, let us decompose the test statistic T into

$$T=\underbrace{\sum_{n=0}^{n_0-1}|x[n]|^2}_{:=T_1}+\underbrace{\sum_{n=n_0}^{N-1}|x[n]|^2}_{:=T_2}. \tag{2.2}$$

Based on (2.2), we shall first derive the conditional detection probability; the average detection probability can then be easily obtained by taking the expectation with respect to n_0.

Let us assume that (i) the signal $s[n]$ and noise $v[n]$ are zero-mean white sequences with variances given by σ_s^2 and σ_v^2, respectively; (ii) $s[n]$ and $v[n]$ are independent. Note that, with T_1 and T_2 defined in (2.2), it is easy to verify $z_1:=T_1/\sigma_v^2\sim\chi^2_{n_0}$ and $z_2:=T_2/(\sigma_v^2+\sigma_s^2)\sim\chi^2_{N-n_0}$, and hence the associated probability density functions (PDF) are

$$f_{z_1}(x)=\frac{x^{(n_0/2)-1}e^{-x/2}}{\sqrt{2^{n_0}}\Gamma(n_0/2)}u(x) \text{ and}$$
$$f_{z_2}(x)=\frac{x^{[(N-n_0)/2]-1}e^{-x/2}}{\sqrt{2^{(N-n_0)}}\Gamma\left((N-n_0)/2\right)}u(x), \tag{2.3}$$

where $u(x)$ is the unit step function. To simplify notation let us consider the equivalent test statistic

$$\begin{aligned}\bar{T}&=\frac{T}{\sigma_v^2}=\frac{1}{\sigma_v^2}\sum_{n=0}^{N-1}|x[n]|^2=\frac{T_1}{\sigma_v^2}+\frac{T_2}{\sigma_v^2}\\&=z_1+(\frac{\sigma_s^2+\sigma_v^2}{\sigma_v^2})z_2=z_1+(1+SNR)z_2,\end{aligned} \tag{2.4}$$

where $SNR:=\sigma_s^2/\sigma_v^2$. Since z_1 and z_2 are independent, the pdf of $\bar{T}$ is given by

$$f_{\bar{T}}(x)=f_{z_1}(x)*f_{z_2}\left((1+SNR)x\right), \tag{2.5}$$

where $*$ denotes the convolution. In terms of Laplace transform, (2.5) reads

$$\begin{aligned}F_{\bar{T}}(s)&=F_{z_1}(s)\times\mathcal{L}\left\{f_{z_2}((1+SNR)x)\right\}\\&=F_{z_1}(s)\times\frac{1}{1+SNR}F_{z_2}\left(\frac{s}{1+SNR}\right),\end{aligned} \tag{2.6}$$

where the second equality follows since $\mathcal{L}\{f(ax)\}=(a)^{-1}F(s/a)$ [11]. To derive an explicit expression for $F_{\bar{T}}(s)$ in (2.6), we need the next lemma.

Lemma 2.1 [11]: For $\lambda>0$, we have $\mathcal{L}\left\{x^{\lambda-1}e^{-ax}u(x)\right\}=\Gamma(\lambda)(s+a)^{-\lambda}$. □

From (2.3) and by means of Lemma 2.1, we immediately have

$$F_{z_1}(s)=\frac{\Gamma\left(n_0/2\right)\left(s+1/2\right)^{-n_0/2}}{\sqrt{2^{n_0}}\Gamma\left(n_0/2\right)}=\frac{\left(s+1/2\right)^{-n_0/2}}{\sqrt{2^{n_0}}} \tag{2.7}$$

and

$$\begin{aligned}F_{z_2}(s)&=\frac{\Gamma\left((N-n_0)/2\right)\left(s+1/2\right)^{-(N-n_0)/2}}{\sqrt{2^{(N-n_0)}}\Gamma\left((N-n_0)/2\right)}\\&=\frac{\left(s+1/2\right)^{-(N-n_0)/2}}{\sqrt{2^{(N-n_0)}}}.\end{aligned} \tag{2.8}$$

