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Abstract
Laser dynamics of asynchronous rational harmonic mode-locked (ARHM) fiber soliton lasers are investigated in detail. In particular, based on the unique laser dynamics of asynchronous mode-locking, we have developed a new method for determining the effective active modulation strength *in situ* for ARHM lasers. By measuring the magnitudes of the slowly oscillating pulse timing position and central frequency, the effective phase modulation strength at the multiplication frequency of rational harmonic mode-locking can be accurately inferred. The method can be a very useful tool for developing ARHM fiber lasers.

(Some figures may appear in colour only in the online journal)

1. Introduction

High repetition rate mode-locked fiber lasers are attractive light sources for many applications. Based on the active harmonic mode-locking (HM) mechanism, the fiber laser can be mode-locked at an applied modulation frequency that is an integer multiple of the cavity fundamental repetition frequency [1, 2]. To further boost the pulse repetition rate, rational harmonic mode-locking (RHM) techniques have also been intensively developed [3–13]. In these laser systems, the modulation frequency is detuned by a rational ratio of the cavity fundamental repetition frequency and the repetition rate multiplication is achieved through frequency-matched higher order modulation harmonics. The intra-cavity active modulation can be implemented either by intensity or phase modulation. Intensity modulation schemes are most commonly used because one can utilize the nonlinear transmission curve of the Mach–Zehnder type intensity modulators to more effectively generate the higher order modulation harmonics. However, typical RHM fiber lasers based on intensity modulation may suffer from the drawback of unequal pulse amplitudes. The pulses circulating inside the laser cavity will experience periodic jumps in the modulation phase which will cause the amplitudes of the pulses to oscillate periodically. Such unequalized

amplitude characteristics can be easily seen in the time domain by means of a fast oscilloscope and can also be clearly seen in the RF spectrum of the output pulse train with unwanted harmonic components. To remedy this problem, additional amplitude equalization schemes may be required to equalize the pulse amplitudes to an acceptable level [7–9]. In contrast, phase-modulated RHM lasers have been demonstrated with very good stability [10, 11]. The unequalized amplitude problem is not present due to the use of phase modulation, this being one of its main advantages.

Asynchronous harmonic mode-locking (AHM) is another active harmonic mode-locking scheme that adopts the intra-cavity phase modulation [14–18]. In contrast to synchronous harmonic mode-locking, the modulation frequency is slightly detuned with respect to the cavity harmonic frequency by a few tens of kHz. Very stable laser operation with high repetition rates and sub-ps pulse widths has been demonstrated experimentally at 10 GHz. This type of laser exhibits many unique dynamic lasing characteristics, including a slowly oscillating optical central frequency and pulse timing position. A preliminary experimental demonstration of asynchronous rational harmonic mode-locking (ARHM) has also been reported recently [19]. However, the laser dynamics of these ARHM lasers are
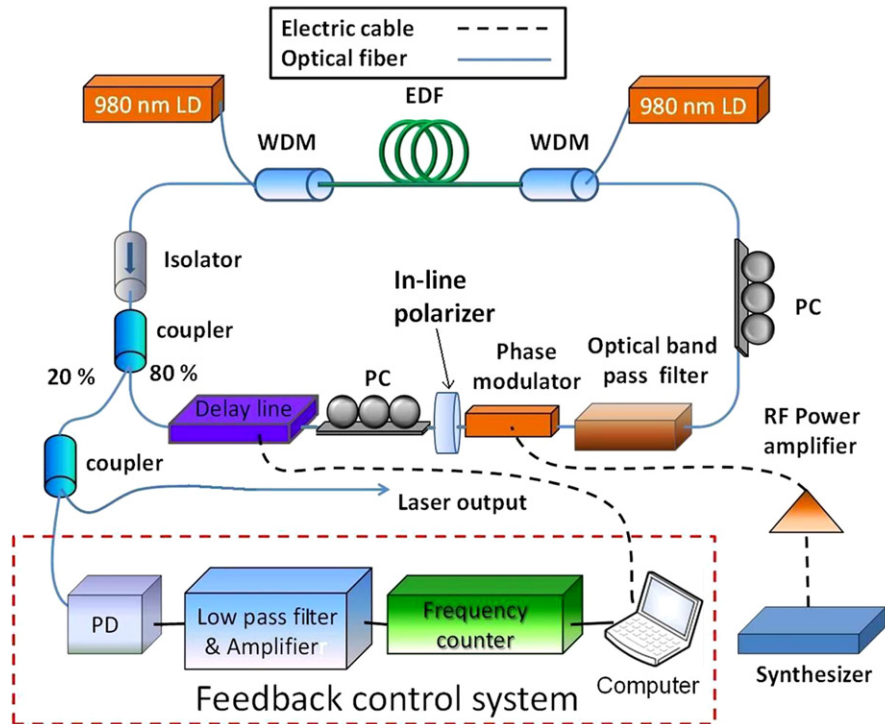
Figure 1. Experimental setup of the asynchronous rational harmonic mode-locked fiber laser.

