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Decision Support

Robust weighted vertex *p*-center model considering uncertain data: An application to emergency management

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ABSTRACT

This paper presents a generalized weighted vertex *p*-center (WVPC) model that represents uncertain nodal weights and edge lengths using prescribed intervals or ranges. The objective of the robust WVPC (RWVPC) model is to locate *p* facilities on a given set of candidate sites so as to minimize worst-case deviation in maximum weighted distance from the optimal solution. The RWVPC model is well-suited for locating urgent relief distribution centers (URDCs) in an emergency logistics system responding to quick-onset natural disasters in which precise estimates of relief demands from affected areas and travel times between URDCs and affected areas are not available. To reduce the computational complexity of solving the model, this work proposes a theorem that facilitates identification of the worst-case scenario for a given set of facility locations. Since the problem is *NP*-hard, a heuristic framework is developed to efficiently obtain robust solutions. Then, a specific implementation of the framework, based on simulated annealing, is developed to conduct numerical experiments. Experimental results show that the proposed heuristic is effective and efficient in obtaining robust solutions. We also examine the impact of the degree of data uncertainty on the selected performance measures and the tradeoff between solution quality and robustness. Additionally, this work applies the proposed RWVPC model to a real-world instance based on a massive earthquake that hit central Taiwan on September 21, 1999.

1. Introduction

The *p*-center model, which aims to locate *p* facilities to minimize maximum distance (or travel time) between demand nodes and their closest facilities (e.g., Kariv and Hakimi, 1979; Albareda-Sambola et al., 2010), has been considered particularly suitable for emergency applications (e.g., Jia et al., 2007a,b; Huang et al., 2010), among various facility location models that have been presented in the literature (e.g., Mirchandani and Francis, 1990; Daskin, 1995; Altiparmak et al., 2006). This work adapts the *p*-center model to locate urgent relief distribution centers (URDCs) in an emergency logistics network that aims to promptly deliver relief supplies from URDCs to all relief or medical service stations in affected areas in the aftermath of quick-onset disasters (e.g., Altay and Green, 2006; Yi and Ozdamar, 2007; Campbell and Jones, 2011).

URDCs play an important role in an emergency logistics network, because they serve as hubs that seamlessly integrate and coordinate inbound and outbound emergence logistics responding to relief demands from affected areas. These hubs also have an inventory management function (i.e., risk pooling)—aggregating relief demands or their forecasts across several affected areas to reduce the adverse impact of relief demand variability and uncertainty on the system. In emergency response to quick-onset disasters, government rescue agencies typically designate existing public buildings (e.g., schools and stadiums) with little or no damage that can be promptly converted to shelters for survivors and/or warehouses for relief supplies as candidate sites of URDCs, instead of establishing new emergency facilities from scratch. Thus, the problem of locating URDCs can be considered as the vertex *p*-center problem which restricts the set of candidate sites to locations of existing public buildings (i.e., facility nodes). Furthermore, when relief demands faced by relief or medical stations are taken into account, the problem becomes a weighted vertex *p*-center (WVPC) problem (e.g., Current et al., 2002) with nodal weights reflecting relief demands from relief stations (i.e., demand nodes) in affected areas and the objective being to minimize maximum demand-weighted travel time between relief stations and their closest URDCs.

The proposed WVPC model for locating URDCs explicitly accounts for uncertain relief demands from relief stations and travel (or delivery) times between URDCs and relief stations, mainly due to poor measurements based on limited information available during a disaster's aftermath or approximations in the modeling process using aggregated demands and choosing a distance norm. Two major categories of approaches have been adopted in the literature to deal with uncertain coefficients in facility location models (Snyder, 2006), namely, stochastic programming (SP) and

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robust optimization (RO). The former has been used typically to deal with decision-making for facility locations in risk situations, in which the values of uncertain coefficients are governed by discrete or continuous probability distributions that are known to a decision-maker. The SP approach has been widely applied to emergency logistics for short-notice disasters (e.g., hurricanes, flooding, and wild fires) by assuming that possible impacts of these disasters can be estimated based on historical and meteorological data. The common goal of these stochastic location models is to optimize the expected value of a given objective function. A classical example of applying SP to disaster relief is the scenario-based, two-stage stochastic model proposed by Mete and Zabinsky (2010), for medical supply pre-positioning and distribution in emergency management. Other examples can be found in, for instance, Chang et al. (2007) and Balcik and Beamon (2008).

On the other hand, the RO approach attempts to optimize the worst-case system performance in uncertain situations that lack any information about the probability distributions of uncertain coefficients (e.g., Kouvelis and Yu, 1997); hence, the RO approach generally describes uncertain data using pre-specified intervals or ranges. Typical robustness measures include mini-max objective value and mini-max regret in an objective value. The RO approach may be more appropriate than the SP approach in emergency response to quick-onset or no-notice disasters (e.g., earthquakes, tsunamis, and landslides). For quick-onset disasters, because of the difficulty in predicting disaster occurrence and impacts as well as a lack of historical data, probability distributions and scenario data are generally unavailable. For example, an extremely large earthquake, 9.1 on the Richter scale, which hit the northeastern coast of Japan on March 11, 2011, was never considered in the nation's preparedness planning for earthquakes, even though Japan is widely regarded as one of the most advanced countries in earthquake preparedness. Thus, in responding to such a disaster, decision-makers may prefer an alternative method for describing uncertain data (i.e., using intervals to represent uncertain data).

In the proposed RWVPC model, uncertain relief demands at relief stations in affected areas and travel times between URDCs and relief stations are represented using prescribed intervals (or ranges), rather than probability distributions. The objective of locating p URDCs is to minimize worst-case deviation in maximum demand-weighted travel time between URDCs and relief stations from the optimal solution. This work proposes a theorem that facilitates identification of the worst-case scenario for a given set of URDC locations, thereby reducing complexity of solving the problem. Since the problem is *NP*-hard (Averbakh, 2003), a local search-based algorithmic framework incorporating the theorem for identifying the worst-case scenarios is developed to find robust solutions within a reasonable amount of computational resources. Then, a specific framework implementation based on simulated annealing (SA) is developed to conduct numerical experiments, including a case study based on the Jiji Earthquake, which hit central Taiwan on September 21, 1999.

The p-center problems with interval-represented uncertain data tend to be very difficult because of the mini-max structure. Therefore, analytical results and exact algorithms for the p-center problems with interval data have only been attained in special cases, such as locating a single facility on general networks or multiple facilities on tree networks (e.g., Averbakh and Berman, 2000; Burkard and Dollani, 2002). To the best of our knowledge, only Averbakh and Berman (1997) reported analytical results for an absolute weighted p-center problem with interval-represented nodal weights. No study has addressed absolute or vertex multi-center problems with both interval-represented edge lengths and nodal weights.

This study contributes significantly to the literature by (i) modeling the URDC location problem as the WVPC problem with interval-represented edge lengths and nodal weights on general networks; (ii) providing an effective and efficient algorithmic framework for solving the problem; and (iii) shedding light on the applicability and potential benefits of the proposed model to real-world instances.