Based on (2.6), (2.7), and (2.8), direct manipulation shows

$$\begin{aligned}F_{\bar{T}}(s)=&\frac{1}{(1+SNR)\sqrt{2^N}}\left(s+\frac{1}{2}\right)^{-n_0/2}\\&\times\left(\frac{s}{1+SNR}+\frac{1}{2}\right)^{-(N-n_0)/2}\\=&\frac{(1+SNR)^{[(N-n_0)/2]-1}}{\sqrt{2^N}}\left(s+\frac{1}{2}\right)^{-n_0/2}\\&\times\left(s+\frac{1+SNR}{2}\right)^{-(N-n_0)/2}.\end{aligned} \tag{2.9}$$

With the aid of (2.9), the pdf $f_{\bar{T}}(x)$ is given by

$$\begin{aligned}f_{\bar{T}}(x)=&\frac{(1+SNR)^{[(N-n_0)/2]-1}}{\sqrt{2^N}}\Big\{\mathcal{L}^{-1}\left\{(s+1/2)^{-\frac{n_0}{2}}\right\}\\&*\mathcal{L}^{-1}\left\{(s+(1+SNR)/2)^{-(N-n_0)/2}\right\}\Big\}\\\overset{(a)}{=}&\frac{(1+SNR)^{[(N-n_0)/2]-1}}{\sqrt{2^N}}\left\{\left[\frac{x^{\frac{n_0}{2}-1}e^{-\frac{x}{2}}u(x)}{\Gamma(n_0/2)}\right]\right.\\&\left.*\left[\frac{x^{[(N-n_0)/2]-1}e^{-(1+SNR)x/2}u(x)}{\Gamma\left((N-n_0)/2\right)}\right]\right\}\\=&\frac{(1+SNR)^{[(N-n_0)/2]-1}}{\sqrt{2^N}\Gamma(n_0/2)\Gamma\left((N-n_0)/2\right)}\int_0^x\tau^{[(N-n_0)/2]-1}\\&\times e^{-(1+SNR)\tau/2}(x-\tau)^{n_0/2-1}e^{-(x-\tau)/2}d\tau\\=&\frac{(1+SNR)^{[(N-n_0)/2]-1}}{\sqrt{2^N}\Gamma(n_0/2)\Gamma\left((N-n_0)/2\right)}e^{-x/2}\times\\&\int_0^x\tau^{[(N-n_0)/2]-1}(x-\tau)^{n_0/2-1}e^{-SNR\tau/2}d\tau,\end{aligned} \tag{2.10}$$

where (a) holds by using Lemma 2.1. Hence, for a given threshold γ determined according to the prescribed false-alarm probability, the conditional detection probability can be computed based on (2.10) as

$$P_D(n_0) = \int_{\gamma}^{\infty} f_{\bar{T}}(x)dx = \frac{(1+SNR)^{[(N-n_0)/2]-1}}{\sqrt{2^N}\Gamma(n_0/2)\Gamma\left((N-n_0)/2\right)} \int_{\gamma}^{\infty} dx \times \underbrace{\left[e^{-x/2}\int_0^x \tau^{[(N-n_0)/2]-1}(x-\tau)^{\frac{n_0}{2}-1}e^{-SNR\tau/2}d\tau\right]}_{:=p(x)}. \quad (2.11)$$

To find a closed-form expression of $P_D(n_0)$ in (2.11), we need the next lemma.

Lemma 2.2 [11]: For $\nu > 0$ and $\mu > 0$, it follows

$$\int_0^x t^{\nu-1}(x-t)^{\mu-1}e^{\delta t}dt = B(\mu,\nu)x^{\mu+\nu-1}\Phi(\nu,\mu+\nu;\delta x), \quad (2.12)$$

where $B(\cdot,\cdot)$ is the beta function, and $\Phi(\cdot,\cdot;\cdot)$ is the confluent hyper-geometric function defined by

$$\Phi(\alpha,\gamma;z) = 1+\frac{\alpha}{\gamma}\cdot\frac{z}{1!}+\frac{\alpha(\alpha+1)}{\gamma(\gamma+1)}\cdot\frac{z^2}{2!}+\frac{\alpha(\alpha+1)(\alpha+2)}{\gamma(\gamma+1)(\gamma+2)}\cdot\frac{z^3}{3!}+\ldots. \quad (2.13)$$