still less studied. In the present work, we have investigated in detail the laser dynamics of an ARHM Er-doped fiber soliton laser that can be operated at up to 33 GHz, which is the highest repetition rate reported for ARHM lasers so far. Based on the unique laser dynamics of asynchronous mode-locking, we have also successfully developed a new method that allows us to determine the effective phase modulation strength *in situ* for ARHM fiber lasers. For phase-modulated RHM fiber lasers, the effective modulation strength is much smaller as compared to phase-modulated HM lasers. It is thus worthwhile to be able to accurately characterize the effective modulation strength so that one can have more information to improve the laser design. In practice, it is not easy to accurately determine the effective modulation strength of ARHM fiber lasers at the multiplication frequency of rational harmonic mode-locking, mainly due to the unknown characteristics of the electro-optic modulator and electrical connection at all high frequencies and operating conditions. It will be better to be able to perform *in situ* measurements when the laser is under operation. In the present work, by measuring the magnitudes of the slowly oscillating pulse timing position and central frequency induced from asynchronous mode-locking, we will demonstrate that the effective phase modulation strength at the multiplication frequency of rational harmonic mode-locking can be accurately inferred. The results obtained also help to clarify the origin of effective modulation in phase-modulated harmonic mode-locked fiber lasers.

2. Experiment and discussion

2.1. Laser configuration