The remainder of this paper is structured as follows. Section 2 describes the RWVPC problem, the representation of data uncertainty, and the property of worst-case scenarios. Section 3 presents the generic heuristic framework and a specific implementation using SA. This is followed by the numerical experiments in Section 4. Section 5 provides a case study demonstrating the applicability of the proposed model to real-world instances. Concluding remarks are given in Section 6.

2. Weighted vertex *p*-center problem with data uncertainty

2.1. The deterministic problem

Consider a connected, undirected network $G = (N, A)$, where N is the vertex set and A the arc (or edge) set. Let U be the set of candidate sites (i.e., facility nodes) for URDC locations and V be the set of relief stations (i.e., demand nodes) in affected areas; $U \cup V = N$, and $U \neq V$. Each possible pair of relief station $i \in V$ and URDC $j \in U$ is connected by an arc $(i, j) \in A$ that is associated with a positive (real or integer) number, t_{ij}, representing travel (or delivery) time between relief station i and URDC j. Although t_{ij} denotes the travel time, it can also be used for other measures of utility/disutility, such as distance or travel cost. Each relief station $i \in V$ faces relief demand ξ_i and is serviced by a single URDC. For a given set of predetermined candidate sites, U, the WVPC problem is to locate p ($p < |U|$) URDCs and assign relief stations to these centers, thereby minimizing maximum demand-weighted travel time between relief stations and URDCs. A mixed integer linear programming (MILP) formulation of the problem is as follows (e.g., Current et al., 2002).

$$\text{(WVPC)} \quad \text{Minimize } z, \tag{1}$$

$$\text{Subject to } z \geqslant \Sigma_{j \in U} \xi_i t_{ij} y_{ij}, \quad \forall i \in V, \tag{2}$$

$$\Sigma_{j \in U} y_{ij} = 1, \quad \forall i \in V, \tag{3}$$

$$y_{ij} - x_j \leqslant 0, \quad \forall i \in V, \quad j \in U, \tag{4}$$

$$\Sigma_{j \in U} x_j = p, \tag{5}$$

$$x_j \in \{0, 1\}, \quad \forall j \in U, \tag{6}$$

$$y_{ij} \in \{0, 1\}, \quad \forall i \in V, \quad j \in U. \tag{7}$$

The decision variables are binary variables x_j, $\forall j \in U$ and y_{ij}, $\forall i \in V$, $j \in U$. $x_j = 1$ if candidate site j is selected; otherwise, $x_j = 0$. Additionally, $y_{ij} = 1$ if relief station i is serviced by URDC j; otherwise, $y_{ij} = 0$. The objective function (1) minimizes maximum demand-weighted travel time between relief stations and URDCs. Constraint (2) defines the lower bound of maximum demand-weighted travel time, which is being minimized. Constraint (3) requires that each relief station be assigned to exactly one URDC. Constraint (4) restricts relief station assignments only to selected URDCs. Constraint (5) stipulates that p URDCs are to be located. Constraints (6) and (7) indicate that location and allocation decision variables are binary. The WVPC problem is also known as the minimum k-supplier problem (Ausiello et al., 1999).

2.2. Representation of data uncertainty and the robust WVPC problem

Uncertain relief demands at relief stations and travel times between relief stations and URDCs are described using intervals or ranges. Specifically, An interval $[\xi l_i, \xi u_i]$, $0 \leqslant \xi l_i < \xi u_i$, represents uncertain relief demand at station i, and an interval $[tl_{ij}, tu_{ij}]$, $0 \leqslant tl_{ij} < tu_{ij}$, captures the uncertainty of travel time between station i and URDC j. Let W be the Cartesian product of intervals $[\xi l_i, \xi u_i]$,

$\forall i \in V$, and $[tl_{ij}, tu_{ij}]$, $\forall i \in V, j \in U$. A scenario $w \in W$ represents a realization of relief demands at relief stations (i.e., $\xi_i(w) \in [\xi l_i, \xi u_i]$, $\forall i \in V$, where $\xi_i(w)$ denotes relief demand at station i in scenario w) and travel times (or distances or costs) between relief stations and URDCs (i.e., $t_{ij}(w) \in [tl_{ij}, tu_{ij}]$, $\forall i \in V, j \in U$).

Let $\boldsymbol{\tau} = (\boldsymbol{x}, \boldsymbol{y})$, where $\boldsymbol{x} = \{x_j, j \in U\}$ and $\boldsymbol{y} = \{y_{ij}, i \in V, j \in U\}$, be a feasible solution to the WVPC problem (i.e., satisfying Constraints (3)–(7), and let Ω be the set of feasible solutions. In this study, $\boldsymbol{\tau} \in \Omega$ is called a location plan of URDCs. For a plan $\boldsymbol{\tau}$ and a relief station $i \in V$, this work defines travel time between i and $\boldsymbol{\tau}$ in scenario w as

$$d(w, i, \boldsymbol{\tau}) = \text{Min}\{t_{ij}(w) | x_j = 1, \forall j \in U\}. \tag{8}$$

For a plan $\boldsymbol{\tau}$ and scenario w, the maximum demand-weighted travel time between plan $\boldsymbol{\tau}$ and relief stations in scenario w, $Z(w, \boldsymbol{\tau})$, is determined as follows:

$$Z(w, \boldsymbol{\tau}) = \underset{i \in V}{\text{Max}}\ \xi_i(w) d(w, i, \boldsymbol{\tau}). \tag{9}$$

Thus, for a given scenario w, the deterministic WVPC problem (1)–(7) can be written as

$$\text{WVPC}(w) = \underset{\boldsymbol{\tau} \in \Omega}{\text{Min}}\ Z(w, \boldsymbol{\tau}). \tag{10}$$

This work defines the robust deviation of plan $\boldsymbol{\tau}$ in scenario w, $DEV(w, \boldsymbol{\tau})$, as the difference between maximum demand-weighted travel time and that of the optimal plan, $\boldsymbol{\tau}^*(w)$, in scenario w.

$$DEV(w, \boldsymbol{\tau}) = Z(w, \boldsymbol{\tau}) - Z(w, \boldsymbol{\tau}^*(w)). \tag{11}$$

Robustness cost (i.e., regret in the worst-case scenario) of a plan $\boldsymbol{\tau}$ can be obtained by solving the following sub-problem:

$$RC(\boldsymbol{\tau}) = \underset{w \in W}{\text{Max}}\ DEV(w, \boldsymbol{\tau}). \tag{12}$$

The RWVPC problem can be stated formally as

$$(\text{RWVPC})\ \ \underset{\boldsymbol{\tau} \in \Omega}{\text{Minimize}}\ RC(\boldsymbol{\tau}). \tag{13}$$

This is equivalent to finding $\boldsymbol{\tau}_{robust} = \text{argMin}_{\boldsymbol{\tau} \in \Omega}\ RC(\boldsymbol{\tau})$; that is, the RWVPC problem is to find a robust solution $\boldsymbol{\tau}_{robust}$ that minimizes the largest (worst-case) deviation of maximum demand-weighted travel time from the optimal solution.