□

Based on Lemma 2.2, equation (2.11) becomes

$$P_D(n_0) = \frac{(1+SNR)^{[(N-n_0)/2]-1}B\left(\frac{N-n_0}{2},\frac{n_0}{2}\right)}{\sqrt{2^N}\Gamma(n_0/2)\Gamma\left((N-n_0)/2\right)} \times \int_{\gamma}^{\infty} e^{-x/2}x^{(N/2)-1}\left[\sum_{i=0}^{\infty}a_i x^i\right]dx, \quad (2.14)$$

where

$$a_0 = 1, a_1 = \frac{(N-n_0)/2}{N/2}\cdot\frac{(-SNR/2)}{1!}, \quad a_2 = \frac{[(N-n_0)/2]\{[(N-n_0)/2]+1\}}{N/2\{[N/2]+1\}}\cdot\frac{(-SNR/2)^2}{2!},\ldots. \quad (2.15)$$

Based on (2.14), the exact form of the conditional detection probability can be obtained as[1]

$$P_D(n_0) = \frac{(1+SNR)^{[(N-n_0)/2]-1}B\left(\frac{N-n_0}{2},\frac{n_0}{2}\right)}{\sqrt{2^N}\Gamma(n_0/2)\Gamma\left((N-n_0)/2\right)} \times \left[\sum_{i=0}^{\infty}a_i\int_{\gamma}^{\infty}e^{-x/2}x^{(N/2)+i-1}dx\right] \overset{(b)}{=} \frac{(1+SNR)^{[(N-n_0)/2]-1}B\left(\frac{N-n_0}{2},\frac{n_0}{2}\right)}{\sqrt{2^N}\Gamma(n_0/2)\Gamma\left((N-n_0)/2\right)} \times \sum_{i=0}^{\infty}a_i\left[2^{(N/2)+i}\Gamma\left(\frac{N}{2}+i,\frac{\gamma}{2}\right)\right], \quad (2.16)$$

[1]In the case of $n_0 = 0$, (2.16) reduces to the widely known result in [6, p-144].

where (b) follows since $\int_{\gamma}^{\infty} x^{\nu-1}e^{-\mu x}dx = \mu^{-\nu}\Gamma(\nu,\mu\gamma)$ [6, 346], and $\Gamma(\alpha,y) := \int_y^{\infty} e^{-t}t^{\alpha-1}dt$ is the incomplete Gamma function. Based on (2.16), we summarize the main result in the following theorem.

Theorem 2.3: The average detection probability of the ED under the hypothesis test (1.2) is given by

$$P_D = \frac{1}{N}\sum_{n_0=0}^{N-1}P_D(n_0) = \frac{1}{N}\sum_{n_0=0}^{N-1}\left\{\frac{(1+SNR)^{[(N-n_0)/2]-1}B\left(\frac{N-n_0}{2},\frac{n_0}{2}\right)}{\sqrt{2^N}\Gamma(n_0/2)\Gamma\left((N-n_0)/2\right)} \times \sum_{i=0}^{\infty}a_i\left[2^{(N/2)+i}\Gamma\left(\frac{N}{2}+i,\frac{\gamma}{2}\right)\right]\right\}, \quad (2.17)$$

where γ is the threshold determined according to the prescribed false-alarm probability. □

B. Low-SNR Regime

While the formula (2.17) appears quite involved, in the low-SNR regime it admits a very simple form that is compatible with the existing study of ED [6]. To see this, we need the next lemma, which provides an upper and lower bounds for the conditional detection probability $P_D(n_0)$.

