Phase Conjugation by Four-Wave Mixing in Single-Mode Fibers

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***Abstract*— We present an exact solution to the coupled-mode equations, governing four-wave mixing in single-mode fibers, under the perfect phase-matching condition. The solution accounts for pump depletion and fiber absorption. Optimum length of fiber is derived and phase conjugation efficiency is calculated and discussed. The calculated results are in good agreement with the reported experimental results.**

I. Introduction

FOUR-WAVE mixing (FWM) in optical fibers [1]–[5] is an important nonlinear process which is useful for phase conjugation and wavelength conversion [6], [7]. Recently, an experiment using four-wave mixing to achieve temporal pulse restoration in optical fibers was reported [8]. In the experiment, an optical pulse was phase conjugated by four-wave mixing in a single mode fiber. Generally speaking, the pulse shape undergoes a chirp reversal as a result of phase conjugation. This leads to a restoration of the pulse shape as the pulse propagates through another fiber and undoes all the chromatic dispersion experienced by the original pulse. Wavelength conversion near the zero-dispersion point of fibers was also reported [7]. As a result of these unique properties, four-wave mixing has potential applications in long-haul fiber optical communication systems and all-optical multiwavelength networks. From a practical point of view, high phase conjugation efficiencies (or wavelength conversion efficiencies in the context of wavelength conversion) with a significant depletion of the pump are desirable. Due to the moderate nonlinearity of fibers, such a depletion occurs only in a fairly long fiber. Under these conditions, fiber absorption can not be neglected. Although a number of special cases of optical four-wave mixing in fibers have been studied, a general theory including pump depletion and fiber absorption is not available. Most analytical solutions were based on the assumption of no pump depletion [1], [2], while others taking into account pump depletion were based on the assumptions of no absorption [5], [10] and no group velocity dispersion (GVD) [9], which are not realistic for long fibers. In this letter, we present an analytical solution for four-wave mixing in single-mode fibers, taking into account both pump depletion and fiber absorption. We show that efficient phase conjugation is possible at reasonable input powers. Optimum fiber length will be derived from the solution so that the maximum efficiency can be obtained.

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Consider the propagation of light in a single-mode fiber. Using notations similar to those of [3], the electric fields of the four waves can be written as, $E_j = A_j(z)F_j(x,y)\exp[i(\omega_j t - k_j z)]$, $j = 1,2,3,4$, where $\omega_j' s$ are the frequencies and k_j's are the propagation constants in the fiber. A_1 through A_4 denote the amplitudes of the two pump waves, probe wave (to be phase conjugated) and the phase conjugate wave, respectively, and $F_j(x,y)$'s are wave functions describing the transverse distribution of the four waves in the fiber. We assume that all four frequencies are near the zero-dispersion point of the single-mode fiber. However, the spacing between the four waves is large enough so that only the phase-matched, four-wave mixing process can be efficiently built up, while all other high-order wave mixing processes between the newly generated waves and the input waves can be neglected. We further assume that the self- and/or cross-phase modulation (SPM–XPM), the phaseshift induced by the intenisty-dependent refractive index, can be neglected. This assumption will be justified later for cases of moderate input powers, which is true for diode lasers operating near 1.3 or 1.5 μm. First we consider the nondegenerate case where all four frequencies are different. Partially degenerate four-wave mixing (PDFWM), which implies $\omega_1 = \omega_2$, will be considered later. The nondegenerate FWM process is governed by the following coupled-mode equations, [3]

$$\begin{aligned}
\frac{dA_1}{dz} &= -i\frac{n_2\omega_1}{c}2f_{1234}A_2^*A_3A_4\exp(i\Delta kz) - \frac{\alpha}{2}A_1\\
\frac{dA_2}{dz} &= -i\frac{n_2\omega_2}{c}2f_{2134}A_1^*A_3A_4\exp(i\Delta kz) - \frac{\alpha}{2}A_2\\
\frac{dA_3}{dz} &= -i\frac{n_2\omega_3}{c}2f_{3412}A_1A_2A_4\exp(-i\Delta kz) - \frac{\alpha}{2}A_3\\
\frac{dA_4}{dz} &= -i\frac{n_2\omega_4}{c}2f_{4312}A_1A_2A_3\exp(-i\Delta kz) - \frac{\alpha}{2}A_4 \quad (1)
\end{aligned}$$

where n_2 is the nonlinear index coefficient, f's are constants describing mode overlapping in the single-mode fiber [3], α is the fiber absorption coefficient, and $\Delta k = k_1 + k_2 - k_3 - k_4$ is the difference of the propagation constants describing the phase mismatch of the FWM process. In (1) the terms responsible for SPM–XPM are neglected as we assumed.