The experimental setup for the ARHM fiber laser is illustrated in figure 1. Basically it is an all-fiber hybrid mode-locked Er-fiber laser configuration, in which the active mode-locking mechanism is through the use of a LiNbO$_3$ EO phase modulator while the passive mode-locking mechanism is through the polarization additive pulse mode-locking (P-APM) technique [20–23]. A section of 5.5 m long Er-doped fiber is used as the gain fiber, which is bi-directionally pumped by two 980 nm pump laser diodes through two wavelength-division multiplexing (WDM) couplers. Two polarizer controllers are used for polarization control. The in-line polarizer before the EO phase modulator performs as the required polarization-dependent element to act together with the polarization controllers as an effective saturable absorber for implementing the P-APM mechanism. A tunable optical band-pass filter with FWHM = 13.5 nm is used to help stabilize the laser through the combined effects of band-pass filtering and nonlinear spectral broadening. It can also be used to select the lasing wavelength. The average cavity length is estimated to be 24.8 m and the fundamental cavity frequency is 8.3 MHz. A synthesizer is used to provide the RF signals for driving the phase modulator at frequencies of 7 and 11 GHz. With a power amplifier, the signals are amplified to 14 dBm in the case of 7 GHz and 17.1 dBm in the case of 11 GHz, before entering the phase modulator. Third-order ARHM has been successfully achieved with final repetition frequencies of 21 and 33 GHz for the purpose of a comparison study.
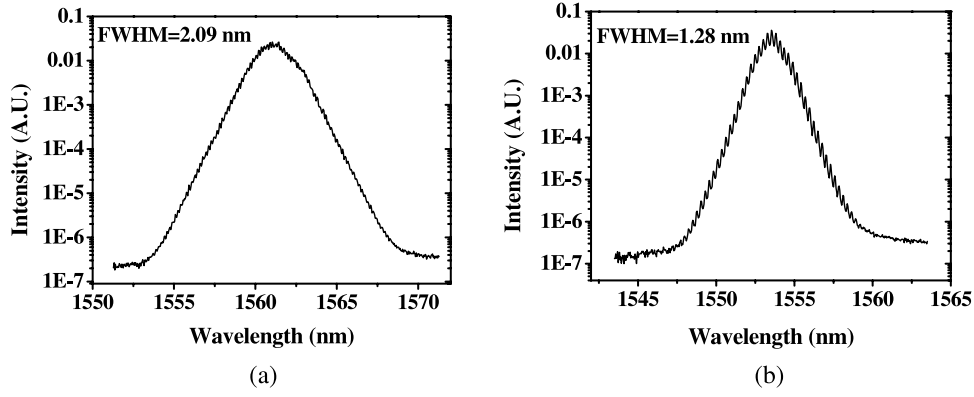
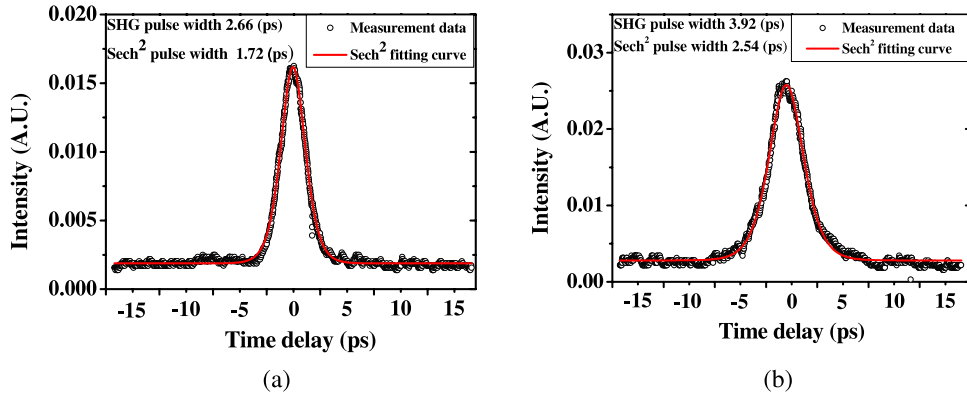**Figure 2.** Output optical spectra at (a) 21 GHz and (b) 33 GHz repetition rates.

Figure 3. Output intensity autocorrelation traces at (a) 21 GHz and (b) 33 GHz repetition rates.
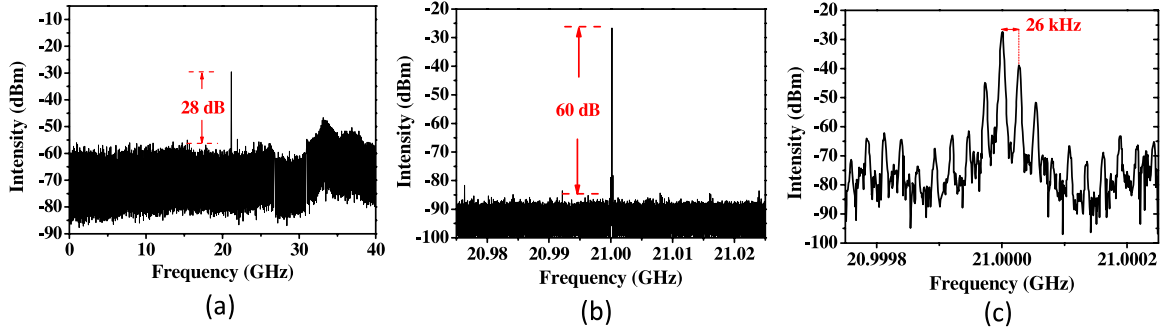

Figure 4. Output RF spectra at 21 GHz (a) 40 GHz-span; (b) 50 MHz-span and (c) 500 kHz-span.

