

Evaluation of bridge instability caused by dynamic scour based on fractal theory

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2013 Smart Mater. Struct. 22 075003

(<http://iopscience.iop.org/0964-1726/22/7/075003>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 140.113.38.11

This content was downloaded on 25/04/2014 at 09:29

Please note that [terms and conditions apply](#).

Evaluation of bridge instability caused by dynamic scour based on fractal theory

Tzu-Kang Lin^{1,3}, Rih-Teng Wu², Kuo-Chun Chang² and Yu Shian Chang¹

¹ Department of Civil Engineering, National Chiao Tung University, Taiwan

² Department of Civil Engineering, National Taiwan University, Taiwan

E-mail: tklin@nctu.edu.tw, r01521250@ntu.edu.tw, ciekuo@ntu.edu.tw and deer.cv99g@nctu.edu.tw

Received 31 October 2012, in final form 18 February 2013

Published 28 May 2013

Online at stacks.iop.org/SMS/22/075003

Abstract

Given their special structural characteristics, bridges are prone to suffer from the effects of many hazards, such as earthquakes, wind, or floods. As most of the recent unexpected damage and destruction of bridges has been caused by hydraulic issues, monitoring the scour depth of bridges has become an important topic. Currently, approaches to scour monitoring mainly focus on either installing sensors on the substructure of a bridge or identifying the physical parameters of a bridge, which commonly face problems of system survival or reliability. To solve those bottlenecks, a novel structural health monitoring (SHM) concept was proposed by utilizing the two dominant parameters of fractal theory, including the fractal dimension and the topothesy, to evaluate the instability condition of a bridge structure rapidly. To demonstrate the performance of this method, a series of experiments has been carried out. The function of the two parameters was first determined using data collected from a single bridge column scour test. As the fractal dimension gradually decreased, following the trend of the scour depth, it was treated as an alternative to the fundamental frequency of a bridge structure in the existing methods. Meanwhile, the potential of a positive correlation between the topothesy and the amplitude of vibration data was also investigated. The excellent sensitivity of the fractal parameters related to the scour depth was then demonstrated in a full-bridge experiment. Moreover, with the combination of these two parameters, a safety index to detect the critical scour condition was proposed. The experimental results have demonstrated that the critical scour condition can be predicted by the proposed safety index. The monitoring system developed greatly advances the field of bridge scour health monitoring and offers an alternative choice to traditional scour monitoring technology.

(Some figures may appear in colour only in the online journal)

1. Introduction

Natural disasters such as earthquakes and flooding, and the inevitable aging process, can cause structures to collapse without warning. Recently, as some economies and societies have been quite seriously affected by natural catastrophes, structural health monitoring (SHM), an interdisciplinary concept that originated from the field of aerospace engineering, has become an emerging issue in civil engineering all around the world. Contrary to the traditional

design concept, structural engineers not only need to provide sufficient structural strength as regulated by building codes to guarantee the safety of structures, but they should also equip structures with a proper SHM system for long-term maintenance.

Over the past decade, vibration-based SHM techniques have been widely studied. By comparing the basic characteristics extracted from measurements such as modal frequencies, damping ratios or mode shapes with the predetermined criteria of the undamaged structure, a global evaluation of the structural health condition can be identified,

³ Author to whom any correspondence should be addressed.

quantified, and localized. For example, a vibration-based SHM methodology was successfully proposed by Lam *et al* to establish the stiffness reduction due to damage, based on the benchmark study set up by the IASC-ASCE Task Group [1]. The measured structural responses from the undamaged and damaged systems were used to identify the modal parameters for a parameterized model updated using a Bayesian system of identification without full-state measurement [2]. Statistical methods of discriminant analysis to detect the structural damage and localization by response changes were also developed [3]. Furthermore, research focusing on utilizing different data sources has been considered [4, 5]. Sara Casciati proposed a differential evolution algorithm by selecting the stiffness parameters as optimization variables to reflect the measured response. The comparison of the identified stiffness matrix with the initial matrix then allowed damage detection and localization [6]. A correlation-based methodology as an effective nonparametric data analysis approach for detecting and localizing structural changes using strain data under operational loading conditions was also proposed recently [7]. However, it is still not possible to determine the exact behavior of the structure while accounting for the inevitable uncertainty using these methods, as the damage criteria are set manually.

As a result of the many hazards that threaten the structural stability of bridges, real-time bridge health monitoring systems have become increasingly important over the past decade. Some bridge health monitoring systems have been proposed, utilizing a network of sensors including thermocouples, humidity sensors, velocity meters, and accelerometers for long-term monitoring [8, 9]. As most of the observed cases of unexpected damage or destruction of bridges have been due to hydraulic issues, monitoring the scour depths of bridge foundations has become one of the most important and difficult aspects of bridge health monitoring. Traditionally, sensors are installed directly onto the substructure to indicate the possible scour depth directly. However, in the complicated conditions faced during flooding or typhoons, the sensors may not work as well as expected. Therefore, the reliability of vibration-based algorithms has become another issue for the purposes of scour depth monitoring.

Recently, pattern recognition techniques have been proposed and widely accepted by SHM researchers to improve the reliability of SHM. By comparing the information collected from the structure and a specific scenario database, the efficiency of the SHM system can be greatly enhanced. For example, the support vector machine (SVM), a branch of informatics, has been effectively applied to a wide variety of subjects, such as damage detection in helicopter propeller blades [10], the improvement of an existing bridge SHM system [11], and the identification of temperature variation on the modal parameters of a bridge structure [12]. In addition, slow variations of decay in stiffness or system parameters by internal damage within nonlinear structures can also be detected [13]. However, there still exist some limitations and difficulties to be faced when applying the SVM algorithm to the problem of SHM. For example, the lack of training samples, which would normally be obtained from field

experiments of structures with different damage conditions, is a real hindrance to its practicability. To solve the problem of this bottleneck, alternative methods are considered.

The core concept of fractal theory (FT), similar to the concept of the pattern recognition technique, is that self-similarity can be found within objects with high irregularity when viewed at different scales. A close correspondence between the structural ultimate strength and the structural surface in terms of fractal geometry was found in the simulation of the surface roughness of a shear wall element [14]. FT was also used as an alternative index to quantify the stability of an aggregate of cultivated soil [15]. In order to predict the cohesive processes in clay soil, the fractal dimensions were used as an indicator for the yielding force of the soil [16]. Moreover, FT was used to study size effects on the stiffness of brittle materials and the fatigue of metals [17, 18]. By verifying tests on a model of an arch bridge, the nonlinear behaviors of structural dynamic responses can be obtained [19]. Furthermore, Qiao *et al* proposed an FT-based method to identify the fissure locations and the numbers of cracks in a beam-type structure [20].

In order to improve the reliability of the traditional bridge SHM systems, an alternative solution is approached by introducing an FT-based algorithm, as shown in figure 1. Similar to the concept conducted by Fabio Casciati and Sara Casciati, where the damaged and undamaged condition associated with different time histories obtained from the same structure can be identified by the Lyapunov exponents [21], the main objective is to extend the original concept on the issue of evaluating the instability condition of dynamic systems under various damage conditions by the proposed FT-based algorithm to further improve the practicability of health condition evaluation and localization. In order to reach the goal, a safety index is proposed to assess the stability of the bridge, and a real-time warning signal reducing the risk to human lives and property can be sent out. The remainder of this paper is organized as follows. First, the basic concepts of fractal theory, including their physical meaning and the potential for practical applications are described. In order to verify the performance of the proposed bridge SHM concept, a series of dynamic scour experiments was conducted, utilizing both single-column and full-bridge specimens. The correlation between the FT parameters and the scour condition was then carefully investigated. Based on the results, a bridge health monitoring equation indicating the safety condition is proposed and examined for its reliability. Finally, a summary is given and conclusions are drawn.