Lemma 2.4: Let $P_D(n_0)$ be defined in (2.16). Then we have

$$\frac{\Gamma\left(\frac{N}{2},\gamma\left(\frac{1+SNR}{2}\right)\right)}{(1+SNR)^{(n_0/2)+1}\Gamma(N/2)} \le P_D(n_0) \le \frac{(1+SNR)^{[(N-n_0)]/2-1}\Gamma\left(\frac{N}{2},\frac{\gamma}{2}\right)}{\Gamma(N/2)}. \quad (2.18)$$

Proof: See appendix. ■

To gain further insight based on (2.18), let us assume without loss of generality that the total number of samples N is even, so that $N/2$ is a positive integer. In this case, we have $\Gamma(N/2) = [(N/2)-1]!$ and $\Gamma(N/2,y) = [(N/2)-1]!e^{-y}\sum_{k=0}^{(N/2)-1}\frac{y^k}{k!}$ [11, p-900]. Hence (2.18) becomes

$$\frac{e^{-\gamma(1+SNR)/2}\sum_{k=0}^{(N/2)-1}\frac{[\gamma(1+SNR)/2]^k}{k!}}{(1+SNR)^{(n_0/2)+1}} \le P_D(n_0) \le (1+SNR)^{[(N-n_0)/2]-1}e^{-\gamma/2}\sum_{k=0}^{(N/2)-1}\frac{[\gamma/2]^k}{k!}. \quad (2.19)$$

When $SNR \to 0$, we have $1+SNR \to 1$ and (2.19) then becomes

$$P_D(n_0) \to e^{-\gamma/2}\sum_{k=0}^{(N/2)-1}\frac{(\gamma/2)^k}{k!} \overset{(c)}{=} Q_{\chi_N^2}(\gamma), \quad (2.20)$$

where (c) holds directly by definition of the right-tail probability of the Chi-square random variable χ_N^2 with an even degree-of-freedom [6, p-25]. With the aid of (2.20) and since the limiting probability is independent of n_0, we have the following asymptotic result.

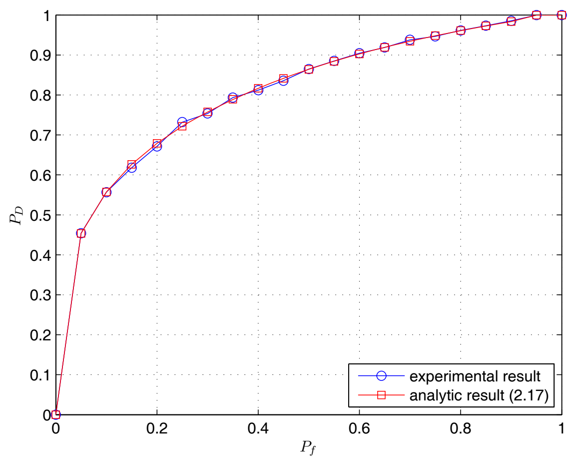

Fig. 1. Analytic and experimental ROC curves of energy detector (SNR = -5 dB).

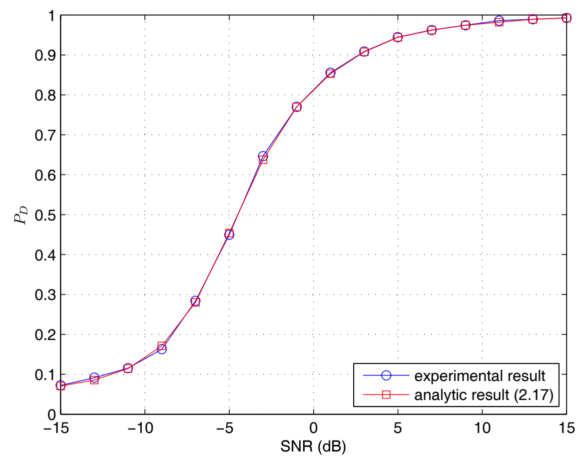

Fig. 2. Detection probability P_D versus SNR ($P_f = 0.05$).

Proposition 2.5: Let P_D be the average detection probability defined in (2.17). Then we have

$$\lim_{SNR \to 0} P_D = Q_{\chi_N^2}(\gamma). \tag{2.21}$$

□

Recall from [6, Sec. 5.3] that $Q_{\chi_N^2}(\gamma)$ is the detection probability for ED when $SNR = \sigma_x^2/\sigma_v^2 \to 0$. In this case, the performance of ED can be very poor since the energy of the received signal in either hypothesis is very close to the noise floor. To further enhance the detection performance when SNR is low and the signal timing mismatch is present, a robust ED scheme based on the Bayesian principle is proposed next.

