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A SEMIANALYTICAL SOLUTION FOR RESIDUAL DRAWDOWN AT A FINITE DIAMETER WELL IN A CONFINED AQUIFER[1]

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ABSTRACT: After the end of pumping the water level in the observation well starts to recover and the reduced drawdown during the recovery period is named as the residual drawdown. Traditional approaches in analyzing the data of residual drawdown for estimating the aquifer hydraulic parameters are mostly based on the application of superposition principle and Theis equation. In addition, the effect of wellbore storage is commonly ignored in the evaluation even if the test well has a finite diameter. In this article, we develop a mathematical model for describing the residual drawdown with considering the wellbore storage effect and the existing drawdown distribution produced by the pumping part of the test. The Laplace-domain solution of the model is derived using the Laplace transform technique and the time-domain result is inverted based on the Stehfest algorithm. This new solution shows that the residual drawdown associated with the boundary and initial conditions are related to the well drawdown and the aquifer drawdown, respectively. The well residual drawdown will be overestimated by the Theis residual drawdown solution in the early recovery part if neglecting the wellbore storage. On the other hand, the Theis residual drawdown solution can be used to approximate the present residual drawdown solution in the late recovery part of the test.

(KEY TERMS: aquifer characteristics; ground water hydrology; drawdown; wells; mathematical models; pumping tests.)

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INTRODUCTION

In general, there are three steps involved in the estimation of hydraulic parameters of confined aquifers, such as the transmissivity and storativity. First, a continuous or instantaneous stress is applied to the test well. The response of the aquifer to the stress is then measured temporally or spatially at the same well and/or observation wells. Finally, the measured response, i.e., the drawdown, is analyzed using the Theis equation (Theis, 1935) or Cooper-Jacob equation (e.g., Batu, 1998).

The groundwater level will rise after the stoppage of pumping. The depth to the rising in water levels during the recovery period minus the depth to the static water level is known as the residual drawdown. The analysis of residual drawdown data can provide an independent check on hydraulic parameters determined from the analysis of data observed in the

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pumping period (e.g., Todd and Mays, 2005). Generally speaking, the residual data is more reliable than the drawdown data because there are no pumping effects involved. Traditional approaches in analyzing residual drawdown data are commonly developed based on the superposition principle and the Theis equation or Cooper-Jacob equation (e.g., Berg, 1975; Goode, 1997; Samani and Pasandi, 2003; Singh, 2003; Zheng *et al.*, 2005). The use of Theis equation in the analysis of residual drawdown data however implies that the effects of well radius and wellbore storage are ignored. Willmann *et al.* (2007) suggested that the designed recovery period should not be shorter than twice the pumping duration if the Theis equation is employed to analyze the recovery data. An equation describing the residual drawdown and taking account of wellbore storage would be helpful in analyzing the early recovery test data.

Picking (1994) proposed a type-curve match method for analyzing recovery data based on tabular well function values computed from of Papadopulos and Cooper's solution (1967). Shapiro *et al.* (1998) introduced a conceptual model for describing the early-time recovering water level following the termination of pumping in a well subject to turbulent head losses. Their approximation to the recovering water level in the test well was obtained according to the principle of superposition and the solutions of Papadopulos and Cooper (1967) and Cooper *et al.* (1967), while the former solution is employed to deal with the response of the formation and the latter is used to handle the turbulent head loss. Moreover, the approximation of Shapiro *et al.* (1998, Equation 5) assumed that the pumping period is sufficiently large and the recovery period is very short when Papadopulos and Cooper's solution (1967) is used. No solution for recovery in finite-diameter wells apart from those utilizing the principle of superposition has been presented before. A residual drawdown solution obtained from theoretical development rather than the superposition principle has its need in engineering applications. Yeh and Wang (2009) introduced a residual drawdown model for the recovery period of the test after a constant head injection. Recently, Mills (2010) presented a method for data analysis based on Papadopulos and Cooper's solution (1967) and Picking's equation (1994) for water level recovery following pumping of confined aquifers.

