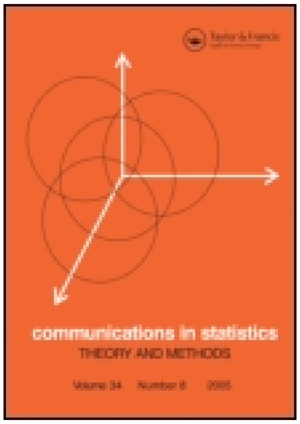

Communications in Statistics - Theory and Methods
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/lsta20

Assessing Profitability of a Newsboy-Type Product with Normally Distributed Demand Based on Multiple Samples

Rung-Hung Su [a] , Dong-Yuh Yang [b] & W. L. Pearn [c]

[a] Electronics Division, SEDA Chemical Product Co., LTD. , Taipei , Taiwan

[b] Institute of Information and Decision Sciences , National Taipei College of Business , Taipei , Taiwan

[c] Department of Industrial Engineering and Management , National Chiao Tung University , Hsinchu , Taiwan

Accepted author version posted online: 14 Apr 2013.Published online: 16 Jul 2013.

To cite this article: Rung-Hung Su , Dong-Yuh Yang & W. L. Pearn (2013) Assessing Profitability of a Newsboy-Type Product with Normally Distributed Demand Based on Multiple Samples, Communications in Statistics - Theory and Methods, 42:18, 3401-3419, DOI: 10.1080/03610926.2011.624245

To link to this article: http://dx.doi.org/10.1080/03610926.2011.624245

Communications in Statistics—Theory and Methods, 42: 3401–3419, 2013

ISSN: 0361-0926 print/1532-415X online
DOI: 10.1080/03610926.2011.624245

Assessing Profitability of a Newsboy-Type Product with Normally Distributed Demand Based on Multiple Samples

RUNG-HUNG SU[1], DONG-YUH YANG[2], AND W. L. PEARN[3]

[1]Electronics Division, SEDA Chemical Product Co., LTD., Taipei, Taiwan
[2]Institute of Information and Decision Sciences, National Taipei College of Business, Taipei, Taiwan
[3]Department of Industrial Engineering and Management, National Chiao Tung University, Hsinchu, Taiwan

This article develops a new index "Achievable Capacity Index," I_A*, which can accurately measure the profitability of newsboy-type product with normally distributed demand. An unbiased and effective estimator of* I_A *is derived to estimate actual* I_A *as the parameters of distribution are unknown. Practically, the market information regarding demand is obtained from multiple samples rather than single sample. Therefore, we estimate and test* I_A *based on multiple samples. A hypothesis testing for examining whether the profitability meets designated requirement is presented. Critical values of the test are calculated to determine the evaluation results. Finally, a real case on the sales of donuts is presented to illustrate the applicability of our approach.*

Keywords Achievable capacity index; Estimating and testing; Multiple samples; Newsboy; Normal demand.

Mathematics Subject Classification Primary 62F03; Secondary 62P30.

1. Introduction

In the traditional newsboy problem, it usually focused on short shelf-life products such as daily newsarticles, monthly/weekly magazines, milks, seasonal products, fresh food, and many others. Since the surplus products are subject to storage for a short period of time, one ought to pay additional costs to dispose these items. If the unsatisfied demand is lost, the opportunity cost may be occurred.

Received April 1, 2011; Accepted September 9, 2011
Address correspondence to Dong-Yuh Yang, Institute of Information and Decision Sciences, National Taipei College of Business, Taipei 100, Taiwan; E-mail: yangdy@webmail.ntcb.edu.tw

Generally, the demand presented in the newsboy problem is unknown and assumed to be a random variable with a known probability distribution. Consequently, the determination of the ordering quantity (or manufacturing quantity) is critical for achieving certain objective function in the newsboy problem. There is an excellent survey of the literature on the various objective functions such as minimizing the expected cost (Nahmias, 1993), maximizing the expected profit (Khouja, 1995), maximizing the expected utility (Ismail and Louderback, 1979; Lau, 1997), and maximizing the probability of achieving a target profit (Ismail and Louderback, 1979; Shih, 1979; Lau, 1980; Sankarasubramanian and Kumaraswamy, 1983). However, so far, existing researches never care about the value of the maximum expected profit and the probability of achieving a target profit. These values can be expressed the product's profitability.

Whenever the demand is uncertain, several researchers always considered that the demand is random and follows common distributions with known parameters. For example, the normal is preferred when the demand per cycle is relatively large, while the Poisson is better for low-demand items because it is discrete. Lau (1997) has pointed that some seasonal or fashion products which have very high demand uncertainties may be more suitably modeled by the exponential distribution. In practical work, the parameter(s) of demand distribution is/are unknown and depend(s) on the estimation technique. Berk et al. (2007) used the frequentist and the Bayesian approaches for demand estimation. Also, most of the research focused on distribution-free newsboy problem, where the form of the demand distribution is unknown but only the mean and variance are specified. It was the pioneer work of Scarf (1958), in which the minimax approach applied to minimize the maximum cost resulting from the worst possible demand distribution. This approach can derive a simple closed-form expression for the ordering quantity that maximizes expected profit. Moon and Choi (1995) studied a distribution-free newsboy problem with balking, in which customers are allowed to balk when inventory level is low. Ouyang and Wu (1998) presented an inventory model with mixture of backorders and lost sales, which relaxes the assumption about the normal distribution of lead-time demand. Ouyang and Chang (2002) modified the continuous review inventory models involving variable lead time with a mixture of backorders and lost sales. They utilized the minimax distribution-free procedure for finding the optimal inventory strategy in the fuzzy sense where information about the lead time demand distribution is partial. Alfares and Elmorra (2005) extended the analysis of the distribution-free newsboy problem to the case when shortage cost is taken into consideration. Mostard et al. (2005) derived a simple closed-form formula to determine the order quantity for the distribution-free newsboy inventory problem with returns. It was shown in Mostard et al. (2005) that the distribution-free order rule performs well when the coefficient of variation (cv) is at most 0.5, but is far from optimal when the cv is large. Recently, Kevork (2010) developed appropriate estimators for the optimal ordering quantity and the maximum expected profit when demand is normally distributed. They investigated the statistical properties for both small and large samples analytically and through Monte Carlo simulation.

The product profitability evaluation is a practical problem frequently occurred in the inventory control. In many real-world inventory systems, the new products are unceasingly introduced. In order to hold the competitive advantage, the managers not only determine the optimal ordering/manufacturing quantity but also measure the old product's characteristic for evaluating whether the old product is worthy of being ordered/manufactured. If the old product's characteristic is not satisfied, it would be curtailed or substituted for new product due to spatial or capacity constraints. In this article, we consider a newsboy-type product with random demand (D), and study the profitability evaluation problem which deals with examining whether the product's profitability meets designated requirement. Note that the product's profitability is defined as the probability of achieving the target profit under the optimal ordering policy. Since the form of the profitability ought to be complex, it is hard to effectively find the statistical estimation of profitability when the parameter(s) of demand distribution is/are unknown. This motivated us to develop a simple index combined with product's profitability. A new index is called "Achievable Capacity Index." To the best of our knowledge, the index depends on demand mean $E(D)$ and demand standard deviation $\sqrt{Var(D)}$ if the selling price and the related costs are given. Therefore, if the normal demand is considered, the achievable capacity index, I_A, is a function of μ and σ. Under the assumptions that μ and σ are unknown, we ought to collect past demand data, and develop an unbiased and effective estimator of I_A to estimate actual I_A. For the demand data, the demand is the sum of the sales volume and the unsatisfied demand. It seems as if the unsatisfied demand is unable to observed or record. Practically, in order to understand the actual demand for controlling inventory and diminishing the lost sale opportunity cost, the retailers not only care about the sales volume but also try to record unsatisfied demand. Some products would appear to fit these conditions such as high-profit products and new products. Another kind of possibility is that the product is purchased by using the order. At this time, the order can be referred to demand. On the other hand, the majority of the results related to the distributional properties of the estimators were obtained based on the assumption of having a single sample. However, from a practical perspective, several stores have observed a weekly-based (or daily-based) demand records for monitoring profitable status such as fast food restaurants, dairy industries, chemical industries, and so on. Therefore, in these particular environments, the demand data is collected from multiple samples rather than single sample. In order to tackle the profitability evaluation problem, we implement the statistical hypothesis testing methodology based on multiple samples. Critical values of the test are calculated to determine the evaluation results. Furthermore, for practitioners' convenience, we provide a simple procedure to use in making decision on whether the profitability meets designated requirement.

