Adiabatically driven Brownian pumps

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We investigate a Brownian pump which, being powered by a flashing ratchet mechanism, produces net particle transport through a membrane. The extension of the Parrondo's approach developed for reversible Brownian motors [Parrondo, Phys. Rev. E **57**, 7297 (1998)] to adiabatically driven pumps is given. We demonstrate that the pumping mechanism becomes especially efficient when the time variation of the potential occurs adiabatically fast or adiabatically slow, in perfect analogy with adiabatically driven Brownian motors which exhibit high efficiency [Rozenbaum *et al.*, Phys. Rev. E **85**, 041116 (2012)]. At the same time, the efficiency of the pumping mechanism is shown to be less than that of Brownian motors due to fluctuations of the number of particles in the membrane.

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I. INTRODUCTION

Mechanisms of active transport are the focus of current research primarily because of their particular relevance to studies of transmembrane transport [1–3]. One of the promising ways to treat the problem is based on the concept of Brownian motors and pumps driven by nonequlibrium fluctuations of a potential or an unbiased force (ratchet effect) [4–7]. Brownian motors model the particle transport along a spatially periodic potential, while Brownian pumps mimic particle transport through a finite system (membrane) from a reservoir at low concentration to one at the same or higher concentration. Most pump models assume that a channel, connecting the reservoirs, can contain only one particle or none. In this case the concentrations of particles on both sides of a membrane are introduced via multipliers in the pseudo-first-order rate coefficients which define transitions between occupied and unoccupied channel states [8–11]. Such a pump can be treated in terms of a flashing periodic potential model [12].

There are alternative pump models designed for transport of noninteracting particles. In these models, the particle concentrations in the reservoirs determine the boundary conditions for the particle concentration in the membrane (see, e.g., Refs. [13,14] and [15–17] for flashing and rocking schemes, respectively) and the number of particles inside the membrane depends not only on the outside concentrations but also on the potential inside the membrane. When the potential fluctuates, this number is not constant, in contrast to that we have seen for Brownian motors where it is fixed on each period of the potential. This fact plays an important role in the working mechanism [14].

Investigation of adiabatic (no heat exchange between a working system and surroundings) driving regime plays a special role in the Brownian ratchet theory and its applications. In this regime, Brownian motors exhibit high efficiency of energy conversion [18–25]. On the other hand, the adiabatic approximation makes the analysis analytically treatable, thus providing a convenient way to better understand regularities of motion-inducing and energy conversion mechanisms. Common and distinct features of adiabatically slow [18–21] and adiabatically fast [22–25] driven Brownian motors have been discussed in Refs. [26,27].

The goal of the present paper is to compare adiabatically slow and adiabatically fast driven Brownian pumps with Brownian motors functioning in the same regimes. In contrast to models for Brownian motors, where one exploits the normalization and periodicity conditions for the particle concentration, here the boundary conditions are determined by externally fixed particle concentrations in the left and right reservoirs. Such a replacement of the boundary conditions introduces physics having important consequences, which are discussed in the subsequent sections. Following in this way, we derive explicit analytical expressions for the main pump characteristics and demonstrate, in particular, that the pumping mechanism becomes especially efficient when the time variation of the potential occurs adiabatically fast or adiabatically slow, in perfect analogy with adiabatically driven Brownian motors. At the same time, a Brownian pump can operate at potential sign fluctuations and its efficiency is less than that of motors due to fluctuations of the number of particles in the membrane.

In Sec. II we formulate the model and show how the main equations for adiabatically driven motors are modified in the case of adiabatically driven pumps. Additionally, we discuss similarities and differences in the operating principles of these devices. In Sec. III an adiabatically fast driven Brownian pump is considered in more detail using a technique of singular

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barriers introduced in Refs. [28,29]. The main findings of the paper and their relevance are discussed in concluding Sec. IV.

II. THE MODEL AND MAIN EQUATIONS

Consider pointlike particles moving in a static fluid filling a channel (a membrane) of length L bounded by two particle reservoirs. The particle concentration is assumed to be small enough such that the influence of the particles on the fluid and interparticle interactions in the bulk can be neglected. We also assume that the channel cross section does not vary along its length, so that we can neglect entropic effects in channel-facilitated transport [30,31] and consider the problem as one dimensional. The channel is assumed wide enough in order to neglect the effects studied in the context of single-file diffusion [32,33], so that the particle motion exhibits normal, rather than anomalous diffusion. Thus to concentrate on the ratchetlike pumping mechanism, the model leaves out additional complexities. The simplifications mentioned above are the price we have to pay to make the problem analytically treatable.

The particles are subjected to the potential $V(x)$, where x is the coordinate along the channel with the origin at the left end $0 \leqslant x \leqslant L$ (the potential is zero outside the membrane). The particle concentration $\rho(x,t)$ and the particle current $J(x,t)$ obey the continuity equation

$$\frac{\partial}{\partial t}\rho(x,t) = -\frac{\partial}{\partial x}J(x,t), \quad J(x,t) = \hat{J}(x)\rho(x,t), \tag{1}$$

where the current operator $\hat{J}(x)$ in the overdamped regime is written as [34]

$$\hat{J}(x) = -De^{-\beta V(x)}\frac{d}{dx}e^{\beta V(x)}. \tag{2}$$

Here $\beta = (k_B T)^{-1}$ (k_B is the Boltzmann constant and T is the absolute temperature) and $D = (\beta\zeta)^{-1}$ is the (potential free) diffusion coefficient (ζ is the friction coefficient).

