



Performance Modeling for Mobile Telephone Networks

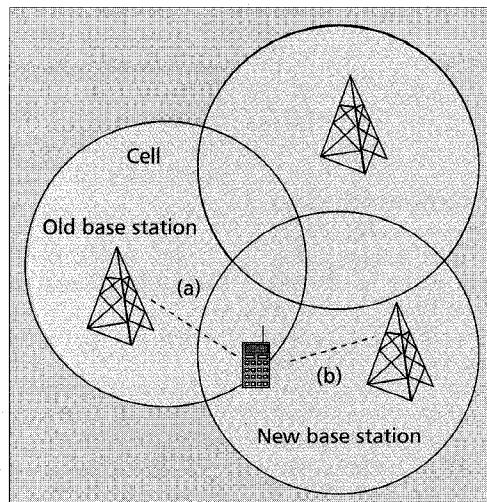
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Abstract

In a mobile telephone network, users may move around the service area during conversations, which can significantly affect the efficiency of radio resource (i.e., radio channels) allocation in the network. The author describes a simple analytic model to study the effect of user mobility on the performance of a mobile telephone network. Throughout the derivation of the model, the intuition behind the equations is provided to explain how user behavior affects network performance. This model can be used to study different handoff schemes with single and mixed types of users with different mobility patterns.

In a mobile telephone network, a service area is populated with a large number of base stations, each providing coverage in its vicinity. The radio coverage of a base station is called a *cell* (Fig. 1). We consider a fixed or quasi-static channel assignment [1] where a group of channels (time slots, frequencies, spreading codes, or a combination of these) are assigned to each base station.

When the network attempts to deliver a call to a handset or the handset attempts to originate a call, the call is connected if a channel is available. Otherwise, the call is blocked (referred to as the *new call blocking*). When a handset moves from one cell to another during a call, in order to maintain call continuation the channel in the old cell is released (see link a in Fig. 1), and a channel is required in the new cell (see link b in Fig. 1). This process is called *hand-off*. If no channel is available in the new base station, then the hand-off call is *force-terminated*. The forced termination of an ongoing call is considered less desirable than blocking a new call. Several models [2-9] have been proposed to study various hand-off schemes for mobile telephone networks. These models provide important insight into understanding the performance of the hand-off schemes under different handset mobility patterns. This article proposes a model to generalize the previous proposed models [2-9] by accommodating handsets with arbitrary mobility patterns



■ Figure 1. Cells, base stations, and handoff.

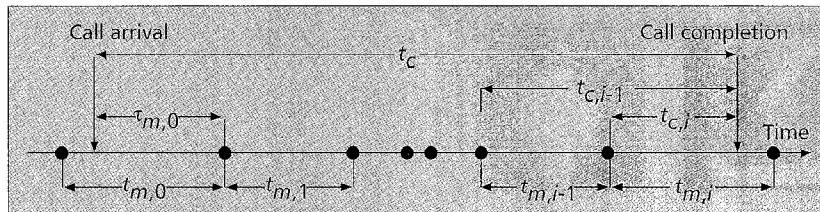
(i.e., with arbitrary residence time distributions). Our model can be used in wireless network planning as follows. In the process of establishing a wireless network, after the base stations are installed, there will be a field trial to test the radio signals of base stations, and the cell residence times can be measured by either the handsets or the base stations.

The cell residence time data can be approximated by a general distribution. Call traffic volumes are estimated for cells during the network planning phase. The call completion probability is also determined, as the performance goal of the network, at network planning. Then our model can be applied to find the number of radio channels to be assigned at the cells. We note that measurement of cell residence times

is not a trivial task, and no measured data have been obtained from any large-scale field trial.

Based on the model described in this article, we have extended our analysis to accommodate handsets with general call holding time distributions [10]. The derivation in [10] involved nonintuitive mathematical operations that are difficult to understand. Similarly, this article will include many mathematical derivations. However, we will try to provide intuitive descriptions for the equations derived in this article.