In order to solve (1), we further assume that $\omega_1 \approx \omega_2 \approx \omega_3 \approx \omega_4$ and all f's are equal to $1/A_{\text{eff}}$, where A_{eff} is the effective area of the fiber [3]. We first solve the equations for cases where phase matching is satisfied, i.e., $\Delta k = 0$. This can be achieved by setting $\omega_1 - \omega_0 = \omega_0 - \omega_2$, where ω_0 is the zero-dispersion frequency of the fiber [8]. When the unit for n_2 is expressed in terms of cm^2/W, A_j's can be substituted by, $A_j = \sqrt{P_j}\exp(i\phi_j) = \sqrt{Q_j}\exp(-\alpha z/2)\exp(i\phi_j)$, where P_j's are the corresponding powers of the four waves [3]. (1) can be rewritten as the following

$$\begin{aligned}\frac{dQ_1}{dz} &= -4\gamma\sqrt{Q_1Q_2Q_3Q_4}\exp(-\alpha z)\sin\Delta\phi\\ \frac{dQ_2}{dz} &= -4\gamma\sqrt{Q_1Q_2Q_3Q_4}\exp(-\alpha z)\sin\Delta\phi\\ \frac{dQ_3}{dz} &= +4\gamma\sqrt{Q_1Q_2Q_3Q_4}\exp(-\alpha z)\sin\Delta\phi\\ \frac{dQ_4}{dz} &= +4\gamma\sqrt{Q_1Q_2Q_3Q_4}\exp(-\alpha z)\sin\Delta\phi\end{aligned} \tag{2}$$

and

$$\begin{aligned}\frac{d\Delta\phi}{dz} &= 2\left(\frac{1}{Q_3}+\frac{1}{Q_4}-\frac{1}{Q_1}-\frac{1}{Q_2}\right)\gamma\\ &\quad\times\sqrt{Q_1Q_2Q_3Q_4}\exp(-\alpha z)\cos\Delta\phi\end{aligned} \tag{3}$$

where $\gamma = n_2\omega/cA_{\text{eff}}$ is a constant describing the four-wave mixing gain and $\Delta\phi = \phi_1+\phi_2-\phi_3-\phi_4$. It can be easily verified from (2) and (3) that there exist four independent invariants: $Q_1+Q_2+Q_3+Q_4$, $Q_3-Q_4-Q_1-Q_2$, and $\sqrt{Q_1Q_2Q_3Q_4}\cos\Delta\phi$.

Equations (2) and (3) are similar to those of phase locking in coupled oscillators and the solutions of (3) can be readily obtained as $\Delta\phi = \pm\pi/2$. This solution can also obtained by the use of the invariants [5]. Using the boundary condition of $P_4(0) = 0$, we know that $\sqrt{Q_1Q_2Q_3Q_4}\cos\Delta\phi = 0$ for all z. Because all four waves are finite when $z \neq 0$, we conclude that $\cos\Delta\phi = 0$, which leads to $\Delta\phi = \pm\pi/2$. According to the boundary conditions and (2), we find that $\Delta\phi = \pi/2$ is a proper solution. Note that we have assumed that the initial input of the fourth (phase conjugate) wave is absent, which is valid for lightwave systems designed for phase conjugation or wavelength conversion. Substituting the solution back into (2), the coupled equations can be integrated for cases of equal pump powers, i.e., $P_1(0) = P_2(0) = P_p(0)$. With the help of the invariants and the boundary conditions, the solution of the coupled equations takes the form of

$$\begin{aligned}Q_1 &= P_p(0)y\\ Q_2 &= P_p(0)y\\ Q_3 &= P_p(0)(1-y)+P_3(0)\\ Q_4 &= P_p(0)(1-y),\end{aligned} \tag{4}$$

which leads to the following differential equation for y $(0 \le y \le 1)$,

$$\begin{aligned}&\frac{dy}{y\sqrt{[(1-y)+P_3(0)/P_p(0)](1-y)}}\\ &\quad= -4\gamma P_p(0)\exp(-\alpha z)dz.\end{aligned} \tag{5}$$

(5) can be exactly integrated and y is given by

$$y = \frac{r+1}{r\cosh^2(\sqrt{r+1}f)+1} \tag{6}$$

where $r \equiv P_3(0)/P_p(0)$ and $f = 2\gamma P_p(0)[1-\exp(-\alpha z)]/\alpha$.