The rest of the article is organized as follows. In the next section, we study the profitability measure for the newsboy-type product with normally distributed demand and devise the achievable capacity index I_A. More noteworthy is that I_A has simple form and can accurately measure the profitability. In Sec. 3, we derive an unbiased estimator $\widetilde{I}_A$ to estimate actual I_A based on multiple samples. The distribution of $\widetilde{I}_A$ is also discussed. In Sec. 4, the critical value of the test is calculated to determine the evaluation results. Section 5 presents an example for donuts to illustrate the practicality of the approach to data collected from a donut store for profitability evaluation. In the final section, concluding remarks are given.

2. Profitability Measurement

In this section, we consider a newsboy-type product. The demand D follows a normal distribution, $N(\mu, \sigma^2)$, and satisfies that the coefficient of variation (cv) is below 0.3 for neglecting the negative tail, i.e., $f(D < 0) = \Phi(-\mu/\sigma) = \Phi(-1/cv) < \Phi(-1/0.3) \approx 0$. However, if $cv \geq 0.3$, the truncated normal distribution is more suitable for modeling the demand instead of normal distribution. In addition, the profitability is defined as the probability of achieving the target profit ($k > 0$) under the optimal ordering policy, in which the target profit is set according to the product property and the sales experience.

2.1. *Achievable Capacity Index*

If the selling price and the related cost (shortage, excess, and purchasing/manufacturing costs per unit) are given, the optimal ordering quantity and the level of profitability depend on the demand mean $E(D)$ and the demand standard deviation $\sqrt{Var(D)}$. We develop a new index, which is a function of $E(D)$ and $\sqrt{Var(D)}$ to express the product's profitability, and so-called "Achievable Capacity Index (ACI)." For the normal demand, the achievable capacity index I_A is defined as follows:

$$I_A = \frac{E(D) - T}{\sqrt{Var(D)}} = \frac{\mu - T}{\sigma},$$

where

p the selling price per unit, $p > 0$;
c the purchasing/manufacturing cost per unit, $p > c > 0$; and
T the target demand which is the minimal demand required for achieving profit, i.e., $T = k/(p - c) > 0$.

The numerator of I_A provides the difference between demand mean and target demand. The denominator gives demand standard deviation. Obviously, it is desirable to have an I_A as large as possible.

2.2. *Interrelationship between Profitability and I_A*

Based on the literature Sankarasubramanian and Kumaraswamy (1983), the profit Z depends on the demand D and the ordering quantity Q, which are formulated as follows:

$$Z = \begin{cases} pD - c_d(Q - D) - cQ = (c_p + c_e)D - c_eQ, & 0 \leq D \leq Q \\ pQ - c_s(D - Q) - cQ = -c_sD + (c_p + c_s)Q, & D > Q \end{cases},$$

where

c_p the net profit per unit (i.e., $c_p = p - c > 0$);

c_d the disposal cost for a surplus product, $c_d > 0$;

c_e the excess cost per unit (i.e., $c_e = c_d + c > 0$); and

c_s the shortage cost per unit, $c_s > 0$.

Note that if the surplus products can be salvaged, the value of c_d is negative and redefine into salvage price. It is well known that in order to possibly achieve the target profit, the ordering quantity must be greater than target demand, i.e., $Q \geq T$. For any $Q \geq T$, Z is strictly increasing in $D \in [0, Q]$ and strictly decreasing in $D \in [Q, \infty)$, and has a maximum at point $D = Q$. The maximum value of Z is equal and higher than k, i.e., $Z = pD - cQ = c_p D = c_p Q \geq c_p T = k$. The target profit will be realized when D is equal to either $LAL(Q)$ or $UAL(Q)$, so the target profit will be achieved in $D \in [LAL(Q), UAL(Q)]$, where

$$LAL(Q) = \frac{c_e Q + k}{c_p + c_e} \quad \text{and} \quad UAL(Q) = \frac{(c_p + c_s)Q - k}{c_s}$$

are the lower and upper achievable limits, respectively, and both are the functions of Q. Under the assumption that the demand is normally distributed, the probability of achieving the target profit is

$$\Pr(Z \geq k) = \Phi\left(\frac{UAL(Q) - \mu}{\sigma}\right) - \Phi\left(\frac{LAL(Q) - \mu}{\sigma}\right), \tag{1}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. Before calculating the profitability, we first find the optimal ordering quantity that maximizes $\Pr(Z \geq k)$. We take the first-order of $\Pr(Z \geq k)$ with respect to Q, and obtain

$$\frac{d\Pr(Z \geq k)}{dQ} = \frac{1}{\sqrt{2\pi}}\left[\frac{c_p + c_s}{c_s} e^{-\frac{1}{2}\left(\frac{UAL(Q)-\mu}{\sigma}\right)^2} - \frac{c_e}{c_p + c_e} e^{-\frac{1}{2}\left(\frac{LAL(Q)-\mu}{\sigma}\right)^2}\right].$$

It is well known that the necessary condition for Q to be optimal must satisfy the equation $\mathrm{d}\Pr(Z \geq k)/\mathrm{d}Q = 0$, which implies

$$\mu = \frac{UAL(Q) + LAL(Q)}{2} - \frac{\omega\sigma^2}{UAL(Q) - LAL(Q)}, \tag{2}$$

where $\omega = \ln[1 + c_p A / c_s c_e]$ and $A = c_p + c_e + c_s$. For $Q \geq T$, we solve Eq. (2), then obtain the unique optimal ordering quantity

$$Q^* = T + \frac{c_s(c_p + c_e)(c_p\mu - k)}{c_p(c_p A + 2c_e c_s)} + \sqrt{\left[\frac{c_s(c_p + c_e)(c_p\mu - k)}{c_p(c_p A + 2c_e c_s)}\right]^2 + \frac{2c_s^2(c_p + c_e)^2\omega\sigma^2}{c_p A(c_p A + 2c_e c_s)}} > T. \tag{3}$$

In addition, the sufficient condition is given by

$$\left.\frac{\mathrm{d}^2\Pr(Z \geq k)}{\mathrm{d}Q^2}\right|_{Q=Q^*} = -\frac{(c_p + c_s)}{\sqrt{2\pi}\sigma^3 c_s^2 (c_p + c_e)} \exp\left[-\frac{1}{2}\left(\frac{UAL(Q^*) - \mu}{\sigma}\right)\right] \times \left\{\frac{[UAL(Q^*) - LAL(Q^*)]\,(c_p A + 2c_e c_s)}{2} + \frac{c_p A\omega\sigma^2}{UAL(Q^*) - LAL(Q^*)}\right\} < 0.$$

We can conclude that the stationary point Q^* is a global maximum. By using Eq. (2) and substituting Eq. (3) into Eq. (1), the profitability, AC, can be obtained as follows:

$$AC = \Phi\left(G + \frac{\omega}{2G}\right) - \Phi\left(-G + \frac{\omega}{2G}\right),$$

where

$$G = \frac{UAL(Q^*) - LAL(Q^*)}{2\sigma} = M\left(\frac{\mu - T}{\sigma}\right) + \sqrt{M^2\left(\frac{\mu - T}{\sigma}\right)^2 + M\omega}$$
$$= MI_A + \sqrt{M^2 I_A^2 + M\omega} > 0,$$

and

$$M = \frac{c_p A}{2(c_p A + 2c_e c_s)} > 0.$$

One can easily sees that AC is a function of I_A. Taking the first-order derivative of $AC(I_A)$ with respect to I_A, we have

$$\frac{\mathrm{d}AC(I_A)}{\mathrm{d}I_A} = \frac{MG}{\sqrt{2\pi(MI_A^2 + M\omega)}}\left[e^{\omega} + 1 + \frac{\omega}{2G^2}(e^{\omega} - 1)\right] e^{-\frac{1}{2}\left(G + \frac{\omega}{2G}\right)^2} > 0.$$

Consequently, $AC(I_A)$ is a strictly increasing function of I_A. Therefore, we can express the product's profitability according to the value of I_A, and the value of I_A is as large as possible.