2. The fractal theory—based SHM concept

The concept of fractal theory (FT) was first proposed by the mathematician Mandelbrot to describe complex nonlinear phenomena in nature, such as the shape of a mountain, turbulent flow, or the length of a coastline, where the characteristics of self-similarity can be found between multiple scales of the structure [22]. As a result of analyzing the self-similarity based on FT at a small scale, where the identification can be easily achieved, the implicit regularity of

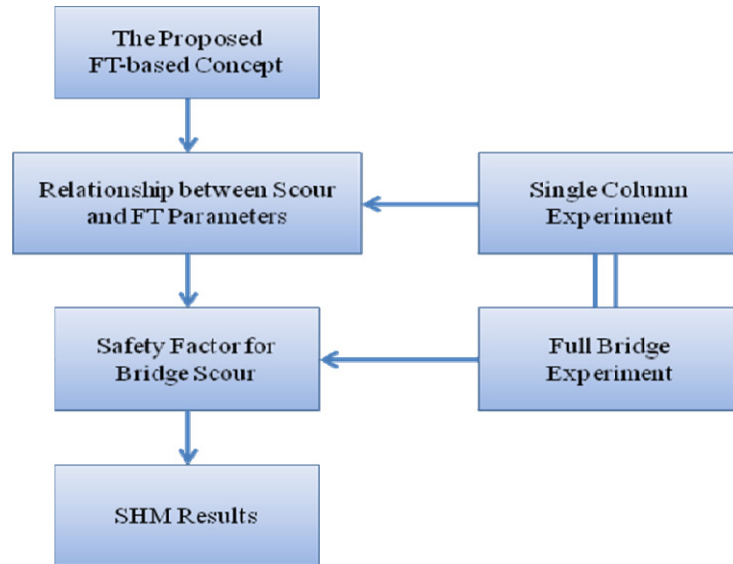


Figure 1. Flowchart of the proposed SHM system.

the whole structure can be estimated with the help of a partial structure. Although no regularity may be observed from the structure at the original scale, the self-similarity characteristic can start to appear on extracting a smaller part of the whole structure. A significant similarity that may not be detected from the original scale observation can then be found.

The fractal dimension and the topothesy, commonly denoted as D_s and G , respectively, are the two dominant parameters in FT. By applying these parameters, an illustration of the variation of a vibration signal has been demonstrated to be clearer than by directly estimating the dominant frequency and amplitude [23]. The derivations of the fractal dimension and the topothesy are described briefly below.

In order to describe the non-differential characteristic and continuity of a fractal curve, the Riemann function $z(x)$ was first proposed [22] as:

$$z(x) = \sum_{n=1}^{\infty} n^2 \cos(n^2 x) \quad (1)$$

where n is a continuous integer. To consider the effects of the amplitudes of the signals, the above equation is rewritten to give the Weierstrass–Mandelbrot fractal function (W–M fractal function) as:

$$z(x) = G^{(D_s-1)} \sum_{n=1}^{\infty} \frac{\cos(2\pi \gamma^n x + \phi_n)}{\gamma^{(2-D_s)n}} \quad (2)$$

where G is the topothesy, D_s is the fractal dimension, and γ^n represents the frequency module, which is the reciprocal of λ_n with the random variable phase ϕ_n . To extract the D_s and G terms from the original signal $z(x)$, a scaling constant C_p related to the signal amplitude is introduced. Based on research conducted by Yan and Komvopoulos [24], the time history of a rough surface can be simulated by introducing the structure function:

$$S(\tau) = 2^{2(4-D)} G^{2(D-2)} (\ln \alpha) (\tau)^{2(3-D)} \quad (3)$$

where τ stands for the sampling interval of instrumentation; D is the fractal dimension in three dimensions, and α expresses the dominant parameter of the frequency density at the surface. The structure function S can also be derived from the power spectrum function related to gamma function Γ as [25]:

$$S(\tau) = \int_{-\infty}^{\infty} P(f) [\exp(if\tau) - 1] df = \frac{2C_p}{\eta - 1} \sin\left(\frac{\pi}{2}(2 - \eta)\right) \Gamma(2 - \eta) |\tau|^{\eta-1} \quad (4)$$

where $f = 1/\tau$, η is a constant, and C_p is the scaling constant from the spectrum function of $P(f) = \frac{C_p}{f^\eta}$.

The function S can be rewritten by calculating the exponential value τ under the condition $\eta = 7 - 2D$ as:

$$S(\tau) = \frac{C_p}{3 - D} \sin\left(\frac{\pi}{2}(2D - 5)\right) \Gamma(2D - 5) \tau^{2(3-D)}. \quad (5)$$

By combining equations (3) and (5), the scaling constant C_p can be solved as:

$$C_p = \frac{(3 - D) 2^{2(4-D)} G^{2(D-2)} \ln \alpha}{\sin\left(\frac{\pi(2D-5)}{2}\right) \Gamma(2D - 5)}. \quad (6)$$

As the relationship between the fractal dimensions in two and three dimensions was proved to be $D_s = D - 1$ [22], the fractal topothesy can then be expressed in the two-dimensional forms as:

$$G = \left\{ \frac{C_p \sin\left[\frac{\pi(2D_s-3)}{2}\right] \Gamma(2D_s - 3)}{(2 - D_s) 2^{2(3-D_s)} \ln \alpha} \right\}^{\frac{1}{(2D_s-2)}}. \quad (7)$$

The theoretical derivation of G is shown in equations (7); however, the complicated coupling relationship still makes their estimation difficult. To estimate D_s and G directly from a measured signal, some algorithms such as the slit

island method, the box counting method, the power spectrum method, and the variation method have been proposed by other researchers [26–28]. Based on the studies of Dubuc *et al* [29], the variation method and the power spectrum method are more suited for calculating fractal parameters when the fractal interval is located between one and two dimensions, which is the case for the vibration signal in this study. Therefore, these two methods are used to derive D_s and G , respectively. By calculating the fractal parameters D_s and G , the variation of the vibration signal causing structural instability can be evaluated more thoroughly.

2.1. Calculation of the fractal dimension

The fractal dimension D_s is first calculated with the variation method to obtain a higher precision. The concept is briefly illustrated here. For a two-dimensional array $h(i)$, $\{i \in 1, \dots, N\}$ with N points, where $h(i)$ indicates the height of the signal at coordinate i , a line segment with its central point lying at $p(i)$ with an extension of length k ($k = 1, \dots, N/2$) to both right and left of the central point, the maximum u_k and the minimum b_k of k of the point i can be defined as:

$$u_k = \max\{h(l); l \in [i \pm k]\} \quad (8)$$

$$b_k = \min\{h(l); l \in [i \pm k]\}. \quad (9)$$

The difference between the local maximum and local minimum values existing on this line segment $v_h(i, k)$, which is the variation of height at point $p(i)$ in the area ε (k/N), is defined as:

$$v_h(i, k) = u_k(i) - b_k(i). \quad (10)$$

In order to calculate all the $v_h(\varepsilon)$ values of height variation for the whole array, the line segment is shifted from the point $p(i)$ to a new point $p(i + n)$, $\{n \in [-N/2, \dots, N/2]\}$; $n \neq 0$, and the local maximum and minimum values of signal amplitude in the area ε are recalculated. Repeating the process mentioned above, the average of all the height variations, named the average ε -variation, representing the area occupied by the local maximum and minimum values of the measured signal $v_h(\varepsilon)$ shown in figure 2, can be obtained:

$$V_h(\varepsilon_n) \approx \left(\frac{1}{N}\right) \sum_i^N v_h(i, \varepsilon_n) \quad (11)$$

where N is the total number of points. The fractal dimension D_s of the measured signal can then be easily derived from the slope of the logarithmic plot of the average ε -variation $V_h(\varepsilon)$ versus ε as:

$$\log \left[\left(\frac{1}{\varepsilon_n}\right)^2 V_h(\varepsilon_n) \right] = D_s * \log \left(\frac{1}{\varepsilon_n}\right); \quad \varepsilon_n = \frac{k}{N}. \quad (12)$$

It should be noted that the main concept of the variation method is to analyze the same vibration signal by switching between different inspection scales. Therefore, the upper- and lower-bound covering segments and the average ε -variation depend on different k values, where a larger k value would result in a greater surface error.

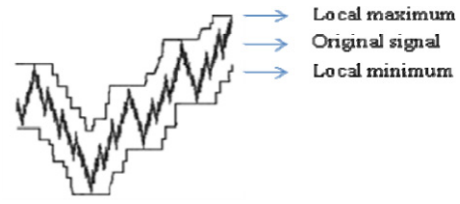


Figure 2. A height array of a two-dimensional signal.