Before we investigate the mobile network handoff modeling problem, we should discuss the distribution of a handset resi-



■ Figure 2. The timing diagram.

dence time in a cell (the time a handset spends in a cell). Many studies have assumed exponential handset residence time distributions. However, handset residence times depend on the user mobility pattern, size of a cell, radio propagation environment, and so on. In particular, propagation plays an interesting role in handoff and makes precise definition of residence time complicated. For example, there are no hard cell boundaries which clearly separate cells. Shadow fading can make these cell boundaries very irregular. What really counts is what the handset perceives based on its measurements of the signals from surrounding base stations. These measurements are never really direct measurements of the local mean signal but are subject to effects of Rayleigh fading. We consider a general distribution function of the residence time which is flexible enough to accommodate these effects. Candidates for general distributions can be log normal, Gamma, or Weibull [11], which have the desirable property of approximating an extensive class of distributions by setting appropriate parameters. Note that this article does not intend to study "what is a reasonable residence time distribution." Our experience indicates that residence time distributions are highly dependent on the specific networks under study, and it is difficult to make general characterizations of user movement patterns; instead, moving patterns should be studied case by case.

The illustrative examples in this article will consider the Gamma handset residence time distribution. A Gamma density function is

$$f(t) = \frac{\beta^\gamma t^{\gamma-1} e^{-\beta t}}{\Gamma(\gamma)} \quad (1)$$

where $\Gamma(\gamma) = \int_{\tau=0}^{\infty} e^{-\tau} \tau^{\gamma-1} d\tau$, and $\gamma > 0$

where β is called the *scale* parameter (which controls the mean and variance of the distribution), and γ is called the *shape* parameter (which controls the shape of the distribution curve). Depending on the values of the parameters, it can be shaped to represent many distributions as well as measured data. For example, an exponential distribution is a Gamma distribution with $\gamma = 1$, a Chi-squared distribution is a Gamma distribution with $\beta = 0.5$ and $\gamma = \nu/2$ (where ν is the degrees of freedom), and an Erlang distribution is a Gamma distribution with a nonnegative integer γ value. One may also measure the handset residence times in a real mobile telephone network, and the measured data can be approximated by a Gamma distribution as the input to our mobile network handoff model. Let

$$f_m^*(s) = \int_{t=0}^{\infty} f_m(t) e^{-st} dt \quad (2)$$

be the Laplace transform of the handset residence time distribution. The Laplace transform $f_m^*(s)$ will appear in most equations derived in this article. Many Laplace transform pairs are already available in the literature [12]. For example, a Gamma distribution with the shape parameter γ and the mean $1/\eta$ (i.e., the scale parameter $\beta = \gamma\eta$) has the Laplace transform

$$f_m^*(s) = \left(\frac{\gamma\eta}{s + \gamma\eta} \right)^\gamma \quad (3)$$

We first describe the assumptions of our model, and derive

the expected channel occupation times and the hand-off traffic. Then we describe the iterative algorithm used in our model. We show how to use the model to study the hand-off effect for handsets with several different mobility patterns. The notation used in this article is listed in Appendix A.

Assumptions and Some Results

This section describes the assumptions and some important results to be used in our mobile network handoff model.

We assume that the call arrivals to a handset form a Poisson process (i.e., the intercall arrival times are exponentially distributed). The Poisson call arrivals have been observed in most telecommunication networks, and the assumption is justified [13].

In Fig. 2, t_c is the call holding time of a handset, which is assumed to be exponentially distributed with the density function

$$f_c(t_c) = \mu e^{-\mu t_c}$$

where the mean call holding time is $E[t_c] = 1/\mu$. The residence time of a handset at a cell i (the time interval that a handset stays in cell i) is $t_{m,i}$. For all i , $t_{m,i}$ are assumed to be independent, identical random variables with a general distribution with the density function $f_m(t_{m,i})$, and the mean value $E[t_{m,i}] = 1/\eta$.

Suppose that a call arrives when a handset is in cell 0. Let $\tau_{m,0}$ be the period between the arrival of the call and when the handset (the user) moves out of cell 0. Note that $\tau_{m,0} \leq t_{m,0}$ as shown in Fig. 2.