3. Estimating I_A Based on Multiple Samples

The historical data of the demand ought to be collected in order to estimate the actual I_A due to unknown μ and σ. For multiple samples of m groups each of size n is given as $\{x_{i1}, x_{i2}, \ldots, x_{in}\}$, where $i = 1, 2, \ldots, m$, let $\bar{x}_i = \sum_{j=1}^{n} x_{ij}/n$ and $s_i^2 = \sum_{j=1}^{n} (x_{ij} - \bar{x}_i)^2/(n-1)$ be the ith sample mean and sample standard deviation, respectively. We first consider the natural estimator $\widehat{I}_A$ which is obtained by replacing the μ and σ by their unbiased estimators $\bar{\bar{x}} = \sum_{i=1}^{m} \bar{x}_i/m$ and $s_p = [\sum_{i=1}^{m} s_i^2/m]^{1/2}$ i.e.,

$$\widehat{I}_A = \frac{\bar{\bar{x}} - T}{s_p}.$$

Furthermore, the natural estimator $\widehat{I}_A$ can be written as

$$\widehat{I}_A = \frac{\bar{\bar{x}} - T}{s_p} = \frac{1}{\sqrt{mn}} \times \frac{\frac{\bar{\bar{x}} - \mu}{\sigma/\sqrt{mn}} + \frac{\mu - T}{\sigma/\sqrt{mn}}}{\sqrt{\frac{m(n-1)s_p^2/\sigma^2}{m(n-1)}}}$$
$$= \frac{1}{\sqrt{mn}} \times \frac{Z + \sqrt{mn}I_A}{\sqrt{\frac{W}{m(n-1)}}} = \frac{1}{\sqrt{mn}} \times \frac{Z_A}{\sqrt{\frac{W}{m(n-1)}}},$$

where $Z_A = Z + \sqrt{mn}I_A \sim N(\sqrt{mn}I_A, 1)$, $Z \sim N(0,1)$, $W = m(n-1)s_p^2/\sigma^2 \sim \chi^2_{m(n-1)}$. Since Z_A and W are independent, the estimator $\widehat{I}_A$ is distributed as $(mn)^{-1/2} t_{m(n-1)}(\theta)$, where $t_{m(n-1)}(\theta)$ is a non central t distribution with $m(n-1)$ degree of freedom and the non-centrality parameter $\theta = (mn)^{1/2} I_A$. Since

$$E(\widehat{I}_A) = \frac{[m(n-1)/2]^{1/2}\Gamma[(m(n-1)-1)/2]}{\Gamma[m(n-1)/2]} \times I_A \neq I_A,$$

the natural estimator $\widehat{I}_A$ is biased. To tackle this problem, we add a correction factor as follows:

$$b = \frac{[2/m(n-1)]^{1/2}\Gamma[m(n-1)/2]}{\Gamma[(m(n-1)-1)/2]}.$$

Then we can obtain unbiased estimator $b\widehat{I}_A$, which is denoted by $\widetilde{I}_A$. Since $\widetilde{I}_A$ is based solely on the complete and sufficient statistics $(\bar{\bar{x}}, s_p^2)$, it leads to the conclusion that the estimator $\widetilde{I}_A$ is the uniformly minimum variance unbiased estimator (UMVUE) of I_A based on multiple samples. The probability density function of $\widetilde{I}_A = R$ is derived as follows (for more details, see Appendix A):

$$f_R(r) = \frac{\sqrt{2mn}\left(\frac{m(n-1)}{2}\right)^{\frac{m(n-1)}{2}}}{b\sqrt{\pi}\Gamma\left(\frac{m(n-1)}{2}\right)} \int_0^\infty v^{m(n-1)} \exp\left\{-\frac{1}{2}\left[\frac{(vr - bI_A)^2}{b^2/mn} + m(n-1)v^2\right]\right\} dv,$$
$$-\infty < r < \infty.$$

Figure 1 plots the probability density function of R, $I_A = 1.0, 1.5, 2.0$, $n = 3, 4, 5$, and $m = 10, 25, 40$ (from bottom to top in plots). From Fig. 1, we can see that (1) for fixed sample sizes m and n, the variance of $\widetilde{I}_A = R$ increases as I_A increases; (2) for a fixed n and I_A, the variance of $\widetilde{I}_A = R$ decreases as m increases; and (3) for a fixed m and I_A, the variance of $\widetilde{I}_A = R$ decreases as n increases.

3.1. *Discussion*

For the case with unequal sample sizes, the natural estimator of I_A can straightforwardly be expressed as:

$$\widetilde{I}'_A = \frac{\bar{\bar{x}}' - T}{s'_p},$$

where $\bar{\bar{x}}' = \sum_{i=1}^m n_i\bar{x}_i/N$ is the grand mean of the overall sample, $N = \sum_{i=1}^m n_i$ is the number of observation in the total sample, and $s'^2_p = \sum_{i=1}^m (n_i - 1)s_i^2/(N-m)$ is the pooled sample variance. The estimator $\widetilde{I}'_A$ can be rewritten as

$$\widetilde{I}'_A = \frac{\bar{\bar{x}}' - T}{s'_p} = \frac{1}{\sqrt{N}} \times \frac{\frac{\bar{\bar{x}}'-\mu}{\sigma/\sqrt{N}} + \frac{\mu - T}{\sigma/\sqrt{N}}}{\sqrt{\frac{(N-m)s'^2_p/\sigma^2}{N-m}}}$$
$$= \frac{1}{\sqrt{N}} \times \frac{Z + \sqrt{N}I_A}{\sqrt{\frac{W'}{N-m}}} = \frac{1}{\sqrt{N}} \times \frac{Z'_A}{\sqrt{\frac{W'}{N-m}}},$$

where $Z'_A = Z + \sqrt{N}I_A \sim N(\sqrt{N}I_A, 1)$, $W' = (N-m)s'^2_p/\sigma^2 \sim \chi^2_{N-m}$. Since Z'_A and W' are independent, the estimator $\widehat{I}'_A$ is distributed as $(N)^{-1/2}t_{N-m}(\theta')$, where $t_{N-m}(\theta')$

