RECURSIVE MINIMUM ENERGY ALGORITHM FOR IMAGE INTERPOLATION

Pei-Chuan Liu, Wen-Thong Chang and Wen-Zen Shen
Department of Electronic Engineering
Chiao-Tung University
Hsinchu 30039 Taiwan, R.O.C.

Abstract

An interpolation method based on the criterion to maximize the smoothness is proposed. In contrast to the conventional approach, where the interpolated signal is made smooth by low-pass filtering, in the proposed method, the smoothness is obtained by minimizing a global energy function. With the minimum energy formulation for interpolation, a recursive method is used to derive the solution and named as the recursive minimum energy algorithm. Because the smoothness is improved by increasing the calculation iteration but not by increasing the tap length, the controllability of this system is increased. Experiments show that the smoothness and the frequency performance obtained by the proposed algorithm are better than those of the conventional method. Also, the corresponding architecture design for the image interpolation is presented and shown to be suit for VLSI implemented.

I. Introduction

Interpolation has been applied in many field such as signal processing, computer graphic, data analysis, image processing, and so on. Algorithms for interpolation have been widely studied in both science and engineering. In digital signal processing, interpolation is usually a reconstruction of a signal from its decimated version. In this field, formulation for interpolation is derived based on the sampling theorem [1-2]. The signal (or function) to be interpolated is assumed to be band-limited in frequency domain and low-pass filtering is used for smoothness. The problem with the interpolation is then the design of the low pass filter. The window method with sinc function is usually used for interpolation. Conventionally the rectangular window, hamming window and hanning window [3] are used for the design of FIR filter.

For the application of image interpolation, the frequency spectrum analysis is not a good criterion for performance comparison. In this field, perception is an important criterion for the judgement of image performance. To get good perception, smoothness is the basic requirement. Maximum smoothness has been used as the principle of interpolation and reconstruction in many processes [4-6]. A common goal of interpolation is to derive a high resolution signal with high degree of smoothness with reasonable computation time.

Minimum energy interpolation is an interpolation method established on the aim to maximize the smoothness of the interpolated sequence under the constraint of preservation of origin signals. In many applications, smoothness is directly measured as the curvature of the resultant signal. In this proposed approach, a global energy function of the summation of second order partial derivation of signal is introduced to measure the smoothness of the interpolated sequence. With this, the interpolation problem is treated as an optimization problem in which a global energy function is to be minimized.

To solve the linear equation of the optimization system, a recursive method called as the recursive minimum energy (RME) algorithm is proposed. As compared to the conventional interpolation method, the smoothness is improved by increasing filter length, in the RME algorithm the smoothness is improved by increasing the calculation iteration. Because the derived method is a noncausal system, it is needed to use the delay element to get the requirement. For the first order derivative of energy measure, the required number of delay is only two samples. Based on the little amount of calculation of RME algorithm the architecture of RME algorithm for image interpolation is proposed. In this architecture parallel process and pipeline operation are used and so the features of regularity, modularity and simplicity can be obtained. The rest of this paper is organized as follows. In the next section, we first introduce the concept of minimized energy problem and then the RME algorithm is derived. In Section III the performance analysis of RME algorithm and the comparison with the conventional method are made. In Section IV, the corresponding architecture of RME algorithm for image interpolation is proposed. Finally, a conclusion is made in Section V.

II. Recursive minimum energy algorithm

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Without loss of generality, the interpolation of one-dimensional sequence is first discussed. Since the application of multi dimensional interpolation is usually an extension of one-dimensional case. The one dimensional interpolation problem is stated as following.