III. Proposed Bayesian Decision Rule

A. The Bayesian Test

To exploit the prior statistical knowledge of n_0 for enhancing the detection performance, a typical approach is the Bayesian philosophy [6]. The conditional joint pdf of the data samples under the two hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$ are

$$p(\mathbf{x}; \mathcal{H}_0) = \frac{1}{(2\pi\sigma_0^2)^{N/2}} \exp\left[\frac{-1}{2\sigma_0^2}\sum_{n=0}^{N-1}|x[n]|^2\right], \tag{3.1}$$

and

$$p(\mathbf{x}; \mathcal{H}_1) = \frac{1}{(2\pi\sigma_0^2)^{n_0/2}} \exp\left[\frac{-1}{2\sigma_0^2}\sum_{n=0}^{n_0-1}|x[n]|^2\right] \times \frac{1}{(2\pi(\sigma_0^2+\sigma_1^2))^{(N-n_0)/2}} \exp\left[\frac{-1}{2(\sigma_0^2+\sigma_1^2)}\sum_{n=n_0}^{N-1}|x[n]|^2\right]. \tag{3.2}$$

The Bayesian test decides $\mathcal{H}_1$ if (3.3), shown at the top of the next page, holds (see [6, Chap. 6]). The performance advantages of the Bayesian test (3.3) over the conventional ED scheme (2.1) under the considered scenario will be illustrated via computer simulation in the next section.

B. Numerical Simulation

In the following simulations we compare the performance of the conventional ED (2.1) and that of the proposed Bayesian ED (3.3), based on the hypothesis signal model (1.2); the total number of samples is set to be $N = 200$, and the signal arrival time n_0 is uniformly distributed within $0 \le n_0 \le 199$. Figure 1 plots the ROC curves of ED (2.1), with SNR set to be -5 dB; Figure 2 plots the probability of detection P_D at various SNR levels, assuming that the false-alarm probability $P_f = 0.05$. As can be seen from the figures, the derived analytic formula (2.17) closely matches the simulated results. Figures 3 and 4, respectively, compare P_D and $1 - P_f$ curves (as a function of SNR) of the ED (2.1) and the robust ED solution (3.3); note that large values of $1 - P_f$ mean better channel utilization efficiency of secondary users [12]. The figures show that the Bayesian based solution (3.3), which takes into account the statistical knowledge of the signal arrival time, not only improves P_D but also leads to larger $1 - P_f$, especially when SNR is low. Since the proposed Bayesian ED (3.3) is optimal in accordance with the criterion of minimizing average cost function, a natural approach to evaluate the performance is to compare the detectors (2.1) and (3.3) in terms of the average probability of error, i.e.,

$$P_e = P(\mathcal{H}_0|\mathcal{H}_1)P(\mathcal{H}_1) + P(\mathcal{H}_1|\mathcal{H}_0)P(\mathcal{H}_0), \tag{3.4}$$

where $P(\mathcal{H}_i|\mathcal{H}_j)$ denotes the probability that $\mathcal{H}_i$ is decided given that $\mathcal{H}_j$ is true. The optimal decision threshold of the Bayesian rule (3.3) for minimizing average error probability is known to be simply $\tau = P(\mathcal{H}_0)/P(\mathcal{H}_1)$ [13, p-20]. In our simulation, the equally likely hypothesis is assumed, thereby $\tau = 1$. Figure 5 compares the achievable P_e of (2.1) and (3.3) at various SNR. As expected, the Bayesian test (3.3) is seen to yield a smaller average error probability.

IV. Conclusion

Spectrum sensing in the presence of unknown arrival time of the primary signal finds applications in many practical system scenarios and is thus an important issue in the study of CR networks. In this paper we have provided the exact formula

$$\frac{\frac{1}{N}\sum_{n_0=0}^{N-1} p(\mathbf{x}; n_0, \mathcal{H}_1)}{p(\mathbf{x}; \mathcal{H}_0)} = \frac{\frac{1}{N}\sum_{n_0=0}^{N-1} \frac{1}{(2\pi\sigma_0^2)^{n_0/2}} \exp\left[\frac{-1}{2\sigma_0^2}\sum_{n=0}^{n_0-1}|x[n]|^2\right] \times \frac{1}{(2\pi(\sigma_0^2+\sigma_1^2))^{(N-n_0)/2}} \exp\left[\frac{-1}{2(\sigma_0^2+\sigma_1^2)}\sum_{n=n_0}^{N-1}|x[n]|^2\right]}{\frac{1}{(2\pi\sigma_0^2)^{N/2}} \exp\left[\frac{-1}{2\sigma_0^2}\sum_{n=0}^{N-1}|x[n]|^2\right]} > \gamma \quad (3.3)$$