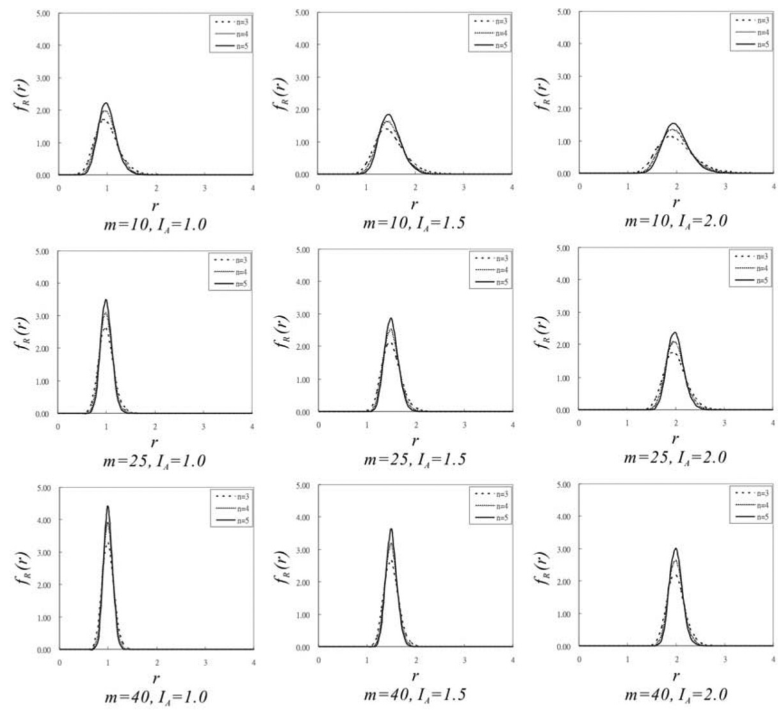

Figure 1. Probability density function plots of r for $n = 3$, 4, 5 and $m = 10$, 25, 40 (from bottom to top in plots).

is a non central t distribution with $N - m$ degree of freedom and the non-centrality parameter $\theta' = (N)^{1/2} I_A$. Similarly, we also obtain the unbiased estimator $\widetilde{I}'_A = b'\widehat{I}'_A$, where $b' = [2/N - m]^{1/2}\Gamma[N - m/2]/\Gamma[(N - m - 1)/2]$ is the correction factor of $\widehat{I}'_A$.

4. Testing I_A Based on Multiple Samples

In order to judge whether the product's profitability meets the designated requirement, the achievable capacity index I_A is adopted to be a criterion. We consider the following hypothesis testing:

$$H_0 : I_A \leq C \quad \text{vs. } H_1 : I_A > C,$$

where C is the designated requirement of I_A. The critical value is used for making decision in profitability performance testing with designated Type I error α (i.e., the chance of incorrectly judging $I_A \leq C$ as $I_A > C$). Since $\widetilde{I}_A$ is distributed as $b(mn)^{-1/2} t_{m(n-1)}(\theta)$, the critical value, c_0, is determined by:

$$\alpha = \Pr\left\{\widetilde{I}_A \geq c_0 \mid I_A = C\right\}$$
$$= \Pr\left\{\frac{b t_{m(n-1)}(\theta)}{\sqrt{mn}} \geq c_0 \mid I_A = C\right\} = \Pr\left\{t_{m(n-1)}(\theta) \geq \frac{\sqrt{mn} c_0}{b} \mid I_A = C\right\}.$$

Thus, we have

$$c_0 = \frac{bt_{m(n-1),\alpha}(\theta)}{\sqrt{mn}},$$

where $c_0 = t_{m(n-1),\alpha}(\theta)$ is the upper α quantile of a non central t distribution with $m(n-1)$ degrees of freedom satisfying $\Pr\{t_{m(n-1)}(\theta) \geq t_{m(n-1),\alpha}(\theta)\} = \alpha$. If the

Table 1
Critical values c_0 for $\alpha = 0.05, 0.025, 0.01$ based on multiple samples with $n = 3(1)5$, $m = 10(2)40$, and $C = 1.0(0.2)2.0$

$\alpha = 0.05$	$C = 1.0$			$C = 1.2$			$C = 1.4$		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	1.445	1.367	1.319	1.690	1.601	1.548	1.938	1.838	1.778
11	1.422	1.348	1.303	1.664	1.580	1.530	1.910	1.815	1.759
12	1.402	1.332	1.289	1.642	1.563	1.515	1.885	1.796	1.742
13	1.385	1.318	1.277	1.623	1.547	1.502	1.864	1.778	1.728
14	1.369	1.305	1.267	1.606	1.533	1.490	1.845	1.763	1.715
15	1.355	1.294	1.257	1.590	1.521	1.479	1.828	1.750	1.704
16	1.343	1.284	1.248	1.577	1.510	1.470	1.813	1.738	1.693
17	1.332	1.275	1.240	1.564	1.500	1.461	1.799	1.727	1.684
18	1.321	1.267	1.233	1.553	1.491	1.453	1.787	1.717	1.675
19	1.312	1.259	1.227	1.543	1.483	1.446	1.775	1.708	1.668
20	1.304	1.252	1.221	1.533	1.475	1.440	1.765	1.700	1.660
21	1.296	1.246	1.215	1.524	1.468	1.434	1.755	1.692	1.654
22	1.288	1.240	1.210	1.516	1.461	1.428	1.746	1.685	1.647
23	1.281	1.234	1.205	1.509	1.455	1.423	1.738	1.678	1.642
24	1.275	1.229	1.200	1.502	1.449	1.418	1.730	1.672	1.636
25	1.269	1.224	1.196	1.495	1.444	1.413	1.723	1.666	1.631
26	1.264	1.219	1.192	1.489	1.439	1.409	1.716	1.660	1.627
27	1.258	1.215	1.188	1.483	1.434	1.405	1.710	1.655	1.622
28	1.253	1.211	1.185	1.478	1.430	1.401	1.704	1.650	1.618
29	1.249	1.207	1.182	1.472	1.426	1.397	1.698	1.646	1.614
30	1.244	1.204	1.178	1.468	1.422	1.394	1.693	1.641	1.610
31	1.240	1.200	1.175	1.463	1.418	1.390	1.688	1.637	1.607
32	1.236	1.197	1.172	1.458	1.414	1.387	1.683	1.633	1.603
33	1.232	1.194	1.170	1.454	1.411	1.384	1.678	1.629	1.600
34	1.228	1.191	1.167	1.450	1.407	1.381	1.674	1.626	1.597
35	1.225	1.188	1.165	1.446	1.404	1.379	1.669	1.622	1.594
36	1.222	1.185	1.162	1.443	1.401	1.376	1.665	1.619	1.591
37	1.218	1.182	1.160	1.439	1.398	1.374	1.662	1.616	1.588
38	1.215	1.180	1.158	1.436	1.396	1.371	1.658	1.613	1.586
39	1.212	1.177	1.156	1.433	1.393	1.369	1.654	1.610	1.583
40	1.210	1.175	1.154	1.429	1.391	1.367	1.651	1.607	1.581

(*continued*)