2.2. Laser characteristics

The main laser characteristics of the studied ARHM laser are presented below. The optical spectra are shown in figure 2, with the FWHM bandwidth = 2 nm at 21 GHz and 1.28 nm at 33 GHz. In the time domain, the pulse autocorrelation traces are shown in figure 3. By assuming a sech^2 pulse-shape, the pulse-widths are 1.72 ps and 2.54 ps respectively and the corresponding time–bandwidth products are 0.45 and 0.4 respectively, which indicates the pulse chirps are small. The stability of the laser can be examined from the RF spectra in figures 4 and 5. The 40 GHz-span RF spectra show that the unwanted lower harmonics are suppressed very well due to the phase modulation approach. No additional amplitude equalization is needed. The 50 MHz-span RF spectra show that super-mode noise is also suppressed very well, especially

at 21 GHz. The asynchronous mode-locking characteristics are clearly exhibited in the 500 kHz-span RF spectra with deviation frequencies of $f_d = 26$ kHz at 21 GHz and 34 kHz at 33 GHz, respectively. The multiple side-peaks signify the successful operation of asynchronous mode-locking, under which the pulse timing position of the output pulse train will oscillate slowly at the deviation frequency f_d : $\Delta t_0(t) = \Delta t_0 \cos(2\pi f_d t + \theta)$. One can estimate the magnitude of the slow pulse timing oscillation Δt_0 according to the following equation [18]:

$$\Delta = |J_0[\omega_m \Delta t_0]/J_1[\omega_m \Delta t_0]|^2. \tag{1}$$

Here J_0 and J_1 are Bessel functions, Δ is the power ratio of the zero-order and first-order peaks in figures 4(c)
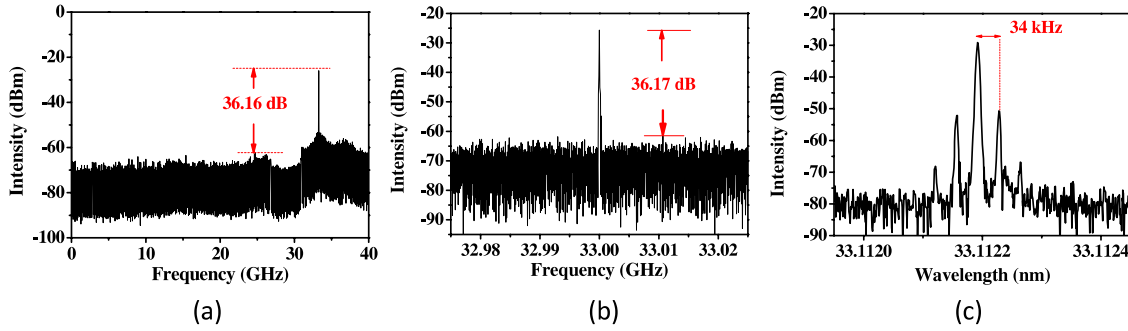
Figure 5. Output RF spectra at 33 GHz (a) 40 GHz-span; (b) 50 MHz-span and (c) 500 kHz-span.
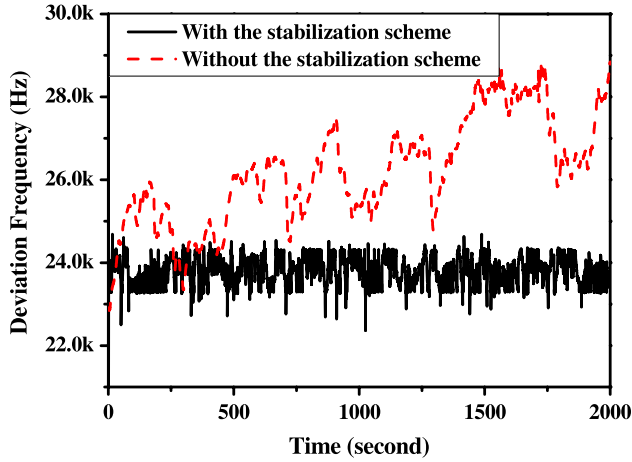