Let ρ_l and ρ_r be the particle concentrations in the left and right reservoirs bounding the channel. The boundary conditions for $\rho(x,t)$ at $x = 0,\ L$ must take into account jumps of the potential at these points

$$\rho(0,t) = e^{-\beta V(0)}\rho_l, \quad \rho(L,t) = e^{-\beta V(L)}\rho_r. \tag{3}$$

At the stationary state [$\partial\rho(x,t)/\partial t = 0$], the solution of Eq. (1) with the boundary conditions (3) can be written as

$$\rho_{\rm st}(x) = e^{-\beta V(x)}\left[\rho_l + (\rho_r - \rho_l)\int_0^x dx' q(x')\right],$$
$$q(x) = e^{\beta V(x)}\Big/\int_0^L dx' e^{\beta V(x')}. \tag{4}$$

The corresponding stationary current is

$$J_{\rm st} = D\frac{\rho_l - \rho_r}{\int_0^L dx' e^{\beta V(x')}}. \tag{5}$$

At $\rho_l = \rho_r$, the stationary solution (4) becomes the equilibrium one and the current (5) vanishes.

Let us assume that the potential inside the membrane instantly changes at $t = 0$ from $V_a(x)$ to $V_b(x)$. So at $t = 0$, the particle concentration and the particle current [designated here and hereafter as $\rho_a(x)$ and J_a, respectively] are defined by Eqs. (4) and (5) with $V(x) = V_a(x)$. Then, the integration of Eq. (1) over large time interval τ_b sufficient for the stationary (or quasistationary) state with the particle concentration $\rho_b(x,\tau_b)$ in the potential $V_b(x)$ to be established gives

$$\rho_b(x,\tau_b) - \rho_a(x) = -\frac{d}{dx}\hat{J}_b(x)\varphi_b(x), \quad \varphi_b(x) = \int_0^{\tau_b} dt\rho(x,t). \tag{6}$$

Here $\hat{J}_b(x)$ is defined by Eq. (2) with $V(x) = V_b(x)$. The integration of Eq. (6) over x yields

$$\Delta\Phi_{ab}(x) \equiv \hat{J}_b(x)\varphi_b(x) = \Delta\Phi_{ab}(x_0) + \int_{x_0}^x dx'[\rho_a(x') - \rho_b(x',\tau_b)]. \tag{7}$$

The quantity $\Delta\Phi_{ab}(x)$ has a simple physical meaning. It is the net fraction of particles crossing the point x during the time τ_b.

Consider a periodic dichotomous process with large durations τ_a and τ_b of alternating states with potentials $V_a(x)$ and $V_b(x)$. The net fraction of particles crossing the point x during the period $\tau = \tau_a + \tau_b$ equals

$$\Delta\Phi_{\rm dich}(x) \equiv \Delta\Phi_{ab}(x) + \Delta\Phi_{ba}(x) = \Delta\Phi_{\rm dich}(x_0) \tag{8}$$

and does not depend on x. Thus the quantity $\Delta\Phi_{ab}(x_0)$ in Eq. (7) at arbitrary point x_0 is to be determined. Using the explicit expression (2) for the current operator $\hat{J}_b(x)$ in Eq. (7), then multiplying both sides of Eq. (7) by $\exp[\beta V_b(x)]$, and finally integrating over x from 0 to L, we obtain with account of the boundary conditions (3)

$$\Delta\Phi_{ab}(x_0) = J_b\tau_b + \int_0^L dx q_b(x)\int_{x_0}^x dx'[\rho_b(x',\tau_b) - \rho_a(x')], \tag{9}$$

where $q_b(x)$ is defined by Eq. (4) with $V(x) = V_b(x)$. As a result, $\Delta\Phi_{\rm dich}$ for the periodic dichotomous process takes the form

$$\Delta\Phi_{\rm dich} = J_a\tau_a + J_b\tau_b + \int_0^L dx[q_b(x) - q_a(x)] \times \int_0^x dx'[\rho_b(x',\tau_b) - \rho_a(x',\tau_a)] \tag{10}$$

[here we put $x_0 = 0$ since $\Delta\Phi_{\rm dich}$ actually is position independent, and use the designation $\rho_a(x',\tau_a)$ instead of $\rho_a(x')$ in order to stress that state "a" is also the quasistationary one].