2.2. Calculation of the topothesy

As shown in equation (2), $z(x)$ is composed of overlapped frequency modules, which indicate the implied physical characteristics of the structure. Alternatively, the multi-scale function can also be examined in terms of the power spectral density, where the characteristics of the measured signal can be checked from the frequency domain. The relationship between the power spectral density function $P(f)$ and $z(x)$ [30] can be written as:

$$P(f) = \frac{1}{L} \left| \int_0^L z(x) \exp(iffx) dx \right|^2. \quad (13)$$

The density function $P(f)$ can also be found from the W–M fractal function [31] as:

$$P(f) = \frac{G^{2(D_s-1)}}{2 \ln \gamma} \frac{1}{f^{(5-2D_s)}} = \frac{C_p}{f^{(5-2D_s)}} \quad (14)$$

where γ is an amplified scale parameter satisfying $z(\gamma x) = \gamma^H z(x)$. By taking the natural logarithms of both sides of equation (14), the relationship between D_s and C_p can be clearly described as:

$$\ln[P(f)] = -(5 - 2D_s) \ln(f) + \ln C_p. \quad (15)$$

By utilizing the C_p calculated from equation (15) and the D_s derived from the variation method, where better precision can be provided, the topothesy G can be estimated through equation (7).

With the support of the fractal dimension and the topothesy, a unique bridge SHM methodology is proposed. As study has shown that the physical characteristics of the bridge column are mainly influenced by the variation of scour depth while the force exerted by the current on the bridge column only has a minor impact [32], the first focus of this study is to process the measured vibration signals of the superstructure and establish the relationship between the two fractal parameters and the scour depth at different scour stages. The advantages of applying FT theory to bridge SHM are also examined by experiments conducted on a scaled-down specimen. Moreover, to enhance the practicability of the proposed system, a formula inferring the safety condition of the bridge is proposed and verified by the experimental database.

3. Relationship between scour characteristics and FT parameters

In order to verify the proposed bridge SHM concept, a series of dynamic scour experiments were conducted. Both

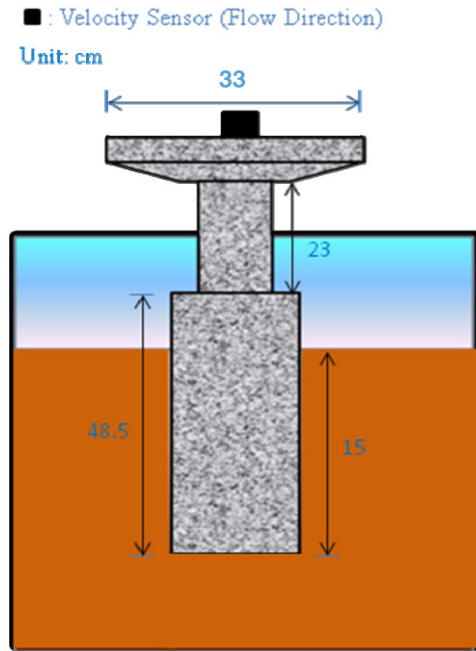


Figure 3. Details of the specimen.

single-column and full-bridge specimens were used, and instrumentation for collecting vibration data was deployed on both the superstructure and the substructure to monitor the scour process for further comparison.

3.1. Single bridge column scour experiment

As shown in figures 3 and 4, a single-column specimen of 25 kg was embedded into the riverbed with sand of a specified depth, 15 cm, and two high-resolution velocity sensors were deployed on the top of the bridge column to capture the responses of the specimen in the flow and vertical directions, respectively. The flow velocity was maintained at a constant speed to negate any unnecessary interference from the environment. In total, 24 min of dynamic response data were recorded at a sampling rate of 200 Hz until the experiment ended with the collapse of the bridge column. Responses from both directions were considered and analyzed during the scour experiment. The entire time history in the transverse (flow) direction is shown in figure 5, where the scour phenomenon was observed from the first minute, and the bridge column finally collapsed after 24 min. As shown in the figure, the amplitude gradually increased for the first five minutes before reaching the stabilization stage, while the scour depth slowly increased. The whole bridge column only vibrated significantly at the final stage of the experiment, and the collapse was observed due to the instability of the structure caused by scour.

In order to investigate the variation of the fractal parameters during the scour event, the 24 min time history of the horizontal vibration response was split into segments of one minute with 12 000 points in each. By analyzing the data collected, the 24 individual groups of fractal parameters D_s and G can be calculated and compared.



Figure 4. Single bridge column test.

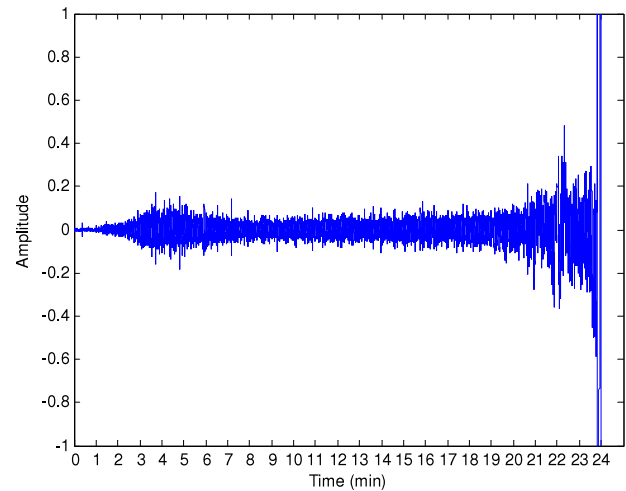


Figure 5. Time history in the transverse direction.

4. The fractal dimension D_s and the signal frequency

The fractal dimension D_s was first calculated. In order to improve the sensitivity and reliability of the D_s value calculated, a special strategy was introduced, where all 12 000 data points in each segment were further divided into 12 sets of 1000 points and plotted on the power spectrum diagram individually. A typical demonstration of the first segment with the 12 power spectrum curves is shown in figure 6. According to equation (15), a fractal interval of frequency between 0.5 and 4 Hz indicated by the red line in the figure was chosen to prevent any distortion on the estimation of fractal parameters. The mean value of the 12 D_s values obtained was then calculated to show the minute variation of D_s related to the time history, as shown in figure 7. Meanwhile, as D_s can be used to evaluate the density variation of the measured signal, the relationship between D_s and the dominant frequency of the signal in each section was also studied. The short-time Fourier transform (STFT) technique was used to rapidly analyze the dominant frequency in the transverse direction of the single bridge column, as shown in figure 8, where the x -axis is the elapsed time in seconds, the y -axis is the frequency, and the power spectral density is expressed by the variation in color. Comparison between figures 7 and 8 demonstrates the basic concept that the density variation exhibited by the fractal dimension D_s can be treated as an alternative to the dominant frequency of a structure.

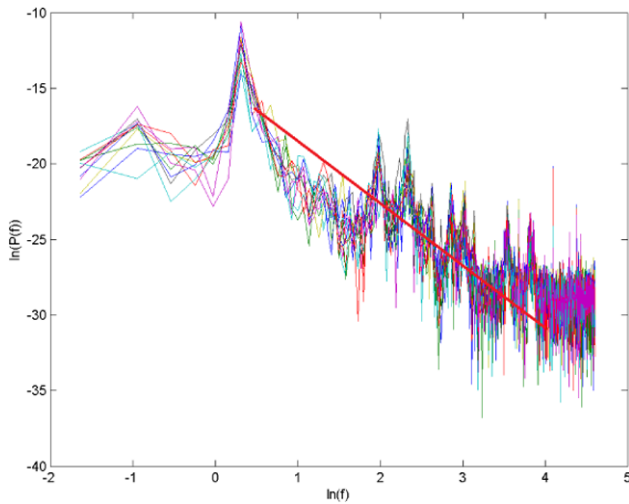


Figure 6. The power spectrum diagram for FT parameters.

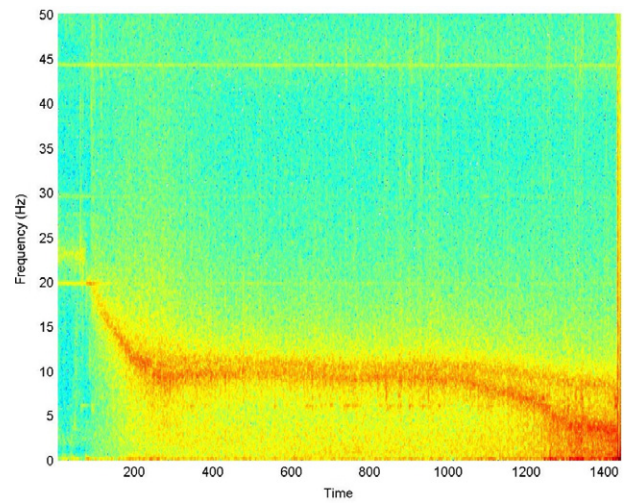


Figure 8. The STFT diagram obtained from the experiment.