Suppose that a call successfully hands over i times. Let $t_{c,i}$ be the period between the time when the handset moves into cell i and the time when the call is completed. The period $t_{c,i}$ is called the excess life of t_c . Let $\Pr[t_c > t_{m,i}]$ ($\Pr[t_{c,i} > t_{m,i}]$) be the probability that a new (handoff) call at the cell is not completed before the handset moves out of the cell. In Appendix B, Eq. 24, we find that $\Pr[t_{c,i} > t_{m,i}]$ can be expressed by the Laplace transform $f_m^*(s)$ of the residence time distribution (the definition of the Laplace transform is given in Eq. 2) with $s = \mu$:

$$\Pr[t_{c,i} > t_{m,i}] = f_m^*(\mu)$$

This simple result is derived directly from the definition of the Laplace transform. For gamma cell residence times, we have

$$\Pr[t_{c,i} > t_{m,i}] = \left(\frac{\gamma\eta}{\mu + \gamma\eta} \right)^\gamma \quad (4)$$

In Eq. 4, when $\eta \gg \mu$, the probability approaches 1, and when $\mu \gg \eta$, the probability approaches 0. This result is consistent with our intuition.

Similarly, the probability $\Pr[t_c > \tau_{m,0}]$ is derived in Appendix B (see Eq. 23) as

$$\Pr[t_c > \tau_{m,0}] = \frac{\eta}{\mu} [1 - f_m^*(\mu)]$$

If the cell residence times are approximated by a gamma distribution, we have

$$\Pr[t_c > \tau_{m,0}] = \frac{\eta}{\mu} \left[1 - \left(\frac{\gamma\eta}{\mu + \gamma\eta} \right)^\gamma \right]$$

For illustration purposes, consider the cases when $\gamma = 2$ and $1/2$. For $\gamma = 2$, the above equation is rewritten as

$$\Pr[t_c > \tau_{m,0}] = \frac{\eta(\mu + 4\eta)}{(\mu + 2\eta)^2} \quad (5)$$

In Eq. 5, when $\eta \gg \mu$, the probability approaches 1 and when $\mu \gg \eta$, the probability approaches η/μ . For $\gamma = 1/2$,

$$\Pr[t_c > \tau_{m,0}] = \left(\frac{2\eta}{2\mu + \eta} \right) \left[1 + \sqrt{\frac{\eta}{2\mu + \eta}} \right]^{-1} \quad (6)$$

Similar to Eq. 5, when $\eta \gg \mu$, the probability in Eq. 6 approaches 1, and when $\mu \gg \eta$, the probability approaches 0.

When a channel is assigned to a new call, the channel is released if the call completes or the handset moves out of the cell. Let t_{do} be the *channel occupation time* of a new call. Then

$$t_{do} = \min(t_c, \tau_{m,0})$$

In Fig. 2, $t_{do} = \tau_{m,0}$. The expected channel occupation time of a new call is derived in Eq. 28 in Appendix B:

$$E[t_{do}] = \frac{1}{\mu} - \frac{\eta}{\mu^2} [1 - f_m^*(\mu)] \quad (7)$$

$$= \frac{1}{\mu} \Pr[t_c < \tau_{m,0}] \quad (8)$$

Equation 8 implies that the longer the call holding time ($1/\mu$), the longer the new call channel occupation time at a cell. However, the channel occupation time is also determined by user mobility (i.e., the term $\Pr[t_c < \tau_{m,0}]$). For a very slow mover, we have $\lim_{\tau_{m,0} \rightarrow \infty} \Pr[t_c < \tau_{m,0}] = 1$ and $E[t_{do}] = 1/\mu = E[t_c]$. For a fast mover, $\lim_{\tau_{m,0} \rightarrow 0} \Pr[t_c < \tau_{m,0}] = 0$ and $E[t_{do}]$ is much shorter than the expected call holding time.

Let t_{dh} be the channel occupation time of a handoff call. If a call successfully hands over i times, then at cell i ,

$$t_{dh} = \min(t_{c,i}, t_{m,i})$$

In Fig. 2, $t_{dh} = t_{m,i-1}$ for cell $i-1$, and $t_{dh} = t_{c,i}$ for cell i . The expected channel occupation time of a handoff call is derived in Appendix B:

$$E[t_{dh}] = \frac{1}{\mu} [1 - f_m^*(\mu)] \quad (9)$$

$$= \frac{1}{\mu} \Pr[t_{c,i} < t_{m,i}] \quad (10)$$

After tedious derivation in Appendix B, we found that Eq. 10 has a simple format similar to Eq. 8. Note that $E[t_{dh}] \neq E[t_{do}]$ except for the case when $f_m(t)$ is an exponential density function.

If the handset residence times are exponentially distributed, then

$$f_m^*(s) = \frac{\eta}{s + \eta} \quad \text{and} \quad E[t_{do}] = E[t_{dh}] = \frac{1}{\eta + \mu}$$

The above equation indicates that the expected channel occupation time is short for high mobility (a large η) or short holding time (a large μ).