2.3. Evaluation of robustness cost

Because the (continuous) interval representation of uncertain demands and travel times may lead to an infinite number of possible scenarios, evaluation of robustness cost $RC(\boldsymbol{\tau})$, which involves identifying the worst-case scenario of plan $\boldsymbol{\tau}$, is a major challenge of solving the RWVPC problem. To efficiently evaluate $RC(\boldsymbol{\tau})$, this work reformulates the sub-problem of $RC(\boldsymbol{\tau})$, defined in Eq. (12), according to Theorem 1 below, indicating that although uncertain relief demands and travel times are described using continuous intervals, the solution to the sub-problem, $RC(\boldsymbol{\tau})$, can be found in a finite subset of scenarios, each of which corresponds to an extreme case. This reformulation increases the tractability of the problem from a combinatorial perspective.

Let $w_k(\boldsymbol{\tau})$ be the scenario *induced* by plan $\boldsymbol{\tau}$ and associated with relief station k, such that (i) demand of station k equals its upper bound (i.e., $\xi_k = \xi u_k$); (ii) demand of any other station i equals its lower bound (i.e., $\xi_i = \xi l_i$, $\forall i \in V, i \neq k$); (iii) travel time between station k and its associated URDC j^* (i.e., k is serviced by j^*) equals its upper bound (i.e., $t_{kj^*} = tu_{kj^*}$, for $y_{kj^*} = 1$); and (iv) travel time between any other pair of stations and URDCs equals its lower bound (i.e., $t_{ij} = tl_{ij}$, $\forall (i, j) \in A$, $i \neq k$, $j \neq j^*$). Suppose $\varpi = \{\xi_i(\varpi), \forall i \in V;\ t_{ij}(\varpi), \forall (i, j) \in A\}$ is the worst-case scenario of plan $\boldsymbol{\tau}$; that is, ϖ is an optimal solution to sub-problem $RC(\boldsymbol{\tau})$. Let relief station $k = \text{argMax}_{i \in V}\ \xi_i(\varpi) \times d(\varpi, i, \boldsymbol{\tau})$. Then, by definition, $Z(\varpi, \boldsymbol{\tau}) = \xi_k(\varpi) d(\varpi, k, \boldsymbol{\tau})$. Additionally, denote j^* as the URDC servicing relief station k in plan $\boldsymbol{\tau}$ and scenario ϖ (i.e., $y_{kj^*} = 1; d(\varpi, k, \boldsymbol{\tau}) = t_{kj^*}(\varpi)$). For simplicity, w_k and $w_k(\boldsymbol{\tau})$ are used interchangeably as follows.

Lemma 1. *With the above definitions of scenarios w_k and ϖ, for a plan $\boldsymbol{\tau}$, $RC(\boldsymbol{\tau}) = Z(\varpi, \boldsymbol{\tau}) - Z(\varpi, \boldsymbol{\tau}^*(\varpi)) = Z(w_k, \boldsymbol{\tau}) - Z(w_k, \boldsymbol{\tau}^*(w_k))$, where $\boldsymbol{\tau}^*(\varpi)$ and $\boldsymbol{\tau}^*(w_k)$ are the optimal plans in scenarios ϖ and w_k, respectively.*

Proof of Lemma 1. Let $\boldsymbol{\tau}^*(\varpi)$ be an optimal solution to the deterministic problem WVPC (ϖ) defined in Eq. (16), and (ϖ, $\boldsymbol{\tau}^*(\varpi)$) is an optimal solution to sub-problem $RC(\boldsymbol{\tau})$.

Main claim: (w_k, $\boldsymbol{\tau}^*(\varpi)$) is also an optimal solution to sub-problem $RC(\boldsymbol{\tau})$; this is equivalent to claiming that w_k is also a worst-case scenario of plan $\boldsymbol{\tau}$, and $\boldsymbol{\tau}^*(\varpi)$ is also an optimal solution to WVPC (w_k). Therefore,

$$\begin{aligned} Z(w_k, \boldsymbol{\tau}) - Z(w_k, \boldsymbol{\tau}^*(w_k)) &= Z(w_k, \boldsymbol{\tau}) - Z(w_k, \boldsymbol{\tau}^*(\varpi)) \\ &= Z(\varpi, \boldsymbol{\tau}) - Z(\varpi, \boldsymbol{\tau}^*(\varpi)) = RC(\boldsymbol{\tau}), \end{aligned}$$

which implies the statement of Lemma 1. The main claim is proved by transforming ϖ into w_k via the following two steps.

Step 1: Replace $\xi_k(\varpi)$ and $t_{kj^*}(\varpi)$ with upper bounds ξu_k and tu_{kj^*}, respectively.
Prior to this transformation of ϖ, since (ϖ, $\boldsymbol{\tau}^*(\varpi)$) is an optimal solution to sub-problem $RC(\boldsymbol{\tau})$, $Z(\varpi, \boldsymbol{\tau})$ - $Z(\varpi, \boldsymbol{\tau}^*(\varpi)) \geqslant 0$. Therefore, $d(\varpi, k, \boldsymbol{\tau}) \geqslant d(\varpi, k, \boldsymbol{\tau}^*(\varpi))$. In the first transformation step, value $Z(\varpi, \boldsymbol{\tau}^*(\varpi))$ cannot increase by more than $(\xi u_k - \xi_k(\varpi)) \times d(\varpi, k, \boldsymbol{\tau}^*(\varpi)) \leqslant (\xi u_k - \xi_k(\varpi)) \times d(\varpi, k, \boldsymbol{\tau})$. Moreover, in Step 1, the value of $Z(\varpi, \boldsymbol{\tau}) - Z(\varpi, \boldsymbol{\tau}^*(\varpi))$ cannot decrease because $\xi u_k \geqslant \xi_k(\varpi)$ and $tu_{kj^*} \geqslant t_{kj^*}(\varpi)$, and cannot increase because (ϖ, $\boldsymbol{\tau}^*(\varpi)$) is an optimal solution to sub-problem $RC(\boldsymbol{\tau})$. Therefore, this value does not change in Step 1. We can conclude that the modification in Step 1 does not change the optimality of (ϖ, $\boldsymbol{\tau}^*(\varpi)$) to sub-problem $RC(\boldsymbol{\tau})$.