The objective of this study is to develop a mathematical model for describing the residual drawdown taking into consideration the existing drawdown distribution introduced by the previous pumping and the effects of well radius and wellbore storage. This model uses the well drawdown after the stoppage of pumping as the boundary condition along the wellbore and the drawdown distribution from prior pumping as the initial condition. The Laplace domain solution for the residual drawdown to such an initial boundary value problem is obtained based on the Laplace transform technique and the time-domain result is evaluated using the Stehfest algorithm (Stehfest, 1970). This solution is applicable at any elapsed times in both the pumping and recovery periods. In addition, this solution can also be employed to investigate the effect of wellbore storage on well residual drawdown or to determine the hydraulic parameters if coupled with an optimization algorithm. The option of using the Theis residual drawdown solution to approximate the present residual drawdown solution is also examined.

ANALYSIS METHODS

A radial groundwater flow equation describing the drawdown distribution in a homogeneous and isotropic confined aquifer of uniform thickness can be written as

$$\frac{\partial^2 s}{\partial r^2}+\frac{1}{r}\frac{\partial s}{\partial r}=\frac{S}{T}\frac{\partial s}{\partial t}, \quad r_w \le r < \infty \quad \text{and} \quad t>0 \tag{1}$$

where $s(r,t)$ is the drawdown; t is the time; r is the radial distance from the centerline of the test well; r_w is the radius of the well screen; and S and T are the storativity and the transmissivity of the aquifer, respectively.

Consider that a test with a constant pumping rate has been conducted at a finite-diameter well for a period of time. The water level begins to recover when the pumping is terminated. Figure 1 shows the schematic diagram of drawdown distributions at the beginning of both the pumping period and recovery period of the test. The drawdown is denoted as $s_1(r,t_1)$ at pumping time t_1 and the residual drawdown is $s_2(r,t_2)$ at recovery time t_2. The well drawdown in the pumping part and the well residual drawdown in the recovery part are expressed as $H_1(t_1)$ and $H_2(t_2)$, respectively. The initial drawdown prior to pumping is zero everywhere (i.e., $s_1(r,0)=0$) while the initial residual drawdown for the recovery part is equal to the final drawdown (i.e., $s_2(r,0)=s_1(r,t_p)$) when the pumping is terminated at $t_{p.}$ A semi-analytical expression for residual drawdown is developed considering the effect of wellbore storage as following.

Drawdown Distribution in Pumping Period

Both the aquifer drawdown and well drawdown before pumping are assumed to be zero. The inner

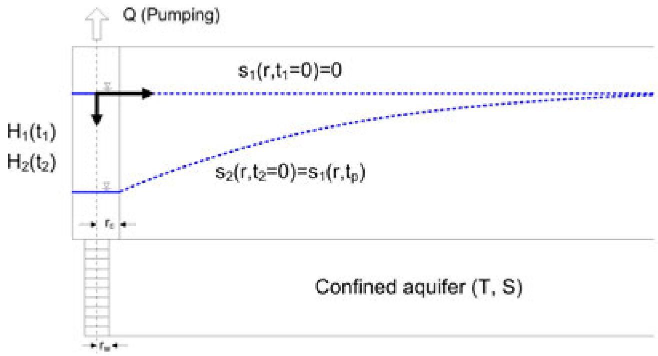

FIGURE 1. Schematic Diagram of Initial Conditions for an Aquifer Test in Pumping and Recovery Periods. The variables with subscript 1 are in the pumping period while those with subscript 2 are in the recovery period.

boundary condition for maintaining a constant pumping rate Q at the test well with the effect of wellbore storage taken into consideration can be expressed as

$$-2\pi r_w T \frac{\partial s_1(r,t_1)}{\partial r}\bigg|_{r=r_w} + \pi r_c^2 \frac{dH_1(t)}{dt}\bigg|_{t=t_1} = Q, \quad t_1 > 0 \tag{2}$$

where r_c is the radius of the well casing. Zero drawdown at infinity is posed as the outer boundary condition during the pumping period. In addition, the well drawdown $H_1(t_1)$ is equal to the drawdown at the wellbore $s_1(r_w, t_1)$ according to the continuity condition. The drawdown solution considering the wellbore storage given by Papadopulos and Cooper (1967) is

$$s_1(\rho,\tau_1) = \frac{2Q\alpha}{\pi^2 T}\int_0^\infty \left[1-\exp\left(-\frac{\tau_1}{\alpha}x^2\right)\right] \times \frac{J_0(\rho x)[xY_0(x)-2\alpha Y_1(x)]-Y_0(\rho x)[xJ_0(x)-2\alpha J_1(x)]}{[xY_0(x)-2\alpha Y_1(x)]^2+[xJ_0(x)-2\alpha J_1(x)]^2}\frac{dx}{x^2} \tag{3}$$