Problem 1 Given a sequence of samples $\{f[i] \mid i=0,....,N\}$, find a new sequence of samples $\{v[j] \mid j=0,....,MN\}$ with the maximum smoothness under the constraints $v[Mi]=f[i]$ for $i=0,....,N$.

where M is the interpolation factor. Those known data $f[i]$, $i=0,....,N$, are samples from a function $f(t)$ with the sampling period T. To model the smoothness of the interpolated sequence, a variation function is introduced. The function is defined inversely proportional to the smoothness of the interpolated function. Usually, for a continuous function $f(t)$, the variation can be defined as the integral of a function of its derivatives. In present discussion, square of second order derivative is used.

$$V(f)=\int(\frac{\partial^2 f}{\partial t^2})dt \tag{1}$$

The idea to use only the second order derivatives comes from the concept of natural spline [7]. This definition can be easily extended to discrete form by replacing the differential operator with finite difference operator.

$$V(v[j])=\sum_{j=0}^{MN}(v[j-1]-2v[j]+v[j+1])^2 \tag{2}$$

where $v[j]=f(jT/M)$. With the introduction of the variation function, interpolation can be formulated as a constrained optimization problem.

Problem 2 Find $\{v[j] \mid j=0,....,MN\}$ such that $V(v[j])$ is minimized subject to $v[Mi]=f[i]$, for $i=0,....,N$.

That is, v[j]'s should be solved to minimized $V(v[j])$ under those $N+1$ equality constraints. Note, in Eqn.(2), the two samples $v[-1]$ and $v[MN+1]$ are outside the domain of interest. They can be not defined in the case. Therefore, proper end conditions are required to define these samples. The second order finite difference outside the domain of interest is set to be zero to get the smooth ending. That is,

$$v[j-1]-2v[j]+v[j+1]=0 \tag{3}$$

for $j<=0$ and $j>=MN$. For example, the end samples can be defined as $v[-1]=2v[0]-v[1]$ and $v[MN+1]=2v[MN]-v[MN-1]$.

To solve the optimization problem, the recursive estimation algorithm could be used. We could handle this problem as a least square error problem. By using the concept of steepest descent adaptive algorithm [8], we could get the updating method of variable $v[j]$ as

$$v^{k+1}[j]=v^k[j]-\varepsilon(\partial V(v[j])/\partial v[j]) \tag{4}$$

where $\partial V(v[j])/\partial v[j]=$

$$\begin{cases} v[j-2]-4v[j-1]+6v[j]-4v[j+1]+v[j+2] & \text{for } j\neq iM,\ i=1,\dots N \\ 0 & \text{for } j=iM,\ i=1,\dots N \end{cases}$$

and in the end condition $\partial V(v[j])/\partial v[j]=$

$$\begin{cases} v[1]-2v[2]+v[3] & \text{for } j=1 \\ -2v[1]+5v[2]-4v[3]+v[4] & \text{for } j=2 \\ v[MN-3]-4v[MN-2]+5v[MN-1]-2v[MN] & \text{for } j=MN-1 \\ v[MN-2]-2v[MN-1]+v[MN] & \text{for } j=MN \end{cases}$$

with k being the calculation iteration and ε being the convergence factor.

III. Performance comparison & Analysis

To study the performance of the RME algorithm proposed above, two sets of experiments are made. In the first set of experiments the frequency analysis is made to exam the frequency performance of the window method and the RME algorithm. The interpolation methods used for comparison are the hamming window method, the hanning window method, the rectangular window method, the kaiser window method and the proposed RME method. For window method the frequency response of filter is gotten from the Fourier transformation of the impulse response. Because the RME algorithm is a nonlinear interpolation method, it is hard to get the impulse response. So for RME algorithm a signal with frequency spectrum of amplitude being one and phase being zero in all frequency is used as the input signal. The all pass signal is first decimated and then interpolated with the RME algorithm. The frequency response of the interpolated signal is then the frequency response of the RME algorithm. The simulation results are shown in Fig.1 and Fig.2. In Fig.1 it shows that the frequency responses of RME algorithm with calculation iteration being 1 to 5. Here the convergence factor is set to be 0.125. According to the simulation results we find for the RME algorithm the frequency performance in transition band and stop band is improved by increasing the calculation iteration. Fig.2 shows the frequency response comparison of four different window methods and the RME algorithm. In this simulation the tap length of the window method is 15 and the calculation iteration of RME algorithm is 3. According to the simulation result we find the frequency performance of RME method is better than that of the window method both in the pass band and stop band. But the cost paid for this is the worse frequency spectrum in transition band.