Now consider a cyclic adiabatically slow process with period τ for which the time dependence of the potential $V(x,t) = V[x;\mathbf{R}(t)]$ can be expressed through a time-periodic vector-function $\mathbf{R}(t) = \{R_1(t), R_2(t), \ldots, R_n(t)\}$, $\mathbf{R}(t+\tau) = \mathbf{R}(t)$. The relation (9) allows us to write the net fraction of particles crossing the point x_0 during the period as the limit of the integral sum over N jumps of the potential with quasiequilibrium states established before each jump

$$\Phi_{\rm slow} = \lim_{N\to\infty}\sum_{m=1}^N \Delta\Phi_{m,m+1}(x_0) = \int_0^\tau dt\, J_{\rm st}[\mathbf{R}(t)] + \oint d\mathbf{R}\int_0^L dx\, q(x;\mathbf{R})\int_0^x dx'\,\nabla_{\mathbf{R}}\rho_{\rm st}(x';\mathbf{R}). \tag{11}$$

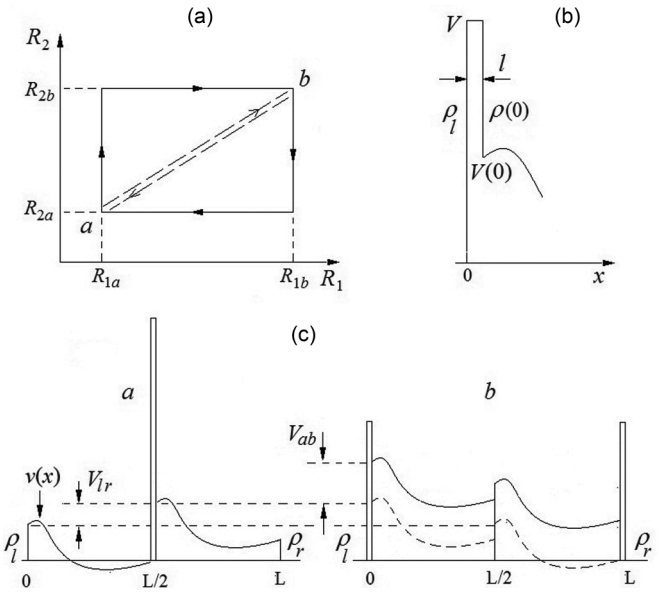

FIG. 1. The phase diagram of functions $R_1(t)$ and $R_2(t)$ describing the time dependence of the two-parametric potential profile $V[x;\mathbf{R}(t)]$ $[\mathbf{R}(t)=\{R_1(t),R_2(t)\}]$. The solid and dashed lines with arrows indicate the directions of loop bypass under adiabatically slow and dichotomic regimes of operating of Brownian pumps, respectively. In the case of a potential with singular barriers [see (c)], the parameters $R_1(t)$ and $R_2(t)$ are related to the time-dependent barrier height and the heights of the smooth regions between the barriers described by the function $v(x)$, respectively (a). The permeable singular $[V\to\infty, l\to 0, \Lambda\equiv l\exp(\beta V)$ is finite] barrier located at the point $x=0$ with different particle concentrations ρ_l and $\rho(0)$ on both sides (b). The potential profiles with singular barriers corresponding to points a and b in (a). Dichotomous switching between these profiles provides the directed motion through the membrane. Parameters V_{lr} and V_{ab} define the energetic shift between the left and right smooth regions and the change of the number of particles in the equilibrium states a and b (c).

The integration in the second term of Eq. (11) is performed over the loop in the $\mathbf{R}$ space [see Fig. 1(a) as a two-dimensional (2D) example with a rectangular loop].

Energetics of Brownian pumps is determined by the input $E_{\rm in}$ and output $E_{\rm out}$ energies for which the following relations hold:

$$E_{\rm in}^{\rm (dich)}=\int_0^L dx[V_b(x)-V_a(x)]\,[\rho_a(x,\tau_a)-\rho_b(x,\tau_b)]\,, \tag{12}$$

$$E_{\rm in}^{\rm (slow)}=\oint d\mathbf{R}\int_0^L dx[\nabla_{\mathbf{R}}V(x;\mathbf{R})]\rho_{\rm st}(x;\mathbf{R})+k_BT\,(\tau_{\rm rel}/\tau)\,,$$

$$E_{\rm out}=k_BT\Phi\ln(\rho_r/\rho_l). \tag{13}$$

The first and the second terms in $E_{\rm in}^{\rm (slow)}$ [Eq. (12)] correspond to the adiabatic and nonadiabatic contributions in the input energy (an explicit formula for the relaxation time $\tau_{\rm rel}$ depending on the potential profile is too cumbersome to be presented here but $\tau_{\rm rel}$ can be estimated as L^2/D). The quantities $\Delta\Phi_{\rm dich}$ or $\Phi_{\rm slow}$ should be substituted for Φ in Eq. (13) if dichotomous or adiabatically slow processes are considered. The efficiency of the Brownian pump is defined as $\eta=E_{\rm out}/E_{\rm in}$.