Let λ_o and λ_h be the new call arrival rate and the handoff call arrival rate to a cell, respectively. Let p_o and p_f be the new call blocking probability and the forced termination probability, respectively. For the moment, we assume p_o and p_f are known (both probabilities will be derived in the next section). Then the handoff call arrival rate can be expressed as a function of λ_o , p_o , and p_f as follows. Consider a homogeneous system where the handoff call arrival rate to a cell is the same as the handoff call departure rate, which is denoted as λ_h . We have

$$\lambda_h = \lambda_h(1 - p_f) \Pr[t_{c,i} > t_{m,i}] + \lambda_o(1 - p_o) \Pr[t_c > \tau_{m,0}] \quad (11)$$

Equation 11 states that a handoff call will overflow from a cell i to its neighbors in two cases:

- The call is a handoff call, which is not force-terminated (with probability $1 - p_f$) at cell i , and the call is not completed before

the handset leaves cell i (with probability $\Pr[t_{c,i} > t_{m,i}]$).

- The call is a new call, which is not blocked (with probability $1 - p_o$) at cell i , and the call is not completed before the handset leaves cell i (with probability $\Pr[t_c > \tau_{m,0}]$).
- After arrangement, Eq. 11 is rewritten as

$$\lambda_h = \frac{(1 - p_o) \Pr[t_c > \tau_{m,0}] \lambda_o}{1 - (1 - p_f) \Pr[t_{c,i} > t_{m,i}]} \quad (12)$$

$$= \frac{\eta(1 - p_o)[1 - f_m^*(\mu)] \lambda_o}{\mu[1 - (1 - p_f) f_m^*(\mu)]} \quad (13)$$

Equation 12 provides the following intuitions. A cell experiences a large handoff traffic if

- p_o is small (a new call is unlikely to be blocked)
- $\Pr[t_c > \tau_{m,0}]$ is large (a new call is unlikely to be completed before the handset leaves the cell)
- p_f is small (a handoff call is unlikely to be force-terminated)
- $\Pr[t_{c,i} > t_{m,i}]$ is large (a handoff call is unlikely to be completed before the handset leaves the cell).

From Eq. 13, it is clear that the handoff rate λ_h and the residence time distribution f_m are highly correlated.

Let p_{nc} be the probability that a call is not completed (either a blocked new call or a force-terminated handoff call). Since an incomplete call may successfully hand over several times before it is force-terminated, it is clear that

$$p_{nc} \neq p_o + p_f$$

The probability p_{nc} was derived in [17]:

$$p_{nc} = 1 - (1 - p_o) \left\{ 1 - \frac{\eta p_f [1 - f_m^*(\mu)]}{\mu [1 - (1 - p_f) f_m^*(\mu)]} \right\} \quad (14)$$

The formal derivation of Eq. 14 in [17] is lengthy and difficult to understand. An intuitive derivation for Eq. 14 is given below. In a period Δt , there are $\lambda_o \Delta t$ new call arrivals to a cell. These new calls generate $\lambda_h \Delta t$ handoff calls. Among these new/handoff calls, the number of blocked calls is $p_o \lambda_o \Delta t + p_f \lambda_h \Delta t$. Thus, p_{nc} is

$$p_{nc} = \frac{p_o \lambda_o \Delta t + p_f \lambda_h \Delta t}{\lambda_o \Delta t} = p_o + \left(\frac{\lambda_h}{\lambda_o} \right) p_f \quad (15)$$

Substituting Eq. 13 into Eq. 15, we have Eq. 14 as expected.

The Iterative Algorithm

Hong and Rappaport [5] proposed an iterative technique for handoff modeling. This technique has been adopted in other handoff models (see [2, 14] and the references therein). This section shows how to use the iterative technique to compute p_o , p_f , and p_{nc} using the equations derived in the previous section. We have experimentally shown that the outputs of the algorithm converge to the true values, and a special case with exponential residence times was shown in [14].