After getting the frequency response comparison, in the second set of experiment the smoothness analysis is made for comparison. There are two sets of image used in this experiment : The image "Lena" and "Baboon". The size of each image is 512×512. These images are first decimated to be size of 256×256 and then interpolated to be size of 512×512. The interpolation is first done row by row and then column by column. The energy function of the summation of second order derivation of image pixels is used for smoothness comparison. The less the energy function is the smoother the interpolated image is. The different tap length for window method and the different calculation iteration for the RME algorithm is used for comparison. The simulation results are shown in Table.1 and Table.2. According to these results we find that the RME algorithm gets the least energy function and so the best smoothness after three iterations.

algorithm	iteration	(1)	(2)
RME algorithm	1	330.915771	2005.260620
	2	264.333923	1931.873779
	3	255.947723	1932.474976
	4	252.600967	1931.997803
	5	250.258713	1929.767578

Table.1 The energy function of the second order derivation of pixels for image (1) Lena (2) Baboon with RME method.

algorithm	length	(1)	(2)
Hamming window	7	1401.665894	3148.213623
	15	300.613770	2093.444824
	25	300.578107	2081.650146
Hanning window	7	1119.169922	2895.490723
	15	264.269012	2036.760742
	25	263.320679	2033.041748
Kaiser window	7	1256.037445	2923.976807
	15	286.377167	2220.202393
	25	284.570221	2212.680908
rectangular window	7	7735.334473	10167.529297
	15	2536.712646	4999.907715
	25	1294.643799	3746.724854

Table.2 The energy function of the second order derivation of pixels for image (1) Lena (2) Baboon with some window methods.

IV. The architecture design

After getting the performance analysis, the architecture of the RME algorithm for image interpolation is proposed. According to Eqn.(4) it could be found the calculation of RME algorithm is very regular and simple. There are six different calculations for the update terms in different position. So we construct six corresponding blocks to complete these calculations.

block 1: $v^{k+1}[j]=v^k[j]-\varepsilon(v[j-2]-4v[j-1]+6v[j]-4v[j+1]+v[j+2])$
block 2: $v^{k+1}[j]=v^k[j]$
block 3: $v^{k+1}[j]=v^k[j]-\varepsilon(v[j]-2v[j+1]+v[j+2])$
block 4: $v^{k+1}[j]=v^k[j]-\varepsilon(-2v[j-1]+5v[j]-4v[j+1]+v[j+2])$
block 5: $v^{k+1}[j]=v^k[j]-\varepsilon(v[j-2]-4v[j-1]+5v[j]-2v[j+1])$
block 6: $v^{k+1}[j]=v^k[j]-\varepsilon(v[j]-2v[j-1]+v[j-2])$

In this architecture block 3 and block 4 are used for the left end points. Block 5 and block 6 are used for the right end points. Block 1 is used to calculate the luminance of general interpolated pixel. Block 2 is used to keep the constrained point to be fixed. The corresponding architectures of block are shown in Fig.3 (a) to Fig.3 (f) named as A1 to A6.

It can be seen that in all blocks the multiplication with number 2, 4, 5 and 6 are replaced by shift operation and addition. So only adder and register are needed. By the way, because there are at most six adders in each block, the architecture is simple and easy to be implemented. The whole system for image interpolation is shown in Fig.4. In this interpolation system, parallel process and pipelined operation are used to get high throughput rate. In the first pipelined step the input image data enters the system sequentially and then is sent to calculators parallel. This operation is known as "series-to-parallel" operation. In the second pipelined step, the interpolated signals are recursively calculated with RME algorithm. After getting the interpolated signals, in the third pipelined step, a parallel-to-series operation is used to send the interpolated signals to memory. Here the memory is used for row-column transposition. These three steps are for row interpolation. After the transposition operation is completed, step 1, 2 and 3 are repeatedly used for the column interpolation and then get the final interpolated image. In this architecture the features of regularity, modularity and simplicity make it easy to be implement by VLSI technology.