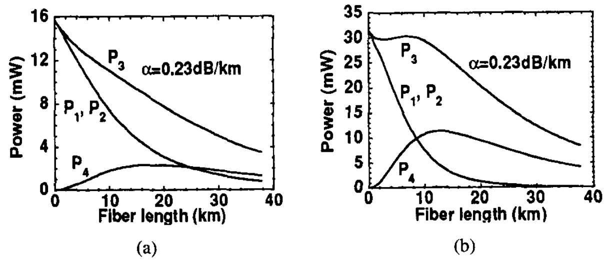


Fig. 1. Power coupling due to nondegenerate FWM in single-mode fibers as functions of fiber length with $\alpha = 0.23$ dB/km. Input powers are (a) 12 dBm and (b) 15 dBm.

In the case of partially degenerate four-wave mixing (PDFWM) where $\omega_1 = \omega_2$, only three distinct waves are present. Then (1) should be modified as

$$\begin{aligned}\frac{dA_1}{dz} &= -i\frac{n_2\omega_1}{c}2f_{1134}A_1^*A_3A_4\exp(i\Delta kz)-\frac{\alpha}{2}A_1\\ \frac{dA_3}{dz} &= -i\frac{n_2\omega_3}{c}f_{3411}A_1A_1A_4^*\exp(-i\Delta kz)-\frac{\alpha}{2}A_3\\ \frac{dA_4}{dz} &= -i\frac{n_2\omega_4}{c}f_{4311}A_1A_1A_3^*\exp(-i\Delta kz)-\frac{\alpha}{2}A_4\end{aligned} \tag{7}$$

where $\Delta k = 2k_1 - k_3 - k_4$. With boundary conditions $P_1|_{z=0} = P_p(0)$, $P_3|_{z=0} = P_3(0)$, and $P_4|_{z=0} = 0$, the solution of (7), assuming $\Delta k = 0$ again, can be similarly obtained as

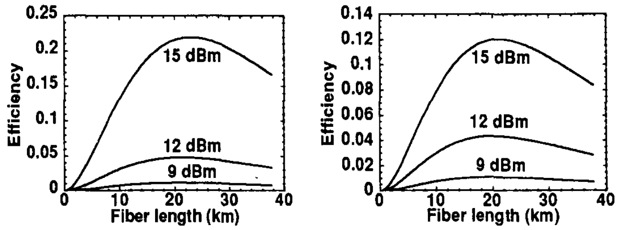
$$\begin{aligned}P_1 &= P_p(0)y\exp(-\alpha z)\\ P_3 &= \left[\frac{1}{2}P_p(0)(1-y)+P_3(0)\right]\exp(-\alpha z)\\ P_4 &= \frac{1}{2}P_p(0)(1-y)\exp(-\alpha z)\end{aligned} \tag{8}$$

where y is given again by (6) with the parameters r and f replaced by $2P_3(0)/P_p(0)$ and $\gamma P_p(0)[1-\exp(-\alpha z)]/\alpha$, respectively.

The magnitude of y is a measure of power transfer between the pump waves and the new wave. Note that in the absence of absorption, the function f in y reduces to $2\gamma P_p(0)z$. With the increase of the fiber length z, y approaches zero, which means a complete power transfer to the new wave. However, fiber absorption can not be neglected from the practical point of view. In that case, y is of the functional form of $[1-\exp(-\alpha z)]/\alpha$. We can conclude that the power coupling from the pump wave to the phase conjugate wave will cease if $z \gg 1/\alpha$. The power coupling described by (4) is plotted in Fig. 1 for $\alpha = 0.23$ dB/km, $n_2 = 3.2\times10^{-20}$ m^2/W, $\lambda = 1.55$ μm and $A_{\text{eff}} = 70\mu\text{m}^2(\gamma = 1.853\times10^{-5}\text{cm}^{-1}\ \text{W}^{-1})$. It shows that when z is greater than a certain value (around 13 km, or $0.7/\alpha$, for $P_p(0) = P_3(0) = 15$ dBm), fiber absorption prevails. Consequently, the power of the phase conjugated wave begins to decrease. Therefore, a proper choice of the fiber length is important.

The optimum length of the fiber can be derived from (4). By setting $dP_4(0)/dz = 0$, we obtain the condition for the optimum length,

$$2y\frac{df}{dz} = \alpha\tanh\sqrt{r+1}f. \tag{9}$$



(a) (b)

Fig. 2. Efficiency of phase conjugation by PDFWM in single-mode fibers as functions of fiber length with $\alpha = 0.23$ dB/km for different pump powers, and (a) $P_3(0) = -6$ dBm, (b) $P_3(0) = 12$ dBm.