The iterative algorithm can be used to model different handoff schemes [5, 14] such as the *nonprioritized* scheme, where the handoff calls and new call attempts are not distinguishable; the *guard channel* scheme, where a number of channels in a base station are reserved for handoff calls; the *queuing prioritizing* schemes (if the new base station does not have any free channel, the handoff call waits in a queue before the handset moves out of the handoff area¹); and the *subrating* scheme (if

¹ The handoff area is an area where a call can be handled by the base station in either the new or the old cell.

the new base station does not have any free channel, an occupied full-rate channel is temporarily divided into two channels at half the original rate: one to serve the existing call and the other to serve the handoff request²). We consider J types of handsets. Handsets of type j ($1 \leq j \leq J$) are distinguished by their residence time distribution $f_{m,j}(t)$ and their mean call holding times $1/\mu_j$. The algorithm is described below.

Input Parameters

The number of channels in a base station is c . For type j handsets ($1 \leq j \leq J$), the following parameters are given: $\lambda_{o,j}$ (the new call arrival rate), μ_j (the call completion rate), and $f_{m,j}(t)$ (the handset residence time density function).

Output Measures

$\lambda_{h,j}$ (the handoff call arrival rate), $p_{o,j}$ (the new call blocking probability), and $p_{nc,j}$ (the call incompleteness probability).

Step 1 — For $1 \leq j \leq J$ select an initial value for $\lambda_{h,j}$.

Step 2 — For $1 \leq j \leq J$ compute the expected channel occupation times $E[t_{do,j}]$ (see Eq. 7) and $E[t_{dh,j}]$ (see Eq. 9) by using $f_{m,j}(t)$ and μ_j .

Step 3a — Consider the nonprioritized handoff scheme where the handoff calls and new calls have the same priority to access channels (modeling of other handoff schemes will be discussed later), and $p_o = p_f$. The system under study is a c -server blocking system with general service times (or an $M/G/c/c$ queue). From the queuing theory [15], the net traffic to a cell is

$$\rho_n = \sum_{1 \leq j \leq J} \{ \lambda_{o,j} E[t_{do,j}] + \lambda_{h,j} E[t_{dh,j}] \}$$

Step 3b — Since the blocking probability for an $M/G/c/c$ queue is the same as an $M/M/c/c$ queue with the same arrival process and the same channel number [16], we have

$$p_f = p_o = B(\rho_n, c) = \frac{(\rho_n^c / c!)}{\sum_{i=0}^c (\rho_n^i / i!)}$$

where $B(\rho_n, c)$ is the Erlang loss equation [16].

Step 4 — For $1 \leq j \leq J$, $\lambda_{h,j,old} \leftarrow \lambda_{h,j}$. This step saves the $\lambda_{h,j}$ values computed in the previous iteration.

Step 5 — For $1 \leq j \leq J$ compute new $\lambda_{h,j}$ values by using Eq. 13:

$$\lambda_{h,j} = \frac{\eta_j (1 - p_o) [1 - f_{m,j}^*(\mu_j)] \lambda_{o,j}}{\mu_j [1 - (1 - p_o) f_{m,j}^*(\mu_j)]}$$

Step 6 — If there exists j such that $|\lambda_{h,j} - \lambda_{h,j,old}| > \delta \lambda_{h,j}$, then go to Step 2. Otherwise, go to Step 7. This step compares the $\lambda_{h,j}$ values in the previous iteration with the values computed in the current iteration. If the difference of the both values is within δ , the algorithm is considered as converge.

Step 7 — The values for $\lambda_{h,j}$ and p_o converge. Compute $p_{nc,j}$ using Eq. 14:

$$p_{nc,j} = p_o \left[1 + \left(\frac{\lambda_{h,j}}{\lambda_{o,j}} \right) \right]$$

² If a channel is released, two subrated channels are switched back to full-rate channels.