where $\rho = r/r_w$ is the dimensionless radial distance, $\tau_1 = Tt_1/r_c^2$ is the dimensionless pumping time, $\alpha = r_w^2 S/r_c^2$ is the coefficient of wellbore storage, J_0 and Y_0 are the Bessel functions of the first and second kinds of order zero, respectively, J_1 and Y_1 are the Bessel functions of the first and second kinds of order one, respectively, and x is a dummy variable. The residual drawdown will, therefore, start with $s_1(\rho, \tau_p)$ for $1 \le \rho < \infty$ when pumping is ended at the dimensionless pumping time τ_p (i.e., $\tau_1 = \tau_p$). The procedure of numerical evaluation for Equation (3) is similar to the one used in Yang *et al.* (2006), which includes a root search scheme, a numerical integration method, and the Shanks method (Shanks, 1955). In the numerical procedure, the Newton method is used to find the root of the integrand and the Gaussian quadrature is employed to perform the numerical integration within the interval between two consecutive roots. The integral is then transformed to an infinite series, which can be accelerated the convergence by the Shanks method when summing up the series.

If the effect of wellbore storage is negligible, the second term on the left-hand side (LHS) of boundary condition (i.e., Equation 2) should be removed. Then, the solution of Papadopulos and Cooper (1967), Equation (3), reduces to (Carslaw and Jaeger, 1959, p. 338)

$$s_1(r,t_1) = \frac{Q}{\pi^2 r_w T}\int_0^\infty \left[1-\exp\left(-\frac{Tt_1}{S}x^2\right)\right] \times \frac{Y_0(rx)J_1(r_w x)-J_0(rx)Y_1(r_w x)}{J_1^2(r_w x)+Y_1^2(r_w x)}\frac{dx}{x^2} \tag{4}$$

If the test well is treated as a line source, the well radius in the first term on the LHS of Equation (2) approaches zero (e.g., $r_w \to 0$) and the second term on the LHS of Equation (2) vanishes. Equation (3) will further reduce to the Theis equation.

Figure 2 shows the curves for the dimensionless well drawdown $s_1(\rho = 1, \tau_1)/(Q/2\pi T)$ *vs.* dimensionless pumping time τ_1 plotted based on Equations (3 and 4) and the Theis equation for the coefficient of wellbore storage α ranging from 10^{-5} to 10^{-1}. Note that Equation (4) can also be evaluated using the same numerical procedure described above. It demonstrates that the difference in dimensionless well drawdown between Equation (4) and the Theis equation decreases rapidly as τ_1 increases and/or α decreases. In addition, the difference in dimensionless well drawdown between Equation (3) and the Theis equation also decreases with increasing τ_1 and/or decreasing α. Papadopulos and Cooper (1967) suggested that the Theis equation can approximate Equation (3) when $\tau_1 > 2.5 \times 10^2$. The difference in dimensionless drawdown between Equation (3) and the Theis equation is 2×10^{-2} when $\tau_1 = 2.5 \times 10^2$ and less than 1×10^{-2} when $\tau_1 > 5 \times 10^2$ for α ranging from 10^{-5} to 10^{-1}.

For real-world well-hydraulics problems, the radius of the well casing generally ranges from 0.05 m to 0.25 m and the hydraulic conductivity for fine sand is about 2 m/day (2.28×10^{-3} cm/s) (Batu, 1998). Accordingly, τ_1 equals 10^3 if $t_1 = 12$ h, $r_c = 0.10$ m (four inches), and the thickness of the confined aquifer is 10 m. In this case, the difference in dimensionless well drawdown between Equation (3) and the Theis equation is less than 1×10^{-3} m when Q is less than 12.5 m^3/day. Under this circumstance, the Theis equation compares reasonably well with Equation (3).