Note that Eq. (11) has a physical meaning only when the first term does not exceed the second one. As the period τ should be large enough for the stationary regime to be established, the second term in Eq. (11) dominates over the first one only if the stationary current tends to zero. To satisfy this condition, we require that $\rho_r\to\rho_l$. Then, substituting Eq. (4) into Eq. (12) for $E_{\rm in}^{\rm (slow)}$, we obtain up to terms of order $(\rho_r-\rho_l)$:

$$E_{\rm in}^{\rm (slow)}\approx k_BT(\rho_r-\rho_l)\rho_l^{-1}\Phi_{\rm slow}^{(0)}+k_BT(\tau_{\rm rel}/\tau),$$

$$\Phi_{\rm slow}^{(0)}=\rho_l\oint d\mathbf{R}\int_0^L dx\,q(x;\mathbf{R})\int_0^x dx'\,\nabla_{\mathbf{R}}\exp[-\beta V(x';\mathbf{R})]. \tag{14}$$

Thus, in the adiabatic limit $(\tau\to\infty)$ and at $\rho_l=\rho_r$, the pump operates without energetic consumption $(E_{\rm in}^{\rm (slow)}=0)$ like Parrondo's reversible ratchet [18,19]. It is easy to show that in the quasiequilibrium regime with accuracy to the quadratic terms $(\rho_r-\rho_l)^2$ and τ^{-2}, the energetic costs per unit time can be written as follows (see, e.g., Ref. [26]):

$$\tau^{-1}E_{\rm out}^{\rm (slow)}\approx-\lambda_{11}(\rho_r-\rho_l)^2+\lambda_{12}(\rho_r-\rho_l)\tau^{-1},$$
$$\tau^{-1}E_{\rm in}^{\rm (slow)}\approx-\lambda_{21}\tau^{-1}(\rho_r-\rho_l)+\lambda_{22}\tau^{-2}, \tag{15}$$

with the kinetic coefficients defined by the expressions

$$\lambda_{11}=\frac{k_BT}{\rho_l^2\tau_{\rm bar}},$$

$$\tau_{\rm bar}^{-1}=D\rho_l\tau^{-1}\int_0^\tau dt\left\{\int_0^L dx\exp\left[\beta V(x;R(t)\right]\right\}^{-1}, \tag{16}$$

$$\lambda_{12}=-\lambda_{21}=k_BT\rho_l^{-1}\Phi_{\rm slow}^{(0)},\quad \lambda_{22}=k_BT\tau_{\rm rel}.$$

The pump efficiency $\eta=E_{\rm out}/E_{\rm in}$ upon substitution of the relations (15) is a simple fractional power function of $(\rho_r-\rho_l)$ with its maximum value η_m specified by the single parameter Z:

$$\eta_m=(\sqrt{1+Z}-\sqrt{Z})^2\approx 1-2Z,$$
$$Z=\frac{\lambda_{11}\lambda_{22}}{\lambda_{12}^2}=\left[\Phi_{\rm slow}^{(0)}\right]^{-2}(\tau_{\rm rel}/\tau_{\rm bar})\,. \tag{17}$$

Thus the maximum efficiency can tend to unity if the potential profile contains high barriers and the time $\tau_{\rm bar}$ [defined by Eq. (16)] is high enough.

Let us compare Eqs. (10) and (11) for dichotomous and adiabatically slow processes at $\rho_l=\rho_r$ (when the particle concentration becomes equilibrium and the stationary current is absent) with the similar equations for a Brownian particle in a spatially periodic potential [18,19] (see also Ref. [27] where the both processes are considered). These equations differ from their motor analogs only by the form of particle concentrations $\rho_{a,b}(x,\tau_{a,b})$ or $\rho_{\rm st}(x;\mathbf{R})$: They are normalized for Brownian motors and not normalized for Brownian pumps. Due to this fact, the symmetry between functions $q(x)$ and $\rho(x)$ with respect to the inversion of the sign of the potential occurring for Brownian motors [27] is broken for Brownian pumps. This leads, in particular, to the possibility of the pump effect induced by dichotomous fluctuations of the potential sign. On the other hand, Brownian motors and pumps operating in adiabatically slow regime behave in a quite similar manner: Both of them are characterized by high efficiency at $\tau_{\rm bar}\ll\tau\to\infty$ and

$\rho_r - \rho_l \to 0$ [with finite $(\rho_r - \rho_l)\tau$]. In the case of Brownian pumps, $(\rho_r - \rho_l)$ plays a role of a load force.

To better understand the difference between the operating mechanisms of motors and pumps, it is necessary to take into account that the number of particles contained in the membrane with given boundary conditions is not fixed in contrast to the fixed number of particles in the spatial period L in the case of periodic boundary conditions. With this in mind, we invoke in the next section a technique of singular barriers (introduced in Refs. [28,29] for description of high-efficient Brownian motors). This allows us to find conditions of high-efficient dichotomous operation of Brownian pumps and to present explicit expressions for main pump characteristics.