If $\gamma P_p(0)$ is not very large such that $\gamma P_p(0)/\alpha \ll 1$, then $y \approx 1$ and $\tanh\sqrt{r+1}f \approx \sqrt{r+1}f$. The optimum fiber length, according to (9), is given by $\alpha z = \ln(1 + 2/\sqrt{r+1}$, For nondegenerate FWM, the optimum length is 0.88/α when $P_p(0) = P_3(0)$. When $P_3(0) \ll P_p(0)$, the optimum length is given by (ln3)/α.

The phase conjugation efficiency (or the wavelength conversion efficiency in the context of the wavelength conversion), defined as $P_4(z)/P_3(0)$, can be written as

$$\eta = \frac{\sinh^2\sqrt{r+1}f}{r\cosh^2\sqrt{r+1}f + 1}\exp(-\alpha z). \quad (10)$$

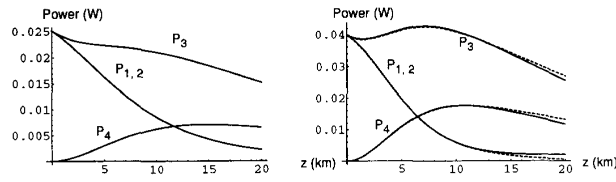
The dependence of the efficiency on the fiber length is plotted in Fig. 2 for different values of pump powers for case of PDFWM. $P_3(0)$ is assumed to be -6 dBm and 12 dBm in Figs. 2(a) and 2(b) respectively. Note that the maximum efficiency occurs around 20.8 km (or $1.1/\alpha$) in Fig. 2(a), and a shorter distance in Fig. 2(b). These optimum fiber lengths are in good agreement with above prediction. Efficiency as high as 22%, or –6.6 dB, can be achieved for $P_p(0) = 15$ dBm and $P_3(0) = -6$ dBm. Using the parameters given in [7] and [8], we find that the calculated results are in excellent agreement with the experimental results.

When $\Delta k \neq 0$, the terms in $\sin\Delta\phi$ and $\cos\Delta\phi$ in (2) and (3) become $\sin(\Delta\phi - \Delta kz)$ and $\cos(\Delta\phi - \Delta kz)$. However, the solution $\Delta\phi = \pi/2$ can still be considered valid provided Δk is small enough such that $\Delta kL \ll 1$. In this case, y is still given by (6) while f is modified to

$$f = 2\gamma P_p(0)\frac{\Delta k e^{-\alpha z}\sin(\Delta kz) + \alpha[1 - e^{-\alpha z}\cos(\Delta kz)]}{(\Delta k)^2 + \alpha^2}. \quad (11)$$

In case of $\Delta k \gg \alpha, f \approx 2\gamma P_p(0)\sin(\Delta kz)/\Delta k$ according to (11). Hence, periodic behavior of the power coupling can be predicted. Furthermore, the optimum fiber length is $\pi/2\Delta k$, or $L_c/4$, [5] according to (9). After this distance, the power in the phase conjugate wave will flow back to the pump waves.

The analytical solutions were obtained under the assumptions that the terms responsible for SPM and XPM can be neglected in the coupled-mode equations. To take into account these terms [3], [5], numerical methods are employed. Fig. 3 shows the numerical results and the analytical results for nondegenerate FWM in fibers with effective area of 70 μm^2 at 1.55 μm for input powers 14 dBm and 16 dBm, respectively. The results indicate that SPM and/or XPM have very little effect on four-wave mixing power coupling for input powers up to 16 dBm. For larger input powers, SPM and XPM may no longer be negligible if the fiber is fairly long. However, the analytical solution still gives a good approximation either by choosing a short fiber or by adjusting Δk to a finite value to compensate the phase modulation effects [3]. Therefore, for most currently available diode lasers operating at 1.5 μm, the effects of SPM–XPM can be reasonably neglected.



(a) (b)

Fig. 3. The effect of SPM/XPM on power coupling by nondegenerate FWM. Solid lines represent the results obtained by numerical methods while the dashed lines represent the analytical resultsfor (a) $P_p(0) = P_3(0) = 14$ dBm and (b) $P_p(0) = P_3(0)$ =16 dBm.

In conclusion, we have derived an analytical solution for phase conjugation via four-wave mixing in single-mode fibers. Pump depletion and fiber absorption have been taken into account. Optimum fiber length as a function of input powers for efficient generation of phase conjugate waves has been derived. It has been shown that SPM–XPM effects can be neglected for input powers up to 16 dBm. The results calculated using the analytical solution are in good agreement with the reported experimental results.

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