Figure 6. Laser feedback stabilization results.

and 5(c), while $\omega_m = 2\pi f_m$ is the mode-locking frequency. Since the deviation frequency f_d is on the order of tens of kHz and is much smaller than the mode-locking frequency (on the order of 10 GHz), we have safely dropped the deviation frequency term in equation (24) of [18] to put equation (1) into a simpler form. The obtained numbers of Δt_0 are around 2.8 ps at 21 GHz and 0.7 ps at 33 GHz, respectively. The measured RF spectra can be slightly asymmetric due to the smaller slow oscillation of the other pulse parameters. One can simply take the average values of the two first-order peaks for the estimation of the timing oscillation.

The laser can be long-term stabilized through the deviation frequency locking scheme with only low-frequency electronics [17]. The stabilization results are shown in figure 6, allowing us to perform the experiments more easily. In order to determine the magnitude of the oscillating center frequency, one needs to measure the magnitudes of the oscillating timing position for the output pulse train after propagating through sections of single-mode fiber with different lengths. The results are shown in figure 7(a). The magnitude of the oscillating central frequency (wavelength) can then be determined by the following equation [18]:

$$\Delta t_0(L) = \sqrt{\Delta t_0^2 + (\Delta\lambda DL)^2}. \qquad (2)$$

Here L and D are the length and dispersion parameters of the single-mode fiber, while $\Delta\lambda = \lambda^2 \Delta\omega/(2\pi c)$ is the

magnitude of the oscillating central wavelength. The obtained number for $\Delta\lambda$ is 0.58 nm for the ARHM laser mode-locked at 21 GHz.

3. Theory and analysis

3.1. Model equations for laser dynamics

A suitable master equation model for the ARHM laser is as follows:

$$\frac{\partial u(T, t)}{\partial T} = \left(\frac{g_0}{1 + \frac{\int |u|^2 \, dt}{E_s}} - l_0 \right) u(T, t)$$

$$+ (d_r + jd_i) \frac{\partial^2}{\partial t^2} u(T, t) + (k_r + jk_i)|u|^2 u$$

$$+ j \sum_m M_m \cos \left[m \left(N + \frac{1}{p} \right) \omega_f(t + RT) \right] u(T, t). \qquad (3)$$