III. SINGULAR BARRIERS

A singular barrier is characterized by a flat top of a large height V and a very small width l so that at $V \to \infty$ and $l \to 0$ the quantity $\Lambda \equiv l \exp(\beta V)$ remains finite and can take arbitrary values. For example, if such a barrier is located at the left boundary point $x = 0$ [see Fig. 1(b)], the current through this barrier is connected with the particle concentrations on both sides of the barrier by the relation

$$J = -\frac{D}{\Lambda}[e^{\beta V(0)}\rho(0) - \rho_l]. \tag{18}$$

This relation is reduced to Eq. (3) when $\Lambda \to 0$. Thus Eq. (18) can be considered as the modified jump boundary condition. The presence of the singular barrier has negligible effect on the factors $\exp[-\beta V(x)]$ but a very significant one on the function $q(x)$ defined in Eq. (4) and entering in Eqs. (10) and (11). This gives us a possibility to distinguish the influence of time-dependent barriers and regions between barriers (wells) on the pump characteristics, which makes the problem analytically treatable.

Consider three singular barriers located at the points $x = 0,\ L/2,\ L$ with time dependencies governed by the first component $R_1(t)$ of the two-component function $\mathbf{R}(t) = \{R_1(t), R_2(t)\}$. The permeabilities of the barriers are defined by $\Lambda_l(R_1)$, $\Lambda_c(R_1)$, and $\Lambda_r(R_1)$, respectively. The integral $\int_0^L dx \exp[\beta V(x;\mathbf{R})]$ is approximately equal to the sum of these quantities, $\Lambda(R_1) = \sum_i \Lambda_i(R_1)$, $i = l,\ c,\ r$, and does not depend on R_2, if $\Lambda(R_1) \gg L$. Then, neglecting the terms of order Λ^{-1}, we obtain

$$\begin{aligned} q(x,R_1) &\approx \lambda_l(R_1)\delta(x) + \lambda_c(R_1)\delta(x - L/2) \\ &\quad + \lambda_r(R_1)\delta(x - L), \\ \lambda_i(R_1) &= \Lambda_i(R_1)/\Lambda(R_1), \end{aligned} \tag{19}$$

where $\delta(x)$ is the Dirac δ function.

Let us assume that the permeabilities of the left and right barriers are equal $[\Lambda_l(R_1) = \Lambda_r(R_1)]$ and the total permeability of the region [inversely proportional to $\Lambda(R_1) = 2\Lambda_l(R_1) + \Lambda_c(R_1)$] does not depend on R_1, i.e. $2\lambda_l(R_1) + \lambda_c(R_1) = 1$. Assume also that in the state "a" there is only the central barrier (without side barriers), $\lambda_l(R_{1a}) = 0$, $\lambda_c(R_{1a}) = 1$, and vice versa in the state "b," i.e., $\lambda_l(R_{1b}) = 1/2$, $\lambda_c(R_{1b}) = 0$ [Fig. 1(c)]. Then, the integration in Eq. (11) over the rectangular contour $d\mathbf{R}$ [see Fig. 1(a)] gives

$$\begin{aligned} \Phi_{\text{slow}} &\approx -D\Lambda^{-1}(\rho_r - \rho_l)\tau \\ &\quad + \frac{1}{2}\int_0^{L/2} dx[\rho_{\text{st}}(x,R_{2a}) - \rho_{\text{st}}(x,R_{2b})] \\ &\quad - \frac{1}{2}\int_{L/2}^{L} dx[\rho_{\text{st}}(x,R_{2a}) - \rho_{\text{st}}(x,R_{2b})]. \end{aligned} \tag{20}$$

Here the second component $R_2(t)$ is used for the description of time dependence of the smooth bare potential $V[x;\mathbf{R}(t)]$ (without additionally introduced singular barriers). Equation (20) containing $\rho_{a(b)}(x,\tau_{a(b)})$ instead of $\rho_{\text{st}}(x,R_{2a(b)})$ can be obtained from Eq. (10) for the periodic dichotomous process with $q_a(x) = \delta(x - L/2)$ and $q_b(x) = (1/2)[\delta(x) + \delta(x - L)]$. Thus, $\Delta\Phi_{\text{dich}}$ is determined by the change of the numbers of particles in the left and right half-regions, $(0,\ L/2)$ and $(L/2,\ L)$.

Important is that in the state b of the dichotomous process the particle exchange between inside and outside membrane regions is hindered due to the presence of the side singular barriers. The corresponding slowdown is characterized by an additional characteristic time τ_{bar} which is an order of $\Lambda L/D$ that is much larger than the relaxation time $\tau_{\text{rel}} \sim L^2/D$. The particle concentration in the quasistationary state a at $\tau_a \gg \tau_{\text{rel}}$ can be written very simply: $\rho_a(x,\tau_a) = \rho_l \exp[-\beta V_a(x)]$ at $x < L/2$ and $\rho_a(x,\tau_a) = \rho_r \exp[-\beta V_a(x)]$ at $x > L/2$. Note that it does not depend on τ_a. On the contrary, the particle concentration in the quasistationary state b does depend on τ_b: $\rho_b(x,\tau_b) = \kappa_b^{-1} N_b(\tau_b)\exp[-\beta V_b(x)]$, where $\kappa_b = \int_0^L dx \exp[-\beta V_b(x)]$ and the number of particles in the membrane $N_b(t)$ at $t = \tau_b \gg \tau_{\text{rel}}$ obeys the differential equation

$$\frac{dN_b(t)}{dt} + \frac{4D}{\kappa_b \Lambda} N_b(t) = \frac{2D}{\Lambda}(\rho_l + \rho_r). \tag{21}$$