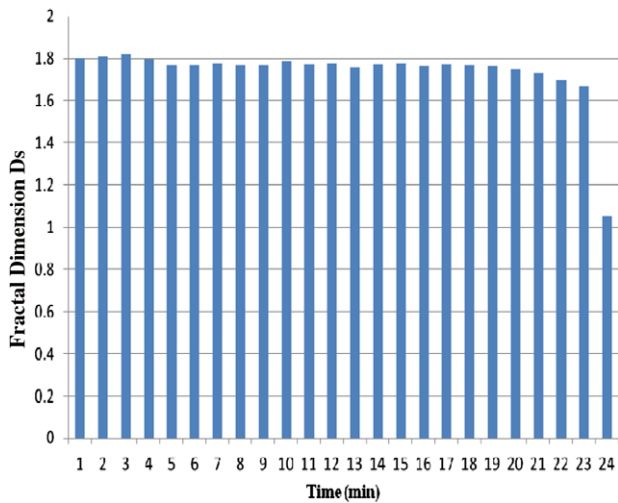


Figure 7. The time history variation of D_s .

The trend of the fractal dimension D_s was then studied. As shown in figure 7. The initial value of D_s was 1.80, and this decreased as the river began to scour the single-column specimen. It remained nearly at 1.80 for the first four minutes, which reflects the condition of the bridge column with only embedded sand. A transition period was observed starting from the fifth minute between the non-scoured and initial scoured conditions due to the sudden change of foundation condition, where D_s gradually changed to the value of 1.77. Similar to the plateau phenomenon found in the modal frequency of the STFT diagram, the D_s value measured from the vibration signal fluctuated slightly between 1.75 and 1.77 between minute 5 and minute 20 as the scour depth continued to change. This follows the self-similarity criterion that the fractal dimension should be located between 1 and 2 when structures are under a stable status. It is inferred that although the boundary condition of the bridge column kept changing, the characteristics of self-similarity kept the fractal dimension constant.

The advantage of the fractal dimension was demonstrated again at the final stage of the scour experiment. As indicated in figure 7, the D_s value fell below 1.70 after 22 min, which was two minutes before the final collapse. Furthermore, D_s then dropped rapidly to 1.05 before the critical collapse of the bridge foundation. This shows that by properly utilizing the characteristic fractal dimension, the stability of a bridge can be effectively evaluated. In contrast, as shown in figure 8, where the first few modal frequencies can be roughly estimated from the STFT diagram, the natural frequency of the single bridge column dropped significantly from the initial value of 20 Hz and spread into a wide range with an approximate central frequency of 10 Hz from minute 5 to minute 20. The frequencies then decreased further to an average region of 5 Hz before the sudden collapse of the bridge column. Although a descending trend was observed for the dominating frequencies, the damage or scour condition cannot be quantified to offer an early warning signal to bridge users. As the main purpose of this research is to develop a simple indicator in evaluating the bridge instability caused by dynamic scour, the extraction of the modal frequencies from STFT processing is not selected.

5. The fractal topothesy G and the signal amplitude

The topothesy G was then examined, as it can be treated as an index representing the vibration amplitude. The variation of horizontal amplitude, represented by the mean absolute value of the signal amplitude at each minute is depicted in figure 9, and the variation of the G values is illustrated in figure 10 for comparison.

As both figures show a similar trend as the scour phenomenon progressed, the basic theory that the topothesy G can be used to reflect the variation of the amplitude of the measured signal is verified. According to FT, the characteristic self-similarity of the vibration signal can be described clearly, especially during events with high nonlinearity, such as the complicated scour problem. This characteristic was observed significantly for the topothesy G . As shown in the first

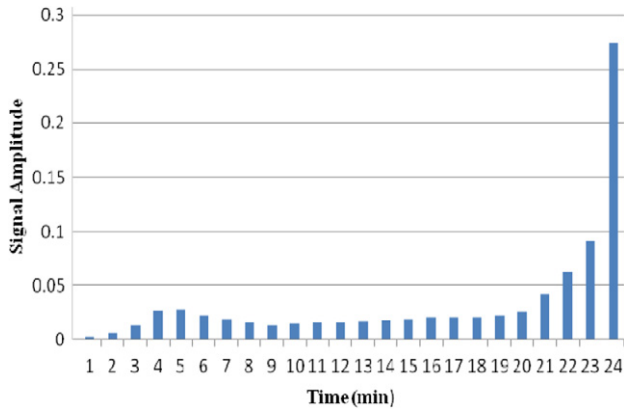


Figure 9. The time history of the variation of the horizontal amplitude.

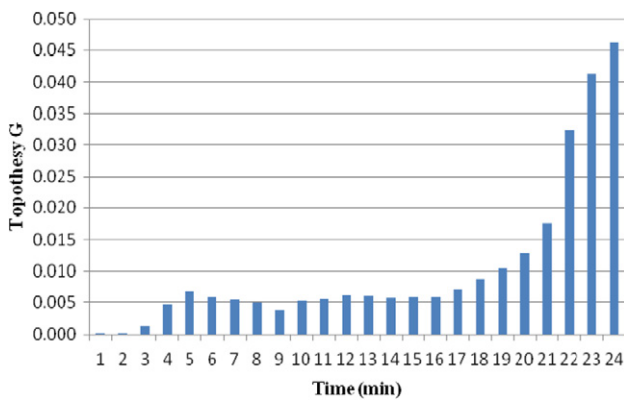


Figure 10. The time history of the topothesy G .

ten minutes, the bridge column was experiencing an initial progression of scour, and a serious variation in the vibration amplitude was found when the water level rose. However, as self-similarity should exist under the stable condition of the whole bridge column, fluctuation on the topothesy G was comparatively minor and kept to almost the same level, near 0.005. Moreover, a clear phenomenon was also observed for

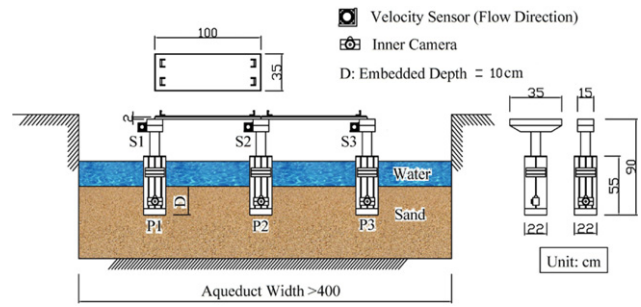


Figure 12. The sensor locations.

the final five minutes of the scour process. The vibration amplitude increased dramatically only during the final stage with the sudden collapse of the bridge column. Conversely, the topothesy G gradually increased from the 20th minute and gave a significant warning signal after 22 min, three minutes before the catastrophe. This vital early warning would be of great help in saving lives and property when applied to actual bridges.

Preliminary investigations on the single bridge column test have shown that the variation in scour depth can be effectively described by D_s , and the variation in the amplitude of the measured signal can be described by the topothesy G . The fractal parameters can be treated as more reliable and stable indices than traditional parameters such as frequency or amplitude for SHM when considering the possible noise or impact caused by floating wood or debris during serious dynamic scour. To verify the feasibility of applying FT in practice, a full-bridge scour experiment, which is a more realistic scenario than the single bridge column condition, was conducted.

6. Full-bridge scour experiment

Following the setup of the single-column experiment, sensors were located on the bridge columns to measure the velocity

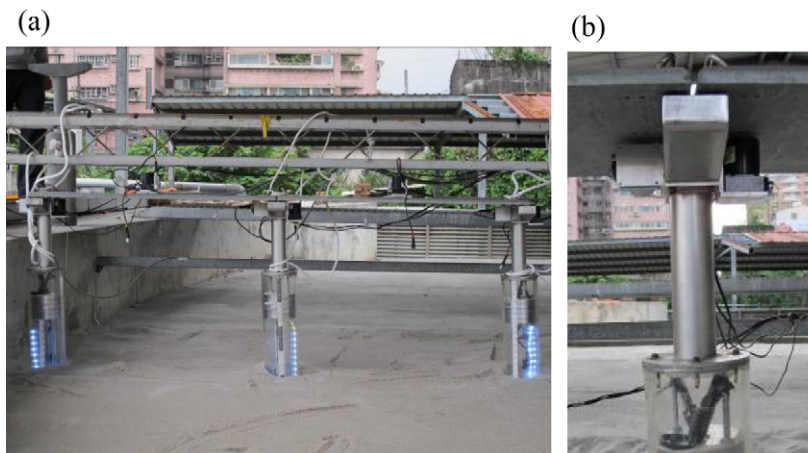


Figure 11. (a) The full-bridge experimental setup. (b) The installed velocity sensor.