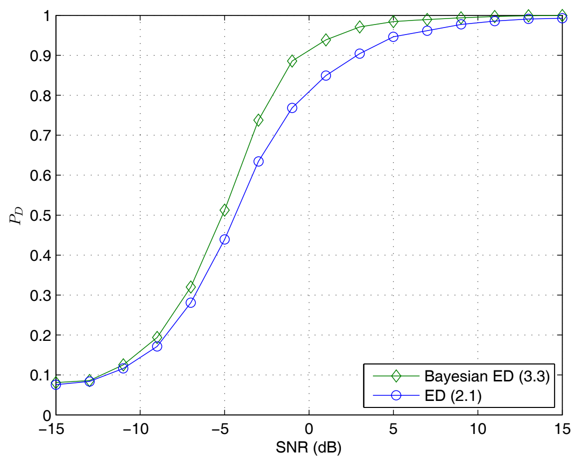

Fig. 3. Detection probabilities of detectors (2.1) and (3.3) versus SNR ($P_f = 0.05$).

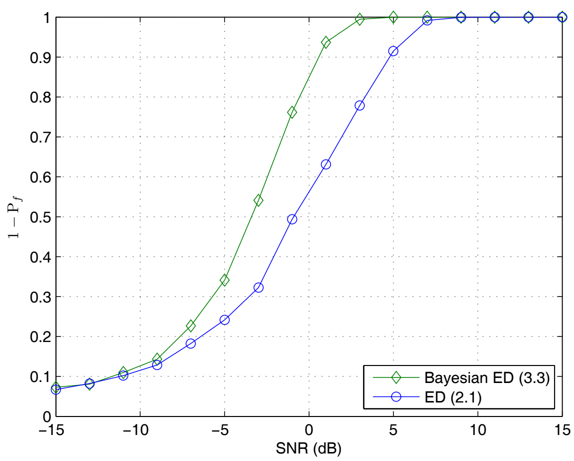

Fig. 4. $1 - P_f$ versus SNR ($P_D = 0.95$).

of the average detection probability for ED when the arrival time of the primary signal is modeled as a uniform random variable over the observation interval. To further improve the detection performance against the timing uncertainty, we have then proposed a Bayesian based detection scheme. Simulation results show that the Bayesian ED reduces the false-alarm probability and thus enhances the spectrum utilization in the considered asynchronous scenario. Future research will be dedicated to extending the current results to the cooperative sensing scenario.

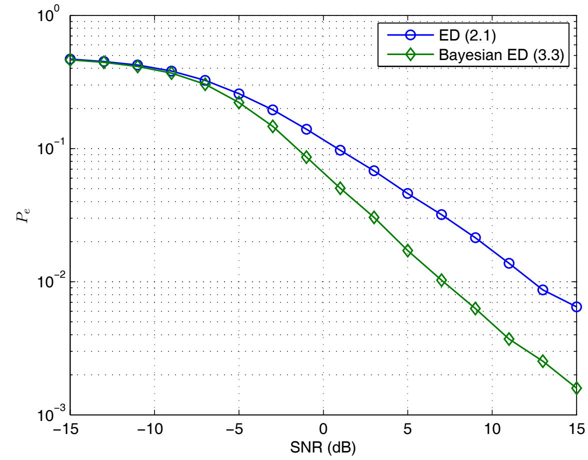

Fig. 5. Average error probabilities of detectors (2.1) and (3.3) versus SNR.