Step 2: Replace $\xi_i(\varpi)$ with the lower bound ξl_i, $\forall i \in V, i \neq k$, and $t_{ij}(\varpi)$ with the lower bound tl_{ij}, $\forall (i, j) \in A$, $i \neq k, j \neq j^*$.
In Step 2, replacing $\xi_i(\varpi)$ with ξl_i, $\forall i \in V$, $i \neq k$, and $t_{ij}(\varpi)$ with tl_{ij}, $\forall (i, j) \in A$, $i \neq k, j \neq j^*$ does not change the values of $Z(\varpi, \boldsymbol{\tau})$ and $Z(\varpi, \boldsymbol{\tau}^*(\varpi))$. Therefore, the modification in Step 2 also does not change the optimality of (ϖ, $\boldsymbol{\tau}^*(\varpi)$) to sub-problem $RC(\boldsymbol{\tau})$. Thus, the main claim is proven, as is Lemma 1. □

Lemma 1 leads to the following theorem, which significantly simplifies the formulation of sub-problem $RC(\boldsymbol{\tau})$.

Theorem 1. *For any $\boldsymbol{\tau} \in \Omega$,*

$$RC(\boldsymbol{\tau}) = \underset{i \in V}{\text{Max}}\{\xi u_i \times tu_{ij^*} - Z(w_i, \boldsymbol{\tau}^*(w_i))\}, \tag{14}$$

where j^* is the distribution center associated with station i in plan $\boldsymbol{\tau}$.

Proof of Theorem 1. The theorem follows in a straightforward manner from the proof of Lemma 1. □

3. Solution algorithm

3.1. A local search-based algorithmic framework

Since the RWVPC problem is *NP*-hard, to obtain robust solutions with a reasonable amount of computational resources for problem instances with practical sizes, heuristics or meta-heuristics are typically adopted. Based on Theorem 1, the following local search-based algorithmic framework is proposed.

Step 1: Initialization
  1.1. Generate randomly an initial solution τ_0; let $\tau = \tau_0$.
  1.2. Evaluate the robustness cost of τ, $RCost(\tau)$, based on Theorem 1.
Step 2: Local search
  2.1. Generate a new solution, $\tau_{\mathbf{new}}$, from the neighborhood of τ.
  2.2. Evaluate the robustness cost of $\tau_{\mathbf{new}}$, $RCost(\tau_{\mathbf{new}})$, based on Theorem 1.
Step 3: Solution acceptance or rejection
  3.1. If the rules (or aspiration rules) of solution acceptance are adopted, then let $\tau = \tau_{\mathbf{new}}$.
  3.2. Otherwise, decline the new solution, $\tau_{\mathbf{new}}$.
Step 4: Convergence check
  4.1. If convergence criteria are satisfied, stop.
  4.2. Otherwise, go to *Step 2*.

In this framework, each candidate solution has more than one neighbor solution, and the choice of which neighbor solution to move is determined using only the information about the neighborhood of the current solution (i.e., local search). When no improvement mechanisms are designed for the neighborhood search, a local search may be stuck at local optima. This issue can be resolved by applying, for instance, restarts with different initial solutions or relatively more sophisticated schemes based on iterations (e.g., iterated greedy) or memory-less stochastic modifications (e.g., SA) in Step 3 of the proposed framework.

3.2. A Specific Implementation based on simulated annealing (SA)

This subsection presents a specific implementation of the proposed framework based on SA, which can escape from being trapped into a local optimum by accepting, with a small probability, worse solutions during iterations. Suman and Kumar (2006) comprehensively reviewed SA-based optimization algorithms. Let T_0 be the initial temperature, T_F the final temperature, Ite_{max} the maximum number of iterations between two different temperatures, Num_{max} the maximum number of temperature reductions, and β the temperature reduction ratio ($0 < \beta < 1$). The SA-based heuristic is presented as follows.

Step 0: Input data and set parameters values T_0, T_F, Ite_{max}, Num_{max}, and β;
Step 1: Initialization
  1.1. Randomly generate the initial solution τ_0; $\tau := \tau_0$;
  1.2. Initialize $Temp := T_0$, $Ite := 0$, $Num := 0$, $\tau_{\text{robust}} := \tau$, $RCost(\tau_{\text{robust}}) := RCost(\tau)$;
Step 2: Generate a solution τ_{new}, and evaluate its robustness cost, $RCost(\tau_{\text{new}})$; $Ite := Ite + 1$;
Step 3: $\Delta E := RCost(\tau_{\text{new}}) - RCost(\tau)$; if $\Delta E \leqslant 0$, go to Step 3.1; otherwise, go to Step 3.2;
  3.1. Let $\tau := \tau_{\text{new}}$;
  3.2. Generate a random number $rand \sim U(0,1)$;
  If $rand < (Temp/(Temp^2 + \Delta E^2))$, then $\tau := \tau_{\text{new}}$;
Step 4: If $RCost(\tau) < RCost(\tau_{\text{robust}})$, then $\tau_{\text{robust}} := \tau$, $RCost(\tau_{\text{robust}})$ - $:= RCost(\tau)$, $Num := 0$;
Step 5: If $Ite = Ite_{\text{max}}$, then $Temp := Temp \times \beta$, $Ite := 0$, $Num := Num + 1$;
Step 6: If $Temp < T_F$ or $Num := Num_{\text{max}}$, then stop; otherwise, go to Step 2.

Note that the SA-based heuristic is provided only as a specific implementation for the framework, presented in Section 3.1. Any other local search-based heuristic can be adopted in the proposed framework, such as iterated greedy. Moreover, while SA is not a new approach, this work demonstrates a successful application of SA (or meta-heuristics) to solve robust combinatorial optimization problems, which is rarely seen in existing literature.

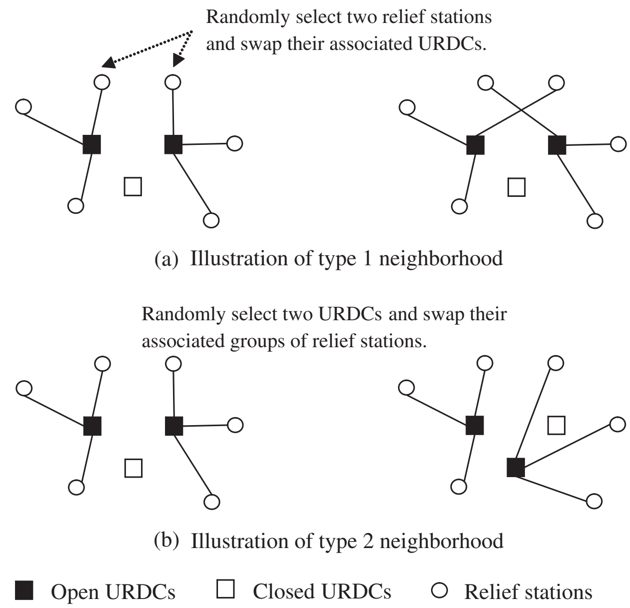

Fig. 1. Illustrations of different neighborhood types.