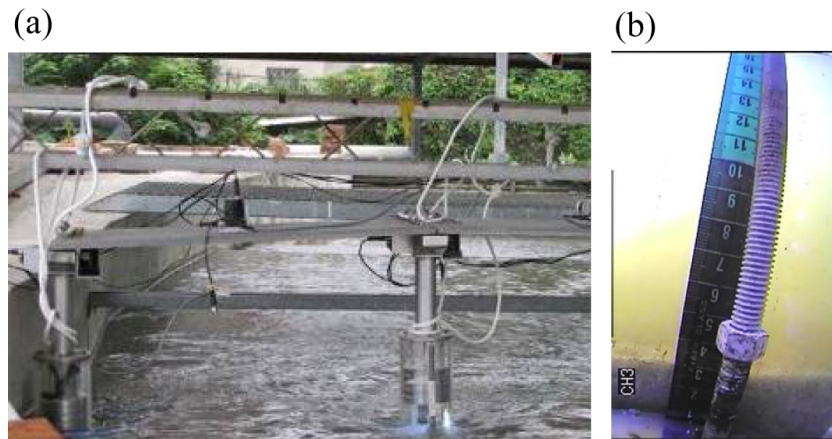


Figure 13. (a) The observed collapse phenomenon. (b) The embedded depth of P_1 recorded by the built-in camera under a major collapse.

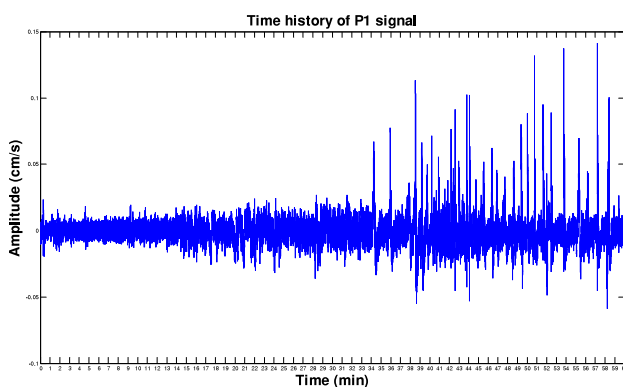


Figure 14. The time history of sensor 1.

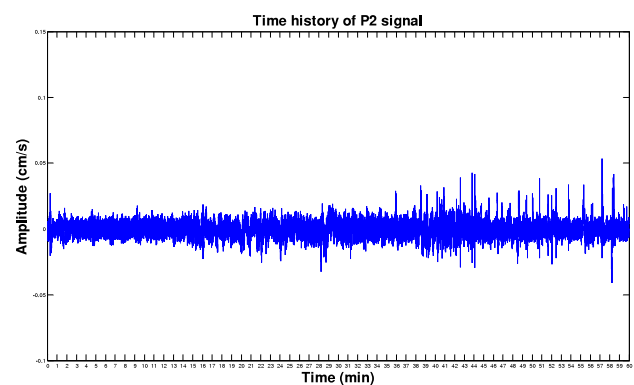


Figure 15. The time history of sensor 2.

response directly in the flow direction. As the dynamic response recorded from the sensors was affected by the scour profile of individual columns, the scour depth of the columns was also investigated using a built-in camera during the entirety of the experiment. Details of the experimental setup and the sensor locations are shown in figures 11 and 12. Some sudden spikes caused by an instantaneous change of the embedded depth during the scour process were observed on both P_1 and P_2 , but no obvious change on embedded depth was found on P_3 . A major collapse phenomenon was identified on column P_1 in the second half stage of the scour process, as shown in figure 13.

In order to figure out the relationship between the scour depth of multiple bridge foundations and the dynamic response of the superstructure of the bridge, data received from sensor 1 relating to the left column (P_1) and sensor 2 relating to the middle column (P_2) were used in the analysis with a sampling rate of 200 Hz. As indicated in figure 12, the initial embedded depth, which gradually decreases during the scour phenomenon, was set at 10 cm to cause a possible collapse of the full bridge.

The 60 min vibration data with different embedded depth data were recorded with an interval of 12000 points every minute per analysis segment for the full-bridge scouring test and the time history of sensors 1 and 2 are depicted in figures 14 and 15, respectively. Moreover, the embedded

depth of the columns P_1 and P_2 is shown in table 1. To clearly illustrate the scour time history, the entire progress was divided into the two main groups shown in different colors. The first scour stage is highlighted in green, where the flood began to scour the bridge column seriously at the beginning. The whole bridge was scoured continuously in the brown section where the embedded depth of P_1 and P_2 changed from 3.8 cm to 0.3 and 1.2 cm, respectively.

The response of sensors 1 and 2 in each section was analyzed using FT. To precisely evaluate the trends associated with the D_s and G parameters in each 1 min section without interference from ambient, the 12000 points set was then divided into 12 subsets, and the mean value was obtained from these subsets as a representative of each time point. Specifically, the trend of the FT parameters and its corresponding embedded depth can be seen.

As in the single bridge column experiment, the trend of the D_s value is first compared with the measured embedded depth, which can be assumed to be correlated to the dominant structural frequency of the full-bridge model. As shown in figure 16, the D_s values calculated from sensor 1 indicate the approximate variation of the embedded depth for bridge column P_1 shown in figure 17. The embedded depth of the bridge foundation reflected the commonly seen scour characteristic and dropped sharply for the first 11 min. Similar to the single-column experiment result shown in figure 7,

Table 1. The time history of the embedded depth (cm) for bridge column P_1 and P_2 .

Sensor location Time (min)	S1 P_1	S2 P_2	Sensor location Time (min)	S1 P_1	S2 P_2
1	10	10	31	2.4	2.9
2	8.1	8.2	32	2.2	2.8
3	6.7	7.0	33	2.1	2.7
4	5.5	5.9	34	2.1	2.7
5	4.8	5.0	35	2.1	2.6
6	4.4	4.6	36	2.0	2.5
7	4.2	4.2	37	1.9	2.6
8	4.0	4.1	38	1.9	2.7
9	3.8	4.2	39	1.8	2.6
10	3.8	4.0	40	1.8	2.6
11	3.9	3.8	41	1.8	2.5
12	3.7	3.8	42	1.6	2.4
13	3.5	3.9	43	1.7	2.4
14	3.4	3.8	44	1.6	2.2
15	3.4	3.8	45	1.5	2.1
16	3.2	3.7	46	1.4	2.1
17	3.2	3.6	47	1.4	2.2
18	3.3	3.7	48	1.3	2.1
19	3.3	3.5	49	1.3	2.0
20	3.1	3.5	50	1.2	1.9
21	2.9	3.4	51	1.2	1.9
22	2.8	3.3	52	1.1	1.8
23	2.8	3.2	53	1.0	1.7
24	2.8	3.2	54	0.8	1.7
25	2.6	3.2	55	0.7	1.6
26	2.5	3.3	56	0.7	1.6
27	2.5	3.2	57	0.6	1.5
28	2.4	3.1	58	0.5	1.4
29	2.3	3.0	59	0.3	1.4
30	2.3	2.8	60	0.3	1.2

where the D_s value followed the change of fundamental frequency to reflect the degradation of structural stiffness caused by the change of embedded depth due to the occurrence of the serious scour phenomenon, the D_s value represents the stable condition of the full bridge and dropped gradually from 1.74 to 1.67. Moreover, another important characteristic of the fractal dimension, which is used mainly to evaluate the stability condition of the system, was observed during the second stage. As shown in figure 14, the time history was observed with some sudden spikes, which were caused by the instantaneous change of embedded depth during the scour process. Although the instantaneous depth change can be shown on vibration data, it might be classified as possible noise or impacts caused by floating wood or debris without any regularity when applied to any real-time monitoring system. In the mean time, the unique phenomenon was reflected by the fractal dimension D_s in the second stage, as shown in figure 16, where the fractal dimension started to fluctuate from minute 28 to 46. In general, the variation of the embedded depth can be translated into the fractal dimension D_s to express the loss of structural stiffness throughout the complex scour process.

The D_s value calculated from sensor 2 and its corresponding embedded depth of column P_2 are shown in figures 18 and 19, respectively. Compared to the time history recorded by sensor 1 shown in figure 14, fewer instantaneous

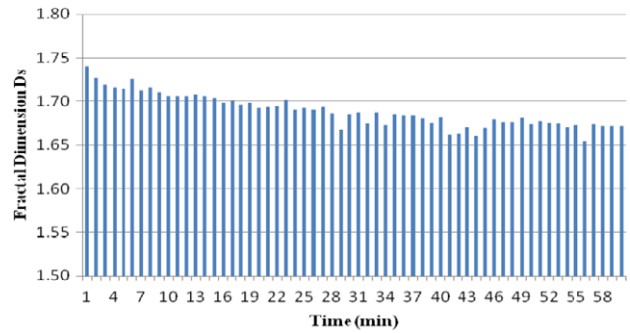


Figure 16. Variation of D_s at sensor 1.