After proposed the architecture of the RME algorithm, the comparisons of calculation amount, access time, throughput rate and hardware requirement are made for different methods. In this comparison the tap length of window method is 15. The results are shown in Table.3. According to the results it could be found that the RME algorithm gets the higher throughput rate and less hardware requirement. This makes it suit for the application of high speed image interpolation.

	algorithm	calculation amount
amount of calculation	window method	8 mults + 8 adds
	RME	7 adds × n
access time	window method	1 mults + 3 adds
	RME	4 adds ×n
throughput rate	window method	(1 mult + 3 adds)/N
	RME	(4 adds × n)/N
hardware requirement	window method	8 mults + 7 adds
for one set of calculator	RME	5 adds

Table.3 The complexity comparison for window method and the RME algorithm. (with n being the calculation iteration and N being the number of set of calculator)

V. Conclusion

A recursive minimum energy algorithm for image interpolation is proposed. Based on which the corresponding architecture is also proposed. Experiments of frequency and smoothness comparisons are made for the performance analysis for the proposed RME algorithm. According to the results the proposed algorithm gets the better smoothness and frequency response as compared to the conventional methods. Similar to the window method of increasing tap length to improve output performance, the new algorithm increases the calculation iteration to improve the smoothness and frequency performance of output signal. With the parallel process and pipelined operation the corresponding architecture can achieve the aim of high throughput rate. For this, it is useful for large scale of applications, especially for high speed image interpolation. Moreover, with the features of regularity, modularity and simplicity, the architecture is suit for VLSI implementation.

Reference

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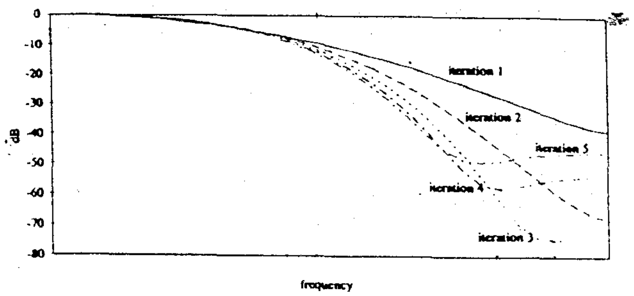

Fig.1 The frequency spectrum of the RME algorithm.

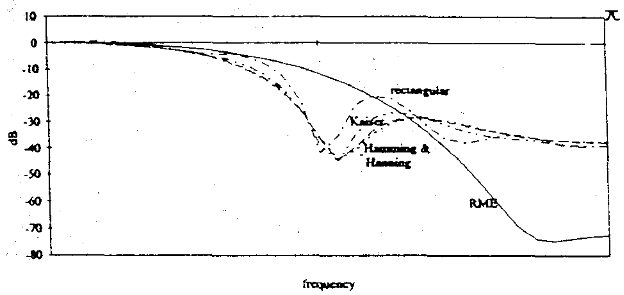

Fig.2 The frequency spectrum comparison of different window methods and the RME algorithm.