3.3. Neighborhood description and evaluation of robustness cost

In Step 2 of the SA-based heuristic, a new solution τ_{new} is generated in each iteration from the neighborhood of the current solution τ. This work defines two neighborhood types. The first is called the *allocation* neighborhood, and involves only changes in allocation decision variables (i.e., y_{ij}, $i \in V, j \in U$). Both location and allocation decision variables (i.e., $x_j, j \in U$, and $y_{ij}, i \in V, j \in U$) change in the second type, called the *location–allocation* neighborhood. Specifically, the allocation neighborhood consists of feasible solutions obtained by randomly selecting two relief stations serviced by different URDCs in the current solution, and swapping their associated URDCs (Fig. 1a). The new solutions in the location–allocation neighborhood are generated by randomly choosing two URDCs and swapping their associated groups of relief stations. In this neighborhood type, at least one chosen URDC must be an open facility. Moreover, if only one chosen URDC is open, this swap is equivalent to moving a group of relief stations from an open facility to another facility that was closed but is now open after the swap (Fig. 1b).

In each iteration of the SA-based heuristic, a new solution τ_{new} is generated from either the allocation (with probability p_1) or location–allocation (with probability p_2) neighborhood of current solution τ. Selection between the two types of neighborhood is based on probabilities p_1 and p_2, and $p_1 + p_2 = 1$.

4. Numerical experiments

4.1. Experimental design

A set of numerical experiments was conducted on a set of test instances, to examine the algorithmic performance of the SA-based heuristic, the tradeoff between robustness and optimality, and the impact of data uncertainty on the solutions. The algorithm was coded using C++ computer language and tested on a personal computer with a Pentium Core 2 Duo 2.4 gigahertz CPU and 2 gigabytes RAM.

To examine the performance of the SA-based heuristic, this work also implemented an enumeration approach and compared the effectiveness (solution quality) and efficiency (computational time) of the SA-based heuristic with those of the enumeration approach. This approach enumerates all possible plans and identifies the plan with minimal objective value. In spite of its computational inefficiency, this enumeration approach guarantees to find exact solutions that serve as the benchmark for evaluating the solution quality of the heuristic.

The test instances have different sizes, represented by the triplet $(|V|, |U|, p)$, where $|V|$, $|U|$, and p denote the numbers of relief stations, candidate URDC sites, and open URDC sites, respectively. Fifteen different sizes of instances were considered. For each problem size, 30 instances were generated, so there were 450 problem instances. For each instance, the two-dimensional coordinates of relief stations and candidate URDC sites were generated from the intervals $(0,100)$ and $(40,60)$, respectively. The nominal travel time t_{ij} between each pair of relief station i and candidate URDC site j is given as the Euclidean distance rounded to the nearest integer. Because of the likely degradation of road condition in the aftermath of disasters, the travel time interval $[tl_{ij}, tu_{ij}]$ is generated as $[t_{ij}, t_{ij} + \alpha_1 \times t_{ij}]$, $\forall i, j$. The relief demand interval $[\xi l_i, \xi u_i]$ is generated as $[\xi_i - \alpha_2 \times \xi_i, \xi_i + \alpha_2 \times \xi_i]$, $\forall i$, since the actual demand at station i could be more or less than the nominal demand (e.g., the relief demand estimated by search-and-rescue teams). The parameters α_1 and α_2 are used to control the degree of data uncertainty; the larger the values of α_1 and α_2, the higher the degree of uncertainty for the data. Each problem instance was tested for nine combinations of (α_1, α_2), where $\alpha_1 = 0.5$, 1.5, and 2.5 and $\alpha_2 = 0.2$, 0.4, and 0.6, so there were 4050 (= 450 × 9) tests.

In addition to the objective values (robustness cost), defined in Eq. (12), the price of robustness $\eta(\boldsymbol{\tau}_{\text{robust}})$ and hedge value $H(\boldsymbol{\tau}_{\text{robust}})$ are the other two performance measures. $\eta(\boldsymbol{\tau}_{\text{robust}})$ is equal to the price that the decision-maker needs to pay for employing the robust plan $\boldsymbol{\tau}_{\text{robust}}$, instead of the optimal nominal plan $\boldsymbol{\tau}_{\text{nominal}}$, in the scenario of nominal travel times, w_{nominal} (i.e., without considering data uncertainties). Specifically,

$$\eta(\boldsymbol{\tau}_{\text{robust}}) = Z(w_{\text{nominal}}, \boldsymbol{\tau}_{\text{robust}}) - Z(w_{\text{nominal}}, \boldsymbol{\tau}_{\text{nominal}}). \quad (15)$$

In Eq. (15), $Z(w_{\text{nominal}}, \boldsymbol{\tau}_{\text{nominal}})$ is the minimum of maximum demand-weighted travel time between URDCs and relief stations in the nominal scenario, w_{nominal}. $H(\boldsymbol{\tau}_{\text{robust}})$ is defined as the value gained from implementing the robust plan $\boldsymbol{\tau}_{\text{robust}}$, instead of the optimal nominal plan $\boldsymbol{\tau}_{\text{nominal}}$, in the worst-case scenarios. Specifically,

$$H(\boldsymbol{\tau}_{\text{robust}}) = RC(\boldsymbol{\tau}_{\text{nominal}}) - RC(\boldsymbol{\tau}_{\text{robust}}). \quad (16)$$

In the definitions, $\eta(\boldsymbol{\tau}_{robust})$ represents the tradeoff between robustness and optimality, while $H(\boldsymbol{\tau}_{\text{robust}})$ can be viewed as the regret of employing the plan $\boldsymbol{\tau}_{\text{nominal}}$ in the worst-case scenario.

Table 1
Comparison of algorithmic performance for the RWVPC instances with $|V| = 10$.

Problem size	(α_1, α_2)	SA-based heuristic		Enumeration	
		Ave. $RC(\boldsymbol{\tau}_{\text{robust}})$	CPU time	Ave. $RC(\boldsymbol{\tau}_{\text{robust}})$	CPU time
(10, 4, 2)	(0.5, 0.2)	2309.90 (4)	6.33	2101.68	29.59
	(0.5, 0.4)	2799.12 (4)		2561.58	
	(0.5, 0.6)	3263.04 (5)		2927.52	
	(1.5, 0.2)	7422.80 (3)		7259.52	
	(1.5, 0.4)	8676.60 (3)		8469.44	
	(1.5, 0.6)	9930.40 (3)		9679.36	
	(2.5, 0.2)	12612.56 (3)		12482.52	
	(2.5, 0.4)	14731.32 (3)		14562.94	
	(2.5, 0.6)	16950.08 (3)		16643.36	
(10, 4, 3)	(0.5, 0.2)	2138.78 (1)	8.92	2105.16	492.57
	(0.5, 0.4)	2598.36 (2)		2498.58	
	(0.5, 0.6)	2855.52 (0)		2855.52	
	(1.5, 0.2)	7355.40 (0)		7355.40	
	(1.5, 0.4)	8623.86 (1)		8401.30	
	(1.5, 0.6)	9807.20 (0)		9807.20	
	(2.5, 0.2)	12612.12 (1)		12575.64	
	(2.5, 0.4)	14371.58 (0)		14371.58	
	(2.5, 0.6)	16816.16 (1)		16767.52	
(10, 5, 2)	(0.5, 0.2)	2234.48 (5)	9.77	1887.96	57.40
	(0.5, 0.4)	2619.82 (6)		2252.18	
	(0.5, 0.6)	2997.12 (6)		2573.92	
	(1.5, 0.2)	6857.34 (3)		6506.58	
	(1.5, 0.4)	7900.23 (3)		7591.01	
	(1.5, 0.6)	8843.12 (2)		8675.44	
	(2.5, 0.2)	11408.26 (3)		11288.48	
	(2.5, 0.4)	13442.97 (3)		13169.87	
	(2.5, 0.6)	15277.68 (3)		15051.28	
(10, 5, 3)	(0.5, 0.2)	2071.70 (1)	12.37	1972.14	2989.96
	(0.5, 0.4)	2390.06 (3)		2200.83	
	(0.5, 0.6)	2529.52 (0)		2529.52	
	(1.5, 0.2)	6399.54 (0)		6399.54	
	(1.5, 0.4)	7933.17 (3)		7816.13	
	(1.5, 0.6)	9120.08 (3)		8932.72	
	(2.5, 0.2)	11265.18 (1)		11161.98	
	(2.5, 0.4)	13542.97 (2)		13372.31	
	(2.5, 0.6)	15420.24 (3)		15082.64	