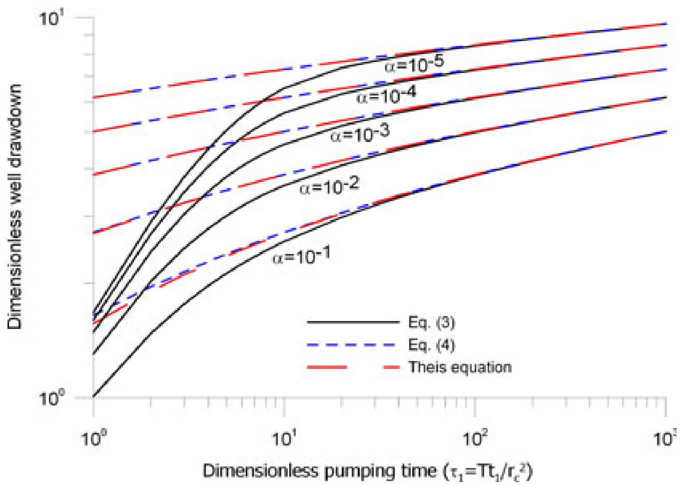

FIGURE 2. The Curves of Dimensionless Well Drawdown *vs.* Dimensionless Pumping Time for α Ranging from 10^{-5} to 10^{-1}.

Residual Drawdown Distribution in Recovery Period

Recovery of water level follows the termination of the pumping. The following relationship is imposed according to the continuity requirement for the flow rate between the aquifer and test well

$$2\pi r_w T \frac{\partial s_2(r,t_2)}{\partial r}\bigg|_{r=r_w} = \pi r_c^2 \frac{dH_2(t)}{dt}\bigg|_{t=t_2}, \quad t_2 > 0 \tag{5}$$

The initial conditions for residual drawdown away from the test well and the water level in the test well are, respectively, written as

$$s_2(r,0) = s_1(r,t_p), \quad r_w \le r < \infty \tag{6}$$

$$H_2(0) = s_1(r_w,t_p) \tag{7}$$

Note that Equation (6) is a function of radial distance and pumping period. The inner and outer boundary conditions for the residual drawdown are, respectively,

$$s_2(r_w,t_2) = H_2(t_2), \quad t_2 > 0 \tag{8}$$

and

$$s_2(\infty,t_2) = 0, \quad t_2 > 0 \tag{9}$$

The detailed derivation for the solution describing the residual drawdown distribution derived using the Laplace transform technique is given in Appendix A and the result is

$$s_2(\rho,\tau_2) = L^{-1}\{A_1(\rho,p) + A_2(\rho,p)\} \tag{10}$$

with

$$A_1(\rho,p) = s_1(1,\tau_p)\frac{K_0(\sqrt{\alpha p}\,\rho)}{h(p)} - \alpha\left[\int_1^\rho f(p,x)dx\right] I_0(\sqrt{\alpha p}\,\rho)$$

$$A_2(\rho,p) = \frac{2\sqrt{\alpha p}I_1(\sqrt{\alpha p}) - pI_0(\sqrt{\alpha p})}{h(p)}\left[\alpha\int_1^\infty f(p,x)dx\right]K_0(\sqrt{\alpha p}\,\rho) + \left[\alpha\int_1^\infty f(p,x)dx\right]I_0(\sqrt{\alpha p}\,\rho) + \left[\alpha\int_1^\rho g(p,x)dx\right]K_0(\sqrt{\alpha p}\,\rho)$$

$$f(p,x) = x\, s_1(x,\tau_p)\, K_0(\sqrt{\alpha p}\,x)$$

$$g(p,x) = x\, s_1(x,\tau_p)\, I_0(\sqrt{\alpha p}\,x)$$

$$h(p) = pK_0(\sqrt{\alpha p}) + 2\sqrt{\alpha p}K_1(\sqrt{\alpha p})$$

where L^{-1} denotes the inverse Laplace transform operator, p is the Laplace variable, $s_1(1,\tau_p)$ and $s_1(x,\tau_p)$ are the drawdowns in the test well and aquifer, respectively, at τ_p, I_0 and K_0 are the modified Bessel functions of the first and second kinds of order zero, respectively, and I_1 and K_1 are the modified Bessel functions of the first and second kinds of order one, respectively.

The first and second terms on the right-hand side (RHS) of Equation (10) are produced from the inner boundary condition and the initial condition, respectively. Equation (10) can be numerically inverted using the Stehfest algorithm (Stehfest, 1970; Chang and Yeh, 2009). Note that this solution can be employed to assess the effect of wellbore storage on well residual drawdown or to determine the hydraulic parameters if coupled with an optimization algorithm as presented, for example, in Chen and Yeh (2009) and Yeh and Chen (2007).