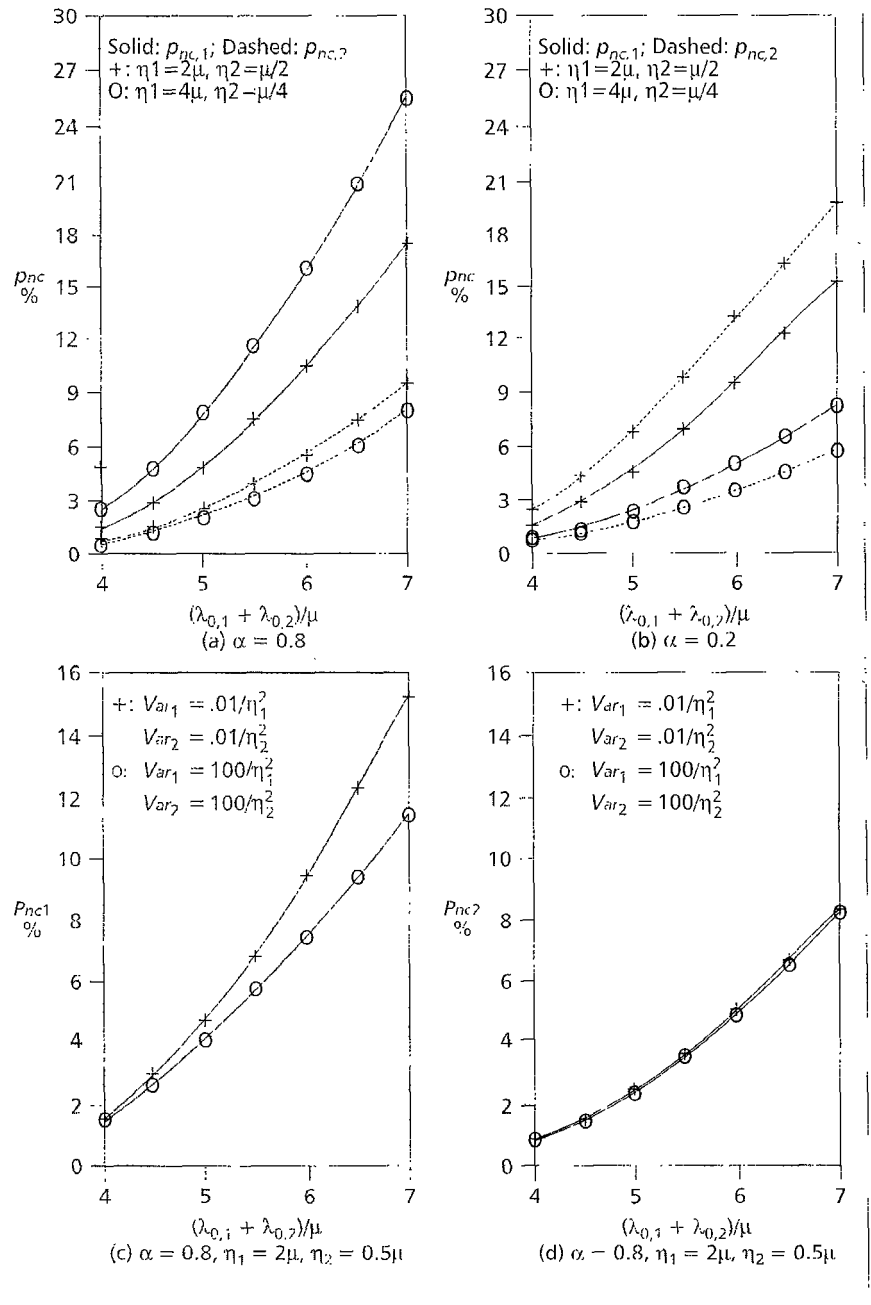


Figure 3. The effect of mixed-type handsets ($c = 10, Var_1 = Var_2 = 1/\eta^2, \mu_1 = \mu_2 = \mu$).

To model priority schemes for handoff, Steps 3a and 3b should be modified. For the exponential residence time distribution, p_o and p_f can be derived by analytic models for the guard channel scheme, the first-in first-out (FIFO) queuing priority scheme with exponential handoff times [17], and the subrating scheme [20]. For a nonexponential residence time distribution, the channel occupation times are not exponential, and it is better to compute p_o and p_f by simulation approaches. A detailed simulation procedure was described in [17, 18, 20].

To illustrate the usage of our analytic model, numerical results are given in Fig. 3 to show the effect of mixed-type handsets on the blocking probabilities. The experiments consider two types of handsets. Type 1 handsets have a Gamma residence time distribution with mobility η_1 and variance Var_1 , new call arrival rate $\lambda_{o,1}$, and call completion rate $\mu_1 = \mu$. Type 2 handsets have a Gamma residence time distribution with mobility η_2 and variance Var_2 , new call arrival rate $\lambda_{o,2}$, and call completion rate $\mu_2 = \mu$. The ratio

$$\alpha = \frac{\lambda_{o,1}}{\lambda_{o,1} + \lambda_{o,2}}$$

is the portion of type 1 handsets in the system. Figures 3a and b (where both types 1 and 2 residence time distributions are exponential) indicate that although the new call blocking probabilities for both types of handsets are the same, the call incompleteness probability for a slow handset is lower than that for a fast handset. Figures 3c and d illustrate the effects of the variance of the residence time distribution. The results indicate that probability p_{nc} is a decreasing function of the variance of the residence time. The figures also indicate that the effect of the variance is more significant on the fast handsets than on the slow ones. From a system point of view, the results indicate that it is necessary to consider different types of users (car drivers and pedestrians) and to predict the performance of each traffic type for network planning.