This equation can be derived by the integration of Eq. (1) over the membrane region and subsequent use of the boundary conditions with the singular barriers at $x = 0,\ L$ [see Eq. (18) for $x = 0$]. Its solution

$$\begin{aligned} N_b(\tau_b) &= N_b(0)e^{-\tau_b/\tau_{\text{bar}}} + \frac{1}{2}\kappa_b(\rho_l + \rho_r)[1 - e^{-\tau_b/\tau_{\text{bar}}}], \\ \tau_{\text{bar}} &= \frac{\kappa_b \Lambda}{4D} \end{aligned} \tag{22}$$

depends on the initial number of particles $N_b(0)$ which equals $N_a(\tau_a) = \rho_l \kappa_a^l + \rho_r \kappa_a^r$ with $\kappa_a^l = \int_0^{L/2} dx \exp[-\beta V_a(x)]$ and $\kappa_a^r = \int_{L/2}^L dx \exp[-\beta V_a(x)]$. Thus

$$\begin{aligned} &N_b(\tau_b \ll \tau_{\text{bar}}) - N_b(\tau_b \gg \tau_{\text{bar}}) \\ &\quad \approx \tfrac{1}{2}(\kappa_a - \kappa_b)(\rho_r + \rho_l) + \tfrac{1}{2}\left(\kappa_a^r - \kappa_a^l\right)(\rho_r - \rho_l) \end{aligned} \tag{23}$$

$(\kappa_a = \kappa_a^l + \kappa_a^r)$ and we are led to the important conclusion that the number of particles in the state b does not depend on τ_b only if $\kappa_a = \kappa_b$ at $\rho_r = \rho_l$ or $\kappa_a = \kappa_b$, $\kappa_a^l = \kappa_a^r$ at $\rho_r \neq \rho_l$. In the first case, Eq. (20) for the slow regime gives $\Phi_{\text{slow}} = \Phi_{\text{slow}}^{(0)} = \rho_l(\kappa_a^l - \kappa_b^l)$. This expression coincides with the expression $\Delta\Phi_{\text{dich}}$ for the dichotomous regime. In all other cases, $N_b(\tau_b)$ changes in time and, due to this fact, the Brownian pump differs from a related Brownian motor.

In order to analyze the efficiency of the Brownian pump operating in the dichotomous regime, consider potential profiles which (as proved for Brownian motors [27,28]) provide minimal energy losses under instantaneous switching. We assume that the shapes of the left and right regions of the potential profiles in both states are the same and are described by the function $v(x)$. This is allowed only at energetic shifts of these regions as a whole [see Fig. 1(c)]. Then, the κ parameters introduced above can be written as follows:

$$\begin{aligned} \kappa_a^l &= \int_0^{L/2} dx e^{-\beta v(x)}, \quad \kappa_a^r = e^{-u}\kappa_a^l, \\ \kappa_b^l &= e^{-u-w}\kappa_a^l, \quad \kappa_b^r = e^{-w}\kappa_a^l, \end{aligned} \tag{24}$$

where parameter $w = \beta V_{ab}$ defines the change of the number of particles in the equilibrium states a and b ($\kappa_b = e^{-w}\kappa_a$), and parameter $u = \beta V_{lr}$ describes the energetic shift V_{lr} between the left and right regions. For such potential profile changes, the expression (12) for $E_{\rm in}^{\rm (dich)}$ is significantly simplified since the difference $V_b(x) - V_a(x)$ equals $V_{ab} + V_{lr}$ or $V_{ab} - V_{lr}$ at $x < L/2$ or $x > L/2$, respectively, and does not depend on x. Due to this fact, $E_{\rm in}^{\rm (dich)}$ is expressible through the same κ parameters, and the final expressions take the form

$$\begin{aligned} \Delta\Phi_{\rm dich} &= \frac{1}{2}\kappa_a^l(\rho_l + \rho_r)\tilde{\Phi}(f), \\ E^{\rm (dich)} &= \frac{1}{2}\kappa_a^l(\rho_l + \rho_r)k_BT\,\tilde{E}^{\rm (dich)}(f), \quad f = \frac{\rho_r - \rho_l}{\rho_l + \rho_r}, \\ \tilde{\Phi}(f) &= -2\tilde{\tau} f + \tilde{\Phi}_0(f), \\ \tilde{\Phi}_0(f) &= e^{-u/2}\frac{\cosh u}{\cosh(u/2)}(\tanh u - f) - \psi(f)\tanh(u/2), \\ \tilde{E}_{\rm out}^{\rm (dich)}(f) &= \tilde{\Phi}(f)\ln\frac{1+f}{1-f}, \\ \tilde{E}_{\rm in}^{\rm (dich)}(f) &= 2u\tilde{\Phi}_0(f) + 2w\psi(f), \quad \eta(f) = \frac{\tilde{E}_{\rm out}^{\rm (dich)}(f)}{\tilde{E}_{\rm in}^{\rm (dich)}(f)}, \\ \psi(f) &= \{e^w[1 - f\tanh(u/2)] - 1\}\tau, \quad \tilde{\tau} = D\tau/\kappa_a^l\Lambda. \end{aligned} \tag{25}$$

Here we have put $\tau_a = \tau_b = \tau/2$ and used the inequality $\tilde{\tau} \ll 1$. The parameter f plays the role of a load force.