Table 2
Comparison of algorithmic performance for the RWVPC instances with $|V| = 15$.

Problem size	(α_1, α_2)	SA-based heuristic		Enumeration	
		Ave. $RC(\boldsymbol{\tau}_{\text{robust}})$	CPU time	Ave. $RC(\boldsymbol{\tau}_{\text{robust}})$	CPU time
(15, 4, 2)	(0.5, 0.2)	2354.78 (3)	13.39	2140.12	223.60
	(0.5, 0.4)	2882.44 (5)		2547.37	
	(0.5, 0.6)	3389.68 (6)		2911.28	
	(1.5, 0.2)	7439.98 (3)		7229.58	
	(1.5, 0.4)	8861.39 (4)		8434.51	
	(1.5, 0.6)	10001.04 (3)		9639.44	
	(2.5, 0.2)	12529.88 (1)		12447.00	
	(2.5, 0.4)	14572.10 (2)		14521.50	
	(2.5, 0.6)	16639.84 (1)		16596.00	
(15, 4, 3)	(0.5, 0.2)	2304.34(N/A)	16.05	2395.22*	5 days
	(0.5, 0.4)	2987.46(N/A)		3315.89*	
	(0.5, 0.6)	3480.88(N/A)		3579.16*	
	(1.5, 0.2)	7161.56(N/A)		7545.24*	
	(1.5, 0.4)	8808.94(N/A)		8983.92*	
	(1.5, 0.6)	10039.84(N/A)		10558.04*	
	(2.5, 0.2)	12629.88(N/A)		13114.62*	
	(2.5, 0.4)	14334.86(N/A)		15392.12*	
	(2.5, 0.6)	16299.84(N/A)		17105.38*	
(15, 5, 2)	(0.5, 0.2)	2386.24(2)	18.36	2112.30	3169.21
	(0.5, 0.4)	2747.72(3)		2470.65	
	(0.5, 0.6)	3215.12(4)		2823.60	
	(1.5, 0.2)	7215.10(2)		7085.70	
	(1.5, 0.4)	8676.81(3)		8266.65	
	(1.5, 0.6)	9753.04(3)		9447.60	
	(2.5, 0.2)	12691.92(4)		12120.58	
	(2.5, 0.4)	14377.39(2)		14148.54	
	(2.5, 0.6)	16333.52(2)		16169.76	
(15, 5, 3)	(0.5, 0.2)	2636.82(N/A)	23.60	2955.48*	5 days
	(0.5, 0.4)	3126.76(N/A)		3205.80*	
	(0.5, 0.6)	3535.04(N/A)		3987.15*	
	(1.5, 0.2)	7182.54(N/A)		7514.44*	
	(1.5, 0.4)	8608.07(N/A)		8796.94*	
	(1.5, 0.6)	9929.52(N/A)		11783.20*	
	(2.5, 0.2)	12339.54(N/A)		12975.36*	
	(2.5, 0.4)	14395.29(N/A)		16003.50*	
	(2.5, 0.6)	16282.72(N/A)		16549.72*	

* The incumbent solution obtained by the enumeration approach in 5 days.

4.2. Numerical results of solving the RWVPC problem

4.2.1. Effectiveness and efficiency of the SA-based heuristic

The computational results of solving the RWVPC problem instances with the number of relief stations $|V| = 10$ and 15 are displayed in Tables 1 and 2, respectively. In the tables, Ave. $RC(\boldsymbol{\tau}_{\text{robust}})$ denotes the average robustness cost, over 30 instances, for each problem size and uncertainty level. For each problem size and each uncertainty level, the number of instances (out of 30 instances) where the SA-based heuristic fails to obtain optimal solution is reported inside the parenthesis in the third column of the tables. As shown in the third and fifth columns of the tables, for most problem sizes and uncertainty levels, the Ave. $RC(\boldsymbol{\tau}_{\text{robust}})$ obtained using the SA-based heuristic is close to that obtained using the enumeration approach. The difference in Ave. $RC(\boldsymbol{\tau}_{\text{robust}})$, between the SA-based heuristic and the enumeration approach is less than 10% in all instances, implying that the SA-based heuristic is able to find close-to-optimal solutions. Note that, since the estimated computational time for the enumeration approach to optimally solve one instance of problem size equal to (15,4,3) or (15,5,3) is more than 1 week, a maximum computational time (5 days) was set and the incumbent solution obtained within the maximal computational time was reported. As shown in Table 2, for the tests on these larger instances, the solution obtained using the SA-based heuristic is significantly better than that obtained using the enumeration approach.

Regarding the computational efficiency of the proposed heuristic, as shown in the fourth and sixth columns of these tables, the SA-based heuristic requires much less computational time than the enumeration approach in all the conducted tests. In particular, although the computational time of the enumeration approach increases dramatically as the problem size gets larger, the increase in computational time of the SA-based heuristic is not significant. For instance, for the tests with the same numbers of $|V| = 10$ and $|U| = 5$, when p increases from 2 to 3, the computational time of the enumeration approach increases by more than 60 times, but the SA-based heuristic requires an average of 2.6 more seconds. In summary, the SA-based heuristic obtains optimal or near-optimal solutions using much less computational time than the enumeration approach.

4.2.2. Impacts of data uncertainty

The tests conducted on the problem instances of larger size, i.e., (30,5,3), (40,8,4), and (50,10,5), aim to examine the impact of data uncertainty on the performance measures of interest. The test results in Table 3 show that both robustness cost and hedge value increase as the uncertainty level becomes higher. Because of the larger relief demand and travel time intervals (i.e., larger α_1 and α_2), the worst-case scenario deviates further from the nominal scenario, resulting in larger robust deviations and the increase in hedge value, which highlights the advantage of implementing robust solutions in the presence of data uncertainty. The robustness price is less than 15% of the optimal nominal objective value (i.e., $Z(w_{\text{nominal}}, \boldsymbol{\tau}_{\text{nominal}})$ reported in the sixth column of Table 3), implying that the robust plans determined by the proposed method do not tradeoff much optimality for robustness.