Traditional approaches in dealing with the recovery test generally assume a hypothetical recharge rate equaling the pumping rate at the termination of pumping (Todd and Mays, 2005). With the application of the Theis equation and the superposition principle, the residual drawdown distribution may be expressed as (Batu, 1998)

$$s_2(\rho,\tau_2) = \frac{Q}{4\pi T}\left[W\left(\frac{\alpha\rho^2}{4(\tau_p+\tau_2)}\right) - W\left(\frac{\alpha\rho^2}{4\tau_2}\right)\right] \tag{11}$$

where W is the Theis well function. Note that Equation (11), referred to as the Theis residual drawdown solution hereinafter, is the same as the one presented in Picking (1994, Equation 2) but in a slightly different form. It is obvious that Equation (11) is much easier to evaluate than the present residual drawdown solution, Equation (10). One can approximate Equation (10) by Equation (11) for all practical purposes if the effect of well radius is negligible. In the next section, we aim to examine the effect of wellbore storage on the well residual drawdown and compare the presented residual drawdown solution with the Theis residual drawdown solution.

ANALYSIS RESULTS

As discussed earlier, if the dimensionless pumping time τ_p is greater than 5×10^2, the drawdown, $s_1(\rho, \tau_p)$ for $1 \leq \rho < \infty$, in Equation (10) can be approximated by the Theis equation. Assuming $\tau_p > 5 \times 10^2$ and $\rho = 1$, the well residual drawdown inferred from Equation (10) can then be simplified as

$$s_2(1,\tau_2) = \frac{Q}{4\pi T} L^{-1} \left\{ W\left(\frac{\alpha}{4\tau_p}\right) \frac{K_0(\sqrt{\alpha p})}{h(p)} + \frac{2\alpha}{h(p)} \int_1^{\infty} x\, W\left(\frac{\alpha x^2}{4\tau_p}\right) K_0(\sqrt{\alpha p}\, x)\, dx \right\} \quad (12)$$

The first and second terms on the RHS of Equation (12) arise from the inner boundary condition and initial condition, respectively. In addition, Equation (12) can be reduced to the well drawdown equation given by Cooper *et al.* (1967, Equation 7) if zero drawdown is assumed as the initial condition.

According to Equation (12), we define a normalized well residual drawdown as $s_2(1,\tau_2)/\lfloor (Q/2\pi T)\ W(\alpha/4\tau_p) \rfloor$, where the denominator $(Q/2\pi T)W(\alpha/4\tau_p)$ is the double of well drawdown at τ_p and the numerator $s_2(1,\tau_2)$ in Equation (12) is evaluated using the Stehfest algorithm.

Figure 3 exhibits the behavior of normalized well residual drawdown as a function of τ_2 for α ranging from 10^{-5} to 10^{-1} when the pumping is ended at $\tau_p = 10^3$. This figure indicates that the present solution (i.e., Equation 12) is close to the Theis residual drawdown solution (i.e., Equation 11) when $\tau_2 > 50$. In addition, this result also suggests that the effect of wellbore storage is negligible because the normalized difference between these two residual drawdown solutions is less than 1×10^{-2} after $\tau_2 > 50$. Note that the curves shown on Figure 3 were computed from a Fortran code we developed based on Equation (12) and the Stehfest algorithm.

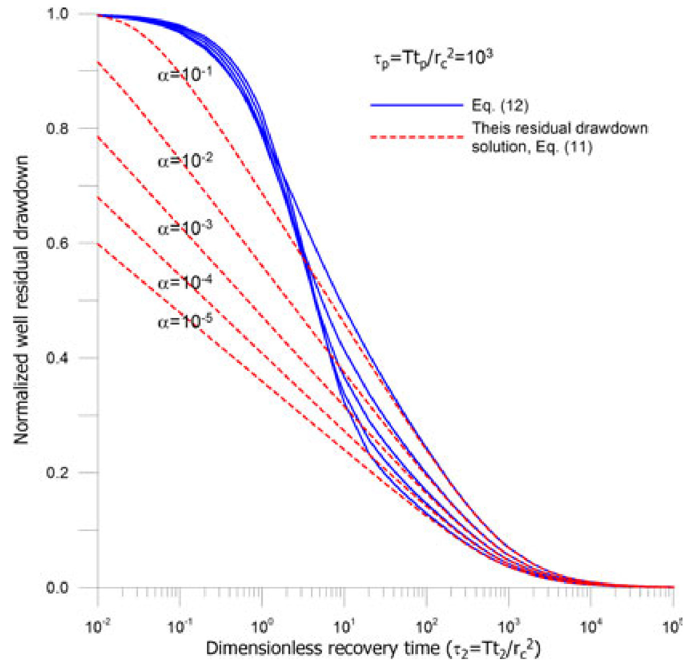

FIGURE 3. The Curves of Normalized Well Residual Drawdowns *vs.* Dimensionless Recovery Time for α Ranging from 10^{-5} to 10^{-1} at $\tau_p = 10^3$.