Neglecting the terms of order $\tilde{\tau}$, we have $\eta(f) \approx [\ln(1+f)/(1-f)]/2u$ with the maximum value equal to unity at $f \approx \tanh u$, i.e., near the stopping point at which an exact cancellation of the ratchet effect takes place. Taking into account the terms of order $\tilde{\tau}$ with $\psi(f) \approx \psi(\tanh u) = (e^w/\cosh u - 1)\tilde{\tau}$ and that $\ln[(1+f)/(1-f)] \approx 2u(1 - \chi\tilde{f})$, where $\tilde{f} = \tanh u - f$ and $\chi = \cosh^2 u/u$, the efficiency is expressible as a simple fractional power function of $\tilde{f}$: $\eta(\tilde{f}) = (1 - \chi\tilde{f})(\tilde{f} - f_1 - f_2)/(\tilde{f} - f_1 + f_3)$. Its maximum value is defined by the equation

$$\begin{aligned} \eta_{\rm max} &= [\sqrt{1 + \chi(f_3 - f_1)} - \sqrt{\chi(f_2 + f_3)}]^2, \\ f_1 &= e^{u/2}[\sinh(u/2)/\cosh u]\,\psi(\tanh u), \\ f_2 &= 2e^{u/2}\tilde{\tau}\tanh u\cosh(u/2)/\cosh u, \\ f_3 &= (w/u)e^{u/2}[\cosh(u/2)/\cosh u]\,\psi(\tanh u). \end{aligned} \tag{26}$$

The dependencies $\eta(f)$ calculated from Eq. (25) are depicted in Fig. 2(a) at different values of parameters u and w. Since the domains of definition are limited by the approximate condition $f < \tanh u$, the families of the curves for given u and different w values are grouped. We can see that the efficiency at $w = 0$ dominates within each group. The maximum efficiencies as functions of u and w calculated from Eq. (26) are presented in Fig. 2(b). The dependence $\eta_{\rm max}(u)$ at $w = 0$ is a monotonic decreasing function which is the envelope of the family $\eta_{\rm max}(u,w)$. On the other hand, the dependencies $\eta_{\rm max}(u)$ at $w \neq 0$ are nonmonotonic functions characterized by asymmetry with respect to the inversion $w \to -w$. Due to this fact, the efficiency at $-w$ dominates over the efficiency at w ($w > 0$) in the region of small u, and vice versa in the region of large u.

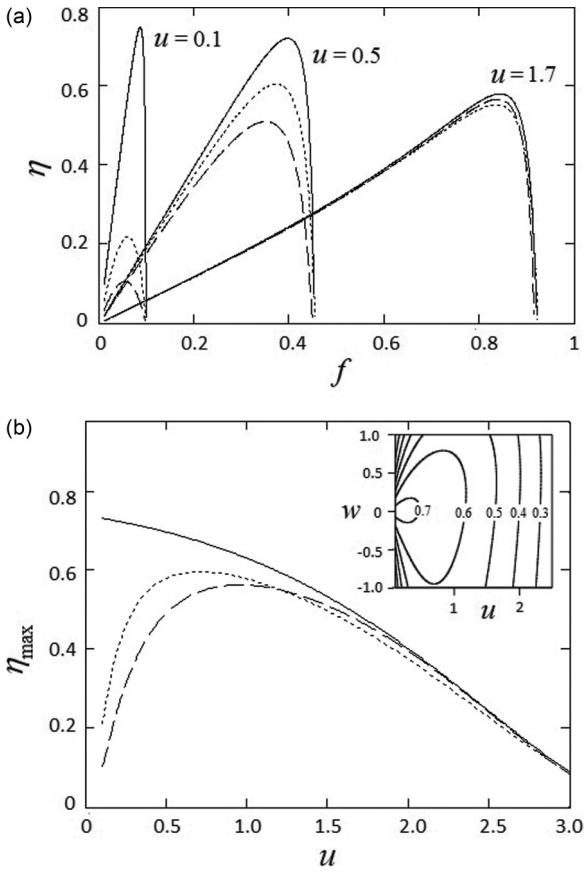

FIG. 2. The efficiency of energy conversion defined by Eq. (25) at $\tau = 0.01$ versus the "load force" of the pump $f = (\rho_r - \rho_l)/(\rho_l + \rho_r)$ at different values of parameter $u = \beta V_{lr}$ (indicated near the corresponding curves) (a), and maximum efficiency of energy conversion [calculated by Eq. (26)] as a function of u at $\tau = 0.01$ (b). The families of solid, dotted, and dashed curves correspond to $w = \beta V_{ab} = 0, -1, 1$, respectively. The inset shows contour plot of the dependence $\eta_{\rm max}(u,w)$.