The first two terms basically represent the standard master equation model for passive mode-locked lasers with an equivalent fast saturable absorber [20]. The coefficients have the following physical meanings: g_0 (linear gain), E_s (gain saturation energy), l_0 (linear loss), d_r (optical filtering), d_i (dispersion), k_r (equivalent fast saturable absorption), k_i (self-phase modulation), T (evolution time in terms of the number of the cavity round trips). The third term is the asynchronous phase modulation term for ARHM lasers. The summation over m indicates that there are multiple harmonic modulation frequencies with different amplitudes. The fundamental modulation frequency is $(N + 1/p)f$, where f is the cavity fundamental repetition frequency, N is the harmonic mode-locking order and p is the rational mode-locking order. Due to the frequency matching requirement, the first effective harmonic modulation frequency is the pth term at the frequency $m = 2\pi f_m = (pN + 1)f$. The effects of the unmatched frequency terms will be very small due to the quick time average and thus can be ignored. This is how the rational mode-locking technique can be used to multiply the laser repetition frequency. At this time, the effective modulation strength will be M_p, under the condition that all the higher order $M_{2p}, M_{3p} \ldots$ terms can be ignored. In this modulation term, R is the linear timing walk-off per round trip due to the modulation frequency offset of asynchronous mode-locking. It can be determined from the offset frequency f_d according to $R = (pN + 1)f_d/f_m^2$.
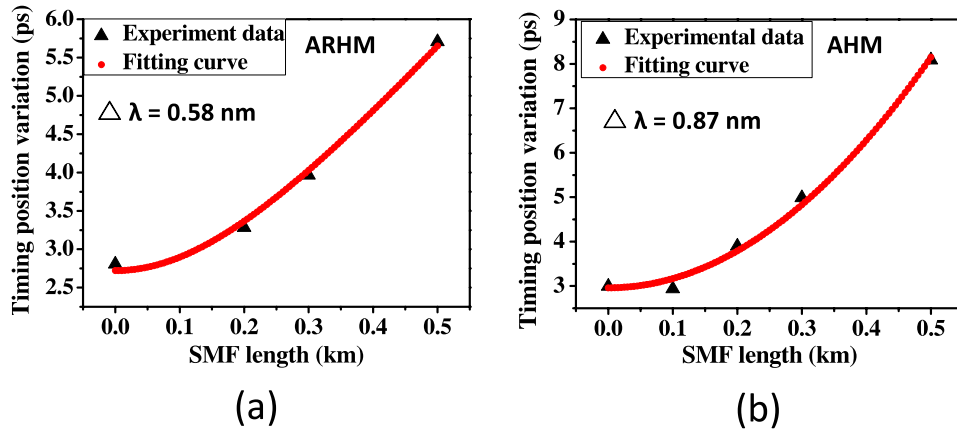
Figure 7. Timing oscillation magnitudes after different lengths of single-mode fiber. (a) ARHM at 21 GHz and (b) AHM at 21 GHz.

After recognizing the fact that theoretically the ARHM fiber soliton laser is similar to the AHM fiber soliton laser with M_p as the effective modulation strength, one can immediately obtain the following approximate expressions for the oscillating magnitudes of the center frequency and pulse timing position under the assumption that the pulse chirp is small [18].

$$\Delta\omega = \frac{M_p}{R}\left[1+\left(\frac{4d_r}{3\tau^2\omega_m R}\right)^2\right]^{-1/2} \quad (4)$$

$$\Delta t_0 = \frac{2d_i M_p}{\omega_m R^2}\left[1+\left(\frac{4d_r}{3\tau^2\omega_m R}\right)^2\right]^{-1/2}. \quad (5)$$

Here τ is the sech^2 pulse-width and is related to the FWHM pulse-width by $\tau = $ FWHM/1.76. Equations (4) and (5) are the main results that will be utilized for *in situ* determining the effective modulation strength M_p and the cavity dispersion parameter d_i in the asynchronous mode-locked laser.

It should be noted that equations (4) and (5) are only approximate solutions to the pulse parameter evolution equations for the central frequency and pulse timing under the decoupling approximation, as has been explained in [18]. If the approximation is not made, it has been proved that the asynchronous to synchronous mode-locking transition will occur when the value of R is below a threshold value dependent on the modulation strength [24]. Therefore equations (4) and (5) are only applicable for asynchronous mode-locking operation when the value of R is above the threshold. When it is below the threshold, synchronous mode-locking occurs and there will be no slow oscillation of the central frequency and the pulse timing.

3.2. Determination of effective modulation strength

First note that on dividing equation (4) by equation (5), one can obtain:

$$d_i = \omega_m R \Delta t_0/(2\Delta\omega). \quad (6)$$

This equation can be used to determine the cavity dispersion coefficient d_i accurately since all the other quantities in the equation can be accurately determined in practice. For the 21 GHz case, the cavity dispersion coefficient d_i is found to be around 0.061 ps^2, which is in good agreement with the direct estimation from the laser configuration.