5. Numerical example

This numerical example demonstrates the application of the proposed RWVPC model to locate URDCs in a relief supply distribution network responding to the massive earthquake which hit central Taiwan on September 21, 1999—the 921 Jiji Earthquake. This earthquake, which measured 7.3 on the Richter scale, mostly affected Taichung and Nantou counties, causing more than 2500 deaths and 8000 injuries and destroying (completely or partially) 39,000 buildings. A three-tier relief supply distribution network was established in Nantou County immediately after this earthquake. Specifically, relief supplies were collected from six unaffected counties (Taipei, Taoyuan, Hsinchu, Changhua, Tainan, and Kaohsiung), transported to two URDCs at Nantou Stadium and Jiji Town Hall, and then delivered to the 51 relief stations in the 11 townships in Nantou County. In this numerical example, in addition to Nantou Stadium and Jiji Town Hall, five other candidate sites for URDCs were selected based on an earthquake prepared-

Table 3
Computational results of solving larger RWVPC problem instances.

Problem size	(α_1, α_2)	$RC(\boldsymbol{\tau}_{\text{robust}})$	$\eta(\boldsymbol{\tau}_{\text{robust}})$	$H(\boldsymbol{\tau}_{\text{robust}})$	$Z(w_{\text{nominal}}, \boldsymbol{\tau}_{\text{nominal}})$	CPU time
(30, 5, 3)	(0.5, 0.2)	3236.86	349.80	1503.68	4988.70	138.24
	(0.5, 0.4)	3874.71	425.07	1655.71		
	(0.5, 0.6)	4461.44	421.45	1859.04		
	(1.5, 0.2)	8836.26	527.37	2826.24		
	(1.5, 0.4)	10430.47	570.95	3175.78		
	(1.5, 0.6)	11920.80	590.95	3629.20		
	(2.5, 0.2)	14780.04	779.02	3860.34		
	(2.5, 0.4)	17203.55	762.19	4543.56		
	(2.5, 0.6)	19661.20	792.19	5192.64		
(40, 8, 4)	(0.5, 0.2)	3652.00	542.16	816.92	4917.40	1266.10
	(0.5, 0.4)	4398.24	553.75	855.82		
	(0.5, 0.6)	5026.56	553.75	978.08		
	(1.5, 0.2)	9212.00	683.41	1720.90		
	(1.5, 0.4)	10855.53	746.37	1899.52		
	(1.5, 0.6)	12406.32	766.33	2170.88		
	(2.5, 0.2)	15033.64	827.84	2522.42		
	(2.5, 0.4)	17605.63	825.92	2876.44		
	(2.5, 0.6)	19020.72	845.92	4387.36		
(50, 10, 5)	(0.5, 0.2)	3886.10	511.13	1000.90	4633.40	13438.63
	(0.5, 0.4)	4648.07	598.65	1053.43		
	(0.5, 0.6)	5298.56	526.32	1217.44		
	(1.5, 0.2)	9593.28	663.76	1888.68		
	(1.5, 0.4)	11303.74	694.54	2091.88		
	(1.5, 0.6)	12918.56	704.54	2390.72		
	(2.5, 0.2)	15480.96	759.06	2595.96		
	(2.5, 0.4)	18076.80	779.06	3012.94		
	(2.5, 0.6)	20659.20	795.36	3443.36		

Table 4
Travel times (minutes) between URDC candidate sites and relief stations.

Township	Relief stations	URDC candidate sites						
		Nantou Stadium	Puli High School	Caotun Middle School	Jhushan Elementary School	Jiji Town Hall	Guoshing Town Hall	Shueili Middle School
Nantou (104,000)[a] (26,000)[b]	NT-A	4	68	22	33	33	57	43
	NT-B	3	65	19	35	35	56	45
	NT-C	6	67	20	35	37	56	47
	NT-D	2	66	20	32	33	56	43
Puli (84,000) (10,500)	PL-A	65	7	54	80	77	38	60
	PL-B	56	4	54	80	78	40	62
	PL-C	69	7	57	84	80	42	64
	PL-D	65	9	53	80	76	38	60
	PL-E	71	20	59	85	91	40	75
	PL-F	61	9	49	76	72	34	56
	PL-G	65	7	53	79	76	38	60
	PL-H	67	6	55	82	78	40	62
Caotun (99,400) (14,200)	CT-A	18	59	6	37	43	42	53
	CT-B	32	42	13	51	57	25	67
	CT-C	36	50	16	55	56	33	71
	CT-D	27	53	8	46	52	35	62
	CT-E	39	54	20	58	60	37	75
	CT-F	48	41	28	67	72	24	85
	CT-G	39	41	20	58	64	25	74
Jhushan (58,000) (29,000)	JS-A	23	71	33	12	22	61	34
	JS-B	27	74	36	6	27	64	40
Jiji (11,700) (3,900)	JJ-A	36	79	50	36	4	78	17
	JJ-B	36	77	50	35	5	75	15
	JJ-C	27	79	41	30	19	67	29
Mingjia (41,000) (8,200)	MJ-A	15	67	29	21	19	57	29
	MJ-B	13	64	32	34	32	64	42
	MJ-C	17	73	35	31	28	63	39
	MJ-D	18	75	37	34	32	65	42
	MJ-E	23	75	37	31	29	62	39
Lugu (19,200)(6,400)	LG-A	41	88	50	19	37	78	44
	LG-B	50	97	59	28	46	87	54
	LG-C	58	103	73	42	34	101	42
Jhongliao (16,000) (8,000)	JL-A	28	65	35	54	38	47	53
	JL-B	21	72	35	39	18	62	33
Yuchih (18,000) (3,000)	YC-A	76	29	65	84	57	49	41
	YC-B	61	47	75	62	35	68	19
	YC-C	91	39	79	98	72	64	55
	YC-D	84	27	72	98	73	56	56
	YC-E	68	40	76	69	43	61	26
	YC-F	92	51	86	93	66	71	50
Guoshing (20,800) (5,200)	GS-A	57	41	38	71	77	1	72
	GS-B	71	37	52	85	60	26	46
	GS-C	75	43	56	89	95	18	90
	GS-D	62	28	42	76	70	17	56
Shueilli (21,000) (3,000)	SL-A	49	61	63	50	24	67	10
	SL-B	71	58	85	72	45	64	31
	SL-C	57	58	71	57	31	64	17
	SL-D	72	86	86	73	46	102	30
	SL-E	65	85	79	66	39	96	24
	SL-F	58	82	72	57	33	91	21
	SL-G	51	65	65	52	25	81	9

[a] Number of survivals in each township.
[b] Relief demand faced by each relief station in a township.

ness report prepared by Taiwan's Ministry of the Interior (Ministry of the Interior, Taiwan, 2000). The triplet $(|V|, |U|, p) = (51, 7, 2)$ denotes the problem size of this numerical example, where $|V|$, $|U|$, and p denote the numbers of relief stations, candidate URDC sites, and selected URDC sites, respectively.