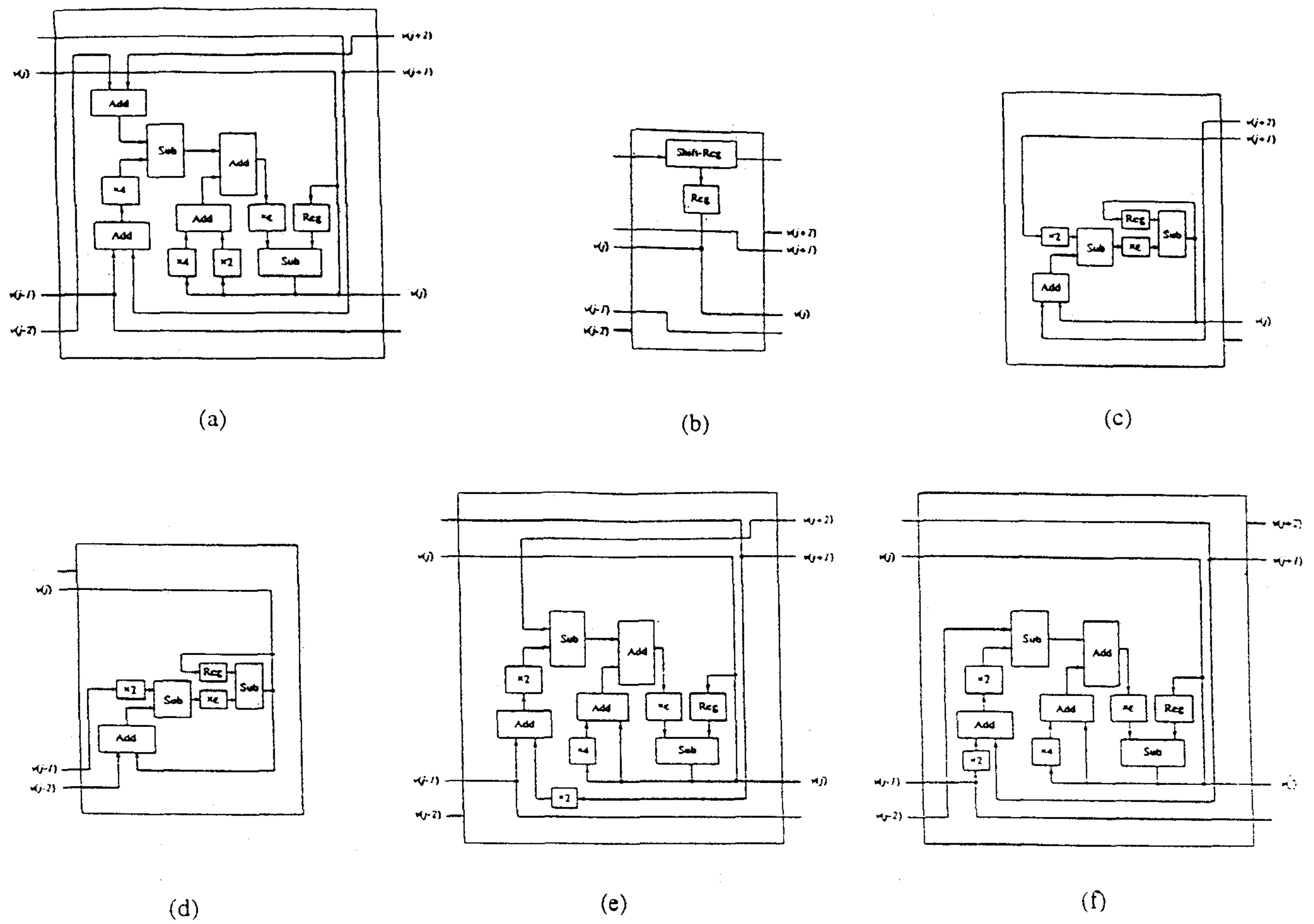

Fig.3 The architecture of (a) A1 (b) A2 (c) A3 (d) A4 (e) A5 (f) A6.

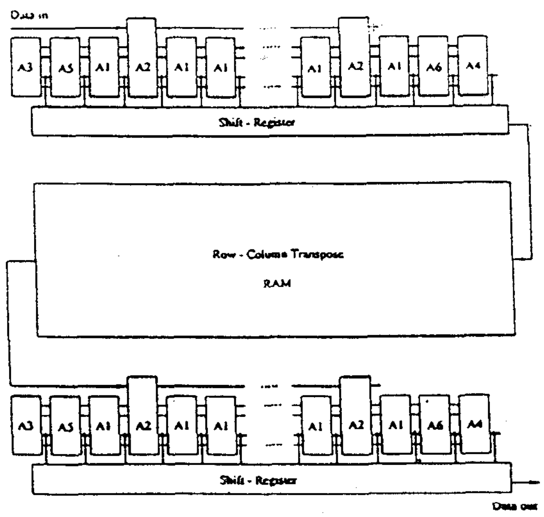

Fig.4 The architecture of RME algorithm for image interpolation.