With the known d_i value, the effective modulation strength M_p can be determined as follows:

$$M_p = \Delta t_0\left[\frac{2d_i}{\omega_m R^2}\right]^{-1}\left[1+\left(\frac{4d_r}{3\tau^2\omega_m R}\right)^2\right]^{1/2}. \quad (7)$$

Note that the square root factor in the equation can also be accurately determined in practice. The coefficient d_r can be estimated from the known filter bandwidth and is found to be around 0.0125 ps^2. The pulse-width FWHM/1.76 can be determined by the autocorrelation measurement. After going through the calculation, we find the effective modulation strength M_p is 0.091 in the 21 GHz case and 0.059 in the 33 GHz case. From the results it is clear that the modulation strength is smaller in the 33 GHz case. This explains why the experimental pulse-width is longer at 33 GHz.

To have more reference data for comparison, we have also applied the whole procedure to the non-rational mode-locking case. The same laser is asynchronous harmonic mode-locked at 21 GHz with a modulation driving power of 13 dBm at 21 GHz. The obtained FWHM pulse-width is 1.38 ps, the deviation frequency is 42 kHz, and the slow timing oscillation magnitude is estimated to be 2.83 ps. The timing oscillation magnitudes after different lengths of single-mode fiber are shown in figure 7(b). The center wavelength oscillation magnitude is determined to be 0.87 nm for this case. Using the same procedure, the effective modulation strength M_p for this non-rational mode-locking case is estimated to be 0.216. When the laser was rational mode-locked at 21 GHz, we observed that the modulation driving signals at 7 GHz also exhibited a 5.3 dBm harmonic component at 21 GHz. By referring to the characterized non-rational mode-locking case, we estimate that a 5.3 dBm harmonic component at 21 GHz will produce an effective modulation strength of 0.089, which is quite close to the previous estimated value

for the ARHM laser mode-locked at 21 GHz. Therefore we can confidently conclude that, for the ARHM laser studied here, the effective modulation at the multiplication frequency should be mainly caused by the harmonics from the nonlinear electric power amplifier that drives the EO phase modulator. Similar conclusions have been made for phase-modulated RHM fiber lasers [9, 10]. This example demonstrates the usefulness of the *in situ* determination method developed here.

4. Conclusion

To conclude, we have experimentally demonstrated an asynchronous rational harmonic mode-locked Er-fiber laser that can be operated up to a 33 GHz repetition frequency with ps output pulse-width. When compared to conventional rational harmonic mode-locked lasers, this new mode of mode-locking exhibits much more equalized pulse amplitudes due to the use of the phase modulation. Currently the highest achievable pulse repetition rate and shortest pulse-width are both limited by the weaker effective modulation strength and the smaller pulse energy (or optical nonlinearity) in the laser cavity. Based on the unique laser dynamics of asynchronous mode-locking, we have developed a new method for *in situ* determination of the effective active modulation strength for these ARHM fiber lasers. By measuring the magnitudes of the slowly oscillating pulse timing position and central wavelength oscillation induced by asynchronous mode-locking, the effective phase modulation strength at the multiplication frequency of rational harmonic mode-locking can be accurately inferred. The method is a very useful tool for further developing ARHM fiber lasers, which are promising for generating uniform ultra-short pulse trains at high pulse repetition rates.

Acknowledgment

References

[1] Bakhshi B and Andrekson P A 2000 40 GHz actively modelocked polarization maintaining erbium fiber ring laser *Electron. Lett.* **5** 411–3

[2] Nakazawa M and Yoshida E 2000 A 40-GHz 850-fs regeneratively FM mode-locked polarization-maintaining erbium fiber ring laser *IEEE Photon. Technol. Lett.* **12** 1613–5

[3] Moores J D, Wong W S and Hall K L 1995 50-Gbit/s optical pulse storage ring using novel rational-harmonic modulation *Opt. Lett.* **20** 2547–9

[4] Ahmed Z and Onodera N 1996 High repetition rate optical pulse generation by frequency multiplication in actively modelocked fiber ring lasers *Electron. Lett.* **32** 455–7

[5] Yoshida E and Nakazawa M 1996 80–200 GHz erbium doped fibre laser using a rational harmonic mode-locking technique *Electron. Lett.* **32** 1370–2