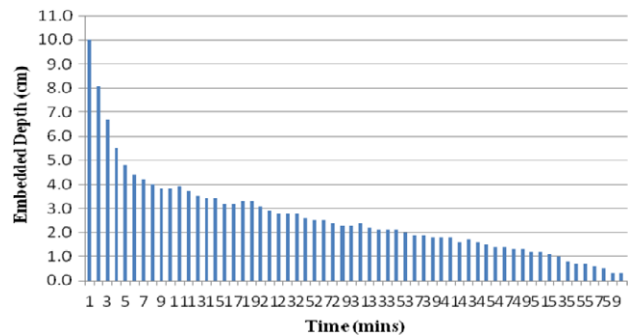


Figure 17. The embedded depth variation for columns P_1 .

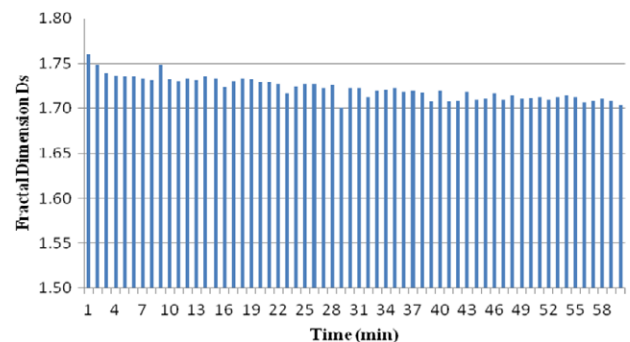


Figure 18. Variation of D_s at sensor 2.

settlements were observed from sensor 2. This phenomenon is also demonstrated by the trend of D_s depicted in figure 18. The D_s value decreases slightly in a more stable condition than what was observed in figure 16. The embedded depth shown in figure 19 also indicates that approximately 8.8 cm of sand was scoured away, expressing a more moderate scour process than column P_1 . As a result, the D_s value dropped from the peak value of 1.76 at the beginning to 1.70 at the end of the experiment, while no obvious collapse was found on column P_2 . In short, the application of D_s to evaluate the system state has been verified again in the full-bridge experiment.

The analysis of the value of fractal topothesy, G , was carried out following the same procedure as for the fractal dimension D_s . To clearly illustrate the relationship between the vibration amplitude and the G value, the entire time history collected by sensors 1 and 2 was calculated at one minute intervals. By applying the power spectrum method

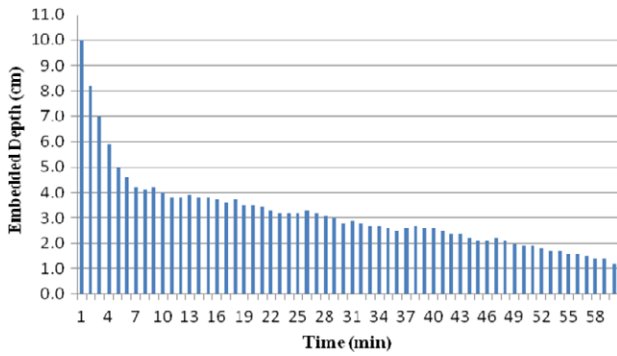


Figure 19. Variation of the embedded depth for columns P_2 .

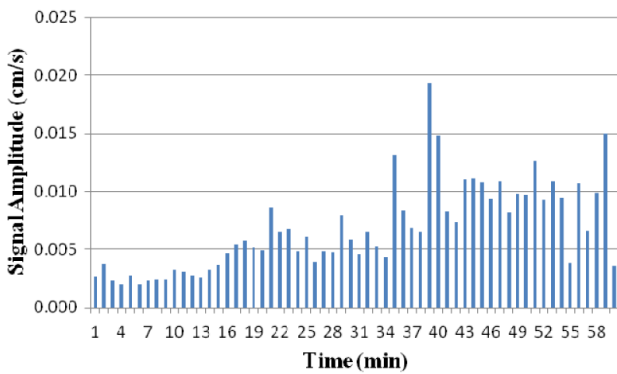


Figure 20. Variation of the signal amplitude from sensor 1.

with equation (15), the variation of the measured amplitude and the corresponding G value can be clearly expressed for comparison.

The vibration data collected by sensor 1 during the complicated condition of full-bridge scour was first analyzed. Similar to the relationship between the amplitude and the fractal topothesy examined from the single-column experiment, a positive correlation was observed, showing that the complicated full-bridge response can be expressed by the fractal topothesy. As shown in figure 20, the whole time history of figure 14 can be simplified by the signal amplitude investigated throughout the entire scour process for the full-bridge experiment. The average amplitude values in one minute are distributed between 0.003 and 0.0195. However, some unique characteristics are implied when checking the fractal topothesy as the embedded depth changed. Unlike the randomly distributed amplitude, which only reflects the dynamic behavior of the full-bridge system, the fractal topothesy oscillated slightly with the basis of 0.000 015 to reveal the stability condition. In the mean time, the sudden scour phenomenon observed in figure 14 after minute 25 can also be illustrated by the spikes of the fractal topothesy shown in figure 21. It is demonstrated that the calculation of the fractal topothesy in equation (2) combines the consideration of the physical quantity of amplitude and also indicates possible dynamic instability.

Similarly, vibration data collected from sensor 2 was also analyzed. The average signal amplitude was distributed randomly between 0.002 and 0.0069, as shown in figure 22.

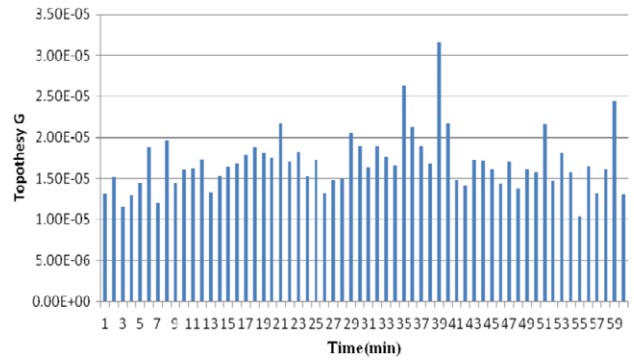


Figure 21. Variation of the G value from sensor 1.

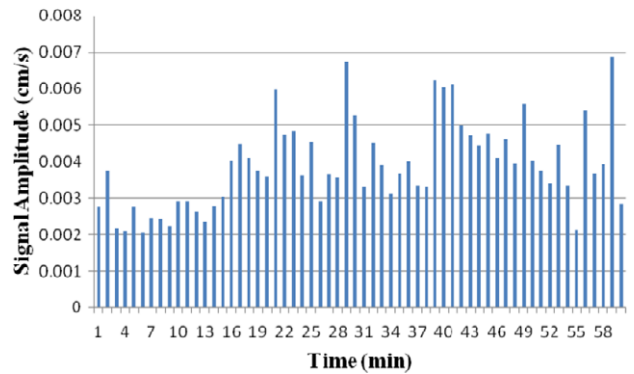


Figure 22. Variation of the signal amplitude from sensor 2.

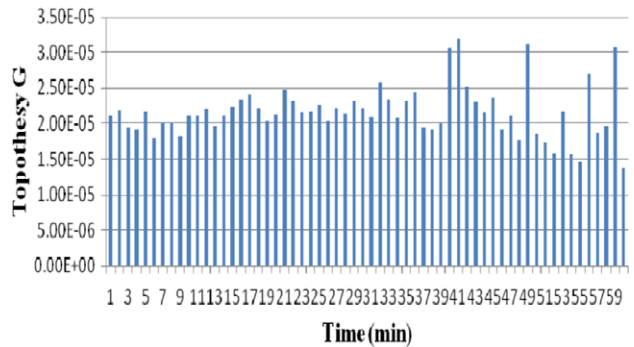


Figure 23. Variation of the G value from sensor 2.