Table 1
Continued

$\alpha = 0.05$	$C = 1.6$			$C = 1.8$			$C = 2.0$		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	2.188	2.076	2.011	2.441	2.316	2.244	2.694	2.558	2.479
11	2.157	2.052	1.990	2.406	2.290	2.222	2.657	2.529	2.454
12	2.130	2.030	1.972	2.377	2.266	2.202	2.625	2.504	2.433
13	2.106	2.012	1.956	2.351	2.246	2.185	2.597	2.482	2.415
14	2.086	1.995	1.942	2.328	2.228	2.170	2.572	2.462	2.398
15	2.067	1.981	1.929	2.308	2.212	2.156	2.550	2.445	2.384
16	2.051	1.967	1.918	2.290	2.198	2.144	2.530	2.430	2.371
17	2.036	1.955	1.908	2.274	2.185	2.133	2.513	2.416	2.359
18	2.022	1.945	1.899	2.259	2.173	2.123	2.497	2.403	2.348
19	2.010	1.935	1.890	2.245	2.162	2.114	2.482	2.391	2.338
20	1.998	1.925	1.882	2.233	2.153	2.105	2.468	2.380	2.329
21	1.988	1.917	1.875	2.221	2.143	2.097	2.456	2.371	2.320
22	1.978	1.909	1.868	2.211	2.135	2.090	2.444	2.361	2.312
23	1.969	1.902	1.862	2.201	2.127	2.083	2.434	2.353	2.305
24	1.960	1.895	1.856	2.191	2.120	2.077	2.424	2.345	2.298
25	1.952	1.889	1.851	2.183	2.113	2.071	2.414	2.337	2.292
26	1.945	1.883	1.845	2.175	2.106	2.065	2.405	2.330	2.286
27	1.938	1.877	1.841	2.167	2.100	2.060	2.397	2.324	2.280
28	1.931	1.872	1.836	2.160	2.094	2.055	2.389	2.317	2.275
29	1.925	1.867	1.832	2.153	2.089	2.050	2.382	2.312	2.270
30	1.919	1.862	1.828	2.147	2.084	2.046	2.375	2.306	2.265
31	1.914	1.857	1.824	2.141	2.079	2.042	2.368	2.301	2.260
32	1.908	1.853	1.820	2.135	2.074	2.038	2.362	2.296	2.256
33	1.903	1.849	1.817	2.129	2.070	2.034	2.356	2.291	2.252
34	1.898	1.845	1.813	2.124	2.065	2.030	2.350	2.286	2.248
35	1.894	1.841	1.810	2.119	2.061	2.027	2.345	2.282	2.244
36	1.889	1.838	1.807	2.114	2.057	2.023	2.340	2.278	2.241
37	1.885	1.834	1.804	2.110	2.054	2.020	2.335	2.274	2.237
38	1.881	1.831	1.801	2.105	2.050	2.017	2.330	2.270	2.234
39	1.877	1.828	1.798	2.101	2.047	2.014	2.326	2.266	2.231
40	1.873	1.825	1.796	2.097	2.044	2.011	2.321	2.263	2.228

(*continued*)

observed value of the statistic $\widetilde{I}_A = w$ is higher than the critical value, the null hypothesis is rejected. We then conclude that the profitability is better than designated requirement with $(1 - \alpha) \times 100\%$ confidence level. Note that the p-value can be also adopted for making decisions in this testing, which presents the actual risk of misjudging $I_A \le C$ as $I_A > C$, i.e.,

$$p - \text{value} = \Pr\left\{\widetilde{I}_A \ge w \mid I_A = C\right\}$$

Table 1
Continued

$\alpha = 0.025$	$C = 1.0$			$C = 1.2$			$C = 1.4$		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	1.559	1.455	1.393	1.817	1.698	1.629	2.079	1.945	1.868
11	1.528	1.431	1.373	1.783	1.672	1.607	2.041	1.915	1.843
12	1.502	1.410	1.355	1.753	1.649	1.587	2.008	1.890	1.822
13	1.479	1.392	1.340	1.728	1.629	1.570	1.980	1.868	1.803
14	1.459	1.376	1.326	1.705	1.611	1.555	1.955	1.849	1.787
15	1.441	1.362	1.314	1.685	1.595	1.542	1.933	1.832	1.772
16	1.425	1.349	1.303	1.668	1.581	1.530	1.913	1.816	1.759
17	1.410	1.337	1.293	1.652	1.569	1.519	1.896	1.802	1.748
18	1.397	1.327	1.284	1.637	1.557	1.510	1.879	1.790	1.737
19	1.385	1.317	1.276	1.624	1.547	1.501	1.865	1.778	1.727
20	1.374	1.308	1.269	1.612	1.537	1.492	1.851	1.768	1.718
21	1.364	1.300	1.262	1.600	1.528	1.485	1.839	1.758	1.710
22	1.355	1.293	1.255	1.590	1.520	1.478	1.828	1.749	1.702
23	1.346	1.286	1.249	1.580	1.512	1.471	1.817	1.740	1.695
24	1.338	1.279	1.244	1.571	1.505	1.465	1.807	1.732	1.688
25	1.330	1.273	1.238	1.563	1.498	1.459	1.798	1.725	1.682
26	1.323	1.267	1.233	1.555	1.492	1.454	1.789	1.718	1.676
27	1.317	1.262	1.229	1.548	1.486	1.449	1.781	1.712	1.670
28	1.310	1.257	1.224	1.541	1.480	1.444	1.773	1.705	1.665
29	1.304	1.252	1.220	1.534	1.475	1.439	1.766	1.700	1.660
30	1.299	1.248	1.216	1.528	1.470	1.435	1.759	1.694	1.655
31	1.293	1.243	1.213	1.522	1.465	1.431	1.753	1.689	1.651
32	1.288	1.239	1.209	1.516	1.461	1.427	1.747	1.684	1.647
33	1.284	1.235	1.206	1.511	1.457	1.423	1.741	1.679	1.643
34	1.279	1.231	1.202	1.506	1.452	1.420	1.735	1.675	1.639
35	1.275	1.228	1.199	1.501	1.449	1.417	1.730	1.671	1.635
36	1.270	1.225	1.196	1.497	1.445	1.413	1.725	1.667	1.632
37	1.266	1.221	1.194	1.492	1.441	1.410	1.720	1.663	1.628
38	1.263	1.218	1.191	1.488	1.438	1.407	1.715	1.659	1.625
39	1.259	1.215	1.188	1.484	1.435	1.404	1.711	1.655	1.622
40	1.255	1.212	1.186	1.480	1.431	1.402	1.707	1.652	1.619

(*continued*)

$$= \Pr\left\{\frac{bt_{m(n-1)}(\theta)}{\sqrt{mn}} \geq w \mid I_A = C\right\}$$

$$= \Pr\left\{t_{m(n-1)}(\theta) \geq \frac{w\sqrt{mn}}{b} \mid I_A = C\right\}. \tag{4}$$

If p-value$< \alpha$, the null hypothesis is rejected. We conclude that the profitability is better than designated requirement with the actual Type I error p-value (rather than

Table 1
Continued

$\alpha = 0.025$	$C = 1.6$			$C = 1.8$			$C = 2.0$		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	2.344	2.194	2.108	2.610	2.445	2.350	2.879	2.697	2.594
11	2.301	2.162	2.081	2.564	2.409	2.321	2.829	2.659	2.562
12	2.266	2.134	2.058	2.525	2.379	2.296	2.786	2.626	2.535
13	2.234	2.110	2.038	2.491	2.353	2.274	2.749	2.598	2.511
14	2.207	2.089	2.020	2.461	2.330	2.255	2.716	2.573	2.490
15	2.183	2.070	2.004	2.434	2.310	2.238	2.687	2.551	2.472
16	2.161	2.053	1.990	2.411	2.291	2.222	2.662	2.531	2.455
17	2.142	2.038	1.977	2.389	2.275	2.208	2.639	2.513	2.440
18	2.124	2.024	1.965	2.370	2.260	2.195	2.618	2.496	2.426
19	2.108	2.011	1.955	2.353	2.246	2.184	2.598	2.482	2.414
20	2.093	2.000	1.945	2.336	2.233	2.173	2.581	2.468	2.402
21	2.080	1.989	1.936	2.322	2.222	2.163	2.565	2.455	2.392
22	2.067	1.979	1.927	2.308	2.211	2.154	2.550	2.444	2.382
23	2.055	1.970	1.920	2.295	2.201	2.146	2.536	2.433	2.373
24	2.044	1.961	1.912	2.283	2.192	2.138	2.523	2.423	2.364
25	2.034	1.953	1.905	2.272	2.183	2.130	2.511	2.413	2.356
26	2.025	1.946	1.899	2.262	2.175	2.123	2.500	2.404	2.349
27	2.016	1.939	1.893	2.252	2.167	2.117	2.490	2.396	2.341
28	2.008	1.932	1.887	2.243	2.160	2.111	2.480	2.388	2.335
29	2.000	1.926	1.882	2.234	2.153	2.105	2.470	2.381	2.328
30	1.992	1.920	1.877	2.226	2.146	2.099	2.461	2.374	2.323
31	1.985	1.914	1.872	2.219	2.140	2.094	2.453	2.367	2.317
32	1.978	1.909	1.867	2.211	2.134	2.089	2.445	2.361	2.312
33	1.972	1.904	1.863	2.204	2.129	2.084	2.437	2.355	2.306
34	1.966	1.899	1.859	2.198	2.124	2.080	2.430	2.349	2.302
35	1.960	1.894	1.855	2.191	2.119	2.076	2.423	2.344	2.297
36	1.954	1.890	1.851	2.185	2.114	2.071	2.417	2.339	2.292
37	1.949	1.885	1.847	2.179	2.109	2.067	2.411	2.334	2.288
38	1.944	1.881	1.844	2.174	2.105	2.064	2.405	2.329	2.284
39	1.939	1.877	1.841	2.169	2.100	2.060	2.399	2.324	2.280
40	1.934	1.874	1.837	2.163	2.096	2.057	2.393	2.320	2.276