[6] Wu C and Dutta N K 2000 High-repetition-rate optical pulse generation using a rational harmonic mode-locked fiber laser *IEEE J. Quantum Electron.* **36** 145–50

[7] Feng X, Liu Y, Yuan S, Kai G, Zhang W and Dong X 2004 Pulse-amplitude equalization in a rational harmonic mode-locked fiber laser using nonlinear modulation *IEEE Photon. Technol. Lett.* **8** 1813–5

[8] Zhu G and Dutta N K 2005 Eighth-order rational harmonic mode-locked fiber laser with amplitude-equalized output operating at 80 Gbits/s *Opt. Lett.* **17** 2212–4

[9] Zhan L, Wang P H, Gu Z C, Luo S Y and Xia Y X 2005 Amplitude-equalized high-order arbitrary numerator rational harmonic mode-locked pulse generation in fiber-ring lasers using nonlinear polarization rotation *Opt. Express* **13** 836–41

[10] Yang S and Bao X 2006 Rational harmonic mode-locking in a phase-modulated fiber laser *IEEE Photon. Technol. Lett.* **12** 1332–4

[11] Yang S, Cameron J and Bao X 2007 Stabilized phase-modulated rational harmonic mode-locking soliton fiber laser *IEEE Photon. Technol. Lett.* **19** 393–5

[12] Li J, Huang T and Chen L R 2011 Rational harmonic mode-locking of a fiber optical parametric oscillator at 30 GHz *IEEE Photon. J.* **3** 468–70

[13] Fukuchi Y and Maeda J 2011 Characteristics of rational harmonic mode-locked short-cavity fiber ring laser using a bismuth-oxide-based erbium-doped fiber and a bismuth-oxide-based highly nonlinear fiber *Opt. Express* **19** 22502–9

[14] Doerr C R, Haus H A and Ippen E P 1994 Asynchronous soliton mode locking *Opt. Lett.* **19** 1958–60

[15] Haus H A, Jones D J, Ippen E P and Wong W S 1996 Theory of soliton stability in asynchronous modelocking *IEEE J. Lightwave Technol.* **14** 622–7

[16] Hsiang W-W, Lin C-Y, Tien M-F and Lai Y 2005 Direct generation of a 10 GHz 816 fs pulse train from an erbium-fiber soliton laser with asynchronous phase modulation *Opt. Lett.* **30** 2493–5

[17] Hsiang W-W, Lin C, Sooi N and Lai Y 2006 Long-term stabilization of a 10 GHz 0.8 ps asynchronously mode-locked Er-fiber soliton laser by deviation-frequency locking *Opt. Express* **14** 1822–8

[18] Hsiang W-W, Chang H-C and Lai Y 2010 Laser dynamics of a 10 GHz 0.55 ps asynchronously harmonic modelocked Er-doped fiber soliton laser *IEEE J. Quantum Electron.* **46** 299n–u

[19] Chang C-H, Jyu S-S, Hsiang W-W and Lai Y 2010 Asynchronous rational harmonic mode-locked Er-doped fiber soliton laser *Conf. on Lasers and Electro-Optics (San Jose, CA, May 2010)*

[20] Haus H A 2000 Mode-locking of lasers *IEEE J. Sel. Top. Quantum Electron.* **6** 1173–85

[21] Ippen P, Haus H A and Nelson L E 1993 77-fs pulse generation from a stretched-pulse mode-locked all-fiber ring laser *Opt. Lett.* **18** 1080–2

[22] Ippen P and Tamura K 1994 Additive-pulse modelocking in fiber lasers *IEEE J. Quantum Electron.* **30** 200–8

[23] Jones D J, Tamura K, Haus H A and Ippen E P 1997 Ultrashort-pulse fiber ring lasers *Appl. Phys.* B **65** 277–94

[24] Jyu S-S and Lai Y 2013 Repetition frequency pulling effects in asynchronous mode-locking *Opt. Lett.* **38** 347–9