Conclusion

The effect of handoff in a mobile telephone network has been intensively studied. Some handoff models [6–8, 17] have assumed that the handset residence time distribution is exponential. Other models [2, 4, 5] with specific residence time distributions (other than exponential) have assumed that both the new calls and the handoff calls have the same channel occupation time distribution. We derived the expected new call channel occupation time and the expected handoff call channel occupation time and proved that they are not equal in general. Based on the derivations of the expected channel occupation times, as well as the derivation of the handoff traffic for an arbitrary handset residence time distribution, we proposed a general yet simple mobile network handoff model which applies to different handset residence time distributions. Numerical experiments are given to illustrate the impact of the variance of handset residence times on the blocking probabilities of the mobile telephone network. Our analysis suggests that performance of a personal communications services network is sensitive to the cell residence time distribution, and it would be a good idea to develop methods and means to measure residence times in real PCS networks. Although the author believes the new methodologies here can be used to improve network provisioning, an analysis of the specific contribution based on commercial field trials has not been done. An assessment of the specific contribution will be possible only after sufficient experience in the field is gained.

Acknowledgments

Thanks to the three anonymous reviewers, the quality of this article has been significantly improved. The derivation of $E[t_{do}]$, $E[t_{dh}]$ was provided by reviewer C.

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Biography

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Appendix A: Notation

This appendix lists the notation used in this article.

- $1/\eta = E[t_{m,i}]$ — mean handset residence time.
- $f_c(t_c)$ — exponential density function of a call holding time t_c .
- $f_{c,i}(t_{c,i})$ — density function of $t_{c,i}$.
- $f_m(t_{m,i})$ — density function of $t_{m,i}$.
- $f_m^*(s)$ — Laplace transform of $f_m(t_{m,i})$.
- $F_m(t_{m,i})$ — distribution of $t_{m,i}$.
- $1/\mu = E[t_c]$ — mean call holding time.
- λ_h — handoff call arrival rate to a cell.
- λ_o — new call arrival rate to a cell.
- p_f — forced termination probability.
- p_{nc} — probability that a call is not completed (either blocked or force-terminated).

- p_o — new call blocking probability.
- $R_m(\tau_{m,0})$ — distribution of $\tau_{m,0}$.
- $r_m(\tau_{m,0})$ — density function of $\tau_{m,0}$.
- $r_m^*(s)$ — Laplace transform of $r_m(\tau_{m,0})$.
- $\tau_{m,0}$ — Suppose that a call arrives when a handset is in cell 0. $\tau_{m,0}$ is the time between the arrival of the call and when the handset moves out of coverage area 0.
- t_c — call holding time of a handset.
- $t_{c,i}$ — Suppose that a call successfully hands over i cells. $t_{c,i}$ is the period during which the handset moves into cell i . $t_{c,i}$ is called the *excess life* of t_c .
- t_{dh} — channel occupation time of a handoff call.
- t_{do} — channel occupation time of a new call. When a channel is assigned to a new call, the channel is released if the call completes or the handset moves out of the cell.
- $t_{m,i}$ — residence time of a handset at cell i .

Appendix B: Derivations of the Expected Channel Occupation Times

This appendix derives the expected channel occupation times. Consider Fig. 2. Assume that the random variable $\tau_{m,0}$ has a distribution function $R_m(\tau_{m,0})$, density function $r_m(\tau_{m,0})$, and Laplace transform $r_m^*(s)$. The function $r_m(t)$ can be derived using $f_m(t)$. Suppose that $f_m(t)$ is *nonlattice*.³ Since the call arrivals to a handset form a Poisson process, a call arrival is a random observer of the time interval $t_{m,0}$. From Eq. 16 we have

$$r_m(t) = \eta \int_{\tau=t}^{\infty} f_m(\tau) d\tau = \eta [1 - F_m(t)] \quad (16)$$