(*continued*)

α). Table 1 displays the critical values for $\alpha = 0.05, 0.025, 0.01$ based on multiple samples $n = 3(1)5$, $m = 10(2)40$, and $I_A = 1.0(0.2)2.0$. Next, we also calculate the β risk. Once the sample size and the α risk are defined, the power function $Power(I_A)$ may be expressed by:

$$Power(I_A) = \Pr\left\{\widetilde{I}_A \geq c_0 \mid I_A > C\right\}$$

Table 1
Continued

$\alpha = 0.01$	$C = 1.0$			$C = 1.2$			$C = 1.4$		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	1.705	1.564	1.484	1.980	1.820	1.730	2.260	2.079	1.978
11	1.663	1.533	1.458	1.934	1.785	1.701	2.208	2.041	1.947
12	1.628	1.506	1.436	1.894	1.756	1.676	2.165	2.008	1.919
13	1.598	1.483	1.416	1.860	1.730	1.655	2.127	1.980	1.896
14	1.571	1.463	1.399	1.831	1.707	1.636	2.094	1.955	1.875
15	1.548	1.444	1.384	1.805	1.687	1.619	2.065	1.933	1.857
16	1.527	1.428	1.370	1.781	1.669	1.604	2.039	1.913	1.840
17	1.508	1.414	1.358	1.760	1.653	1.590	2.016	1.895	1.825
18	1.491	1.400	1.347	1.742	1.638	1.578	1.995	1.879	1.812
19	1.476	1.388	1.336	1.724	1.625	1.567	1.976	1.864	1.799
20	1.462	1.377	1.327	1.709	1.613	1.556	1.959	1.851	1.788
21	1.449	1.367	1.318	1.694	1.601	1.547	1.943	1.838	1.777
22	1.437	1.357	1.310	1.681	1.591	1.538	1.928	1.827	1.768
23	1.425	1.348	1.303	1.668	1.581	1.530	1.914	1.816	1.759
24	1.415	1.340	1.296	1.657	1.572	1.522	1.902	1.806	1.750
25	1.405	1.333	1.289	1.646	1.564	1.515	1.890	1.797	1.742
26	1.396	1.325	1.283	1.636	1.556	1.508	1.879	1.788	1.735
27	1.388	1.319	1.277	1.627	1.548	1.502	1.868	1.780	1.728
28	1.380	1.312	1.272	1.618	1.541	1.496	1.859	1.772	1.722
29	1.372	1.306	1.267	1.610	1.535	1.490	1.849	1.765	1.716
30	1.365	1.301	1.262	1.602	1.529	1.485	1.841	1.758	1.710
31	1.359	1.295	1.257	1.594	1.523	1.480	1.833	1.752	1.704
32	1.352	1.290	1.253	1.587	1.517	1.475	1.825	1.746	1.699
33	1.346	1.285	1.249	1.581	1.512	1.470	1.817	1.740	1.694
34	1.340	1.281	1.245	1.574	1.507	1.466	1.810	1.734	1.689
35	1.335	1.276	1.241	1.568	1.502	1.462	1.804	1.729	1.685
36	1.330	1.272	1.237	1.562	1.497	1.458	1.797	1.724	1.680
37	1.325	1.268	1.234	1.557	1.493	1.454	1.791	1.719	1.676
38	1.320	1.264	1.230	1.551	1.488	1.451	1.785	1.714	1.672
39	1.315	1.260	1.227	1.546	1.484	1.447	1.780	1.710	1.669
40	1.311	1.257	1.224	1.541	1.480	1.444	1.774	1.706	1.665

(continued)

$$= \Pr\left\{\frac{bt_{m(n-1)}(\theta)}{\sqrt{mn}} \geq c_0 \mid I_A > C\right\}$$

$$= \Pr\left\{t_{m(n-1)}(\theta) \geq \frac{c_0\sqrt{mn}}{b} \mid I_A > C\right\}.$$

The power of the test for $C = 1.0, 1.4, 1.8$ vs. various values of I_A, $n = 3, 4, 5$, $m = 10(10)40$, and $\alpha = 0.05$ is showed in Figure 2. It is seen that the larger the sample size, the larger the power of test, and consequently, the smaller the β risk.

Table 1
Continued

α = 0.01	C = 1.6			C = 1.8			C = 2.0		
	n			n			n		
m	3	4	5	3	4	5	3	4	5
10	2.543	2.341	2.229	2.829	2.606	2.482	3.117	2.872	2.737
11	2.486	2.299	2.195	2.767	2.560	2.444	3.049	2.822	2.696
12	2.438	2.263	2.165	2.714	2.520	2.412	2.991	2.779	2.661
13	2.396	2.232	2.139	2.668	2.486	2.384	2.942	2.742	2.630
14	2.360	2.205	2.116	2.629	2.456	2.359	2.899	2.710	2.604
15	2.328	2.180	2.096	2.594	2.430	2.337	2.860	2.681	2.580
16	2.300	2.159	2.078	2.562	2.406	2.318	2.827	2.656	2.559
17	2.274	2.139	2.062	2.534	2.385	2.300	2.796	2.633	2.540
18	2.251	2.122	2.047	2.509	2.366	2.284	2.769	2.612	2.522
19	2.230	2.105	2.034	2.486	2.348	2.269	2.744	2.593	2.506
20	2.211	2.091	2.021	2.465	2.332	2.256	2.721	2.575	2.492
21	2.194	2.077	2.010	2.446	2.318	2.244	2.700	2.559	2.478
22	2.177	2.065	1.999	2.429	2.304	2.232	2.681	2.544	2.466
23	2.162	2.053	1.989	2.412	2.291	2.221	2.663	2.531	2.455
24	2.148	2.042	1.980	2.397	2.279	2.212	2.647	2.518	2.444
25	2.135	2.032	1.972	2.383	2.268	2.202	2.631	2.506	2.434
26	2.123	2.023	1.964	2.369	2.258	2.194	2.617	2.495	2.424
27	2.112	2.014	1.956	2.357	2.248	2.185	2.603	2.484	2.416
28	2.101	2.005	1.949	2.345	2.239	2.178	2.591	2.474	2.407
29	2.091	1.997	1.942	2.334	2.231	2.170	2.579	2.465	2.399
30	2.082	1.990	1.936	2.324	2.223	2.164	2.567	2.456	2.392
31	2.073	1.983	1.930	2.314	2.215	2.157	2.557	2.448	2.385
32	2.064	1.976	1.924	2.305	2.208	2.151	2.547	2.440	2.378
33	2.056	1.970	1.919	2.296	2.201	2.145	2.537	2.433	2.372
34	2.048	1.964	1.914	2.287	2.194	2.139	2.528	2.425	2.366
35	2.041	1.958	1.909	2.279	2.188	2.134	2.519	2.419	2.360
36	2.034	1.952	1.904	2.272	2.182	2.129	2.511	2.412	2.355
37	2.027	1.947	1.900	2.264	2.176	2.124	2.503	2.406	2.349
38	2.021	1.942	1.895	2.257	2.170	2.119	2.495	2.400	2.344
39	2.015	1.937	1.891	2.251	2.165	2.115	2.488	2.394	2.339
40	2.009	1.932	1.887	2.244	2.160	2.111	2.481	2.389	2.335

4.1. *Profitability Evaluation Procedure*

In the following, we develop a simple step-by-step procedure for the practitioners to use for judging whether the profitability meets the designated requirement.