As no significant damage condition was monitored on column P_2 throughout the experiment, the scour condition is not reflected by the variation of amplitude. In contrast, the unique characteristic of the fractal topothesy was investigated with respect to the vibration time history. As shown in figure 23, the G value moves more steadily compared to the fluctuation observed in figure 22 from the beginning to approximately minute 40. Some peak values then occurred, corresponding to some instantaneous scour phenomena. The result once again supports the concept of using the fractal topothesy in dynamic monitoring.

7. Safety factor for bridge scour

Investigation on FT parameters conducted through the single bridge column and full-bridge scour tests has demonstrated the applicability of utilizing FT parameters to describe the

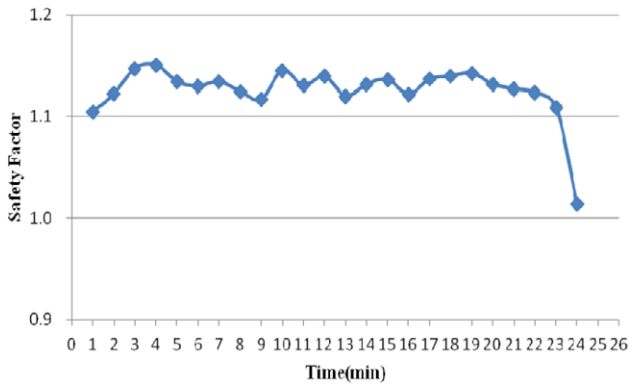


Figure 24. The SFs of the single bridge column in the horizontal direction.

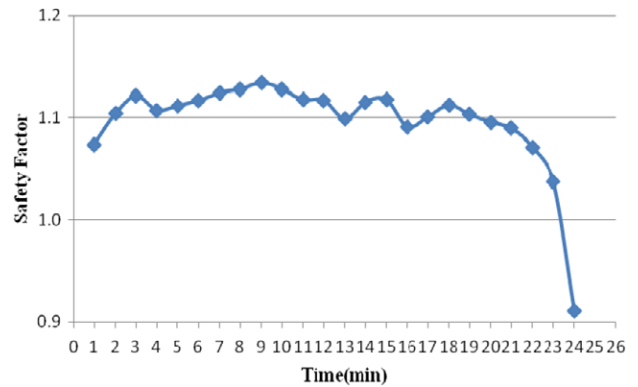


Figure 25. The SFs of the single bridge column in the vertical direction.

complex scour phenomenon of bridge health monitoring, since both of the parameters were shown to be good indicators of scour depth. Based on these results, a customized bridge health monitoring equation to estimate the safety condition is proposed.

Currently, most bridge failures happening as a result of bridge column collapse are classified as a failure of bearing capacity [33]. The boundary conditions can be further considered in three cases as:

$$\begin{aligned}
 \text{Case 1: } & \frac{h_{\text{left}}}{D} \leq 0.3 \\
 \text{Case 2: } & 0.3 \leq \frac{h_{\text{left}}}{D} \leq 1.3 \\
 \text{Case 3: } & \frac{h_{\text{left}}}{D} \geq 1.3
 \end{aligned} \tag{16}$$

where h_{left} is the embedded depth of a bridge column and D is the bridge column diameter.

By selecting the appropriate case for evaluation, the safety factor under a bearing capacity failure mode can be expressed as:

$$(\text{SF})_{\text{B}} = \frac{q_u}{q} \tag{17}$$

where q_u is the vertical ultimate bearing capacity of the soil, and q is the vertical resultant force of the soil.

As the procedure for solving q is very complicated and no dynamic effects are considered to reflect the practical condition of bridge scour, an empirical equation based on FT is attempted to rapidly evaluate the safety factor and improve the practicability of traditional methods. Since the system is using both the fractal dimension, representing an alternative for the fundamental frequency of the bridge system, and fractal topothesy, representing an alternative for the vibration amplitude of the system, the concept of the proposed safety factor is similar to calculating the entropy or complexity, which is commonly used to detect the health condition of humans, while any instability condition of the bridge system can be rapidly reflected by the change of either parameter [34]. Therefore, an empirical equation to indicate the safety factor (SF) of bridge scour is proposed using these two parameters as:

$$\text{SF} = (D_s - 1)(\sqrt{G} + 1) + 0.3. \tag{18}$$

As indicated in equation (18), the first term $(D_s - 1)$ is used to shift the D_s value, as it should be located between 1 and 2 for the vibration response under stable conditions. By introducing the shifting process, the existing bias can be eliminated and the sensitivity of D_s can be emphasized. Moreover, based on the experimental verification of the single bridge column and full-bridge experiments, the topothesy G has been demonstrated to reflect the instability condition in advance. However, based on the experimental result, the value of G is relatively small compared to the calculated D_s value. In order to enhance the contribution of G to stability identification in the formula, the second term $(\sqrt{G} + 1)$ is applied. Combining both D_s and G , the SF value can be used as an important reference for practitioners to easily identify the health condition of a bridge.

The performance of the proposed SF equation is first verified by the single-column experiment for both the horizontal and vertical directions, where the vertical direction only was measured and treated as an auxiliary source of vibration signal in the case of an emergency, and the results are shown in figures 24 and 25. The SF is expected to remain within a specific range when the structure is stable, and a warning signal should be given when the structure is in a critical condition. As shown in figure 24, the SF detected by the horizontal sensor was located at 1.1 from the beginning of the scour experiment, and increased slightly as the water began to strike the single column. The safety factor then remained at about 1.1 for the first 19 min before the start of the unstable phenomenon investigated in figure 5. A descending trend was found in the SF between minutes 19 and 23, as the scour situation became critical. Similar to the calculated fractal topothesy, which is capable of giving a warning signal before a catastrophe, the SF approached the critical value of 1 at minute 22, three minutes before the collapse of the bridge column. Finally, at minute 24, the SF dropped below 1, with the final collapse of the single bridge column. A similar tendency was also shown on the SF in the vertical direction. The SF was remained stable at approximately 1.1, and as it declined to 0.9, the critical condition was clearly reflected using the FT theory again.

As column P_1 was observed with major damage with collapse, and minor scour was investigated on column P_2

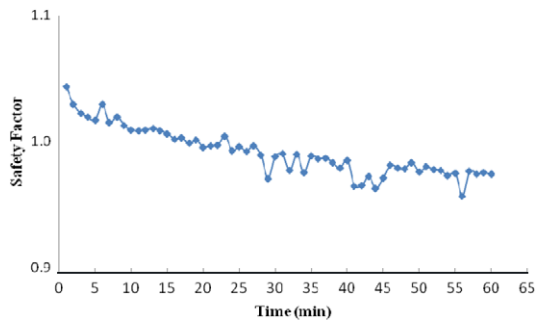


Figure 26. The SFs of the full bridge from sensor 1.

during the scour experiment, respectively, the proposed SF equation was then applied to the full-bridge experiment to verify the applicability and reliability of the bridge SHM system. The SFs for the full bridge were estimated using the same process of evaluating the scour experiment at intervals of one minute. As shown in figure 26, even if the fractal dimension and topothesy corresponding to the scour depth oscillated throughout the experiment, the SF calculated by sensor 1 described the actual scour condition of the full-bridge specimen observed by the inner camera. The SF gradually dropped from 1.05 at the beginning to the margin of 1.0 at minute 22, where the residual embedded depth shown in figure 13(b) was 2.8 cm as column P_1 started to decline with a significant angle. Comparing the time history recorded by sensor 1 shown in figure 14, the embedded depth measured by the inner camera shown in figure 17, and the amplitude shown in figure 20, a more rapid advance warning signal can be provided by the SF. Moreover, the sudden drop in the embedded depth was also reflected on the SF between minutes 25 and 60. The serious scour damage was successfully illustrated by the trend of SF.

Moreover, the minor scour damage that occurred on column P_2 was also expressed by the SF calculated from sensor 2, as shown in figure 27. A higher value of 1.07 was investigated at the beginning of monitoring, which decreased smoothly with some spikes, reflecting again the possible sudden scour at the bottom of column P_2 . Following the observed result that no collapse occurred on the bridge column, the SF remained above the critical value 1.0. The practicability of the proposed bridge SHM system is confirmed.

8. Summary and conclusions

Currently, most approaches to scour monitoring focus primarily on installing sensors on the bridge substructure. In contrast, the novel bridge health monitoring system proposed here analyzes vibration data measured from the superstructure of bridge by exploiting the advantages of FT, which checks for similarity at all levels of magnification in complex problems. By utilizing the two dominant parameters, the fractal dimension and the topothesy, the health condition of a bridge structure can be quickly evaluated. To verify the concept of the system, a series of experiments including single-column and full-bridge tests have been carried out.