Appendix
Proof of Lemma 2.4

We first observe that $p(x)$ in (2.10) satisfies

$$\begin{aligned} &e^{-x/2} \times e^{-SNRx/2} \int_0^x \tau^{[(N-n_0)/2]-1}(x-\tau)^{n_0/2-1} d\tau \\ &\leq p(x) \\ &\leq e^{-x/2} \int_0^x \tau^{[(N-n_0)/2]-1}(x-\tau)^{n_0/2-1} d\tau. \end{aligned} \quad (A.1)$$

Since

$$\begin{aligned} &\int_0^x \tau^{[(N-n_0)/2]-1}(x-\tau)^{n_0/2-1} d\tau \\ &= x^{[(N-n_0)/2]-1}u(x) * x^{n_0/2-1}u(x), \end{aligned} \quad (A.2)$$

we have

$$\begin{aligned} &\mathcal{L}\left\{\int_0^x \tau^{[(N-n_0)/2]-1}(x-\tau)^{n_0/2-1} d\tau\right\} \\ &= \mathcal{L}\left\{x^{[(N-n_0)/2]-1}u(x)\right\} \times \mathcal{L}\left\{x^{n_0/2-1}u(x)\right\} \\ &= \frac{\Gamma((N-n_0)/2)}{s^{(N-n_0)/2}} \times \frac{\Gamma(n_0/2)}{s^{n_0/2}} \\ &= \frac{\Gamma((N-n_0)/2)\,\Gamma(n_0/2)}{s^{N/2}}. \end{aligned} \quad (A.3)$$

By taking the inverse Laplace transform of both sides of (A.3) we have

$$\begin{aligned} &\int_0^x \tau^{[(N-n_0)/2]-1}(x-\tau)^{n_0/2-1} d\tau \\ &= \Gamma((N-n_0)/2)\,\Gamma(n_0/2)\mathcal{L}^{-1}\left\{\frac{1}{s^{N/2}}\right\} \end{aligned}$$

$$= \frac{\Gamma\left((N-n_0)/2\right)\Gamma(n_0/2)}{\Gamma(N/2)} x^{N/2-1}, \tag{A.4}$$

where the last equality holds due to Lemma 2.1. With the aid of (A.4), (A.1) becomes

$$\begin{aligned} &\frac{\Gamma\left((N-n_0)/2\right)\Gamma(n_0/2)}{\Gamma(N/2)} x^{N/2-1} e^{-(1+SNR)x/2} \\ &\leq p(x) \\ &\leq \frac{\Gamma\left((N-n_0)/2\right)\Gamma(n_0/2)}{\Gamma(N/2)} x^{N/2-1} e^{-x/2}. \end{aligned} \tag{A.5}$$

Based on (A.5), we have

$$\begin{aligned} P_D &= \frac{(1+SNR)^{[(N-n_0)/2]-1}}{\sqrt{2^N}\Gamma(n_0/2)\Gamma((N-n_2)/2)} \int_\gamma^\infty p(x)dx \\ &\geq \frac{(1+SNR)^{[(N-n_0)/2]-1}}{\sqrt{2^N}\Gamma(N/2)} \int_\gamma^\infty x^{\frac{N}{2}-1} e^{-(1+SNR)x/2} dx \\ &\overset{(a)}{=} \frac{(1+SNR)^{[(N-n_0)/2]-1}}{\sqrt{2^N}\Gamma(N/2)} \left(\frac{1+SNR}{2}\right)^{-N/2} \times \\ &\quad \Gamma\left(\frac{N}{2}, \gamma\left(\frac{1+SNR}{2}\right)\right) \\ &= \frac{\Gamma\left(\frac{N}{2}, \gamma\left(\frac{1+SNR}{2}\right)\right)}{(1+SNR)^{(n_0/2)+1}\Gamma(N/2)}, \end{aligned} \tag{A.6}$$

where (a) follows since $\int_\gamma^\infty x^{\nu-1}e^{-\mu x}dx = \mu^{-\nu}\Gamma(\nu,\mu\gamma)$ [11, p-346]. Similarly we have

$$P_D \leq \frac{(1+SNR)^{[(N-n_0)/2]-1}}{\Gamma(N/2)} \Gamma\left(\frac{N}{2}, \frac{\gamma}{2}\right). \tag{A.7}$$

The assertion follows from (A.6) and (A.7).

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