Step 1. Determine the value of the designated requirement C, α-risk, and sample size (m, n).

Step 2. Calculate the value of the estimator, $\widetilde{I}_A$, form the given sample.

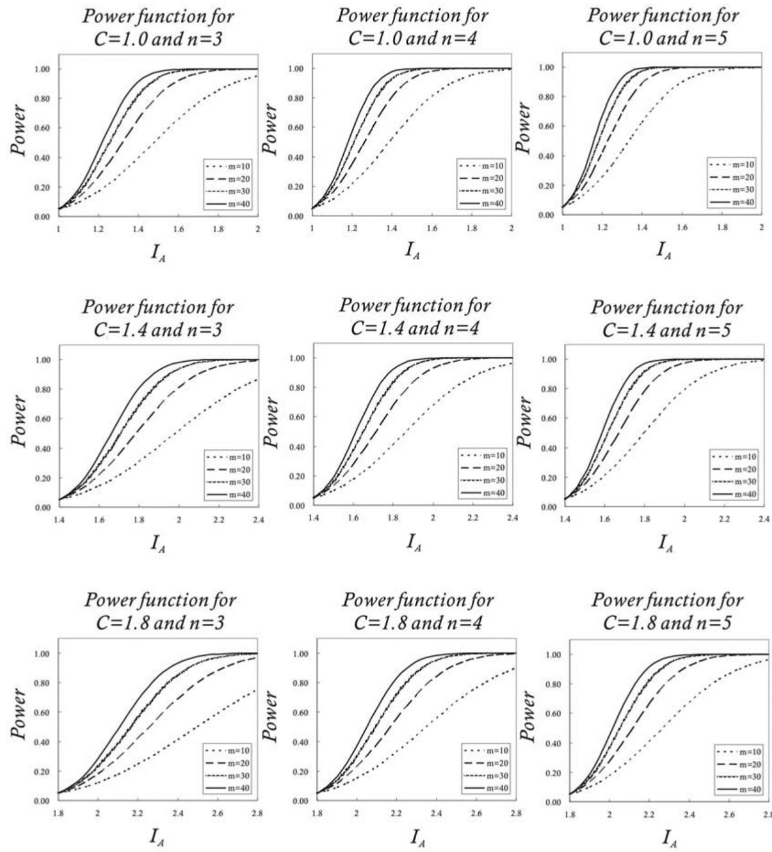

Figure 2. Power curves for $C = 1.0$, 1.4, 1.8, with sample sizes $n = 3$, 4, 5 and $m = 10$, 20, 30, 40.

Step 3. Find the corresponding critical value, $\widetilde{I}_A$, based on α, C, m, and n form the Table 1. Also, we calculate the p-value from the Eq. (4) based on C, m, and n.

Step 4. Conclude that the profitability meets the designated requirement if $\widetilde{I}_A > c_0$ (or p-value $< \alpha$). Otherwise, the profitability does not meet the designated requirement.

5. Application Example

We consider a dessert store, which provides delicious donuts made fresh daily in Taipei, Taiwan. This store is a Japanese-owned incarnation of a donut franchise formerly out of America. Fifty varieties of donuts are offered, one half of them are American style and another half of them are Japanese style. All of the donuts range from NT$20–35. Besides, each donut comes with a label indicating its level of sweetness. However, these donuts only have approximate 12-h shelf-life due to texture deterioration. In order to provide the best texture, this store prepares the donut each day and disposes the overdue donuts after closing store. If the

Table 2
The profitability for $(p, c, c_d, c_s, k) = (25, 10, 1, 3, 2500)$ and $I_A = 0.00(0.01)3.09$

I_A	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.4249	0.4285	0.4322	0.4359	0.4395	0.4432	0.4469	0.4506	0.4543	0.4580
0.1	0.4617	0.4654	0.4691	0.4728	0.4765	0.4802	0.4839	0.4876	0.4913	0.4950
0.2	0.4987	0.5024	0.5061	0.5099	0.5136	0.5173	0.5210	0.5246	0.5283	0.5320
0.3	0.5357	0.5394	0.5431	0.5467	0.5504	0.5541	0.5577	0.5614	0.5650	0.5687
0.4	0.5723	0.5759	0.5795	0.5831	0.5868	0.5903	0.5939	0.5975	0.6011	0.6046
0.5	0.6082	0.6117	0.6152	0.6188	0.6223	0.6258	0.6293	0.6327	0.6362	0.6396
0.6	0.6431	0.6465	0.6499	0.6533	0.6567	0.6601	0.6634	0.6668	0.6701	0.6734
0.7	0.6768	0.6800	0.6833	0.6866	0.6898	0.6931	0.6963	0.6995	0.7026	0.7058
0.8	0.7090	0.7121	0.7152	0.7183	0.7214	0.7245	0.7275	0.7305	0.7335	0.7365
0.9	0.7395	0.7425	0.7454	0.7483	0.7512	0.7541	0.7570	0.7598	0.7627	0.7655
1.0	0.7683	0.7710	0.7738	0.7765	0.7792	0.7819	0.7846	0.7873	0.7899	0.7925
1.1	0.7951	0.7977	0.8002	0.8028	0.8053	0.8078	0.8103	0.8127	0.8151	0.8176
1.2	0.8200	0.8223	0.8247	0.8270	0.8293	0.8316	0.8339	0.8361	0.8384	0.8406
1.3	0.8428	0.8449	0.8471	0.8492	0.8513	0.8534	0.8555	0.8575	0.8596	0.8616
1.4	0.8636	0.8655	0.8675	0.8694	0.8713	0.8732	0.8751	0.8769	0.8788	0.8806
1.5	0.8824	0.8841	0.8859	0.8876	0.8893	0.8910	0.8927	0.8944	0.8960	0.8976
1.6	0.8992	0.9008	0.9024	0.9039	0.9054	0.9070	0.9084	0.9099	0.9114	0.9128
1.7	0.9142	0.9156	0.9170	0.9184	0.9197	0.9211	0.9224	0.9237	0.9250	0.9262
1.8	0.9275	0.9287	0.9299	0.9311	0.9323	0.9335	0.9346	0.9358	0.9369	0.9380
1.9	0.9391	0.9401	0.9412	0.9423	0.9433	0.9443	0.9453	0.9463	0.9473	0.9482
2.0	0.9492	0.9501	0.9510	0.9519	0.9528	0.9537	0.9545	0.9554	0.9562	0.9570
2.1	0.9579	0.9587	0.9594	0.9602	0.9610	0.9617	0.9625	0.9632	0.9639	0.9646
2.2	0.9653	0.9660	0.9666	0.9673	0.9680	0.9686	0.9692	0.9698	0.9704	0.9710
2.3	0.9716	0.9722	0.9728	0.9733	0.9739	0.9744	0.9749	0.9754	0.9759	0.9764
2.4	0.9769	0.9774	0.9779	0.9784	0.9788	0.9793	0.9797	0.9801	0.9806	0.9810
2.5	0.9814	0.9818	0.9822	0.9826	0.9830	0.9833	0.9837	0.9841	0.9844	0.9848
2.6	0.9851	0.9854	0.9857	0.9861	0.9864	0.9867	0.9870	0.9873	0.9876	0.9879
2.7	0.9881	0.9884	0.9887	0.9889	0.9892	0.9894	0.9897	0.9899	0.9902	0.9904
2.8	0.9906	0.9908	0.9911	0.9913	0.9915	0.9917	0.9919	0.9921	0.9923	0.9925
2.9	0.9926	0.9928	0.9930	0.9932	0.9933	0.9935	0.9937	0.9938	0.9940	0.9941
3.0	0.9943	0.9944	0.9945	0.9947	0.9948	0.9949	0.9951	0.9952	0.9953	0.9954

manufacturing quantity can not satisfy the demand, then the manager must pay the lost sale opportunity cost. Therefore, the donut exactly belongs to the newsboy-type product. Now, the manager would like to know whether the profitability of the designated donut is higher than some level. If it is incapable, the manager is going to plan a sale promotion. The selling price of the donut is NT$25 per unit, the manufacturing cost is NT$10 per unit, and the target profit is NT$2500. In addition, the lost sale opportunity cost is NT$3 per unit. The disposal cost for overdue donut is NT$1 per unit. Table 2 displays the profitability for $(p, c, c_d, c_s, k) = (25, 10, 1, 3, 2500)$ and $I_A = 0.00(0.01)3.09$. For the demand data, because of Saturday and Sunday are always have high demand. In order to avoid these extreme values, we only consider the demand on Monday-Friday. Note that the unsatisfied demand is record. Twenty samples of size five (i.e., 20-weeks demand)