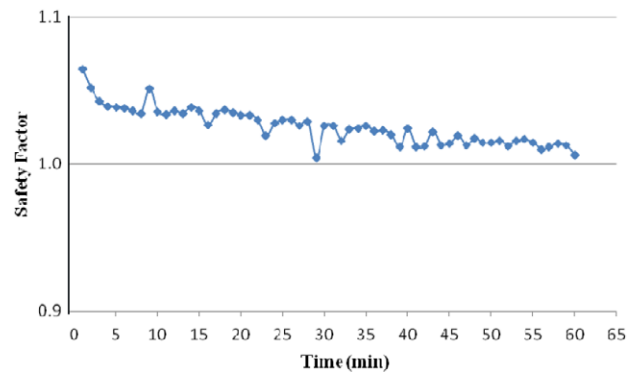


Figure 27. The SFs of the full bridge from sensor 2.

The relationship between the fractal dimension D_s , the embedded depth of soil, the vibration signal amplitude, and the topothesy G was first clarified through the single-column scouring experiment while the STFT analysis was treated as the reference for comparison of the vibration signal in the frequency domain. High correlation was found between D_s and the dominant frequency, and the vibration amplitude can be detected with better sensitivity using the topothesy G . A preliminary study has shown that the fractal parameters can be treated as an index with better sensitivity to detect the structural stability than the traditional parameters.

Based on the results obtained from the single bridge column test, a full-bridge scour experiment, which is a more realistic representation of an actual bridge scour condition, was conducted to verify the feasibility of the proposed FT-based bridge SHM system. The fractal dimension and topothesy were again shown to be good alternative indicators for the traditional parameters. The variation of embedded depth can be transferred into the fractal dimension D_s to express the loss of structural stiffness throughout the complex scour process, and the sudden scour phenomenon can also be illustrated by the spikes of the fractal topothesy.

With the support of these two parameters, a safety index equation to predict a possible trend associated with the critical scour condition was developed. The experimental results have demonstrated that the critical scour condition can be successfully predicted by the safety index proposed in both the single-column and full-bridge experiments. The collapse that occurred on the single-column structure and the specific column P_1 of the full-bridge model can be predicted several minutes before the event, and the stable condition of the specific column P_2 of the full-bridge specimen can also be precisely described throughout the experiment. The monitoring system developed greatly advances the field of bridge scour health monitoring and offers an alternative choice to traditional scour monitoring technology.

Acknowledgments

The authors would like to thank the Taiwan Area National Freeway Bureau, Ministry of Transportation and Communications (MOTC), Taiwan, for financially supporting this research under Contract No. 098A22C003. The experimental

data provided by Professor Chin-Hsiung Loh, National Taiwan University, is also gratefully acknowledged.

References

- [1] Lam H F, Katafygiotis L S and Mickleborough N C 2004 Application of a statistical model updating approach on phase I of the IASC-ASCE structural health monitoring benchmark study *J. Eng. Mech.* **130** 34–48
- [2] Yuen K-V, Au S K and Beck J L 2004 Two-stage structural health monitoring approach for phase I benchmark studies *J. Eng. Mech.* **130** 16–33
- [3] Faravelli L and Casciati S 2004 Structural damage detection and localization by response change diagnosis *Prog. Struct. Eng. Mater.* **6** 104–15
- [4] Ching J, Muto M and Beck J L 2006 Structural model updating and health monitoring with incomplete modal data using Gibbs sampler *Comput. Aided Civ. Infrastruct. Eng.* **21** 242–57
- [5] Gul M and Catbas F 2008 Ambient vibration data analysis for structural identification and global condition assessment *J. Eng. Mech.* **134** 650–62
- [6] Casciati S 2008 Stiffness identification and damage localization via differential evolution algorithms *Struct. Control Health Monit.* **15** 436–49
- [7] Catbas F N, Gokce H B and Gul M 2012 Nonparametric analysis of structural health monitoring data for identification and localization of changes: concept, lab, and real-life studies *Struct. Health Monit.* **11** 613–26
- [8] Cheung M M S, Noruziaan B and Yang C-Y 2007 Health monitoring data in assessing critical behaviour of bridges *Struct. Infrastruct. Eng.* **3** 325–42
- [9] Fraser M, Elgamal A, He X and Conte J P 2010 Sensor network for structural health monitoring of a highway bridge *J. Comput. Civ. Eng.* **24** 11–24
- [10] Pawar P M and Jung S N 2008 Support vector machine based online composite helicopter rotor blade damage detection system *J. Intell. Mater. Syst. Struct.* **19** 1217–28
- [11] Park S, Yun C-B, Roh Y and Lee J-J 2006 PZT-based active damage detection techniques for steel bridge components *Smart Mater. Struct.* **15** 957–66
- [12] Ni Y Q, Hua X G, Fan K Q and Ko J M 2006 Correlating modal properties with temperature using long-term monitoring data and support vector machine technique *Eng. Struct.* **27** 1762–73 SPEC. ISS.
- [13] Bornn L, Farrar C R and Park G 2010 Damage detection in initially nonlinear systems *Int. J. Eng. Sci.* **48** 909–20
- [14] Mistakidis E S and Panagouli O K, Strength evaluation of retrofit shear wall elements with interfaces of fractal geometry *Eng. Struct.* **24** 649–59
- [15] Pirmoradian N, Sepaskhah A R and Hajabbasi M A 2005 Application of fractal theory to quantify soil aggregate stability as influenced by tillage treatments *Biosyst. Eng.* **90** 227–34
- [16] Son M and Hsu T J 2009 The effect of variable yield strength and variable fractal dimension on flocculation of cohesive sediment *Water Res.* **43** 3582–92
- [17] Carpinteri A and Puzzi S 2009 The fractal-statistical approach to the size-scale effects on material strength and toughness, *Probab. Eng. Mech.* **24** 75–83
- [18] Carpinteri A, Spagnoli A and Vantadori S 2002 An approach to size effect in fatigue of metals using fractal theories *Fatigue Fract. Eng. Mater. Struct.* **25** 619–27
- [19] Shaofei J, Feng X and Chun F 2001 Intelligent damage identification model of an arch bridge based on box-counting dimension and probabilistic neural network *J. Comput. Inf. Syst.* **6** 1185–92
- [20] Pizhong Q and Maosen C 2008 Waveform fractal dimension for mode shape-based damage identification of beam-type structures *Int. J. Solids Struct.* **45** 5946–61
- [21] Casciati F and Casciati S 2006 Struct structural health monitoring by Lyapunov exponents of non-linear time series *Struct. Control Health Monit.* **13** 132–46
- [22] Mandelbrot B 1982 *The Fractal Geometry of Nature* (New York: Freeman)
- [23] Liou J L, Tsai C M and Lin J F 2010 The microcontact model developed for sphere-based fractal body in contact with a rigid flat surface *Wear* **268** 431–42
- [24] Yan W and Komvopoulos K 1998 Contact analysis of elastic-plastic fractal surface *J. Appl. Phys.* **84** 3617–24
- [25] Berry M V 1979 Diffractals *J. Phys. A: Math. Gen.* **12** 781–97
- [26] Mandelbrot B, Passoja D and Paullay A 1984 Fractal character of fracture surface of metals *Nature* **308** 721
- [27] Chester S, Wen H Y, Lundin M and Kasper G 1989 Fractal-based characterization of surface texture *Appl. Surf. Sci.* **40** 185–92
- [28] Nogues J, Costa J L and Rao K V 1992 Fractal dimension of thin film surfaces of gold *Physica A* **182** 532–41
- [29] Dubuc B, Quiniou J F, Roques-Carmes C, Tricot C and Zucker S W 1989 Evaluating the fractal dimension of profiles *Phys. Rev. A* **39** 1500–12
- [30] Papoulis A 1965 *Probability, Random Variables and Stochastic Process* (New York: McGraw-Hill)
- [31] Berry M V and Berman D H 1980 On the Weierstrass–Mandelbrot fractal function *Proc. R. Soc. Lond. A* **370** 459–84
- [32] Lin T K, Wang Y P, Huang M C and Tsai C A 2012 Bridge scour evaluation based on ambient vibration *J. Vibroeng.* **14** 1113–21
- [33] Zaghi A E and Saiidi M 2011 Saiid source bearing and shear failure of pipe-pin hinges subjected to earthquakes *J. Bridge Eng.* **16** 340–50
- [34] Jiang Y, Peng C-K and Xu Y 2011 Hierarchical entropy analysis for biological signals *J. Comput. Appl. Math.* **236** 728–42