As mentioned above, the number of particles in the membrane (of the length L) may change ($w \neq 0$) at transitions between the states of the dichotomous process in contrast to the fixed number of particles ($w \equiv 0$, in view of periodical boundary conditions) in the spatial period L for the motor driven by the same dichotomous process. Since $w = 0$ corresponds to the maximum pump efficiency, one can infer that the closer pump to the motor in operating regime, the higher is the pump efficiency.

Note that in contrast to the adiabatically slow regime taking place at $\tau_{\rm bar} \ll \tau \to \infty$, the high-efficient adiabatically fast (dichotomous) regime is realized at $\tau_{\rm rel} \ll \tau \ll \tau_{\rm bar}$. High efficiency is reached due to smallness of the parameter $\tilde{\tau} \sim \tau/\tau_{\rm bar}$. The leading terms in Eq. (25) proportional to small product uf at $\tilde{\tau} \to 0$ have the form $E_{\rm out}^{\rm (dich)} \to \lambda_{12} f u$, $E_{\rm in}^{\rm (dich)} \to -\lambda_{21} u f$, where $\lambda_{12} = \lambda_{21} = 2\kappa_a^l \rho_l k_B T$. Thus, unlike the case of the adiabatically slow regime [see Eq. (16)], the symmetry of kinetic coefficients arises for the dichotomic process near the equilibrium.

IV. DISCUSSION AND CONCLUSIONS

In the present paper we have analyzed a pumping mechanism powered by fluctuations of the potential inside the membrane, with emphasis on the regimes of adiabatically slow and adiabatically fast driving. Although our analysis neglects some precise details about the specific particle-particle and particle-membrane interactions, it provides a physically and mathematically consistent picture of noise-induced transport through a membrane.

A biological relevance of the pump model with the given particle concentration at the membrane sides was first demonstrated in Ref. [15] which is devoted to numerical modeling ratchetlike transport of glycerol in an aquaglyceroporin channel, fueled by the cell's metabolism. Molecular dynamics studies [35–38] established that for biologically relevant periplasmic glycerol concentrations, correlation effects between consecutive glycerol molecules are negligible due to their large spatial separation [15]. The idea to exploit the ratchet effect in a pumping mechanism can be exemplified not only by biological applications but also a chemically driven molecular electron pump [39], a nonadiabatic electron heat pump [40], and superconducting fluxon pumps and lenses [41]. Thus, the pump model used in this paper is relevant to analysis of different systems.

The theoretical significance of the results obtained consists in that Parrondo's approach [18,19] developed for adiabatically driven motors is extended here to adiabatically driven pumps. In particular, Eqs. (10) and (11) determine net fraction of particles crossing an arbitrary point inside the membrane at adiabatically fast (dichotomous) and adiabatically slow potential changing. They differ from those of Brownian motors by unnormalized particle concentrations with given values at the membrane sides. This leads to the break of the symmetry existing for adiabatically driven motors and to the new properties of pumps. At the same time, we have shown that the noise-induced transport through a membrane can be efficient at adiabatically slow and adiabatically fast (dichotomous) regimes of changing the potential, just as in the case of Brownian motors. The lowering of the energy loss is achieved due to (i) adiabaticity of the potential change process, (ii) diffusion-free directed motion generation (provided the potential extrema shift in time), and (iii) an effective rectification mechanism taking place (when the characteristic amplitude of the potential is much larger then thermal energy $k_B T$).

The technique of singular barriers [28,29] is extremely useful in studies of adiabatically driven ratchets (both motors and pumps). A virtue of this method is that it allows us to separate parameters determining the time variation of the potential, which are responsible for driving the system away from equilibrium and for the rectification of the emergent motion into directed motion. In this way, a set of useful analytical regularities can be established. By introducing the singular barriers with low permeabilities [$\Lambda \equiv l \exp(\beta V) \gg L$], an additional characteristic time of the system arises, namely $\tau_{\rm bar} \sim \Lambda L/D$, which is much larger than the diffusive relaxation time $\tau_{\rm rel} \sim L^2/D$. High efficiency of Brownian pumps can be reached in adiabatically slow and dichotomous regimes of cyclic potential changes with the period τ if $\tau_{\rm bar} \ll \tau$ and $\tau_{\rm rel} \ll \tau \ll \tau_{\rm bar}$, respectively.

A replacement of the periodic boundary conditions by the conditions of given concentrations at the membrane boundaries leads to the symmetry breaking of the average current through the membrane with respect to the inversion of the sign of the potential and to the dependencies of the pump characteristics on the number of particles in the membrane. As seen from Fig. 2, if this number is not changed during the dichotomic switching between potential profiles, the efficiency reaches the maximum values. Thus, in the general case, the efficiency of adiabatically fast driven pumps is less than that of motors with the same characteristics of fluctuations of the potential (the spatial period of which is equal to the membrane length). Only under the special condition of the constant number of particles, the properties of pumps and motors become similar.

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