Table 3
The 5-sample data each of 20 observations

Demand units/day	Observations in sample of size five				
Group (Week)	MON	TUE	WED	THU	FRI
1	185	169	189	201	192
2	221	220	191	180	203
3	208	213	217	212	196
4	224	195	208	214	224
5	202	218	208	197	189
6	189	198	212	204	225
7	219	196	190	229	198
8	188	215	188	191	185
9	189	206	194	191	186
10	215	225	198	191	212
11	178	173	186	224	212
12	183	214	244	212	217
13	221	194	187	194	174
14	172	217	205	216	214
15	191	199	183	196	179
16	187	223	183	219	198
17	176	205	211	216	198
18	199	184	235	186	184
19	187	183	206	212	203
20	192	178	210	180	195

are displayed in Table 3. Due to the store's propertied restriction, the prices, costs, and sample data were modified. If the designated requirement of the I_A value is $C = 1.8$, we implement the hypothesis testing: $H_0 : I_A \leq 1.8$ vs. $H_1 : I_A > 1.8$. We first use the Kolmogorov-Smirnov test for the sample data from Table 3 to confirm if the data is normally distributed. A test result in p-value> 0.05, which means that data is normally distributed. For the data displayed in Table 3, we calculate the overall sample mean, pooled sample variance, and sample estimator, and obtain that $\bar{\bar{x}} = 200.48$, $s_p^2 = 237.10$, and $\widetilde{I}_A = 2.1753$. If the Type I error α-risk set to 0.05, the critical value is 2.1050 form Table 1. Since $\widetilde{I}_A = 2.1753 > 2.1050 = c_0$, we conclude that the profitability meets the designated requirement, than it is unnecessary to plan a sale promotion. For calculating the p-value, we obtain p-value=0.0244<0.05. Therefore, it suggests the same evaluation result.

6. Conclusions

In this article, we developed a new index, achievable capacity index, I_A, which has a simple-form to measure the profitability of the newsboy-type product with normally distributed demand. In practical application, the demand data is collected from multiple samples rather than single sample. Hence, we considered an unbiased and effective estimator of I_A based on multiple samples. The evaluation testing of I_A is investigated, i.e., $H_0 : I_A \leq C$ vs. $H_1 : I_A > C$, where C is the designated requirement of I_A. The critical value of the test is calculated to determine evaluation result under the preset risk (Type I error). The implementation of the existing statistical theory for the profitability of Newsboy-type product makes it possible to apply the complicated theoretical results to the actual productions. For convenience, we also provided a simple step-by-step procedure for the practitioners to use in making decisions. Finally, a real-world example on the sales of donuts is presented to illustrate the practicality of the exact approach.

Appendix A

We first define $R = b(\bar{\bar{x}} - T)/s_p = Y/V$, where $Y = b(\bar{\bar{x}} - T)/\sigma$ and $V = s_p/\sigma$. It is easy to see that if the demand is normally distributed, $D \sim N(\mu, \sigma^2)$, we have $Y \sim N(b(\mu - T)/\sigma, b^2/mn)$. Since $m(n-1)s_p^2/\sigma^2$follows the chi-squared distribution with $m(n-1)$ degree of freedom, we then have $V^2 \sim \Gamma(m(n-1)/2, 2/m(n-1))$. By using the technique of change-of-variable, the probability density function of V is derived as follows:

$$f_V(v) = \frac{2v^{m(n-1)-1}}{\Gamma\left(\frac{m(n-1)}{2}\right)\left(\frac{2}{m(n-1)}\right)^{\frac{m(n-1)}{2}}} \exp\left\{-\frac{m(n-1)}{2}v^2\right\}.$$

Because Y and V are independent continuous random variables, the probability density function of R can be obtained by the *Jacobian approach*, i.e.,

$$\begin{aligned} f_R(r) &= \int_0^\infty f_Y(vr) f_V(v)\,|v|\,dv \\ &= \frac{\sqrt{2mn}\left(\frac{m(n-1)}{2}\right)^{\frac{m(n-1)}{2}}}{b\sqrt{\pi}\Gamma\left(\frac{m(n-1)}{2}\right)} \int_0^\infty v^{m(n-1)} \exp\left\{-\frac{1}{2}\left[\frac{(vr - bI_A)^2}{b^2/mn} + m(n-1)v^2\right]\right\} dv. \end{aligned}$$

References

Alfares, H. K., Elmorra, H. H. (2005). The distribution-free newsboy problem: Extensions to the shortage penalty case. *Int. J. Prod. Econ.* 93–94:465–477.

Berk, E., Gurler, U., Levine, R. A. (2007). Bayesian demand updating in the lost sales newsvendor problem: a two-moment approximation. *Eur. J. Oper. Res.* 182:256–281.

Ismail, B., Louderback, J. (1979). Optimizing and satisfying in stochastic cost-volume profit analysis. *Decis. Sci.* 10:205–217.

Kevork, I. S. (2010). Estimating the optimal order quantity and the maximum expected profit for single-period inventory decisions. *Omega* 38:218–227.

Khouja, M. (1995). The newsboy problem under progressive multiple discounts. *Eur. J. Oper. Res.* 84:458–466.

Lau, H. (1997). Simple formulas for the expected costs in the newsboy problem: An educational note. *Eur. J. Oper. Res.* 100:557–561.

Lau, H. S. (1980). The newsboy problem under alternative optimization objectives. *J. Oper. Res. Soc.* 31:525–535.

Moon, I., Choi, S. (1995). The distribution free newsboy problem with balking. *J. Oper. Res. Soc.* 46:537–542.

Mostard, J., Koster, R., Teunter, R. (2005). The distribution-free newsboy problem with resalable returns. *Int. J. Prod. Econ.* 97:329–342.

Nahmias, S. (1993). *Production and Operations Management*. Boston: Irwin.

Ouyang, L. Y., Chang, H. C. (2002). A minimax distribution free procedure for mixed inventory models involving variable lead time with fuzzy lost sales. *Int. J. Prod. Econ.* 76:1–12.

Ouyang, L. Y., Wu, K. S. (1998). A minimax distribution free procedure for mixed inventory model with variable lead time. *Int. J. Prod. Econ.* 56–57:511–516.

Sankarasubramanian, E., Kumaraswamy, S. (1983). Optimal order quantity for predetermined level of profit. *Manage. Sci.* 29:512–514.

Scarf, H. (1958). A min-max solution of an inventory problem. In: Arrow, K., Karlin, S., Scarf, H., eds. *Studies in the Mathematical Theory of Inventory and Production*. Stanford: Stanford University Press, pp. 201–209.

Shih, W. (1979). A general decision model for cost-volume-profit analysis under uncertainty. *Account. Rev.* 54:687–706.