and from Eq. 2, the Laplace transform of $r_m(t)$ is

$$\begin{aligned} r_m^*(s) &= \int_{t=0}^{\infty} \eta [1 - F_m(t)] e^{-st} dt \\ &= \eta \left[\int_{t=0}^{\infty} e^{-st} dt - \int_{t=0}^{\infty} F(t) e^{-st} dt \right] \end{aligned} \quad (17)$$

$$= \frac{\eta}{s} [1 - f_m^*(s)] \quad (18)$$

where Eq. 18 is derived from Eq. 17 and the following identity [12]

$$\int_{t=0}^{\infty} \int_{\tau=0}^t f_m(\tau) e^{-st} d\tau dt = \frac{f_m^*(s)}{s} \quad (19)$$

The probabilities $\Pr[t_c > \tau_{m,0}]$ and $\Pr[t_{c,i} > t_{m,i}]$ are derived as follows.

$$\begin{aligned} \Pr[t_c > \tau_{m,0}] &= \int_{t_c=0}^{\infty} \int_{\tau_{m,0}=0}^{t_c} r_m(\tau_{m,0}) \mu e^{-\mu t_c} d\tau_{m,0} dt_c \\ &= \mu \int_{t_c=0}^{\infty} R_m(t_c) e^{-\mu t_c} dt_c \end{aligned} \quad (20)$$

³ A nonnegative random variable is said to be lattice if it only takes on integral multiples of some nonnegative number.

$$= \mu \left[\frac{r_m^*(s)}{s} \right]_{s=\mu} \quad (21)$$

$$= r_m^*(\mu) \quad (22)$$

$$= \frac{\eta}{\mu} [1 - f_m^*(\mu)] \quad (23)$$

Note that Eq. 21 is derived from Eqs. 20 and 19, and Eq. 23 is derived from Eqs. 18 and 22. Note that Eqs. 18 and 23 were derived in [20]. We include the derivations for the reader's benefit.

From the memoryless property of the exponential distribution, the density function of $t_{c,i}$ is $f_{c,i}(t) = \mu e^{-\mu t}$, and

$$\begin{aligned} \Pr[t_{c,i} > t_{m,i}] &= \int_{t_{c,i}=0}^{\infty} \int_{t_{m,i}=0}^{t_{c,i}} f_m(t_{m,i}) \mu e^{-\mu t_{c,i}} dt_{m,i} dt_{c,i} \\ &= \mu \left[\frac{f_m^*(s)}{s} \right]_{s=\mu} \\ &= f_m^*(\mu) \end{aligned} \quad (24)$$

The expected values $E[t_{do}]$ and $E[t_{dh}]$ are derived as follows. Let CDFs of t_{do} , t_c , and $\tau_{m,0}$ be $F_{t_{do}}$, F_{t_c} , and R_m , respectively. Since

$$t_{do} = \min(t_c, \tau_{m,0})$$

we have

$$F_{t_{do}}(\tau) = F_{t_c}(\tau) + R_m(\tau) [1 - F_{t_c}(\tau)] \quad (25)$$

Since t_{do} is a nonnegative random variable, we have

$$E[t_{do}] = \int_0^{\infty} [1 - F_{t_{do}}(\tau)] d\tau \quad (26)$$

From Eq. 25, Eq. 26 is re-written as

$$E[t_{do}] = \int_0^{\infty} [1 - F_{t_c}(\tau)] d\tau - \int_0^{\infty} R_m(\tau) [1 - F_{t_c}(\tau)] d\tau \quad (27)$$

Since t_c is exponentially distributed with parameter μ , Eq. 27 is rewritten as

$$\begin{aligned} E[t_{do}] &= E[t_c] - \int_0^{\infty} R_m(\tau) e^{-\mu \tau} d\tau \\ &= \frac{1}{\mu} - \frac{r_m^*(\mu)}{\mu} \\ &= \frac{1}{\mu} - \frac{\eta}{\mu^2} [1 - f_m^*(\mu)] \end{aligned} \quad (28)$$

From the memoryless property of the exponential distribution, $f_{c,i}(t_{c,i}) = \mu e^{-\mu t_{c,i}}$, and for $i > 0$, similar to the derivation for $E[t_{do}]$ we have

$$E[t_{dh}] = \frac{1}{\mu} [1 - f_m^*(\mu)]$$