CONCLUSIONS

Traditionally, the analysis of residual drawdown from a recovery test involves applying the superposition principle and Theis equation, which in fact ignores the effects of well radius and wellbore storage. In this study, we develop a semi-analytical model for describing the recovery (or residual drawdown) taking into consideration the effect of wellbore storage, where the existing drawdown distribution from the pumping part of the test is treated as the initial condition. The Laplace-domain solution of this model is obtained based on the Laplace transform technique and the time-domain result is inverted by the Stehfest algorithm. This solution can be applied to the problems with any elapsed times since the pumping stopped and recovery began. Based on the derived solution, the well residual drawdown is contributed from two parts; one is the inner boundary condition related to the well drawdown while the other is the initial condition related to the aquifer drawdown produced during the pumping part of the test. The presented residual drawdown solution reduces to the solution of Cooper *et al.* (1967) if a zero drawdown is used as the initial condition. In addition, this residual drawdown solution can be approximated by the Theis residual drawdown solution in the case of large recovery time, when the effect of wellbore storage is negligible. When the dimensionless recovery time exceeds 50, the difference in normalized well residual drawdown between the presented residual drawdown solution and Theis residual drawdown solution will be less than 1×10^{-2} for the dimensionless pumping time equal to 10^3.

APPENDIX A: DERIVATION OF EQUATION (10)

By taking the Laplace transform with respect to time, the subsidiary equations of Equation (1) and Equations (5-9) can be written as

$$\frac{d^2\overline{s_2}}{dr^2}+\frac{1}{r}\frac{d\overline{s_2}}{dr}=\frac{S}{T}\left[p\overline{s_2}-s_1(r,t_p)\right] \qquad (A1)$$

$$\left.\frac{d\overline{s_2}(r,p)}{dr}\right|_{r=r_w}=\frac{r_c^2}{2r_wT}\left[p\,\overline{s_2}(r_w,p)-s_1(r_w,t_p)\right] \qquad (A2)$$

$$\overline{s_2}(\infty,p)=0 \qquad (A3)$$

where $\overline{s_2}(r,p)$ denotes the Laplace transform of $s_2(r,t)$.

The general solution to (A1) to (A3) can be expressed as (Kreyszig, 2006)

$$\overline{s_2}(r,p)=[c_1I_0(qr)+c_2K_0(qr)]+[\phi_1I_0(qr)+\phi_2K_0(qr)] \qquad (A4)$$

with

$$q=\sqrt{\frac{S}{T}p}$$

$$c_1=\frac{S}{T}\int_{r_w}^{\infty}xs_1(x,t_p)K_0(qx)dx$$

$$c_2=\frac{\frac{r_c^2}{2r_wT}s_1(r_w,t_p)}{\frac{r_c^2}{2r_wT}pK_0(qr_w)+qK_1(qr_w)}+\frac{qI_1(qr_w)-\frac{r_c^2}{2r_wT}pI_0(qr_w)}{\frac{r_c^2}{2r_wT}pK_0(qr_w)+qK_1(qr_w)}\left[\frac{S}{T}\int_{r_w}^{\infty}xs_1(x,t_p)K_0(qx)dx\right]$$

$$\phi_1=-\frac{S}{T}\int_{r_w}^{r}xs_1(x,t_p)K_0(qx)\,dx$$

$$\phi_2=\frac{S}{T}\int_{r_w}^{r}xs_1(x,t_p)\,I_0(qx)\,dx$$

Equation (A4) is a Laplace-domain solution which can be reduced to that of Cooper *et al.* (1967, Equation 7) if $s_1(r,t_p)=0$ for $r_w \le r < \infty$. Being expressed in terms of dimensionless variables and taking the inverse Laplace transform, the time-domain solution for the residual drawdown is then given by Equation (10).

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