Table 4 lists the travel times between the seven candidate sites and 51 relief stations. For instance, the travel time between Nantou Stadium and relief station NT-A in Nantou township was 4 minutes. These travel time data were collected by Sheu (2007, 2010) to evaluate an emergency logistics distribution approach. In addition, because it is very difficult to precisely estimate relief demand faced by each relief station, this work divided number of survivals (provided in the leftmost column of Table 4), equal to population minus number of deaths, by number of relief stations of each township, to obtain an approximate estimate of relief demand faced by each relief station in that township. For example, the number of survivals in Nantou was 104,000 and there were four relief stations in this township, so each relief station faced a relief demand 26,000.

In the earthquake's aftermath, the maximum demand-weighted travel time between the two URDCs (i.e., Nantou Stadium and Jiji Town Hall) and the 51 relief stations was 783,000 persons-minutes

Table 5
Computational results of the numerical example based on Jiji earthquake.

Problem size	(α_1, α_2)	$RC(\tau_{robust})$	Selected URDCs	Max. weighted travel time
(51, 7, 2)	(0.5, 0.2)	93,619	Nantou Stadium, Puli High School	837,800 = 59 × 14,200 (between Puli High School and CT-A)
	(0.5, 0.4)	587,837	Nantou Stadium, Puli High School	783,000 = 27 × 29,000 (between Nantou Stadium and JS-B)
	(0.5, 0.6)	1,477,709	Puli High School, Jhushan Elementary School	910,000 = 35 × 26,000 (between Jhushan Elementary School and NT-B)
	(1.5, 0.2)	940,858	Nantou Stadium, Puli High School	783,000 = 27 × 29,000 (between Nantou Stadium and JS-B)
	(1.5, 0.4)	1,859,069	Nantou Stadium, Guoshing Town Hall	783,000 = 27 × 29,000 (between Nantou Stadium and JS-B)
	(1.5, 0.6)	2,934,605	Puli High School, Jhushan Elementary School	910,000 = 35 × 26,000 (between Jhushan Elementary School and NT-B)
	(2.5, 0.2)	1,883,309	Nantou Stadium, Puli High School	783,000 = 27 × 29,000 (between Nantou Stadium and JS-B)
	(2.5, 0.4)	3,400,291	Puli High School, Jhushan Elementary School	910,000 = 35 × 26,000 (between Jhushan Elementary School and NT-B)
	(2.5, 0.6)	4,356,480	Nantou Stadium, Puli High School	783,000 = 27 × 29,000 (between Nantou Stadium and JS-B)

(from either one of the two URDCs to relief station JS-B with relief demand equal to 29,000 persons and travel time equal to 27 min). Without considering relief demand and travel time uncertainties, this work solved a deterministic WVPC problem based on the data in Table 4. In the solution, Caotun Middle School and Jhushan Elementary School are selected as the URDCs, with a maximum demand-weighted travel time equal to 619,500 persons-minutes (from Caotun Middle School to relief station PL-E with relief demand equal to 10,500 persons and travel time equal to 59 minutes). Setting up the URDCs at this two sites significantly reduces the maximum demand-weighted travel time (783,000 − 619,500 = 163,500 persons- minutes), which indicates a potential improvement in relief distribution efficiency by locating URDCs at the sites suggested by the deterministic WVPC model.

To represent data uncertainty in the RWVPC problem, travel time interval $[tl_{ij}, tu_{ij}]$ was generated as $[t_{ij}, t_{ij} + \alpha_1 \times t_{ij}]$, $\forall i, j$, while relief demand interval $[\xi l_i, \xi u_i]$ was generated as $[\xi_i - \alpha_2 \times \xi_i, \xi_i + \alpha_2 \times \xi_i]$, $\forall i$. This numerical example was tested for nine combinations of (α_1, α_2), where α_1 = 0.5, 1.5, and 2.5 and α_2 = 0.2, 0.4, and 0.6. The average CPU time for the SA-based heuristic to solve one instance was 73 seconds. Table 5 presents the computational results of solving the nine RWVPC problem instances. As expected, the robustness cost increases as the uncertainty level, (α_1, α_2), increases. The URDCs selected in the RWVPC model differ significantly from those in the deterministic WVPC model. Particularly, while an URDC is located at Caotun Middle School in the WVPC model, this site is not included in the RWVPC model for different levels of data uncertainty. Moreover, Puli High School is selected in the RWVPC model for different levels of data uncertainty, but not in the WVPC model. The difference in selected URDCs highlights the importance of considering data uncertainty in choosing URDC sites. The results also suggest that Puli High School is an appropriate site for locating an URDC while considering uncertain relief demands and travel times.

6. Concluding remarks

With particular emphasis on supporting the decision of locating URDCs in an emergency logistics network responding to quick-onset natural disasters, this work developed the RWVPC model and its solution algorithm. The model aims to minimize the worst-case deviation in maximum demand-weighted travel time between URDCs and relief stations from the optimal solution. Rather than using probability distributions to describe data uncertainty, the model represents uncertain relief demands and travel times using prescribed fixed intervals. A reformulation was proposed in Theorem 1 to facilitate identifying the worst-case scenario among an infinite number of possible scenarios, and evaluating robustness costs. The generic algorithmic framework, which incorporates the reformulation in Theorem 1, was developed to obtain robust solutions in a reasonable amount of computational time.

A large number of problem instances, with various problem sizes and different degrees of data uncertainty, were generated and solved using the SA-based heuristic. The numerical results show that the proposed heuristic is able to efficiently obtain optimal, or near-optimal, solutions. It was also found that the robust solutions determined by the proposed method do not trade off much quality (or optimality) for robustness. To demonstrate the applicability of the proposed RWVPC model to real-world instances, a case study based on Jiji Earthquake, which hit central Taiwan on September 21, 1999, was conducted. In this case study, the URDCs selected in the RWVPC model differ from those in the deterministic WVPC problem, highlighting the importance of considering data uncertainty when choosing URDC sites.

This work contributes significantly to the growing body of literature developing robust optimization approaches to emergency logistics. A number of interesting studies can be conducted based on the developed robust emergency facility location model and algorithm. First, one can compare the effectiveness and efficiency of the proposed SA-based heuristic with those of other meta-heuristics (e.g., Genetic Algorithm and Tabu Search). Second, some practical constraints (e.g., capacity and budget constraints on facilities) can be considered when determining emergency facility locations. Moreover, a convex combination of pessimistic and optimistic decision making can provide a sensitivity analysis on different degrees of utility or satisfaction for the WVPC problem with interval weights on transportation networks under general conditions (e.g., Majumdar and Bhunia, 2011).

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